

# **Modeling charged defects and defect levels in semiconductors and oxides with DFT: *An improved inside-out perspective***

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**(With special thanks to Art Edwards at Air Force Research Laboratory)**

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Unlimited Release

# Why model defects in semiconductors and oxides?

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**Radiation effects in electronics**

**Process modeling for semiconductors**

**Radiation detectors**

**Defect chemistry in nuclear fuels and nuclear waste**

Goals:

(1) Qualitative understanding - *Forensics*

Augment experiments

- incomplete, inconclusive, unavailable, expensive

(2) Quantitative characterization - *Predictive*

Predictive simulations, inform coarser models

- not just publishable, but defensible to engineers

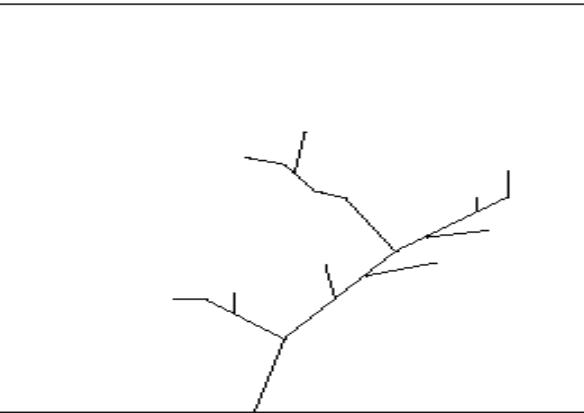
# Defects: from atoms to devices

Initial defect distribution

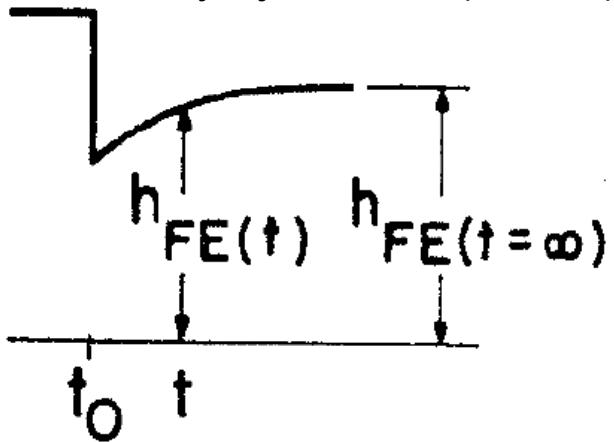
**Radiation creates displacement damage:**



**and charge carriers (electrons and holes)**

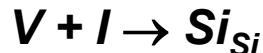


Thx: Harry Hjalmarson (Sandia)

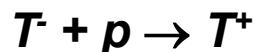


Defect evolution

**Defects react with each other, and with other dopants and impurities:**

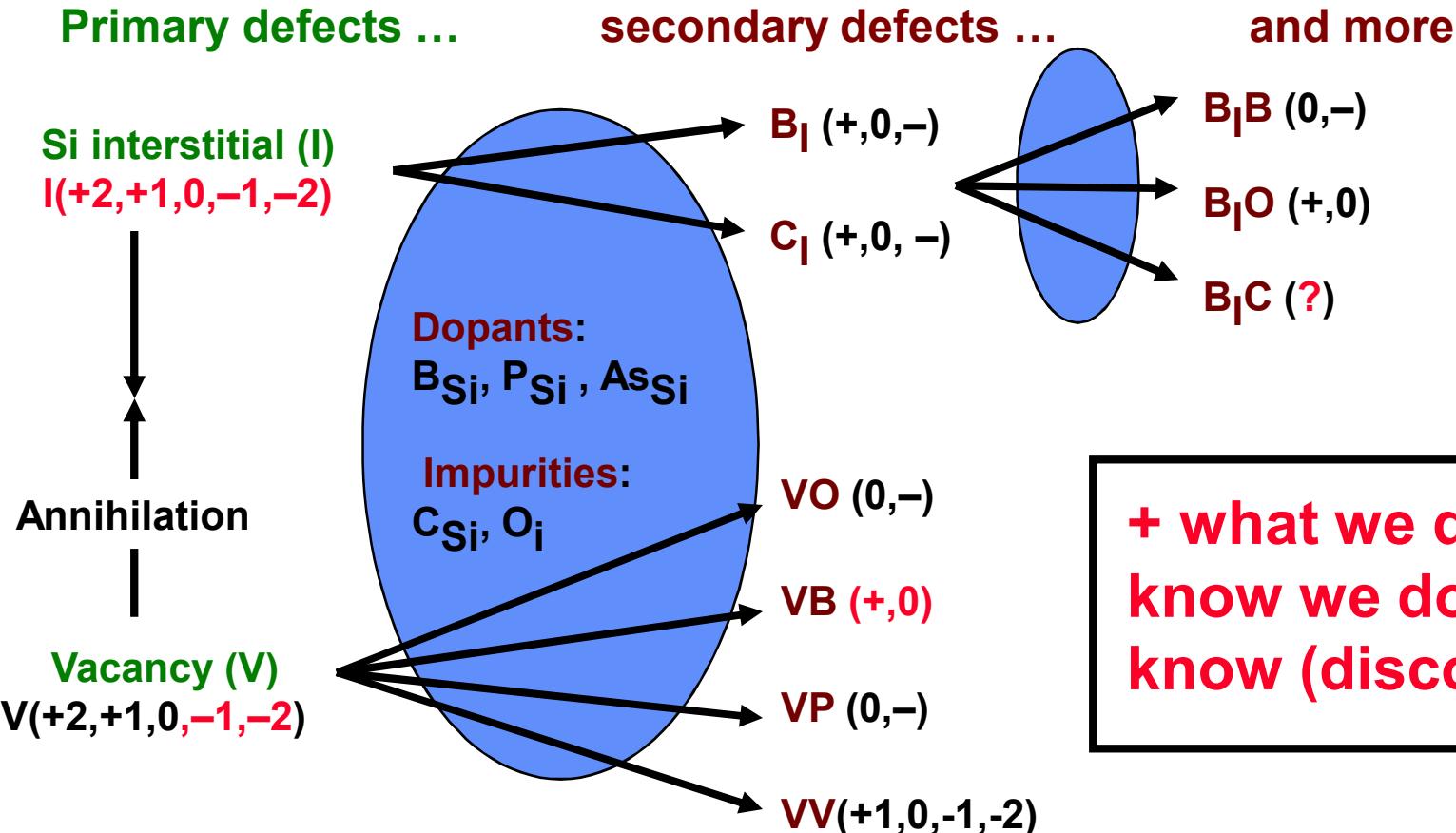


**Defects recombine electrons and holes, modifying currents:**



**Radiation/implant/processing creates evolving chemistry of defects. Those defects govern the performance of electronic devices.**

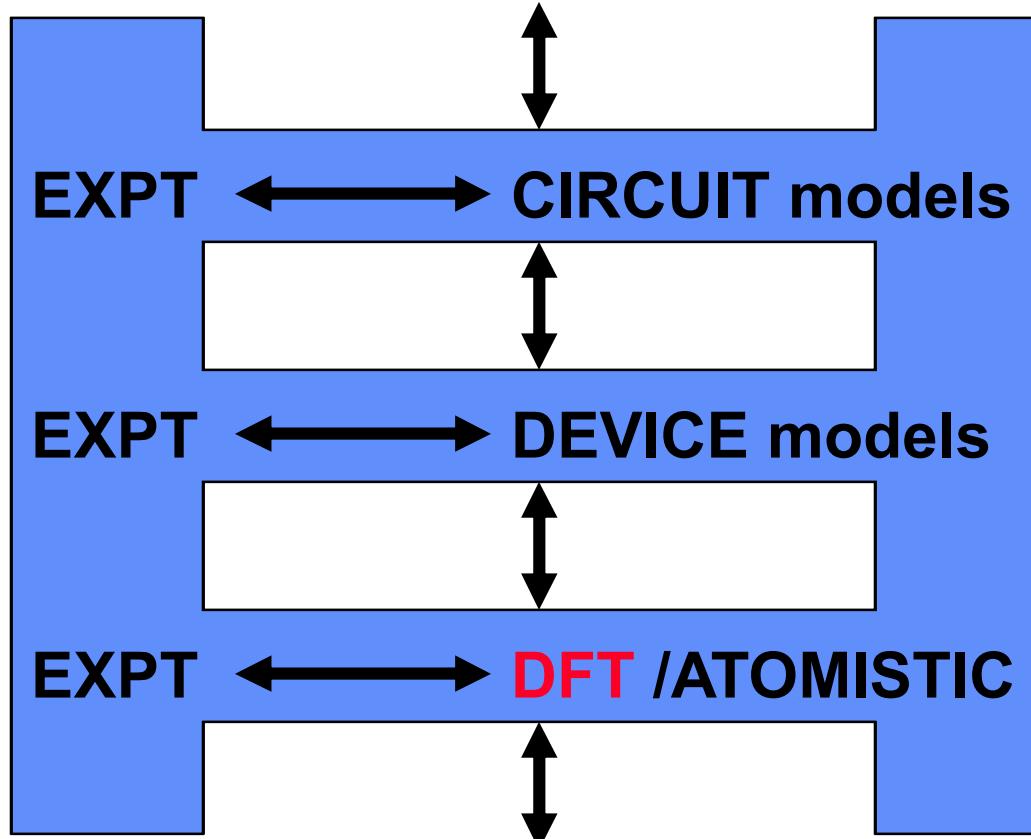
# The radiation effect defect universe: Si



Need DFT - density functional theory - to fill gaps in defect physics: defect band gap energy levels, diffusion activation

# Multiscale ladder for radiation damage

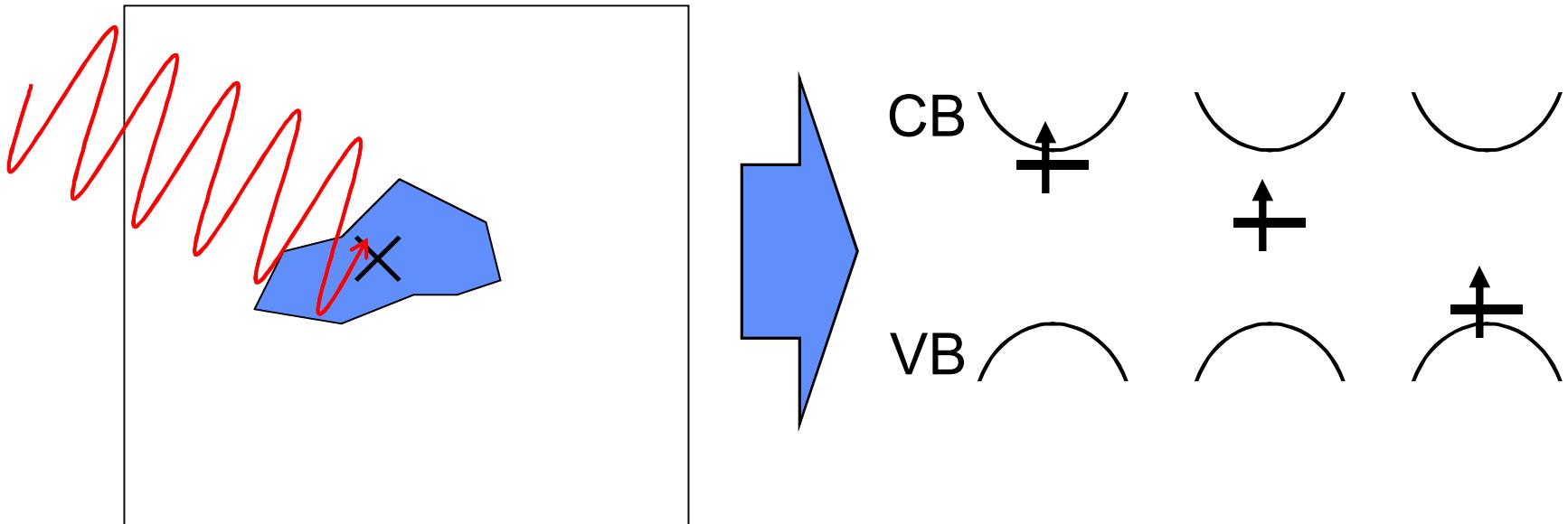
## Electrical system response



Require: quantitative confidence  
Verification, validation, uncertainty

# Radiation damage and defect levels

Radiation damage ...

