

Stochastic Short-term High-resolution Prediction of Solar Irradiance and Photovoltaic Power Output

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Abstract—The increased penetration of solar photovoltaic (PV) energy sources into electric grids has increased the need for accurate modeling and prediction of solar irradiance and power production. Existing modeling and prediction techniques focus on long-term low-resolution prediction over minutes to years. This paper examines the stochastic modeling and short-term high-resolution prediction of solar irradiance and PV power output. We propose a stochastic state-space model to characterize the behaviors of solar irradiance and PV power output. This prediction model is suitable for the development of optimal power controllers for PV sources. A filter-based expectation-maximization and Kalman filtering mechanism is employed to estimate the parameters and states in the state-space model. The mechanism results in a finite dimensional filter which only uses the first and second order statistics. The structure of the scheme contributes to a direct prediction of the solar irradiance and PV power output without any linearization process or simplifying assumptions of the signal's model. This enables the system to accurately predict small as well as large fluctuations of the solar signals. The mechanism is recursive allowing the solar irradiance and PV power to be predicted online from measurements. The mechanism is tested using solar irradiance and PV power measurement data collected locally in our lab.

Index Terms— Photovoltaics, solar variability, distributed energy resources, stochastic prediction, state-space model, Kalman filter, expectation-maximization algorithm.

I. INTRODUCTION

The deployment of distributed energy resources (DERs) has increased dramatically over the last decade, especially solar photovoltaics (PV). The use of solar PV has many benefits that include no emissions and pollution from electric power production, abundant resource, silent operation, long lifetime, and little maintenance [1]. However, the application of solar PV to the electric grid also has drawbacks, in particular the variability of power output that adds stress and uncertainty to the system. The variable nature of solar PV

could hamper further deployment due to the increase in reserves needed on the electric grid to compensate for fluctuations in the power output. In addition, the solar energy conversion systems have a fast and nonlinear response to incident radiation [2]. Thus, the knowledge of temporal variability of solar radiation is essential for the study of these systems and the fluctuating nature of solar radiation should be taken into consideration [3].

Real-time high-resolution predictions of solar irradiance and power output are necessary to overcome the temporal variability challenges of solar radiation. Accurate real-time high-resolution predictions allow for the development of optimal controllers that improve the stability, reliability, and efficiency of the electric grid and microgrids. Previous attempts to model solar irradiance can be classified into three general types [4]: physical, frequency distribution, and stochastic. The physical approach studies the physical processes occurring in the atmosphere and influencing solar radiation. It is exclusively based on physical considerations, allowing that the radiant energy exchanges take place within the earth-atmosphere system. This approach dictates models that account for the estimated solar irradiation at the ground in terms of a certain number of physical parameters, such as, water vapor content, dust, aerosols, clouds and cloud types, etc. [5]. The frequency distribution approach provides descriptive statistical analysis, for each place and period of the year, of the main quantities of interest such as hourly or daily global, diffuse or beam solar irradiation and statistical modeling of the observed empirical frequency distributions. The stochastic modeling approach is more flexible having the ability to incorporate any non-deterministic influences such as cloud movement and pollution levels into the model, and any non-standard features such as shading specific to a particular location [4]. Thus, stochastic modeling is capable for capturing the intrinsically non-deterministic (uncertain) nature of irradiance fluctuations. For example, understanding the sub-minute behavior of photovoltaic generation will be

necessary to develop realistic predictive models (controllers) of small microgrids on the order of single houses to single buildings.

Several stochastic modeling and prediction methods of solar irradiance and power have been reported in the literature. The works in [6], [7], [8] represent solar irradiance using hidden Markov models (HMMs). The works in [4], [9], [10] represent solar irradiance using autoregressive integrated moving average (ARIMA) model. While, the works in [11], [12] provide a framework to predict solar irradiance using artificial neural networks (ANNs). Lastly, the work in [13] presents a stochastic solar irradiance forecasting framework based on conditional random fields (CRFs). These works provide accurate models and predictions that capture the hourly, daily, and seasonal solar PV trends. However, they lack accurate high-resolution modeling and prediction of fast and large fluctuations of solar irradiance/PV power that often occur as illustrated in Fig. 1 for a typical summer day.

