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6-DOF Mechanical Shock Failure Predictions of a Cantilever Structure Using Energy Response Spectra Methods

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Outline

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- Rotational Shock Response Spectra
- Development of composite shock spectra
  - Pseudo-velocity SRS
  - Energy Response Spectra
- Test setup
- Test Results
  - Typical shock input and SRS
  - 6-DOF results and failure comparisons
- Conclusions and Future Work

Introduction & Motivation

- Shock Response Spectra and Energy Spectra are both valid for quantifying mechanical shock severity in single axis testing
  - SRS methods have been around for a long time
  - Energy spectra methods have been around almost as long
- Want to find out how these methods extend to multi-axis shock testing
  - Destructive testing is the best way to evaluate this since the failure load and mechanism are known
  - Simple structures are used to ensure that the results are not corrupted by other influences

Present Objectives

- Build on previous single-axis shock tests of the same components and extend the work to multi-axis shock
  - Previous testing performed on drop table
  - Test and finite element analysis correlated well
  - Energy Spectra and SRS both predict failure well for SDOF shock
- Determine how to convert multi-axis shocks to SRS and Energy Spectra
  - 6-DOF shaker shock testing was performed on the same types of components tested in the single axis shock tests
  - Most 6-DOF shaker work to-date has focused on vibration
- Fundamental Question:
  - How to analyze six spectra curves and relate them to one failure

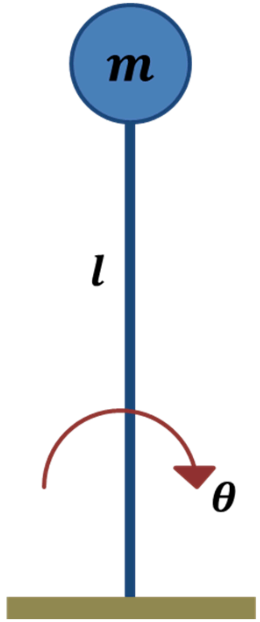
Shock Response Spectra

- SRS is traditionally applied to translational shocks
  - Inherently assumes no foundation rotation
  - 6-DOF testing can develop significant foundation rotation
- Concept of a rotational SRS has been developed previously
  - Follows the same derivation as translational shock
  - Assumes small angle approximation
  - Thus:

$$m\ddot{x} + kx = -m\ddot{\theta}l$$

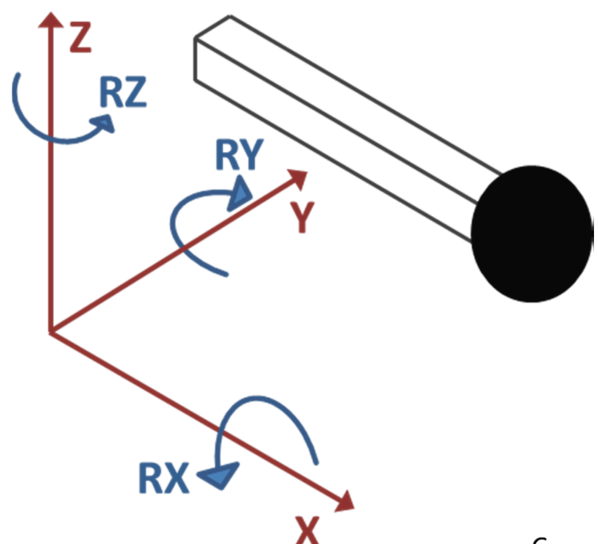
  - Assuming no initial displacement and velocity

$$x(t) = \frac{l}{\omega}\int_0^t \ddot{\theta}(\tau)\sin\omega(t-\tau)\,d\tau$$



Composite SRS

- What is the correct methodology for predicting failure from multi-axis shock?
  - Three translational SRS and three rotational SRS
  - Failure should not be equal to any single axis response but to the composite of all inputs loading the structure
- Since there is a known relationship between pseudo-velocity and stress, assume a stress based solution
- Each of the six shocks develops stress in the structure, bending or axial.