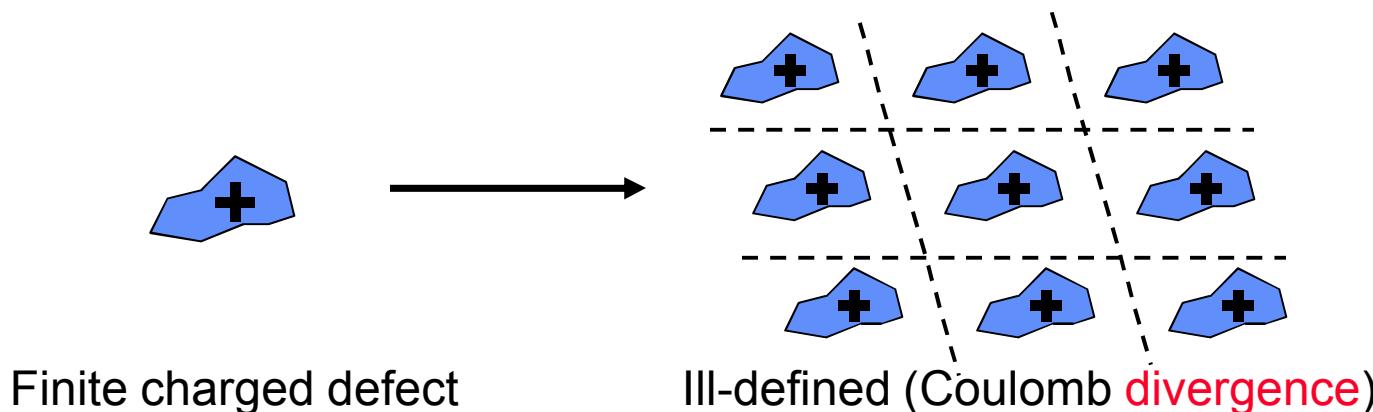


produces defects ... and introduces electronic transitions

... and we need to quantify these transitions; DFT

# Challenges for density functional theory

- **Conventional DFT fails for defect levels in semiconductors**
  - (1) Physical accuracy: e.g., “band gap problem”
  - (2) Computational model size limitations
  - (3) Shortage of good data for validation
  - (4) Supercell problem for charged defects:



Lots of DFT calculations, no robust, predictive method

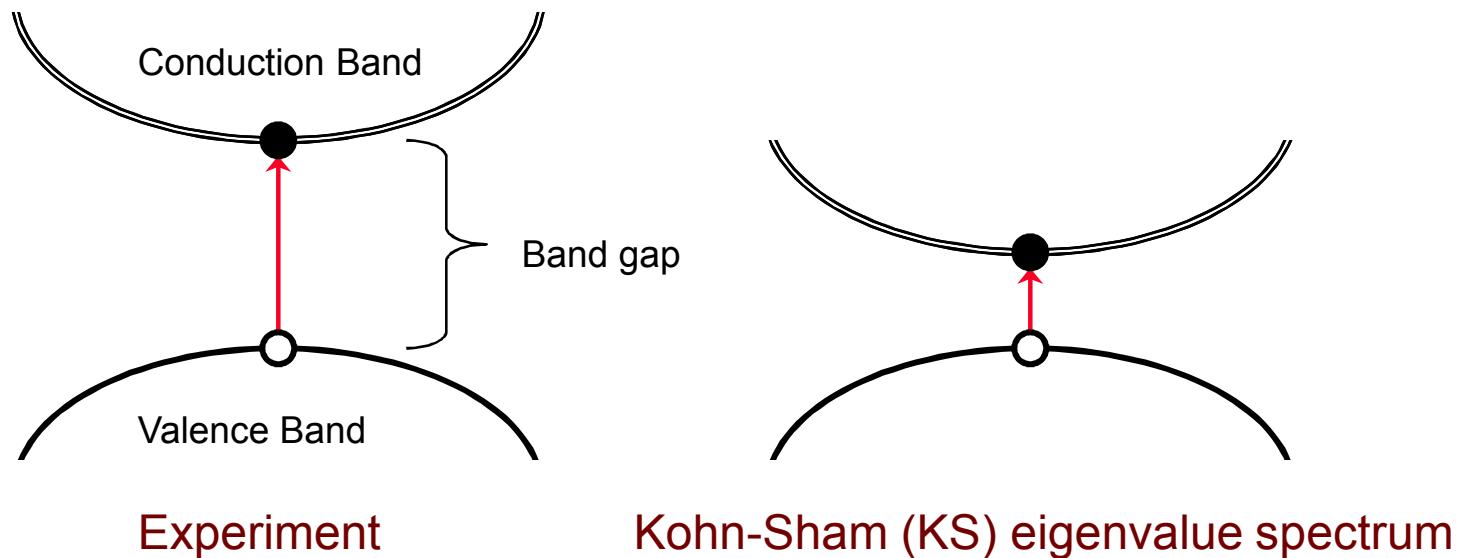
# DFT “band gap problem”

DFT gap. i.e., in KS eigenvalues, significantly underestimates experiment

[L.J. Sham and M. Schlüter, PRL **51**, 1888 (1983); PRB **32**, 3883 (1985)]

Si: expt: 1.2 eV, DFT/LDA: 0.5 eV

GaAs: expt. 1.5 eV, DFT/LDA: 0.5 eV



The band gap defines the energy scale for defect levels

Fundamental impediment to quantitative predictions?

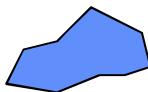
# The Supercell Approximation

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Fast Fourier Transforms are convenient means to solve 3D Poisson Equation.

DFT codes typically assume periodic boundary conditions.

However, our finite defect is not periodic ...



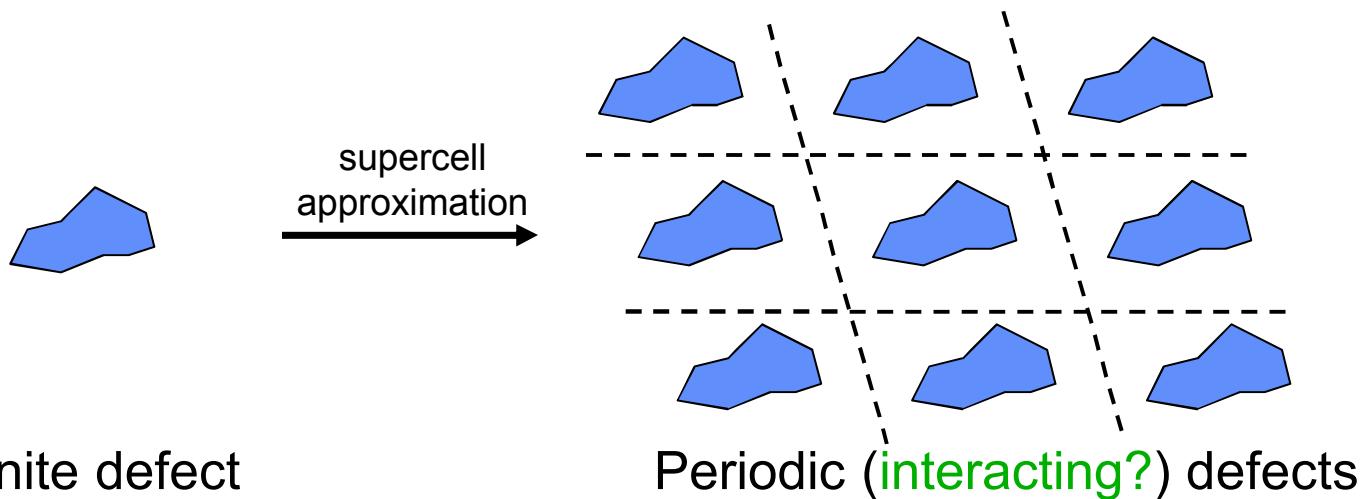
Finite defect

# The Supercell Approximation

Fast Fourier Transforms are convenient means to solve 3D Poisson Equation.

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However, our finite defect is not periodic ...



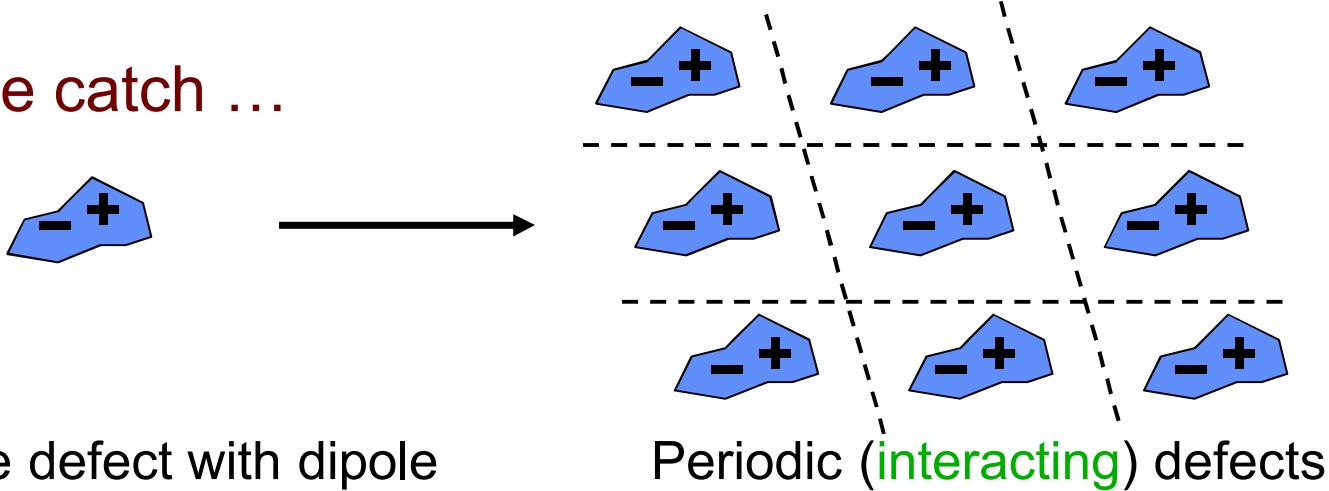
## The supercell Idea:

Surround perturbed defect region with enough material to buffer defects.

In the limit of *large enough* supercells ... approach an isolated defect.

# The Supercell Approximation

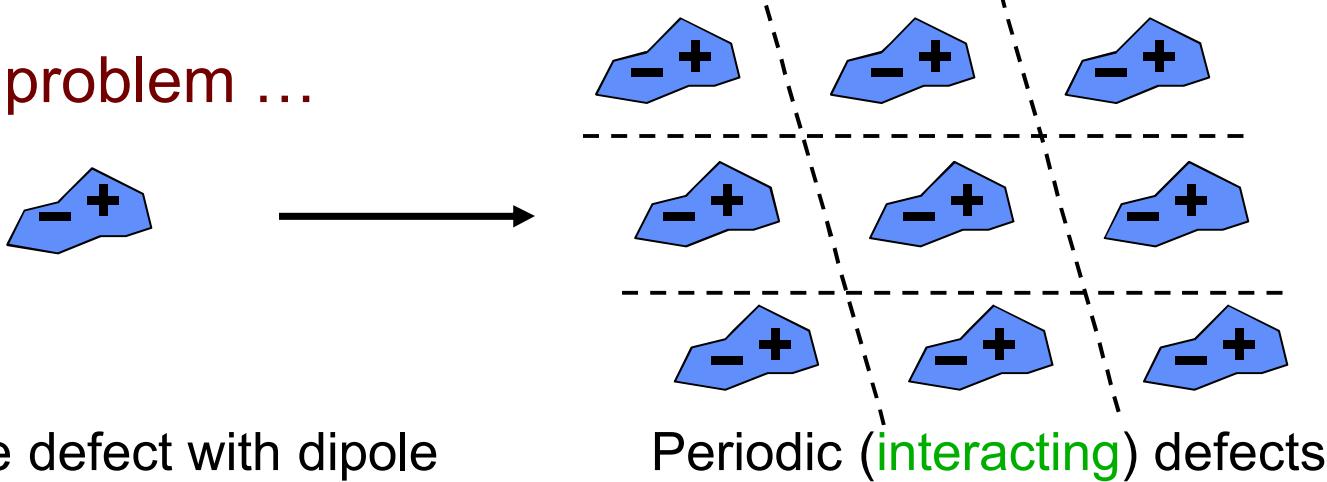
the catch ...



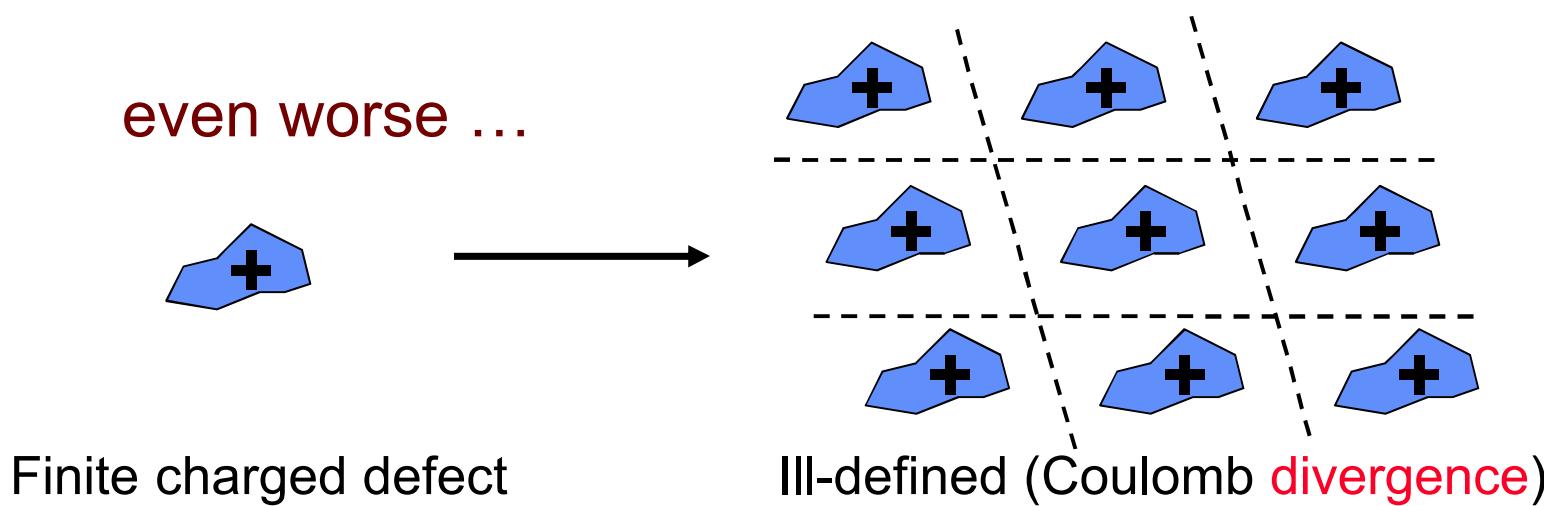
DFT expense limits size of supercell - defects interact

# The supercell approximation

A problem ...



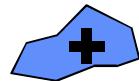
even worse ...



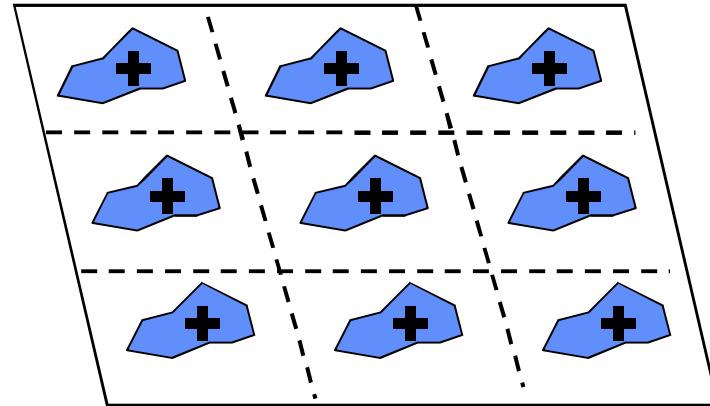
Interactions and divergence are key issues

# Jellium to eliminate divergence?

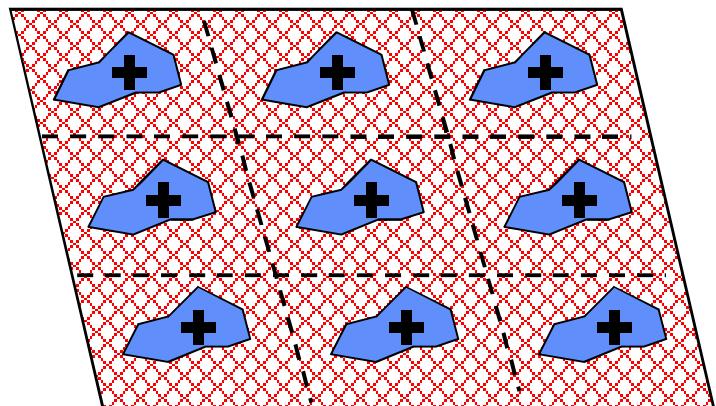
Isolated defect ...