This paper presents and evaluates a state-space method for the short-term high-resolution solar irradiance and power prediction. We employ a filter-based expectation maximization (EM) algorithm [14], [15] and Kalman filtering [16] to estimate the parameters and states, respectively, in the state-space model. The proposed mechanism results in a finite dimensional filter which only uses the first and second order statistics. The structure of the method contributes to a direct prediction of the solar irradiance and PV power output without any linearization process or simplifying assumptions of the signal's model. This enables the system to accurately predict small as well as large fluctuations of the PV signal. This mechanism has been initially employed in [17] for wireless communication channel prediction, however, this paper introduces its first employment for solar PV prediction. The mechanism is recursive allowing the irradiance and power to be predicted online from measurements. The mechanism is tested using solar irradiance and PV power measurement data collected locally in our laboratory. This method of short-term high-resolution predictions of the PV variability enables the design of optimal controllers for PV power output management that can better respond to the fast variations in solar irradiance.

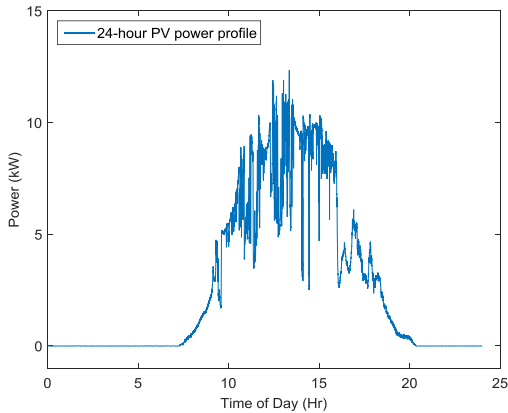


Fig. 1. The solar PV power profile for a typical summer day collected from 13kW PV panel and showing the fast and large fluctuations of PV power output.

The rest of this paper is organized as follows. Section II describes the PV signal (irradiance/power) model and the formulation of the filter equations where the state-space model, as well as the parameter estimation, is introduced. Section III discusses the simulation results of the proposed approach for prediction of solar irradiance (or power output) from real measurements. Finally, Section IV highlights the main conclusions and contributions of this paper.

II. PREDICTION OF PV IRRADIANCE/POWER VIA THE EM ALGORITHM AND KALMAN FILTERING

In this section, we will first briefly discuss the state-space model and apply it to modeling solar irradiance and PV power output signals. After that, the EM algorithm [14], [15] and Kalman filter [16] will be introduced to estimate the model parameters and states, respectively, from received PV irradiance/power measurements.

A. Stochastic State-Space Model

The state-space model has been widely employed in control systems and signal processing since its parameters and states can be updated online after receiving new observations. It consists of a state (or system) equation and a measurement (or output) equation as described in (1). The solar irradiance (or PV power) can be described by the following discrete-time stochastic linear time-variant state-space model

$$\begin{aligned} x_{k+1} &= A_k x_k + B_k w_k \\ y_k &= C_k x_k + D_k v_k \end{aligned} \quad (1)$$

where subscript k is the discrete-time index and it belongs to the set $\{0, 1, 2, \dots\}$, $x_{k+1} \in \mathbb{R}^{n \times 1}$ ($\mathbb{R}^{n \times 1}$ denotes the space of real vectors of dimension $n \times 1$) is the state vector at time k that characterizes the solar signal; it is described by the previous state x_k and the state noise process $w_k \in \mathbb{R}^{m \times 1}$ introduced at each k ; while $A_k \in \mathbb{R}^{n \times n}$ and $B_k \in \mathbb{R}^{n \times m}$ are the corresponding coefficients. $y_k \in \mathbb{R}^1$ is the discrete-time solar irradiance (or power) measurement vector; it is described by the previous state x_k and the measurement noise process $v_k \in \mathbb{R}^{m \times 1}$; while $C_k \in \mathbb{R}^{1 \times n}$ and $D_k \in \mathbb{R}^{1 \times m}$ are the corresponding coefficients.