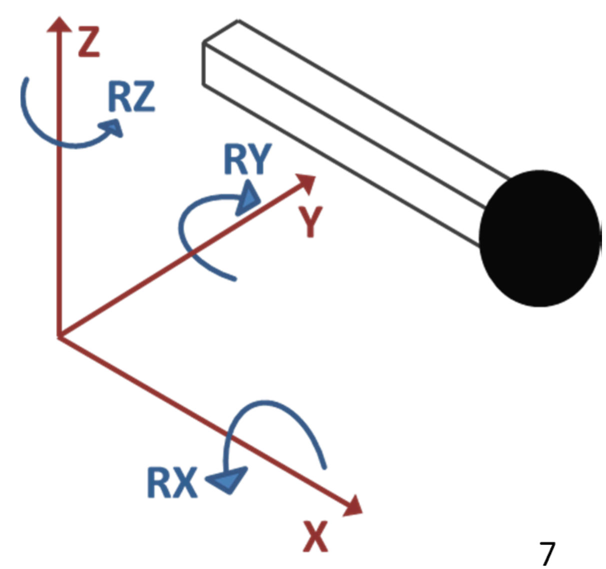
Composite SRS Derived from Stress

- From the coordinate system picture shown here
  - Translational shock in y- or z-direction produces y- or z-bending stress
  - Rotational shock about z- or y-axis yields y- or z-bending stress
  - Axial stress from x-direction shock
- Thus:

$$\sigma_x = \frac{F_x}{A}$$

$$\sigma_y = \frac{F_y l c_y}{I_{zz}} + \frac{M_z c_y}{I_{zz}} + \frac{M_x c_y}{I_{zz}}$$

$$\sigma_z = \frac{F_z l c_z}{I_{yy}} + \frac{M_y c_z}{I_{yy}}$$



Composite SRS Derived from Stress

- The previous stress relations can be directly represented in terms of pseudo-velocity

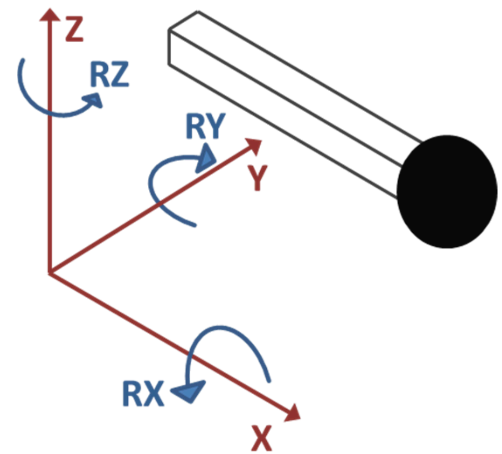
$$PV_{xeff} = PV_x$$

$$PV_{yeff} = PV_y + PV_{rz}l + PV_{rx}h$$

$$PV_{zeff} = PV_z + PV_{ry}l$$

- Combing stresses, and hence PVSRS yields:

$$PV_{composite} = \frac{r}{4l}PV_{xeff} + \sqrt{\left(PV_{yeff}\right)^2 + \left(PV_{zeff}\right)^2}$$



- Where the factor on PV_{xeff} is a scale factor derived from the axial to bending stress ratio

$$\frac{\sigma_{Axial}}{\sigma_{Bending}} = \frac{F/_A}{Mc/_I} = \frac{r}{4l}$$

Composite Energy Spectrum

- The absorbed energy spectrum is related to the pseudo-velocity SRS by:

$$E_{Am} = \frac{1}{2}PV^2$$

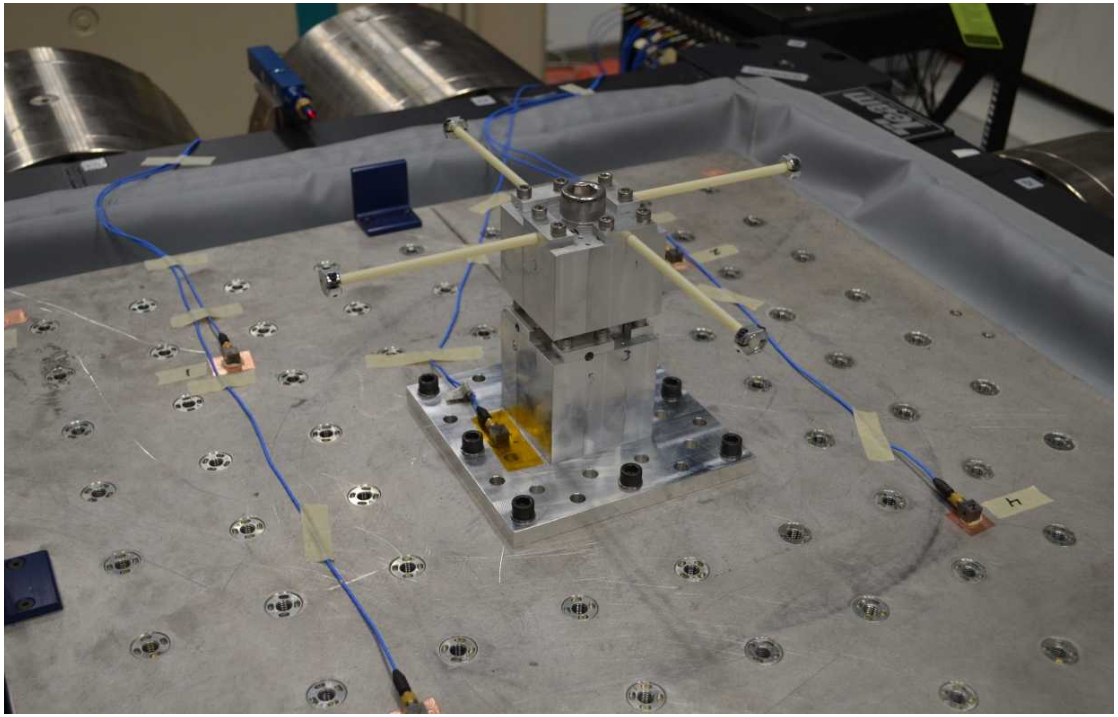
- Thus:

$$E_{Am} = \frac{1}{2}PV_{composite}^2 = \frac{1}{2}\left[PV_{xeff}\frac{r}{4l} + \sqrt{\left(PV_{yeff}\right)^2 + \left(PV_{zeff}\right)^2}\right]^2$$

- The above equation is tedious to expand to a closed form solution due to the preponderance of cross terms
  - The terms in the bracket are actually sums as shown previously
  - Simpler to calculate the SDOF energy, convert to PVSRS, calculate the composite PVSRS, then convert back to energy
    - Not very satisfying but it works
- Hypothesize that this formulation will work for input energy

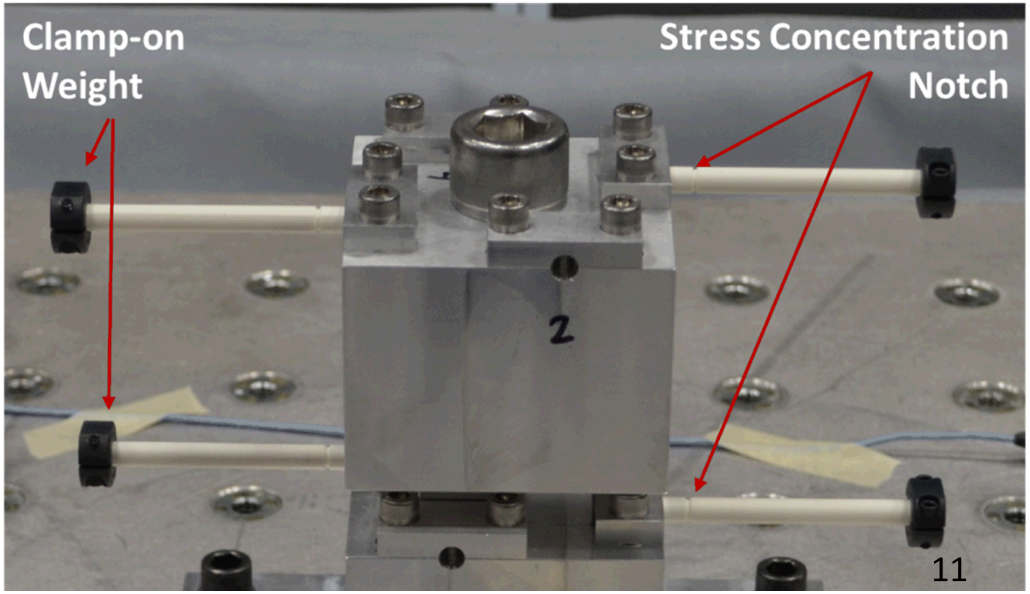
Shock Test Setup

- Tested numerous cantilever beams on the 6-DOF shaker table in sets of four beams per tests
- First passage failures
  - Stepped up input load incrementally until all beams failed