Apply supercell approximation ...

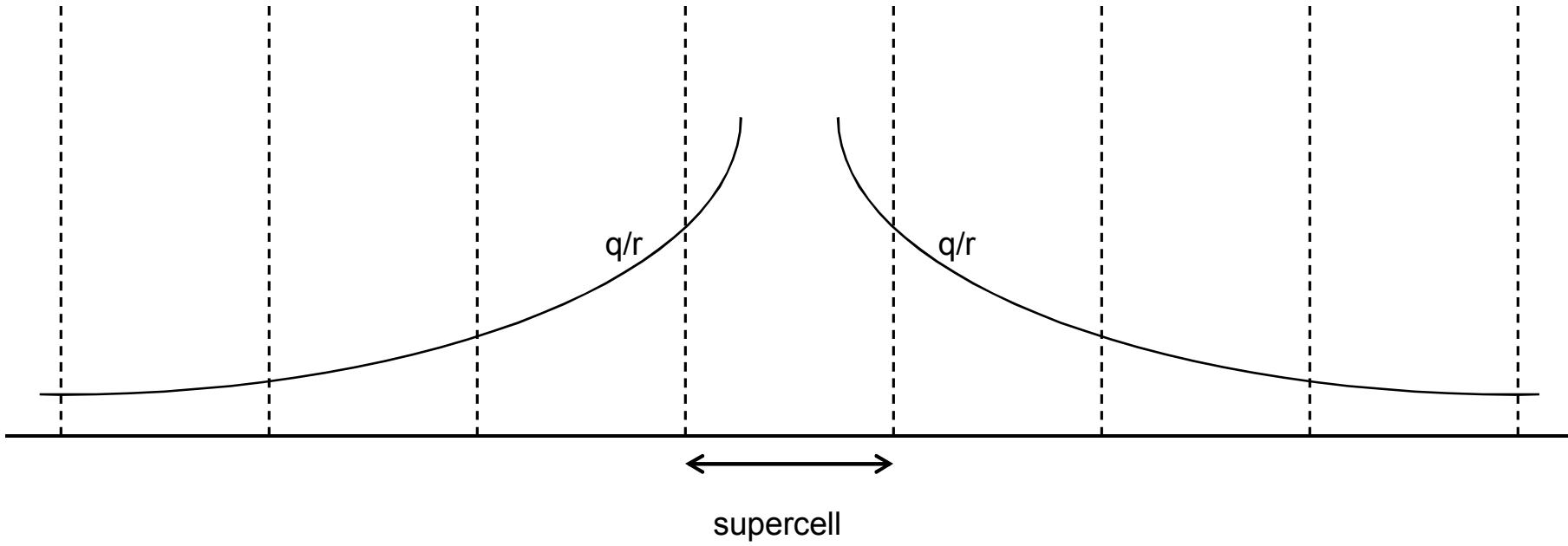


Neutralize with flat  
background charge:  
“jellium”

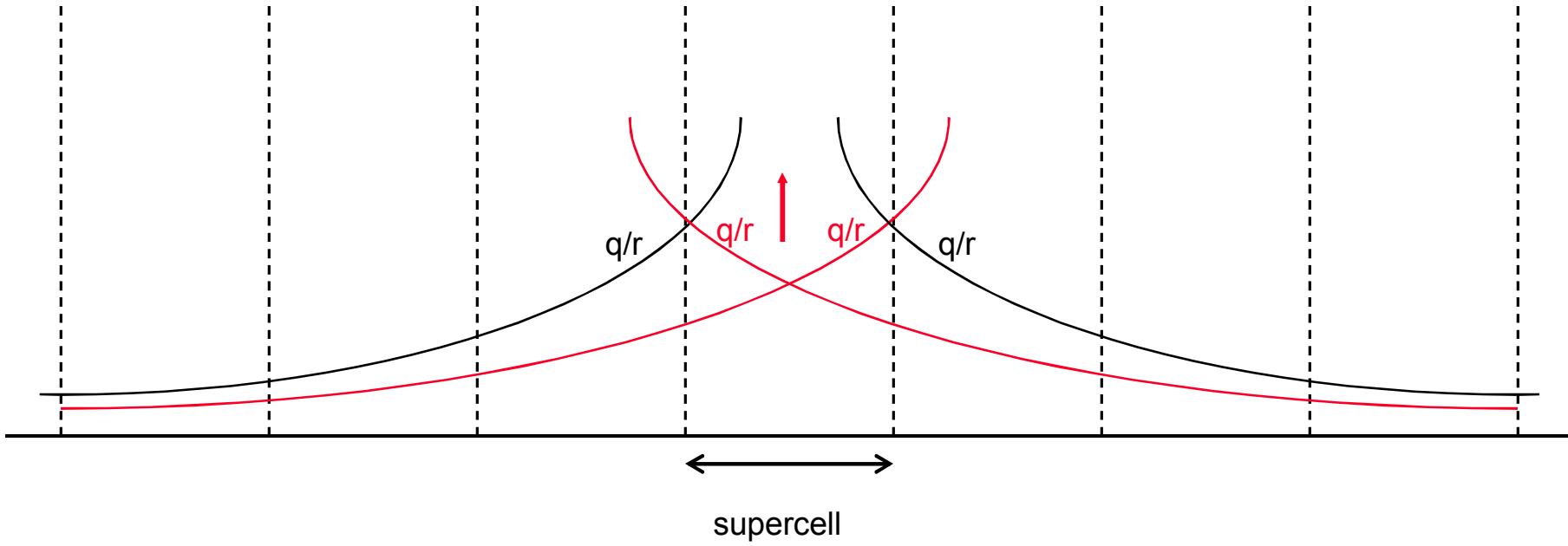


# Whence the divergence?

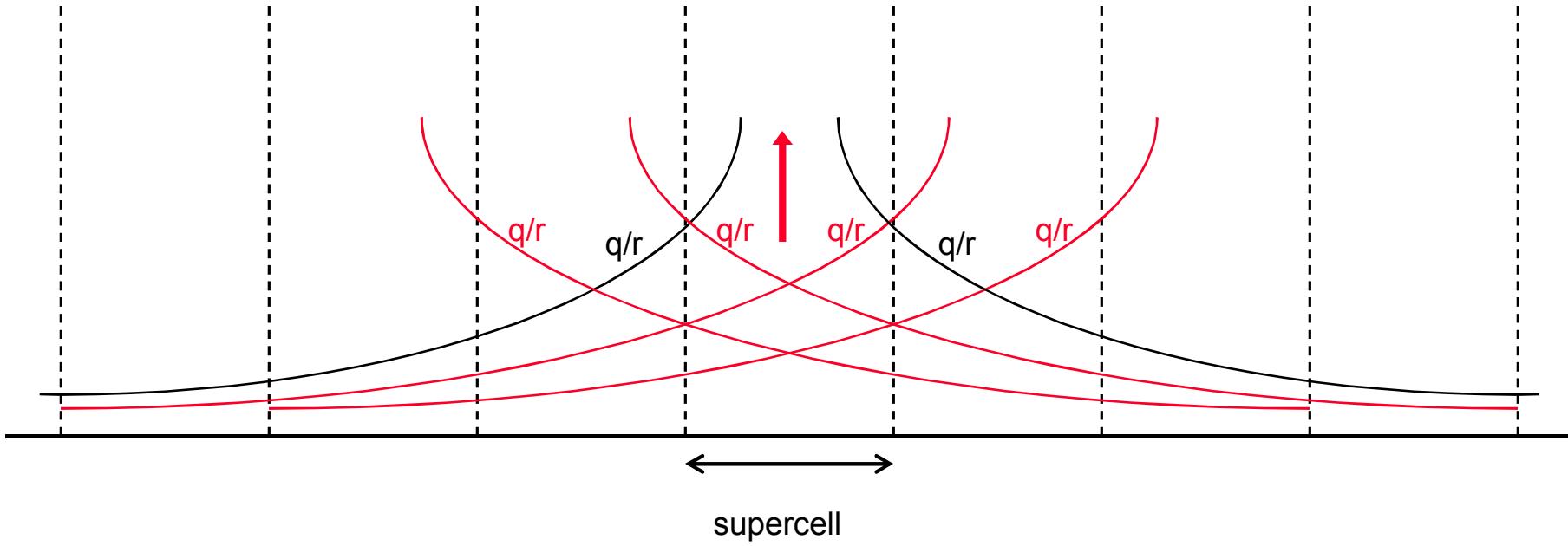
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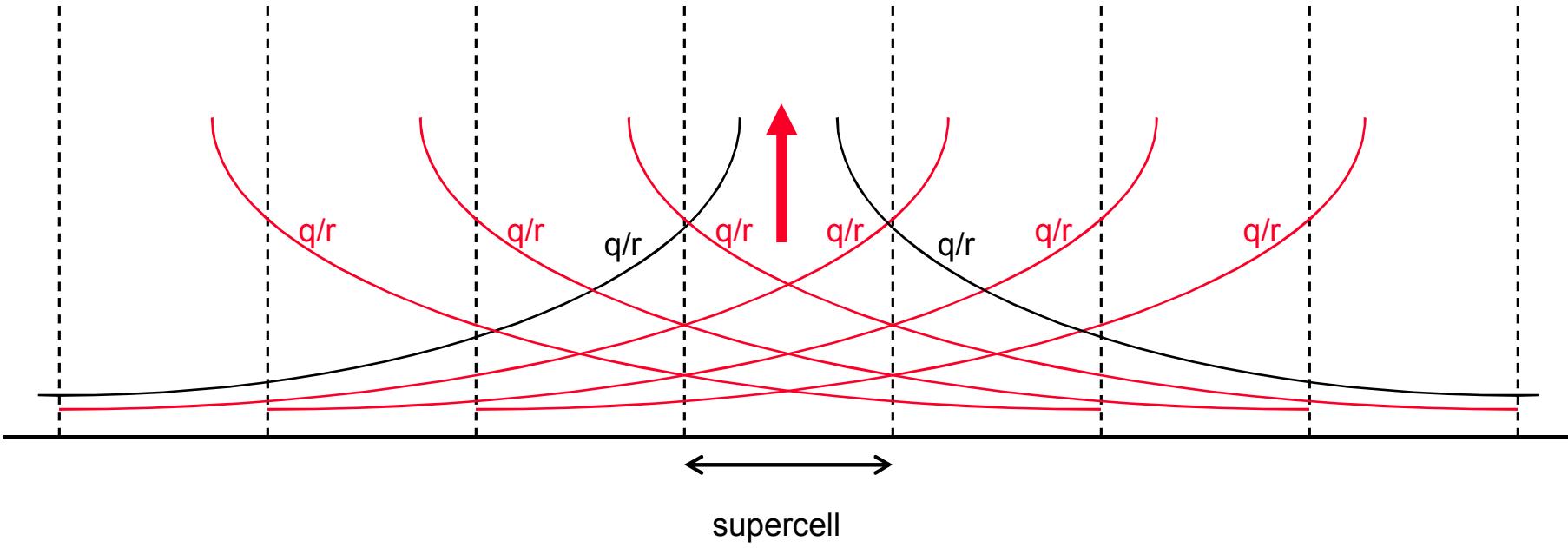
# Whence the divergence?



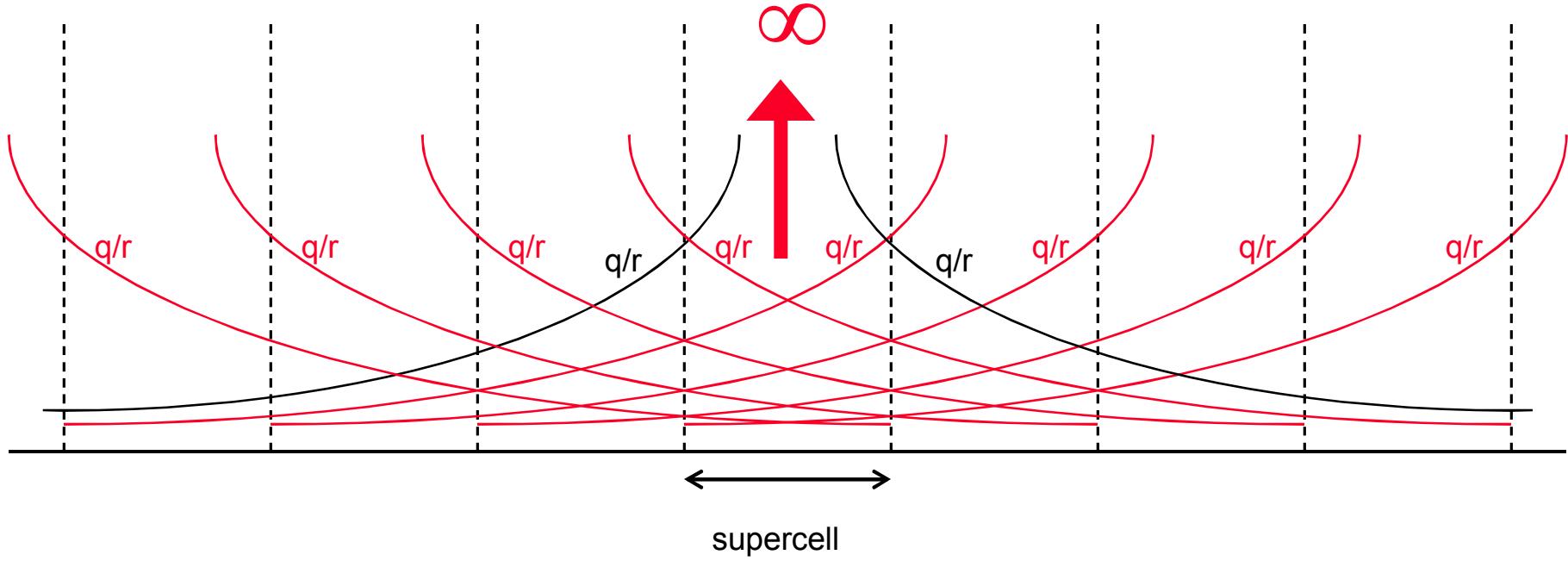
# Whence the divergence?