The time-varying property of the parameters renders the state-space model to be able to adapt dynamically to a variety of irradiance/ PV power values. The noise processes w_k and v_k are able to capture small perturbations or uncertainties introduced at each time k that improves the flexibility of the model. In this paper, the noise processes are assumed to be independent zero-mean and unit-variance Gaussian processes, and their covariances are described by the coefficients B_k and D_k . Due to these properties, we propose to use the state-space model to track and predict the behaviors of solar irradiance and PV power. The unknown system parameters $\theta_k = \{A_k, B_k, C_k, D_k\}$ and the system states x_k can be estimated through a finite set of received signal measurement data $Y_N = \{y_1, y_2, \dots, y_N\}$. Future irradiance/ PV power values

can be predicted recursively based on these estimated parameters. The parameters are identified using a filter-based EM algorithm [14], [15] and the system states are estimated using the Kalman filter [16]. It has been previously proven that the Kalman filter could optimally estimate the system state in the mean square sense and that the filter-based EM algorithm yields a maximum likelihood (ML) parameter estimate [18]. The Kalman filter is introduced next.

B. State Estimation: The Kalman Filter

The Kalman filter estimates the system states x_k for given system parameter θ_k and measurements Y_k . It is described by the following equations [16]

$$\begin{aligned}\hat{x}_{k|k} &= A_{k-1}\hat{x}_{k-1|k-1} + P_{k|k}C_{k-1}^TD_{k-1}^{-2}(y_k - C_{k-1}A_{k-1}\hat{x}_{k-1|k-1}) \\ \hat{x}_{k|k-1} &= A_{k-1}\hat{x}_{k-1|k-1}\end{aligned}\quad (2)$$

where $k=0,1,2,\dots,N$, any variable denoted with “ $\wedge$ ” refers to estimated value, and $P_{k|k}$ is given by

$$\begin{aligned}\bar{P}_{k|k}^{-1} &= P_{k-1|k-1}^{-1} + A_{k-1}^TB_{k-1}^{-2}A_{k-1} \\ P_{k|k}^{-1} &= C_{k-1}^TD_{k-1}^{-2}C_{k-1} + B_{k-1}^{-2} - B_{k-1}^{-2}\bar{P}_{k|k}A_{k-1}^TB_{k-1}^{-2} \\ P_{k|k-1} &= A_{k-1}P_{k-1|k-1}A_{k-1}^T + B_{k-1}^2\end{aligned}\quad (3)$$

where $B_{k-1}^2 = B_{k-1}B_{k-1}^T$, and $D_{k-1}^2 = D_{k-1}D_{k-1}^T$. The model parameters $\theta_k = \{A_k, B_k, C_k, D_k\}$ are estimated using the EM algorithm which is introduced next.

C. Parameter Estimation: The EM Algorithm

The filter-based EM algorithm uses a bank of filters to yield a ML parameter estimate of the Gaussian state space model. The EM algorithm is an iterative numerical algorithm for computing the ML estimate. Each iteration consists of two steps: the expectation and the maximization steps. The filtered expectation step only uses filters for the first and second order statistics. The memory costs are modest and the filters are decoupled and hence easy to implement in parallel on a multi-processor system [15].

Let $\theta_k = \{A_k, B_k, C_k, D_k\}$ denotes the system parameters in (1) and $\{P_{\theta_k}; \theta_k \in \Theta\}$ denotes a family of probability measures induced by the system parameters θ_k . The EM algorithm computes the ML estimate of the system parameters θ_k , given the data Y_k . The expectation step evaluates the conditional expectation of the log-likelihood function given the complete data as

$$\Lambda(\theta_k, \hat{\theta}_k) = E_{\hat{\theta}_k} \left\{ \log \frac{dP_{\theta_k}}{dP_{\hat{\theta}_k}} | Y_k \right\} \quad (4)$$

where $\hat{\theta}_k$ denotes the estimated system parameters at time step k . The maximization step finds

$$\hat{\theta}_{k+1} \in \arg \max_{\theta_k \in \Theta} \Lambda(\theta_k, \hat{\theta}_k) \quad (5)$$