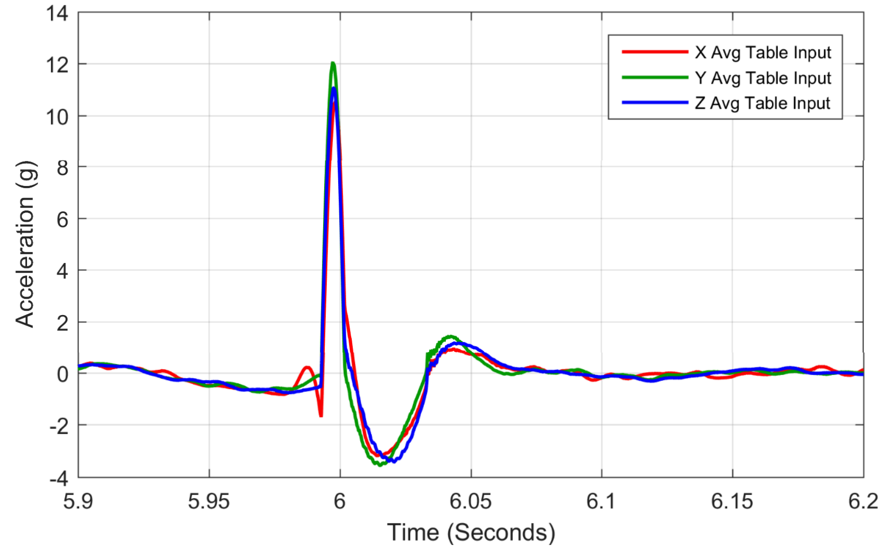
Shock Test Setup

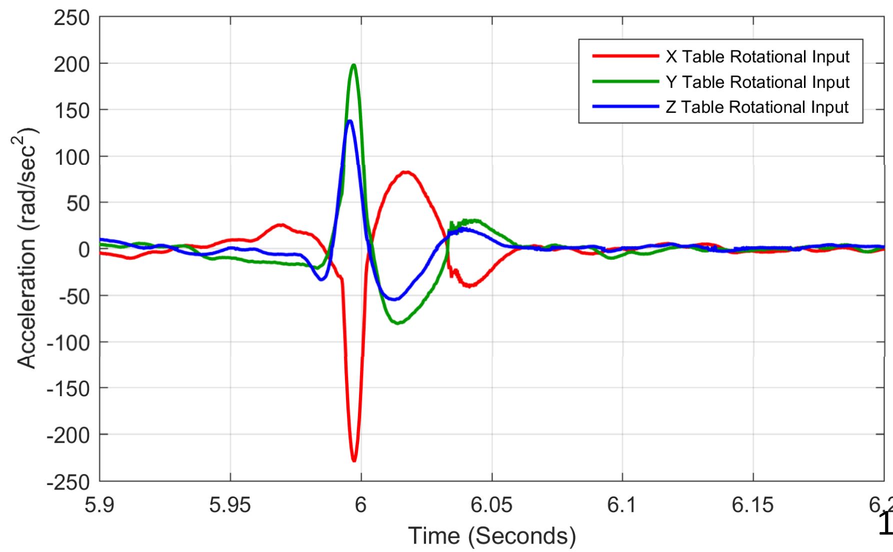
- Cantilever beams were 3-D printed from ABS plastic by the SNL/NM Additive Manufacturing Group
- Beam Features
  - 0.25inch round beams used for this testing
  - Print layers were oriented perpendicular to the beam long axis to intentionally generate brittle failures in the material
  - Stress concentration notches from 0.01 to 0.05 inch were included near the base of all beams
  - Clamp-on weights used to tailor frequency and stress under shock load