# Whence the divergence?



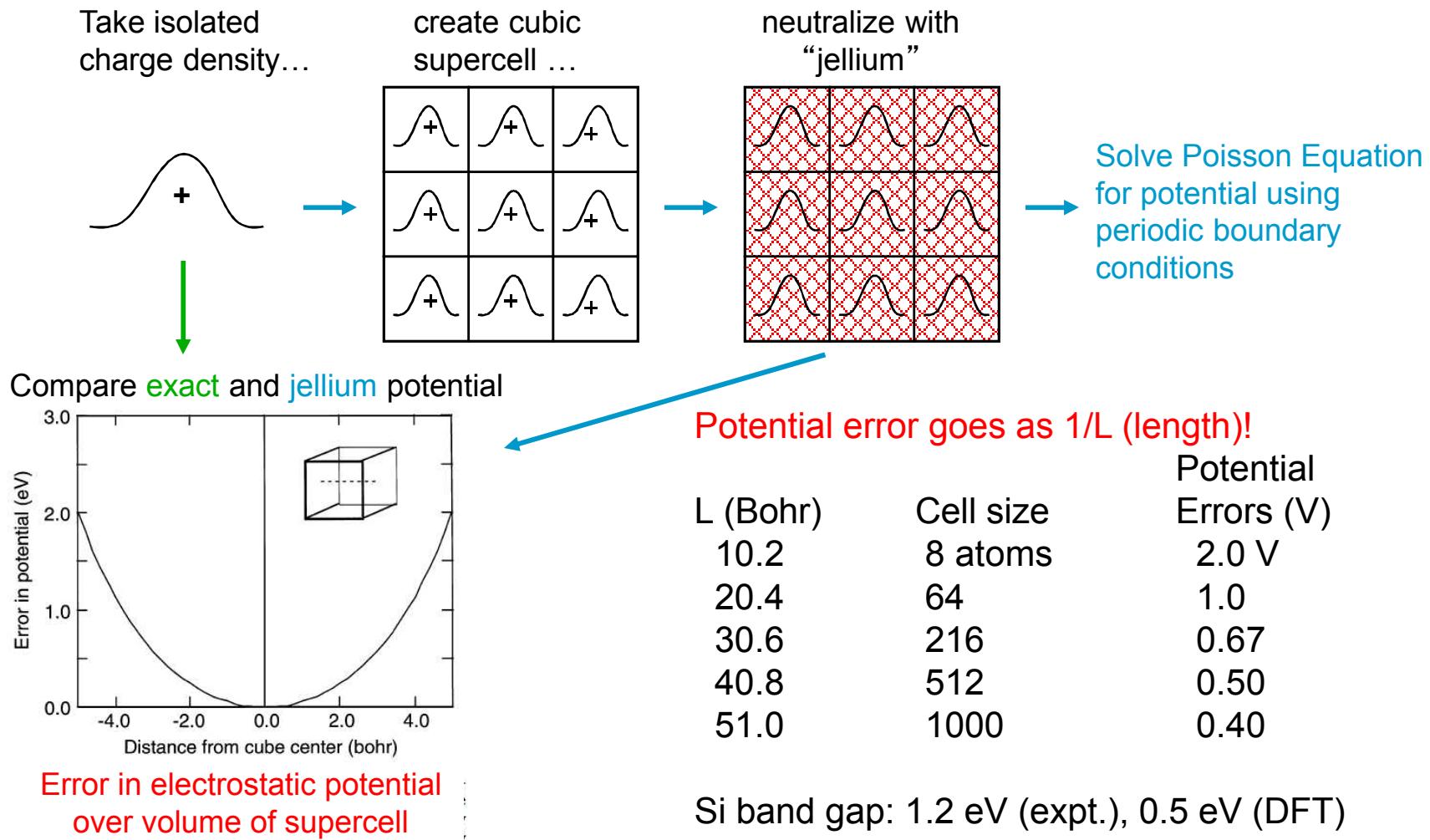
# Whence the divergence?



Divergence arises from infinite-ranged  $q/r$  potentials from periodic images

**Divergence is not flat**

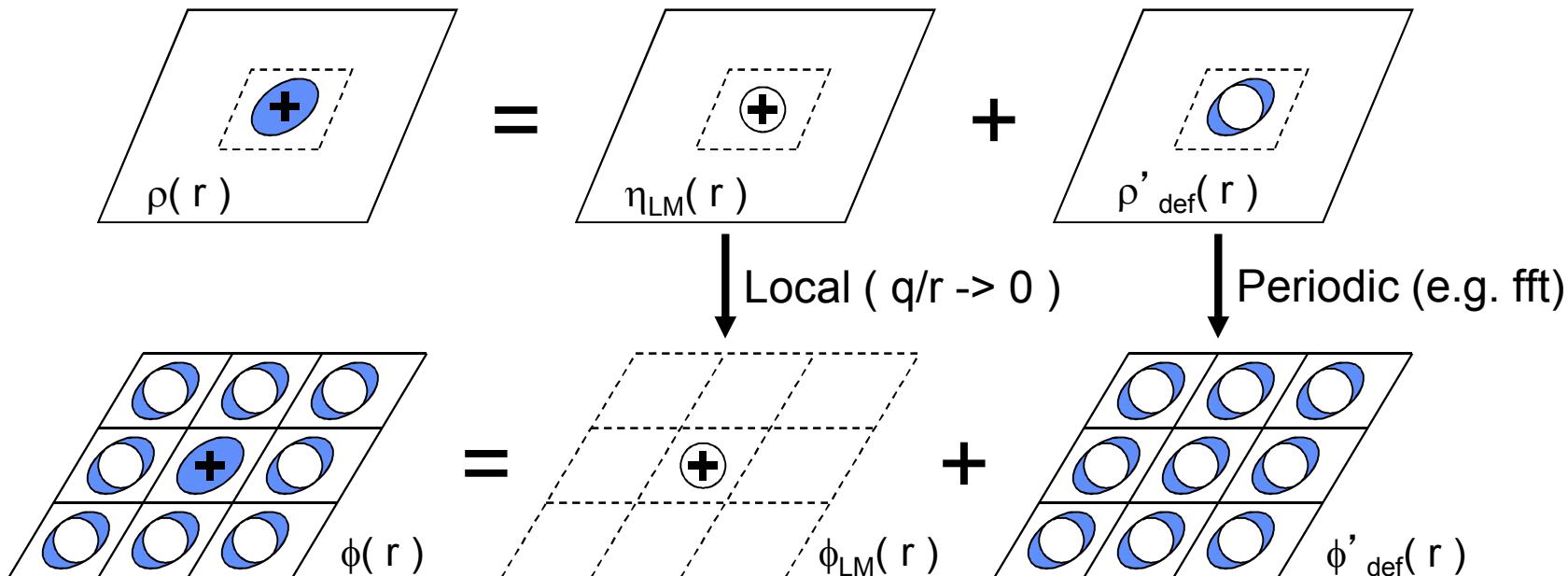
# Net charge boundary conditions - jellium



# Local Moment CounterCharge (LMCC)

P.A. Schultz, PRB **60**, 1551 (1999)

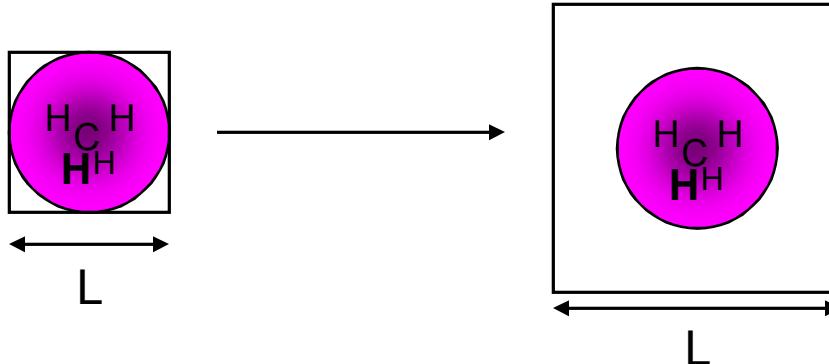
- Solution of Poisson Equation is linear in the density
- LMCC: split total density  $\rho(r)$  into two pieces ...
  - (1) model local density  $\eta_{LM}(r)$  matching multipole (charge) of  $\rho(r)$
  - (2) remainder (momentless) density  $\rho'_{def}(r) = \rho(r) - \eta_{LM}(r)$



Gives proper  $r \rightarrow \infty$  asymptotic boundary condition  
Avoid (not ignore!) Coulomb divergence

# Charged cell convergence - LMCC

P.A. Schultz, PRB **60**, 1551 (1999)



Charged, no dipole:  $\text{CH}_4 \rightarrow \text{CH}_4[+]$  ... Ionization Potential

$L = 18.0 - 30.0$  bohr (9.5-15.9 Å) IP varies  $< 10^{-5}$  eV

Dipole, no charge:  $\text{Na}-\text{Cl}$  diatomic molecule ... Total Energy

$L = 16.8 - 30.0$  bohr (8.9-15.9 Å) TE varies  $< 10^{-5}$  eV

Dipole, charge:  $\text{OH} \rightarrow \text{OH}[-]$  ... Electron Affinity

$L = 18.0 - 30.0$  bohr (9.5-15.9 Å) EA varies  $< 10^{-3}$  eV

Total energy, levels, i.e. full Hamiltonian are all immediately converged.  
-> electrostatic *potential* correctly represented by LMCC, not just energy

# A supercell theory of defect energies

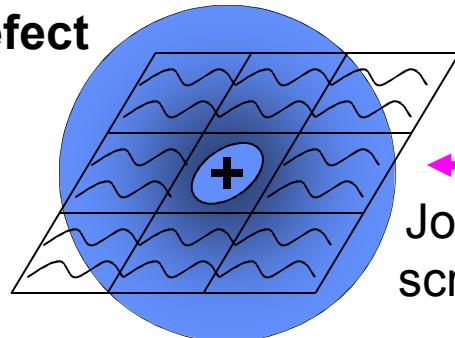
Peter A. Schultz, Phys. Rev. Lett. **96**, 246401 (2006).

**Target system:**  
isolated defect

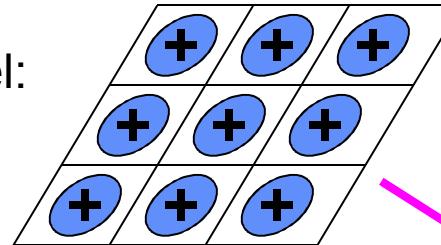
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**Computational  
model for  
isolated defect**

( + DDO  
for defect  
banding)



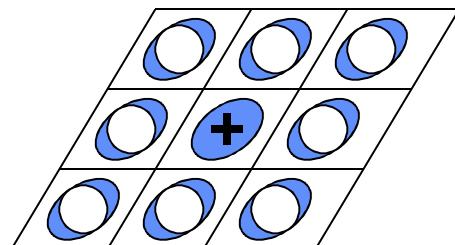
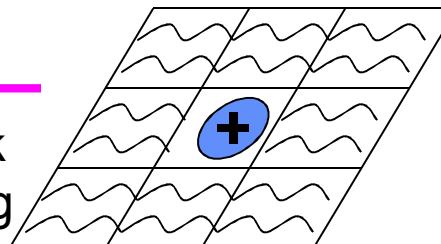
Standard  
DFT model:  
Supercell



LMCC to fix  
boundary  
conditions

## Finite Defect Supercell Model

Jost Bulk  
screening



Crystal embedding  
to fix  $\mu_e$

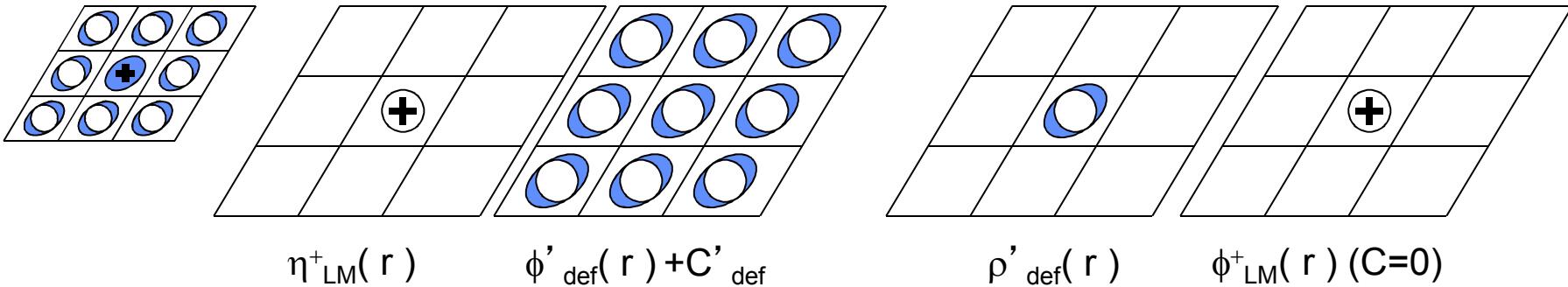
*“Ab initio” computational model – connect model to physics  
Calculations with rigorous control of charge boundary conditions*

(i.e., not jellium-based)