The expectation and maximization steps are repeated until the sequence of model parameters converge to the real parameters. The EM algorithm is described by [15]

$$\begin{aligned}\hat{A}_k &= E \left(\sum_{i=1}^k x_i x_{i-1}^T | Y_k \right) \cdot \left[E \left(\sum_{i=1}^k x_i x_i^T | Y_k \right) \right]^{-1} \\ \hat{B}_k^2 &= \frac{1}{k} E \left(\sum_{i=1}^k \left((x_i - A_{i-1}x_{i-1})(x_i - A_{i-1}x_{i-1})^T \right) | Y_k \right) \\ &= \frac{1}{k} E \left(\sum_{i=1}^k \left((x_i x_i^T) - A_{i-1}(x_i x_{i-1}^T)^T - (x_i x_{i-1}^T)A_{i-1}^T \right. \right. \\ &\quad \left. \left. + A_{i-1}(x_{i-1} x_{i-1}^T)A_{i-1}^T \right) | Y_k \right) \\ \hat{C}_k &= E \left(\sum_{i=1}^k y_i x_i^T | Y_k \right) \cdot \left[E \left(\sum_{i=1}^k x_i x_i^T | Y_k \right) \right]^{-1} \\ \hat{D}_k^2 &= \frac{1}{k} E \left(\sum_{i=1}^k \left((y_i - C_{i-1}x_i)(y_i - C_{i-1}x_i)^T \right) | Y_k \right) \\ &= \frac{1}{k} E \left(\sum_{i=1}^k \left((y_i y_i^T) - (y_i x_i^T)C_{i-1}^T - C_{i-1}(y_i x_i^T)^T \right. \right. \\ &\quad \left. \left. + C_{i-1}(x_i x_i^T)C_{i-1}^T \right) | Y_k \right)\end{aligned}\quad (6)$$

where $E(\cdot)$ denotes the expectation operator and $E(\cdot | Y_k)$ is the conditional expectation given the measurable set Y_k . Thus, the filter-based EM algorithm that computes the ML estimate of θ can be summarized as follows: Choose an initial parameter estimate $\hat{\theta}_0$, then compute $\hat{\theta}_k$ at each iteration according to system (6) for the model. Furthermore, since $\Lambda(\theta_k, \hat{\theta}_k)$ is continuous in both θ_k and $\hat{\theta}_k$, the EM algorithm converges to a stationary point in the likelihood surface [15], [19]. The system parameters $\hat{\theta}_k$ can be computed from the following conditional expectations [14]

$$\begin{aligned}I_k^{(1)} &= E \left\{ \sum_{i=1}^k x_i^T Q x_i | Y_k \right\}, \\ I_k^{(2)} &= E \left\{ \sum_{i=1}^k x_{i-1}^T Q x_{i-1} | Y_k \right\}, \\ I_k^{(3)} &= E \left\{ \sum_{i=1}^k [x_i^T R x_{i-1} + x_{i-1}^T R^T x_i] | Y_k \right\}, \\ I_k^{(4)} &= E \left\{ \sum_{i=1}^k [x_i^T S y_i + y_i^T S^T x_i] | Y_k \right\},\end{aligned}\quad (7)$$

where Q, R and S are given by

$$Q_{ij} = \left\{ \frac{e_i e_j^T + e_j e_i^T}{2} \right\}, R_{ij} = \left\{ \frac{e_i e_j^T}{2} \right\}, S_i = \left\{ \frac{e_i}{2} \right\}; \quad i, j = 1, 2, \dots, 2n \quad (8)$$

in which e_i is the unit vector in the Euclidean space; that is $e_i = 1$ in the i th position, and 0 elsewhere. For instance, consider the case $2n = 2$, then

$$E \left(\sum_{i=1}^k x_i x_{i-1}^T \mid Y_k \right) = \begin{bmatrix} L_k^{(3)}(R_{11}) & L_k^{(3)}(R_{21}) \\ L_k^{(3)}(R_{12}) & L_k^{(3)}(R_{22}) \end{bmatrix} \quad (9)$$

where $R_{ij} = \{e_i e_j^T / 2; i, j = 1, 2\}$. The other terms in (6) can be computed similarly from (7). For more details on how to estimate the conditional expectations $\{L_k^{(1)}, L_k^{(2)}, L_k^{(3)}, L_k^{(4)}\}$ from measurements Y_k , we refer the reader to [17].