Typical 6-DOF Shock Input

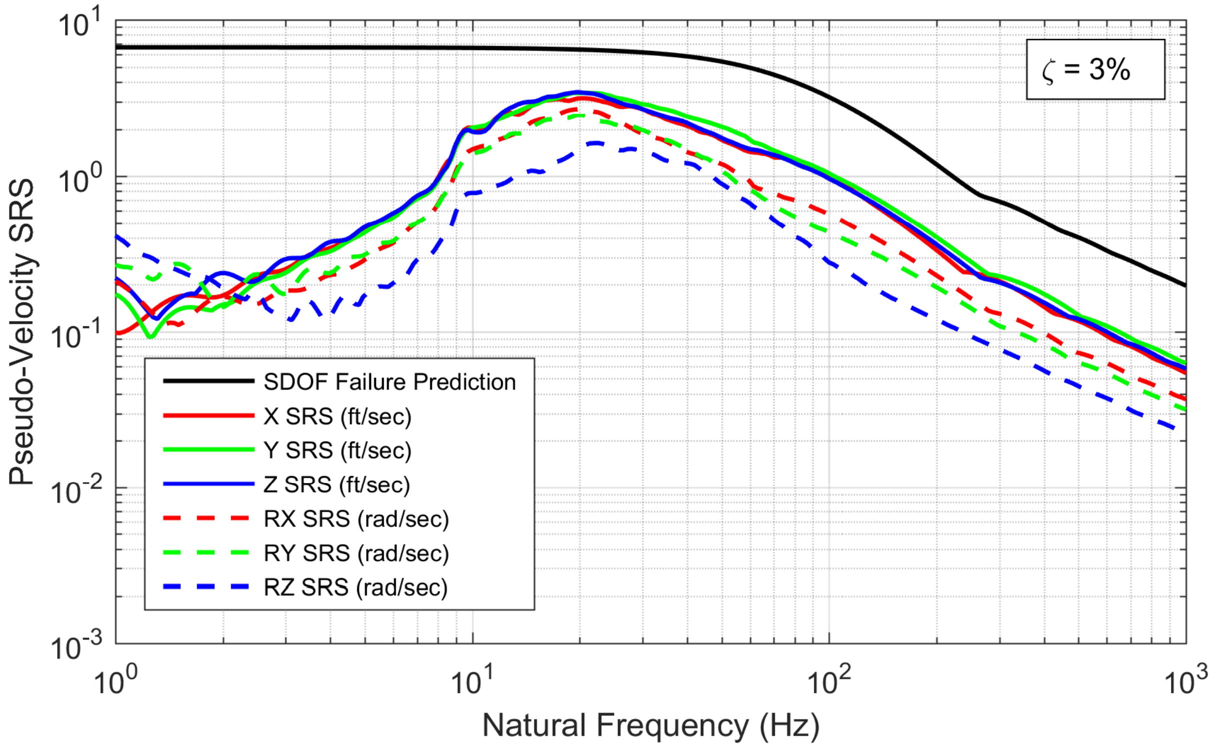
- Typical table translational and rotational acceleration
  - Pulse width and shape were constant throughout testing
  - Amplitude was varied to increase shock severity
- Translational acceleration was calculated as the average of the four corners
- Rotational acceleration was derived from tri-axial accelerometers mounted on the table corners