# A fixed chemical potential $\mu_e$

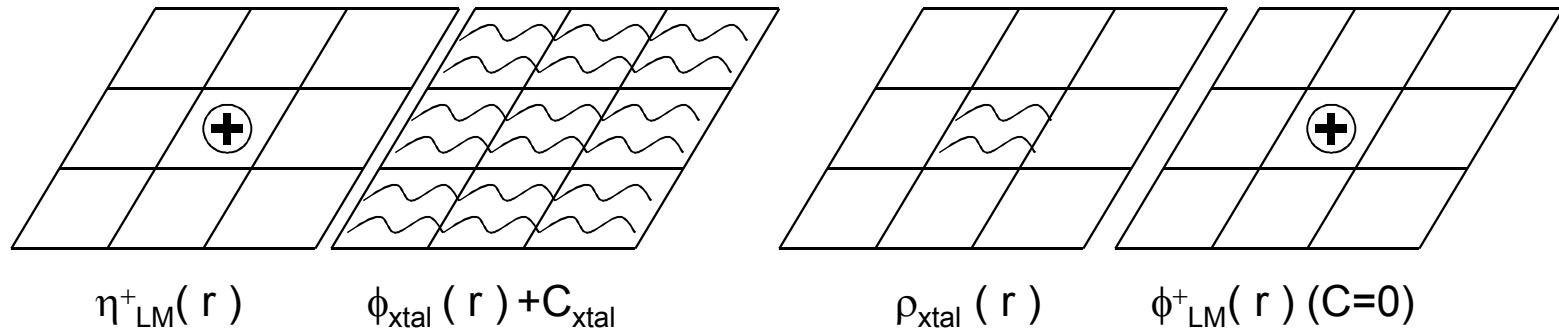
Replace interaction of net charge with periodic defect potential ...

$$E_{\mu_0} = - \int dr \eta^+_{LM} (\phi'_{def} + C'_{def}) + \int_{UC} dr \phi^+_{LM} \rho'_{def}$$



... with crystal potential:

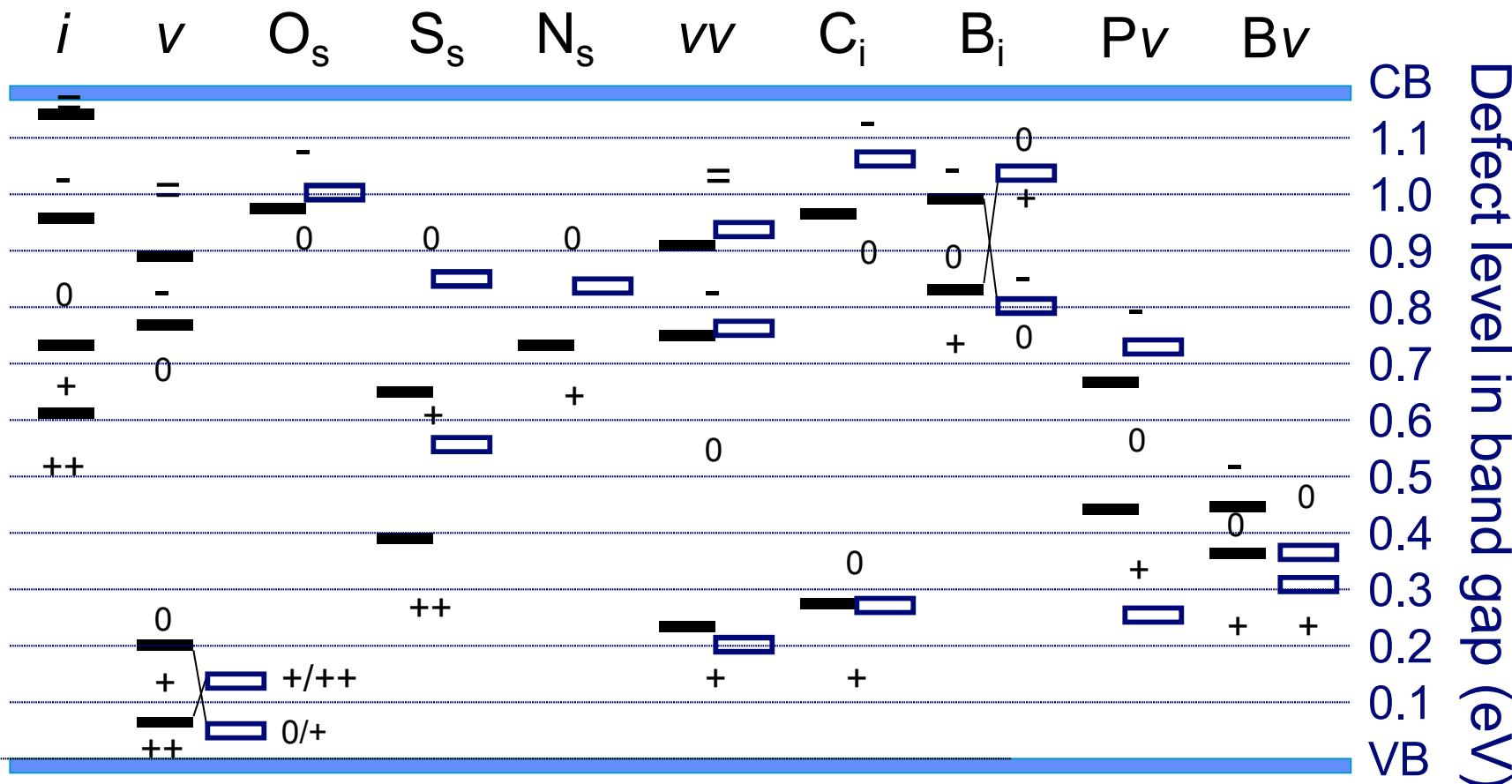
$$+ \int dr \eta^+_{LM} (\phi_{xtal} + C_{xtal}) - \int_{UC} dr \phi^+_{LM} \rho_{xtal}$$



Replace **variable** defect cell  $C'_{def}$ , with **fixed** crystal  $C_{xtal}$  reference  
 Chemical potential equivalent to matching potential at  $R=\infty$

# Si: DFT/LDA vs. Experimental Levels

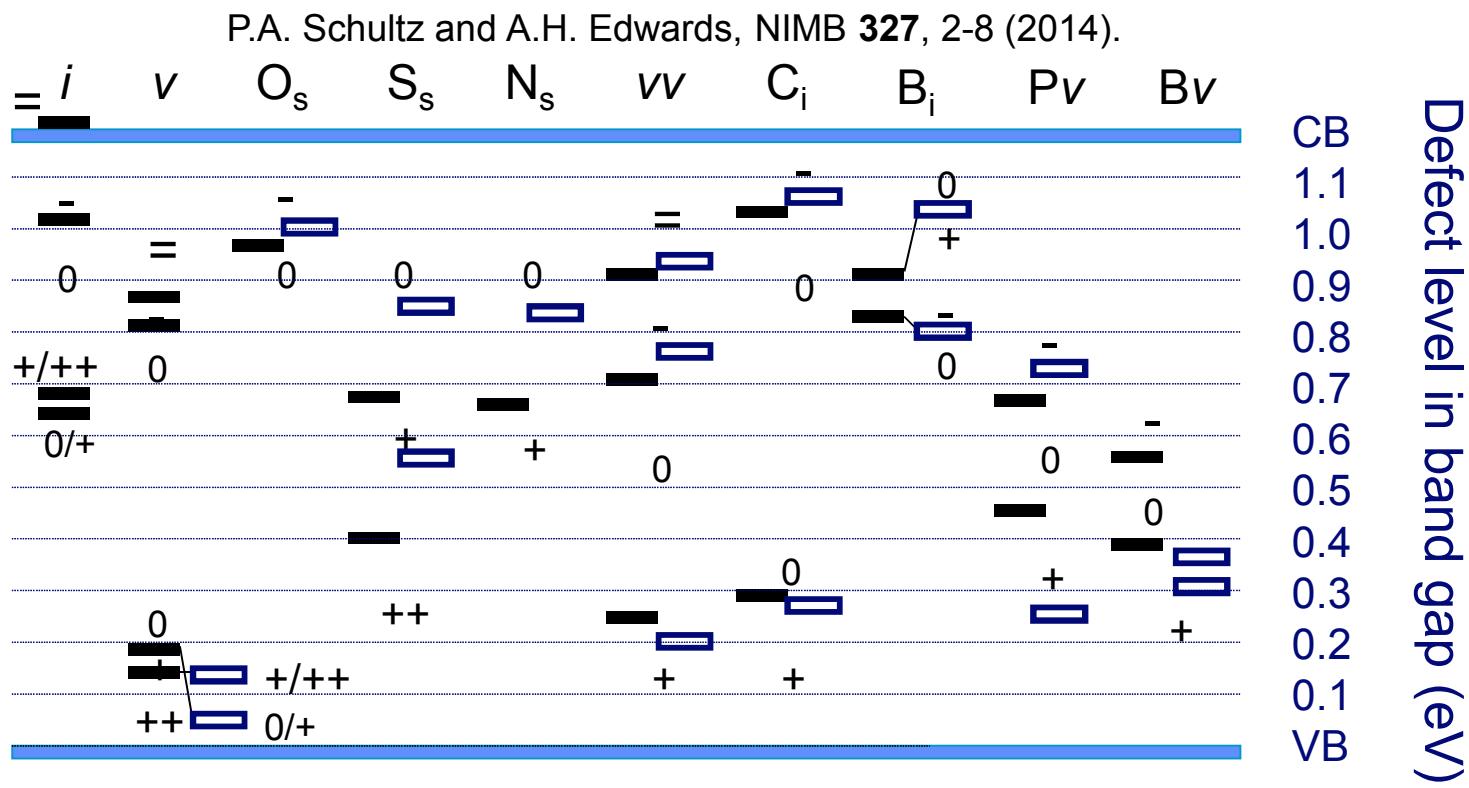
Peter A. Schultz, Phys. Rev. Lett. **96**, 246401 (2006).



LDA: max error=0.25 eV, mean |error|= 0.10 eV

Intrinsic, first-row, second-row, and complexes across gap  
LDA Kohn-Sham gap is only 0.5 eV

# Si: DFT/PBE vs. Experimental Levels



DFT/PBE defect level max error=0.20 eV, mean |error|=0.10 eV  
DFT “defect gap” matches experiment (KS gap: 0.6 eV)  
Band gap problem not seen in **total-energy-based** defect levels

# Computational methods – III-V's

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- General purpose DFT code SeqQuest (<http://dft.sandia.gov/Quest>)
  - well-converged (contracted-Gaussian) local orbital basis
  - both LDA and PBE functionals
  - converged norm-conserving pseudopotentials (Ga,In both  $Z_{\text{val}}=3,13$ )
  - full force relaxed (<1 meV total energies)
  - full FDSM ... robust control of boundary conditions
- Large bulk simulation supercells
  - $a_0=a_0(\text{theory})$ ; GaAs: 5.60 Å(LDA), 5.63 Å(3d), 5.74 Å(PBE);  $a_0(\text{expt})=5.65$  Å
  - Cubic supercells: 64-, 216-, 512-, 1000-site
  - $k$ -sampling:  $3^3$  for 64-site cells,  $2^3$  for 216-, 512-, 1000-site cells,
  - fully calibrated polarization model
  - all these computational parameters are tested for convergence

Comparable method that yielded 0.1 eV accuracy in Si

# Simple intrinsic defects in GaAs: LDA

P.A. Schultz and O.A. von Lilienfeld, MSMSE 17, 084007 (Dec. 2009).