After estimating all the parameters in the state-space model, we can achieve a one-step prediction of the solar irradiance and PV power by [18]

$$\hat{y}_{k+1} = \hat{C}_k \left(\hat{A}_k \hat{x}_{k|k} + \hat{K}_k (y_k - \hat{C}_k \hat{x}_{k|k}) \right) \quad (10)$$

where $\hat{y}_{k+1}$ denotes the predicted solar irradiance (or PV power) at time $k + 1$ and $\hat{K}_k$ is the Kalman gain described by

$$\hat{K}_k = \left(\hat{A}_{k-1} P_{k|k} \hat{C}_{k-1}^T \right) \left(\hat{C}_{k-1} P_{k|k} \hat{C}_{k-1}^T + \hat{D}_{k-1}^2 \right)^{-1} \quad (11)$$

The algorithm yields parameter estimates with non-decreasing values of the likelihood function, and converges under mild assumptions [19].

III. NUMERICAL RESULTS

In this section, we present numerical results that highlight the performance of the proposed EM algorithm and the Kalman filter in predicting the solar irradiance and PV power from measurements. We have tested the proposed prediction mechanism on 12-month measurement of solar irradiance and PV power output data collected from 13kW PV panel located on the roof of our lab at Oak Ridge National Laboratory (ORNL) in Oak Ridge, Tennessee. It is shown in [20] that there is minimal energy delivered by PV systems in variations faster than one hertz. So, the collected irradiance (in W/m^2) and power (in kW) data is recorded at one sample per second (sampling rate is 1 Hz).

Figure 2 demonstrates the performance of the proposed one-step prediction algorithm acting on real solar irradiance data for one sample day selected at random using a 4th order state-space model. Observe that the irradiance has been predicted with very high accuracy. It takes only a few iterations (about 5 iterations) for the prediction algorithm to converge. The absolute prediction error is illustrated in Figure 2 (b). Figure 3 demonstrates the performance of the proposed

one-step prediction algorithm acting on real solar PV power output data for the same day using a 4th order state-space model. The same observations for the solar irradiance prediction case hold here for the PV power prediction. In addition, it can be concluded that a 4th order stochastic model is sufficient to capture the dynamical fluctuations of PV measurements.

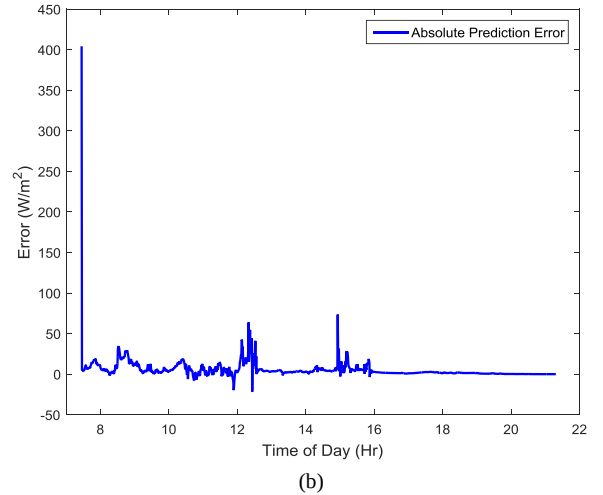
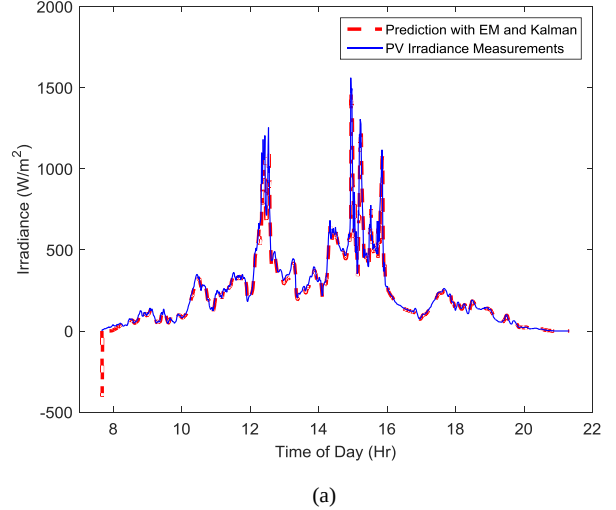


Fig. 2. (a) Numerical results illustrating the performance of the developed EM algorithm together with the Kalman filter in one-step prediction of solar irradiance and (b) its corresponding absolute prediction error.