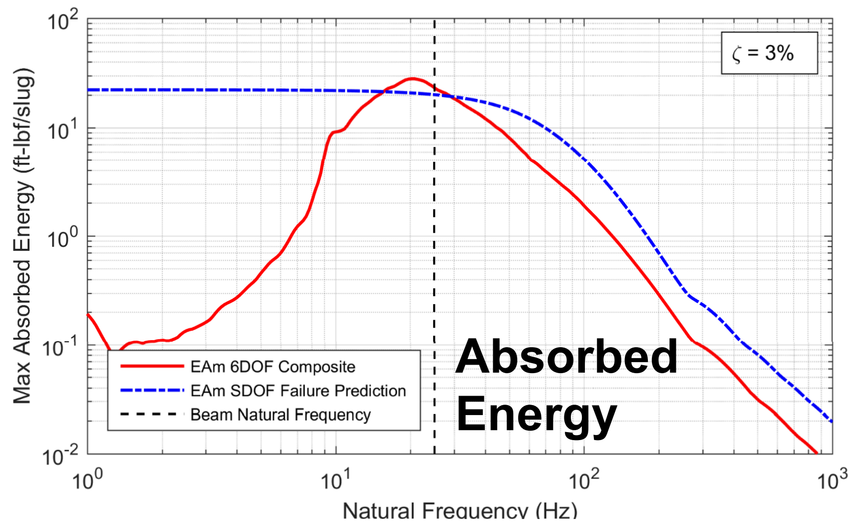
Typical 6-DOF Shock Input

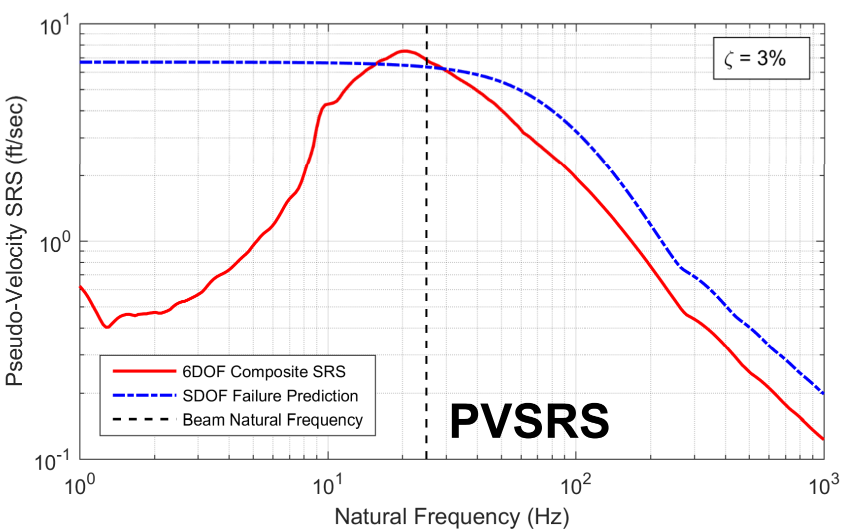
- PVSRS calculated from each 6-DOF time history and compared to SDOF failure prediction
  - SDOF failure prediction validated with drop table testing
  - Individual PVSRS curves do not predict failure, yet the beams failed

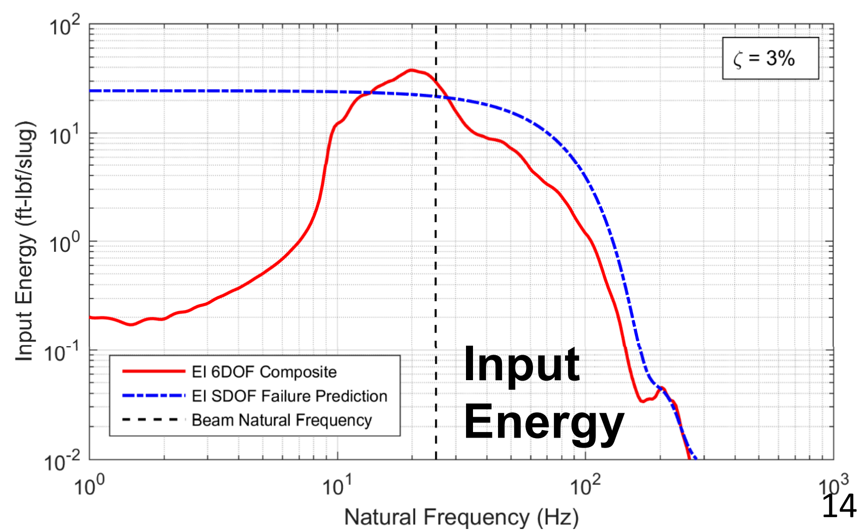


Typical 6-DOF Shock Input

- Using the composite method described here, test spectra cross the failure spectra at the first natural frequency







Conclusions and Future Work

- Conclusions
  - Cannot simply take the maximum SRS from multi-axis testing to predict failure
  - Composite SRS must be representative of stress state developed in the component
  - Energy spectra results were good, as expected, but not straightforward to calculate
- Future Work
  - Work needs to be performed on structures more representative of real world components