$216^- = 512^- = 1000$ -site

Verification: cell-converged

LDA-3d = LDA to  $\leq 0.1$  eV

Verification: PP converged

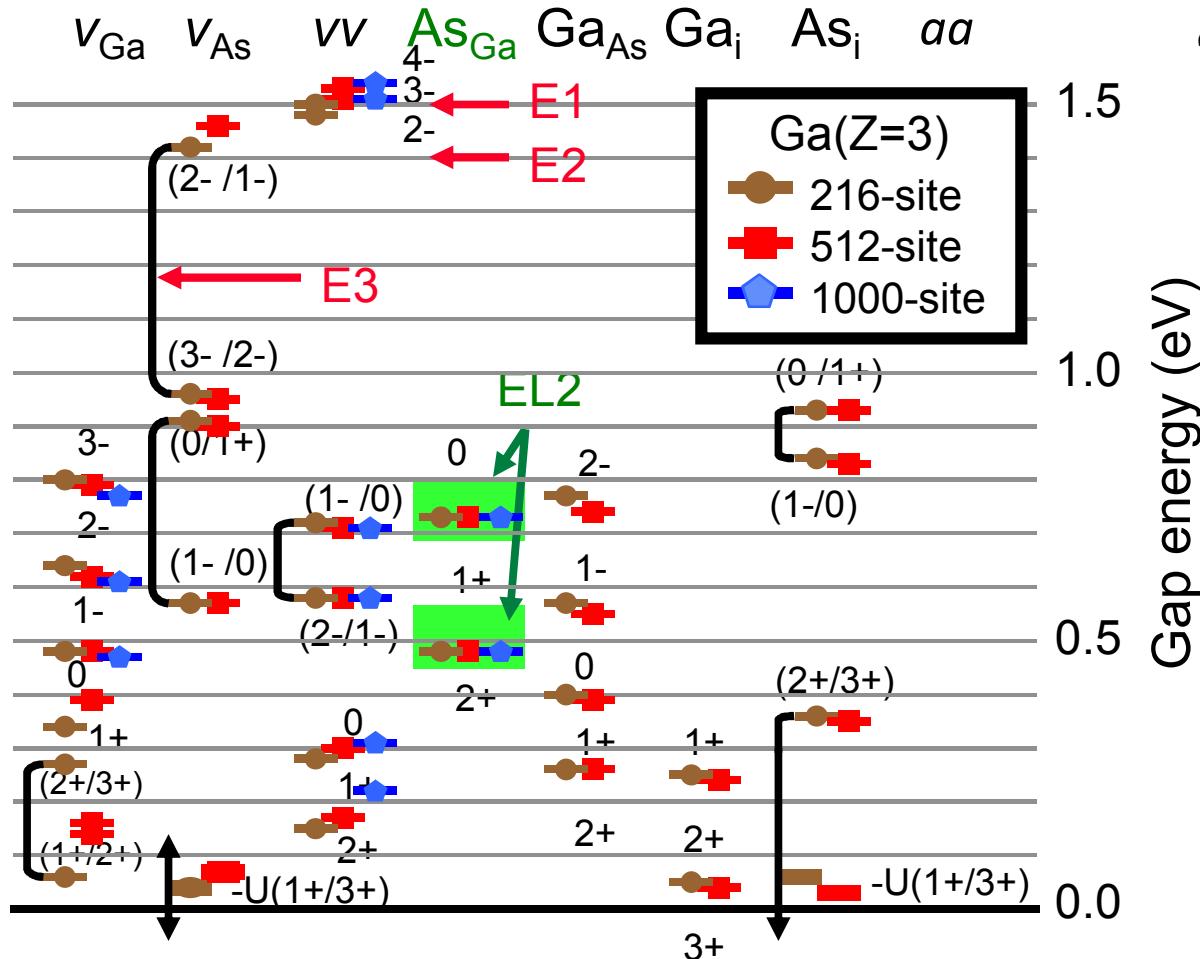
LDA~PBE; spin  $< 0.05$  eV

Verification: functionals

$V_{Ga}$  levels = EL2 levels

$V_{As}$  levels below midgap

Validation: levels  $< 0.1$  eV

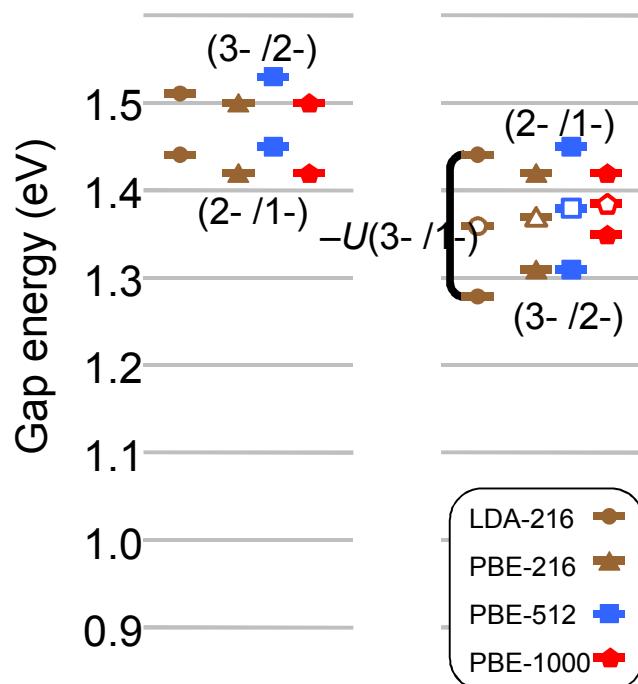


DFT+FDSM: Apparent accuracy of  $\sim 0.1$  eV

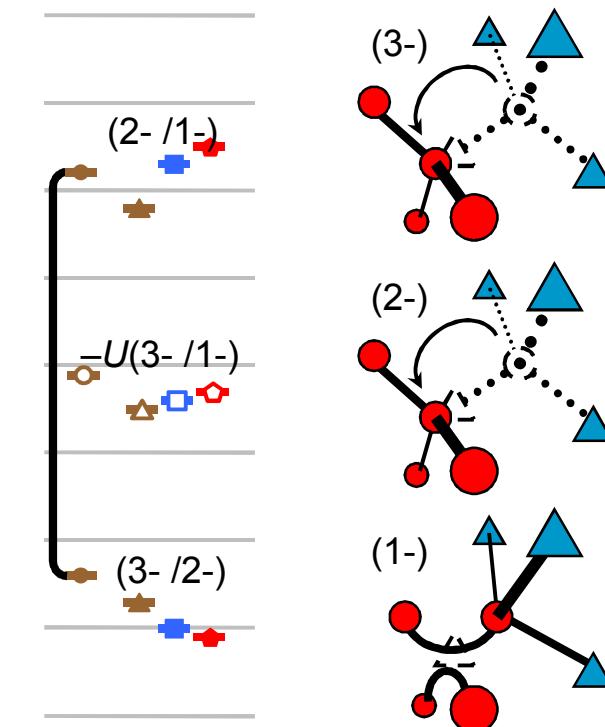
# The $v_{As}$ is *not* the E1-E2 center

*Simple*      *Complex*      *Site-shift*

$v'_{As}$   
 $T_d$        $v'_{As}$   
 $pD_{2d} \leftrightarrow rD_{2d}$        $v_{As}$   
 $v'_{As} \leftrightarrow v^*_{As}$



Incompatible with  $E1-E2$   
positron annihilation

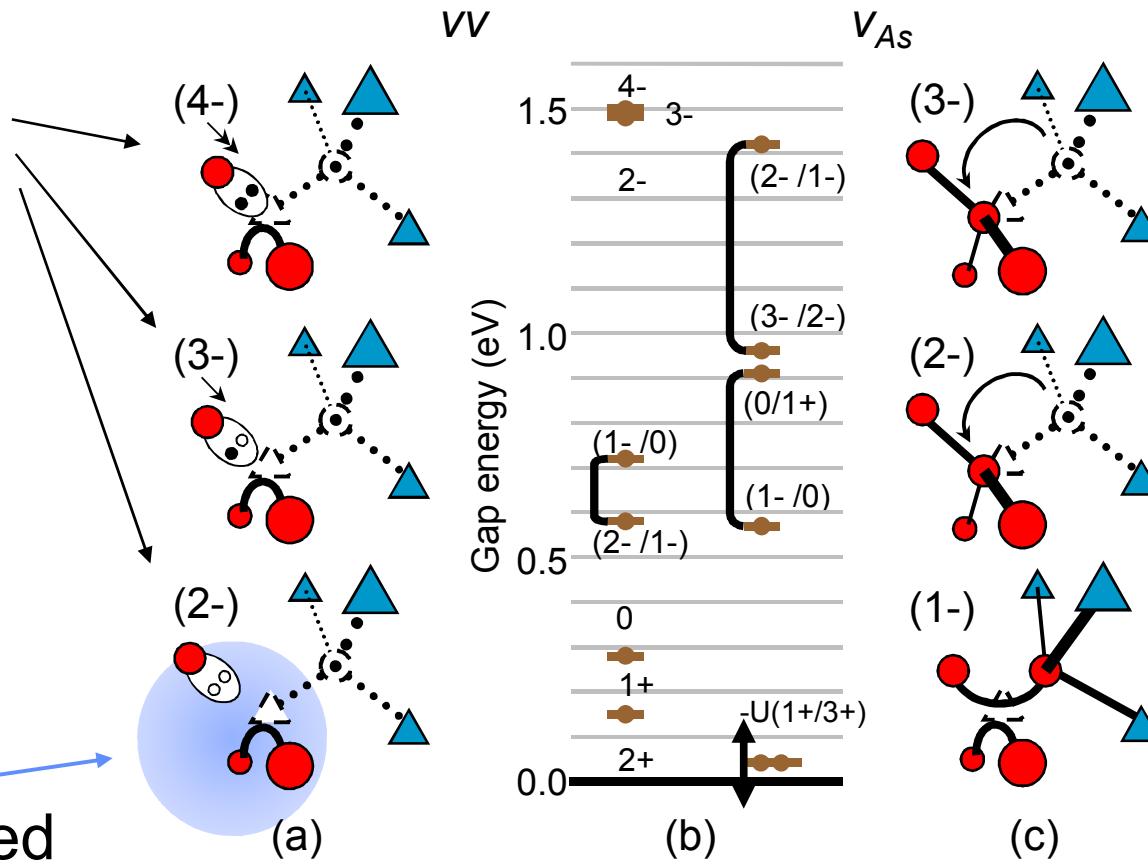


Matches  $E3$   
(DX-like,  
level position)

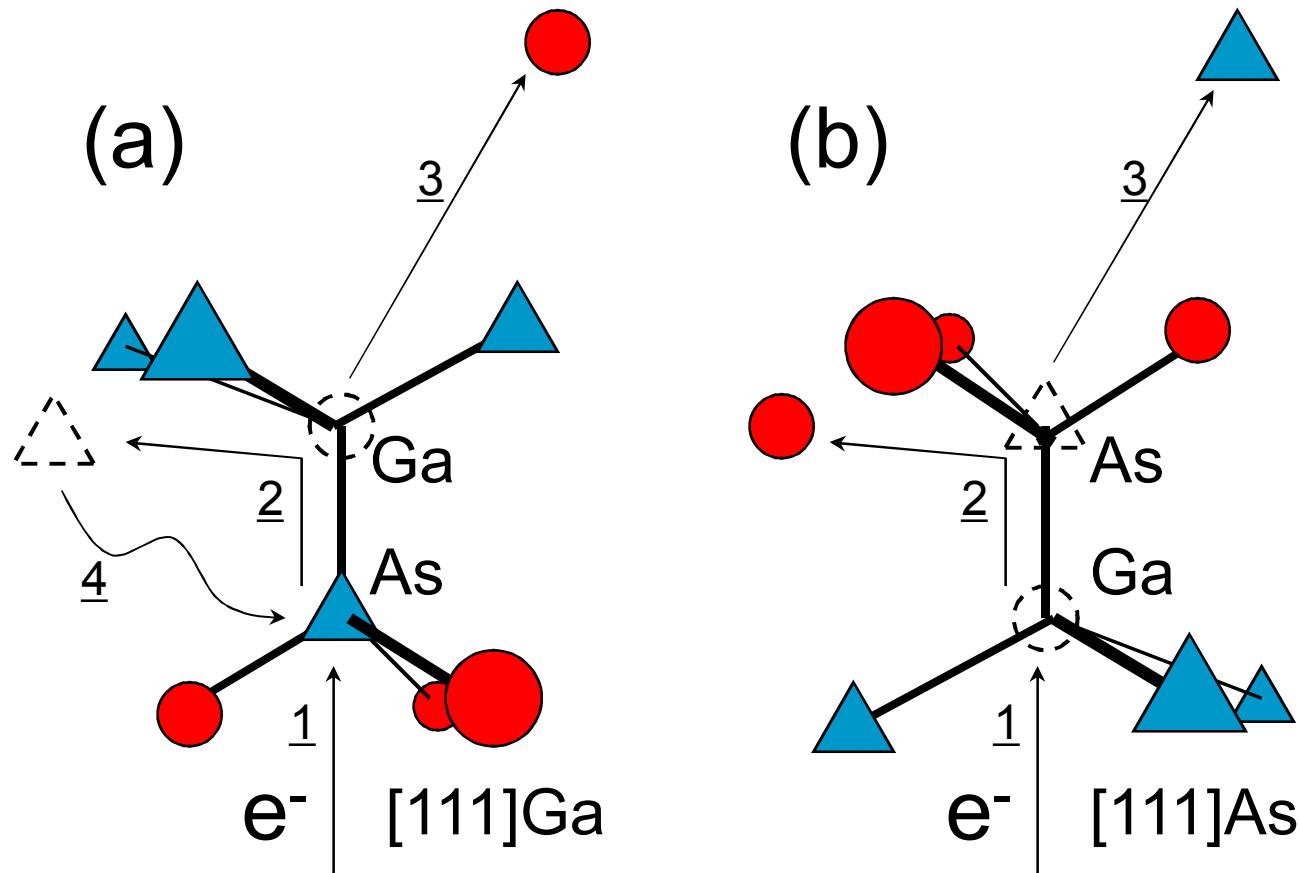
# The GaAs divacancy *is* the $E_1$ - $E_2$ center

Larger charge states

PAS explained on  $V_{As}$  side



# The GaAs divacancy *is* the E1-E2 center



$\nu\nu$  appears to be a threshold defect, on the As site  
(same annihilation kinetics cf.  $As_i$ , and  $\nu_{Ga}$  invisible)

# The GaAs divacancy is the E1-E2 radiation center

P.A. Schultz, J. Phys.: Condens. Matter **27**, 075801 (2015).

Peter A. Schultz

## Old (experimental) lore, back to 1988:

E1, E2 center =  $v_{As}(-/0)$ ,  $v_{As}(0/+)$

$$E_3 = V_{AS} + i$$

vv is dismissed

## Level structure reassigned with DFT:

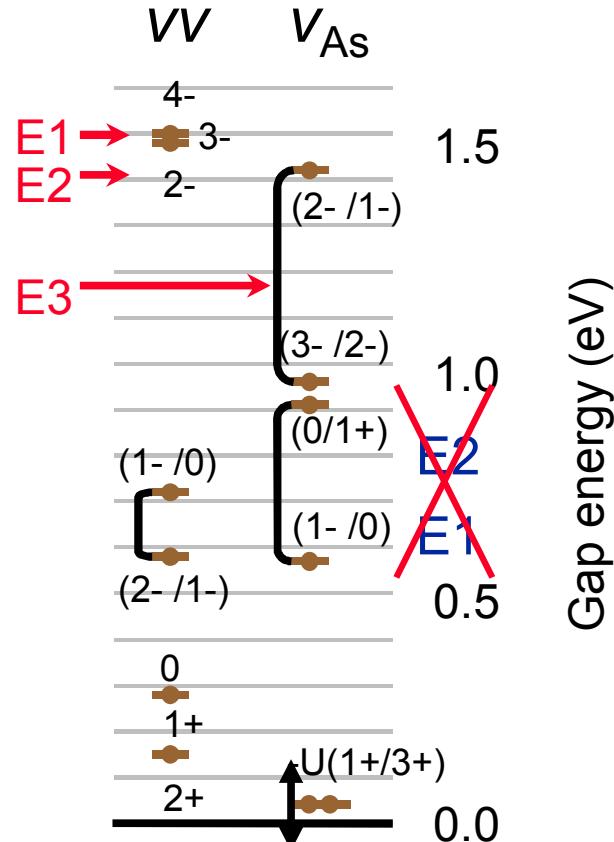
$v_{\text{As}}(-/+)$  is mid-gap negative-U (only one level)

$v_{\text{As}}$ (3-/1-) is upper-gap -U (one level)

## vv(4-/3-/2-) near conduction band

vv is major radiation defect: E1-E2

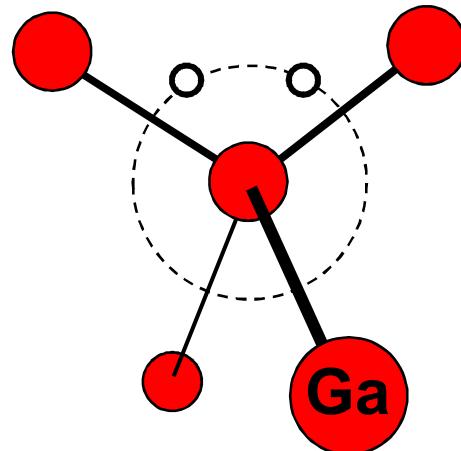
$\nu_{\text{As}}$ (3-/1-) transition is the E3



DFT-SeqQuest+FDSDM levels good enough to identify defects strictly on ***quantitative*** defect level calculations



# Discriminating a deep defect from shallow acceptors in supercell calculations: **Gallium antisite in GaAs**



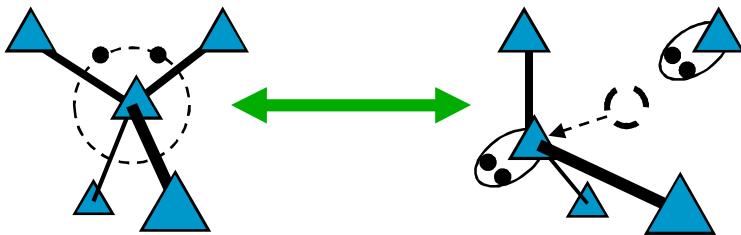
PAS, PRB **93**, 125201 (2016)

UUR  
SAND2016-2676C

# GaAs - defect physics poorly known

$\text{As}_{\text{Ga}} = EL2$

Well characterized – deep double donor



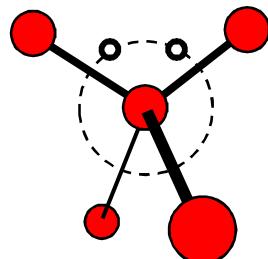
Theory crucial in characterization:

Dabrowski & Scheffler, PRL 1988

Chadi & Chang, PRL 1988

$\text{Ga}_{\text{As}} = ???$

Remains elusive



Experiment: Never definitively identified

Theory: Never definitively characterized  
- supercell problem: deep or shallow?