Table 1 illustrates the normalized root mean square error (RMSE) performance for one, ten, and one hundred steps prediction of the proposed EM algorithm combined with the Kalman filter approach and compared with the traditional autoregressive (AR) prediction method [21]. It is observed that the performance of the proposed prediction approach outperforms the baseline AR prediction method by an order of magnitude, in particular at one-step (high-resolution) prediction period. Based on these observations, it can be concluded that the EM algorithm combined with the Kalman filter can dynamically track the short-term high-resolution stochastic behavior of future solar irradiance and PV power measurements.

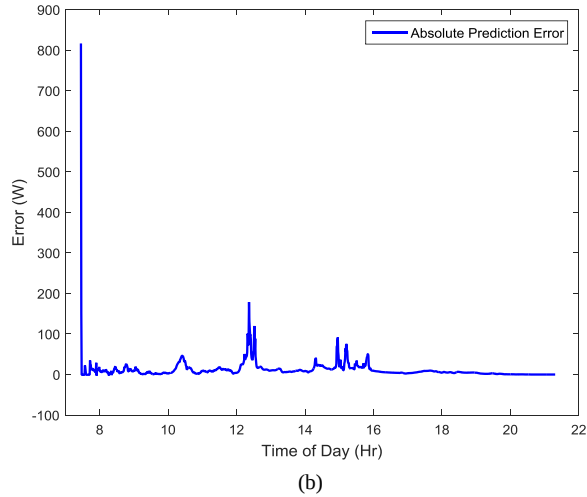
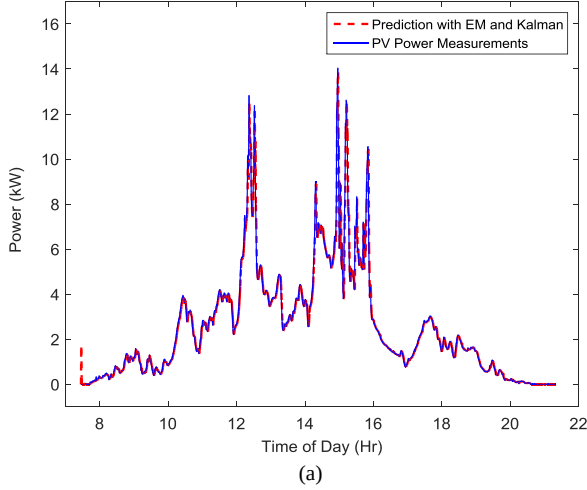


Fig. 3. (a) Numerical results illustrating the performance of the developed EM algorithm together with the Kalman filter in one-step prediction of PV power output and (b) its corresponding absolute prediction error.

Table 1. Normalized RMSE of the proposed EM-KM prediction approach with different prediction steps and compared with the traditional autoregressive prediction method.

Algorithm	Steps	Irradiance (%)	Power (%)
EM-KF	1	1.3	1.0
	10	6.1	4.6
	100	11.9	8.8
AR	1	5.6	4.7
	10	10.4	7.8
	100	15.2	13.2

VI. CONCLUSION

In this paper, we have considered the modeling and prediction of short-term high-resolution solar irradiance and PV power output using stochastic state-space model. The EM algorithm and Kalman filtering were proposed for dynamically predicting the behavior of solar irradiance and PV power output signals. We validated the stochastic models using real measurement data. The proposed mechanism was able to achieve good prediction performance of solar irradiance and PV power output using a 4th order stochastic model. Notice that, for simplicity and data availability issues, our attention was restricted to only solar irradiance and PV power. The suggested state-space model and prediction mechanism for solar irradiance and PV power forecasting, however, can be further enhanced by taking into account other important factors such as cloud cover, temperature, satellite data, and physical location.

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