Crucial to distinguish if defect shallow or deep

# Ga<sub>As</sub> history

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Experiment: Ga-rich GaAs (>0.53 Ga) → “78/203” defect

Elliott 1982, 1983; Yu 1982

- *p*-type
- **residual shallow double acceptor at 78 meV and 203 meV**
- no distortion from  $T_d$
- **Ga-rich and shallow double acceptor → Ga antisite**

Doubts about 78/203: inadvertent B-contamination?

Kiessling, et al. 2008 – grown boron-free

- **semi-insulating, no residual acceptor**
- **shallow double acceptor = B<sub>As</sub> antisite**

# Defect levels in GaAs: LDA

P.A.S. and O.A. von Lilienfeld, MSMSE **17**, 084007 (2009)

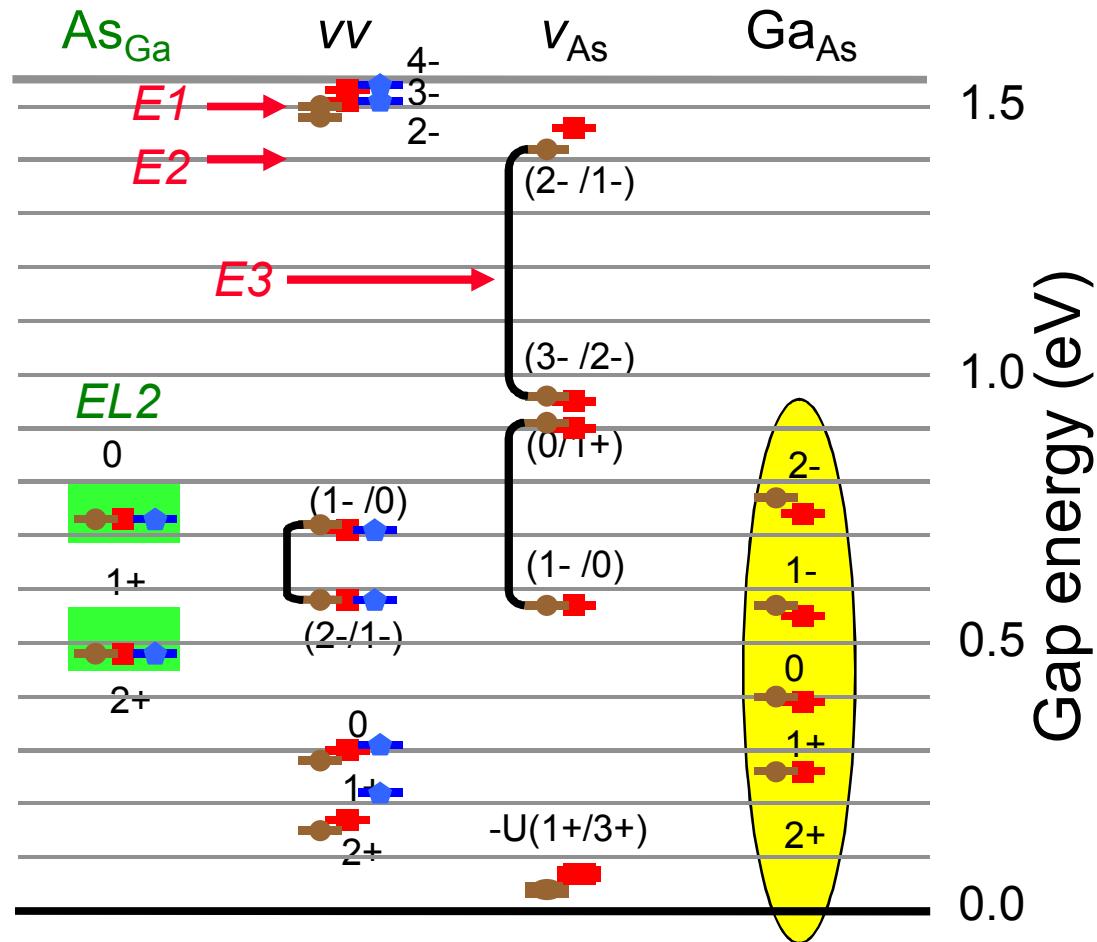
P.A.S., JPCM **27**, 075801 (2015)

## DFT with SeqQuest

Gaussian basis pseudopotentials  
LDA and PBE  
216,512,1000-atom supercells  
Converged model parameters  
LMCC charge boundary conditions  
216-, = 512-, = 1000-atom levels

## DFT matches experiment

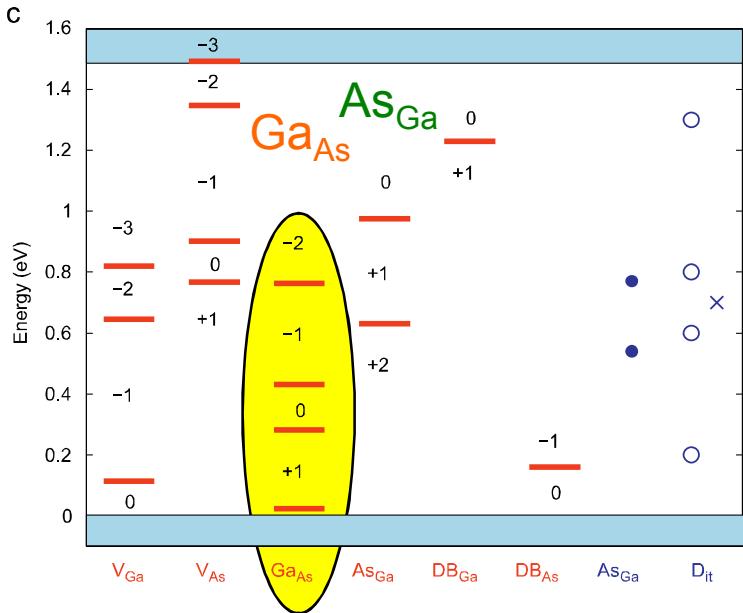
$EL2$  =  $As_{Ga}$   
 $E1-E2$  = divacancy  
 $E3$  =  $v_{As}(3-1-)$   
Accuracy  $\sim 0.1$  eV



**Ga antisite: two deep acceptors, two donors?**

# Theory also inconclusive

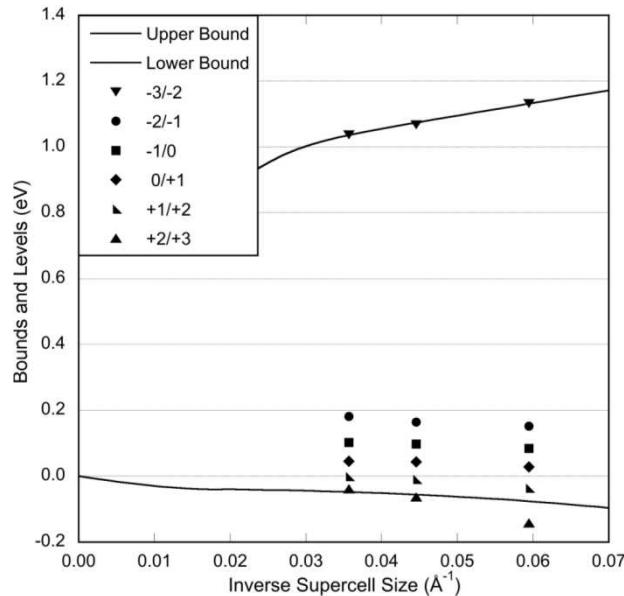
Komsa&Pasquarello, Physica B (2012)



HSE, 64-atom cells

Deep: two acceptor & two donor states

Wright&Modine, PRB (2015)



LDA, 216-, 512-, 1000-atom cells

Shallow double acceptor

HSE study agrees ...

... newer bounds analysis does not

**Is  $Ga_{As}$  deep defect a supercell artifact?**

# Discriminating shallow/deep acceptors

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**Shallow:** state and its charge delocalized, **supercell fails**

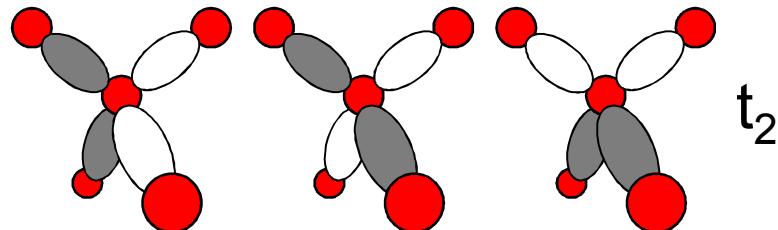
- defect banding, states entangled in VB
- less/no structural distortions

**Deep:** state and its charge localized, **DFT is valid**

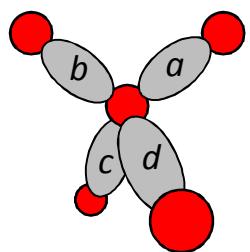
- eigenstates distinct from VBE
- greater structural distortions
- greater spin polarization

# A conceptual model of $\text{Ga}_{\text{As}}$

$\text{Ga}_{\text{As}}$ : LCAO-MO model  
(thank you, George Watkins!)

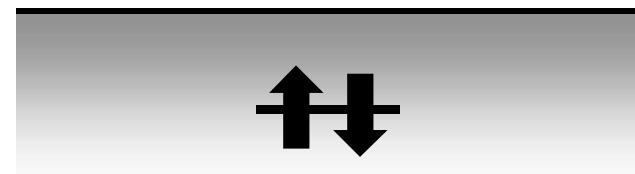
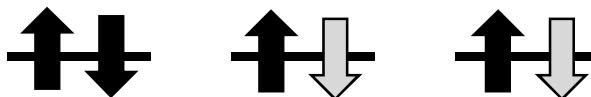


$t_2$



$a_1$

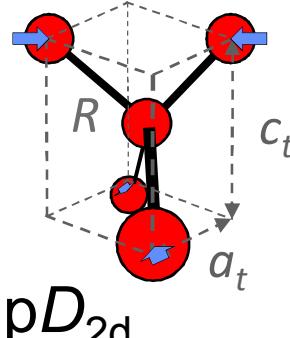
$\text{Ga}_{\text{As}}(0)$



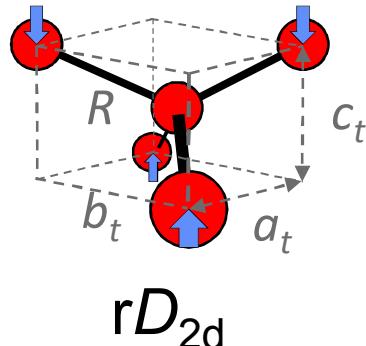
VBE

Jahn-Teller instabilities

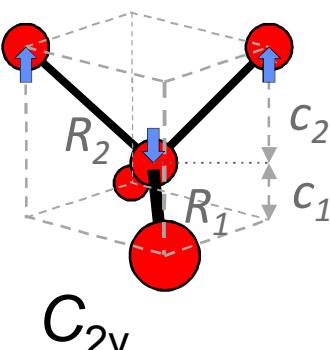
Occupation level patterns determine proper distortion/spin states



$pD_{2d}$



$rD_{2d}$



$C_{2v}$

# Ga<sub>As</sub>(q) ground states

	Ground state		$\Delta E(T_d, \text{ meV})$
(2-)		$T_d$	0
(1-)		$rD_{2d}$	-71
(0)		$pD_{2d}$	-102
(1+)		$T_d$	-95
(2+)		$rD_{2d}$	-25

(1000-atom PBE)

$rD_{2d}(1-)$

Relaxation energy (meV)

3x3x3	-15
4x4x4	-23
5x5x5	-30

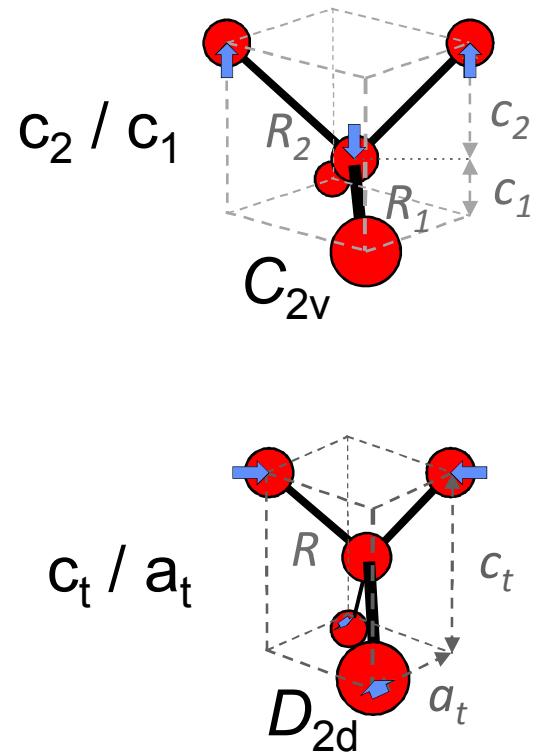
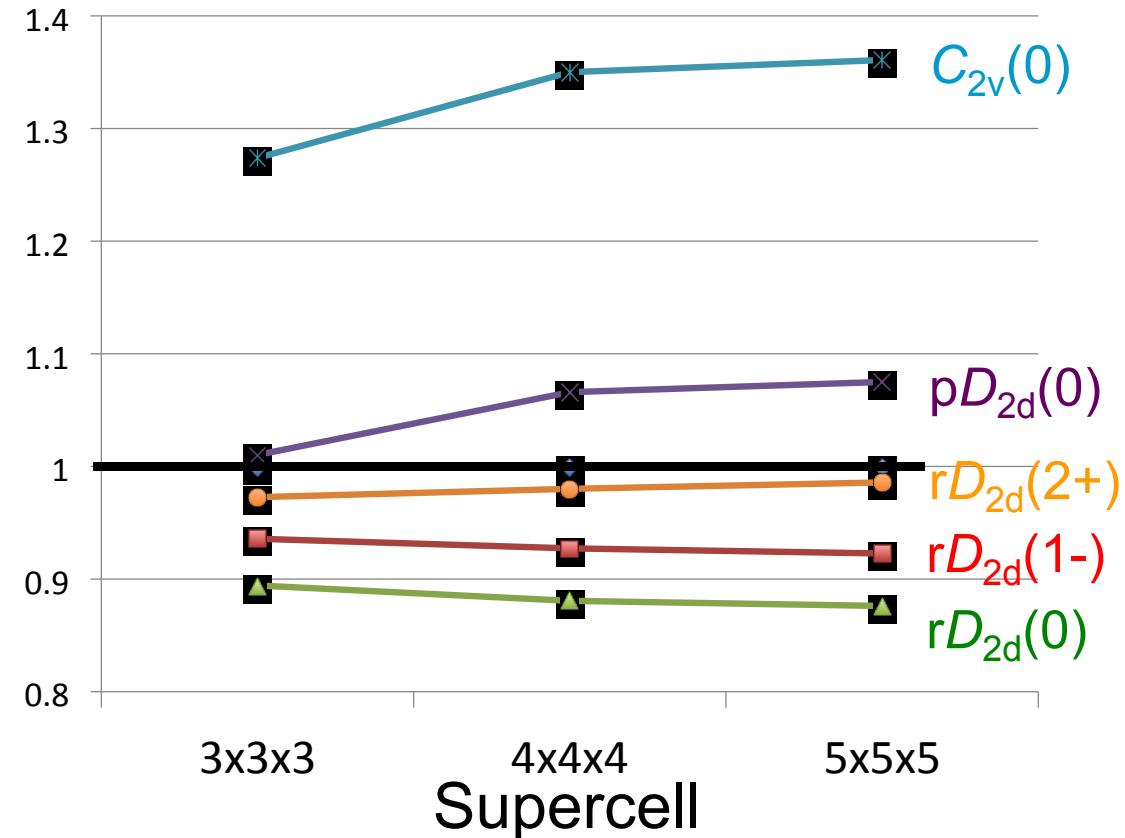
Spin energy (meV)

3x3x3	-33
4x4x4	-38
5x5x5	-41

All ground states distort from (spinless)  $T_d$   
 Relaxation, and spin polarization energy increase with cell size

# Ga<sub>As</sub> distortions magnify with cell size

## Distortion ratios



... except for  $pD_{2d}(2+)$ , which is only barely distorted

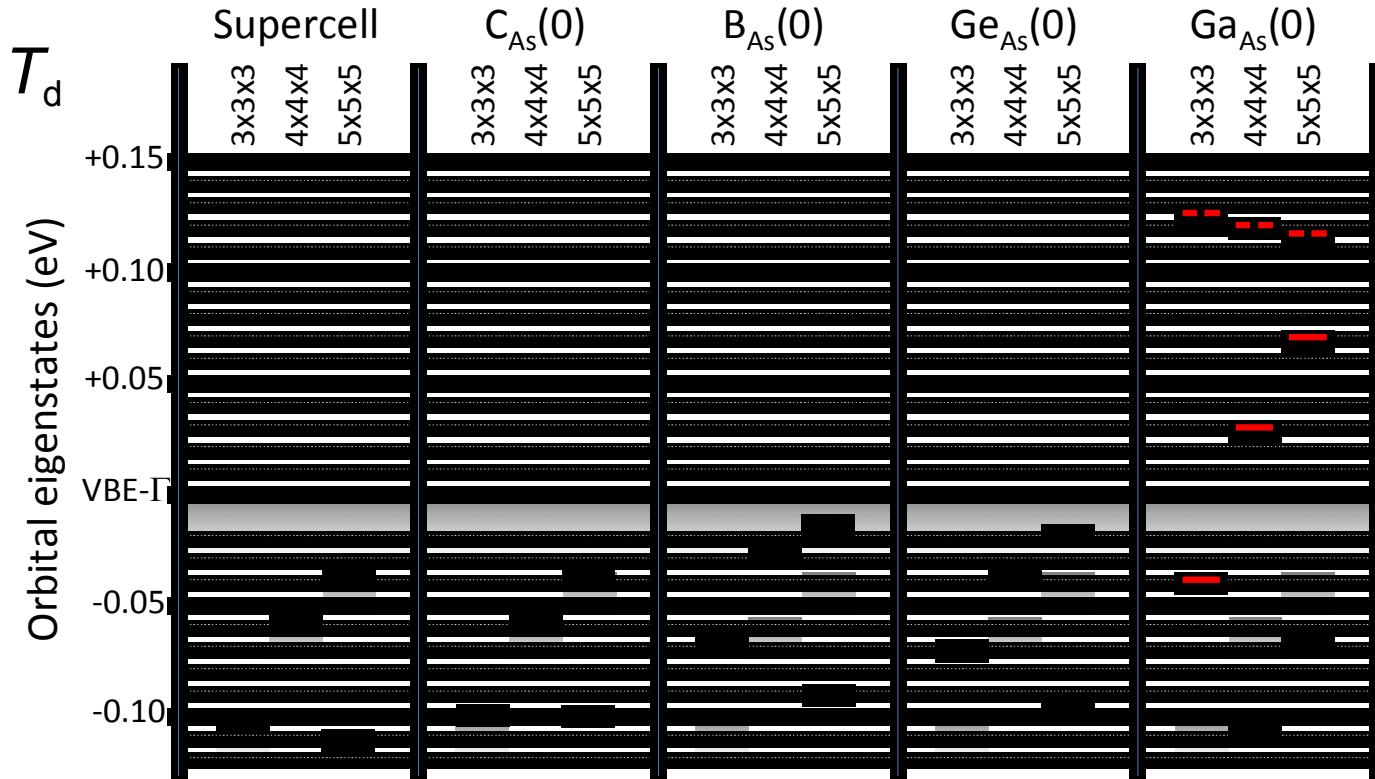
# What do $X_{\text{As}}$ shallow acceptor supercells look like?

$C_{\text{As}}$  = shallow acceptor

$B_{\text{As}}$  = shallow double acceptor

$Ge_{\text{As}}$  = shallow acceptor

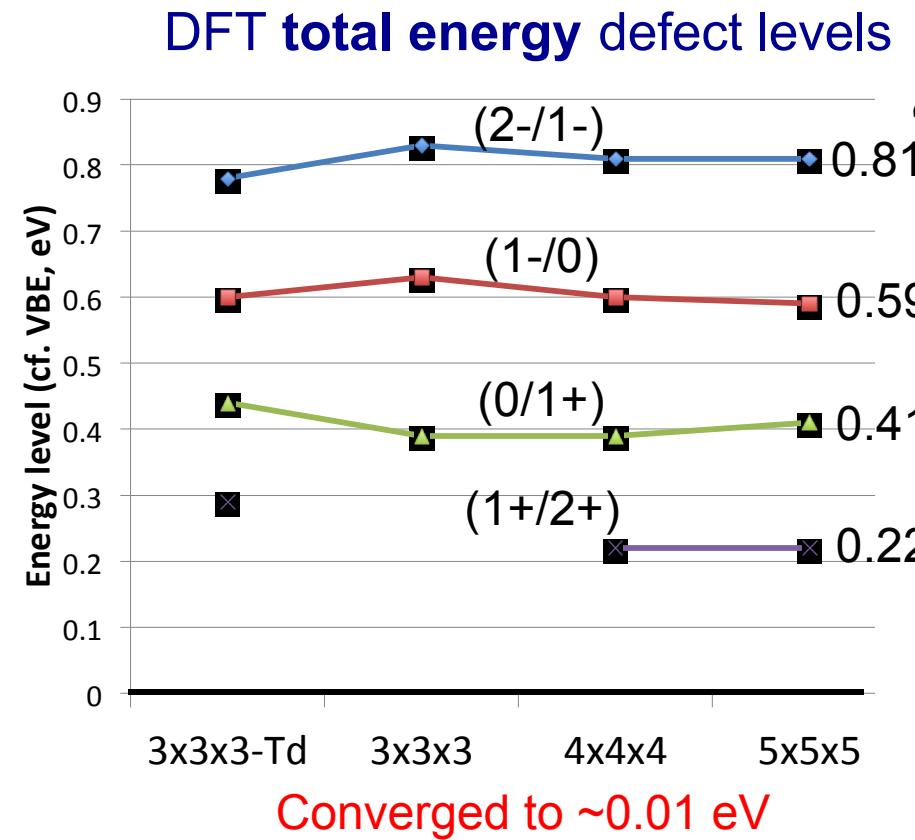
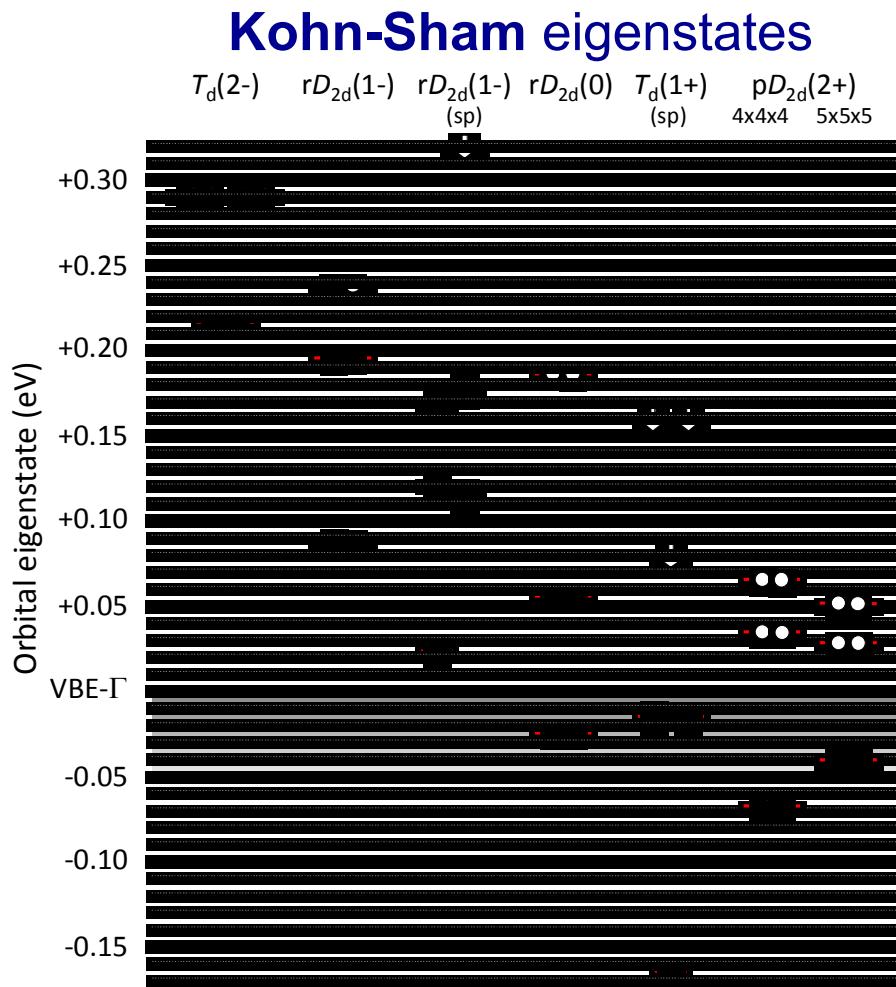
$Ga_{\text{As}}$  = ????



$C_{\text{As}}$ ,  $B_{\text{As}}$ ,  $Ge_{\text{As}}$ : no distortions (<1 meV)  
no spin polarization (<3 meV)  
no new defect eigenstates in “small” cells

$Ga_{\text{As}}$  is different –  $Ga_{\text{As}}$  is not an effective mass state

# Ga<sub>As</sub> eigenstates and energy levels



KS eigenstates cleanly within global Kohn-Sham gap  
Energy levels converged to ~0.01 eV with supercell

**Eigenstates  $\neq$  energy levels: levels expanded**

# The Ga antisite summarized

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Recently published: P.A.S. PRB **93**, 125201 (2016)

## Shallow states in supercells: $\text{C}_{\text{As}}$ , $\text{B}_{\text{As}}$ , $\text{Ge}_{\text{As}}$

- no distortions (<1 meV)
- small, decreasing spin energy (< 3 meV)
- KS states entangled in VB in small supercells, “small” > 1000 atoms!

## $\text{Ga}_{\text{As}}$ is localized: two deep donor, two acceptor states

- large, increasing distortions, increasing spin, clean KS spectrum
- distinct from behavior of known shallow acceptors

## $\text{Ga}_{\text{As}}$ is not the 78/203 shallow double acceptor

- shallow- $\text{Ga}_{\text{As}}$  is high-energy defect in Ga-rich p-type GaAs
- low-energy defects  $\text{Ga}_i$ ,  $\text{v}_{\text{As}}$ , and deep-  $\text{Ga}_{\text{As}}$  are donors → semi-insulating

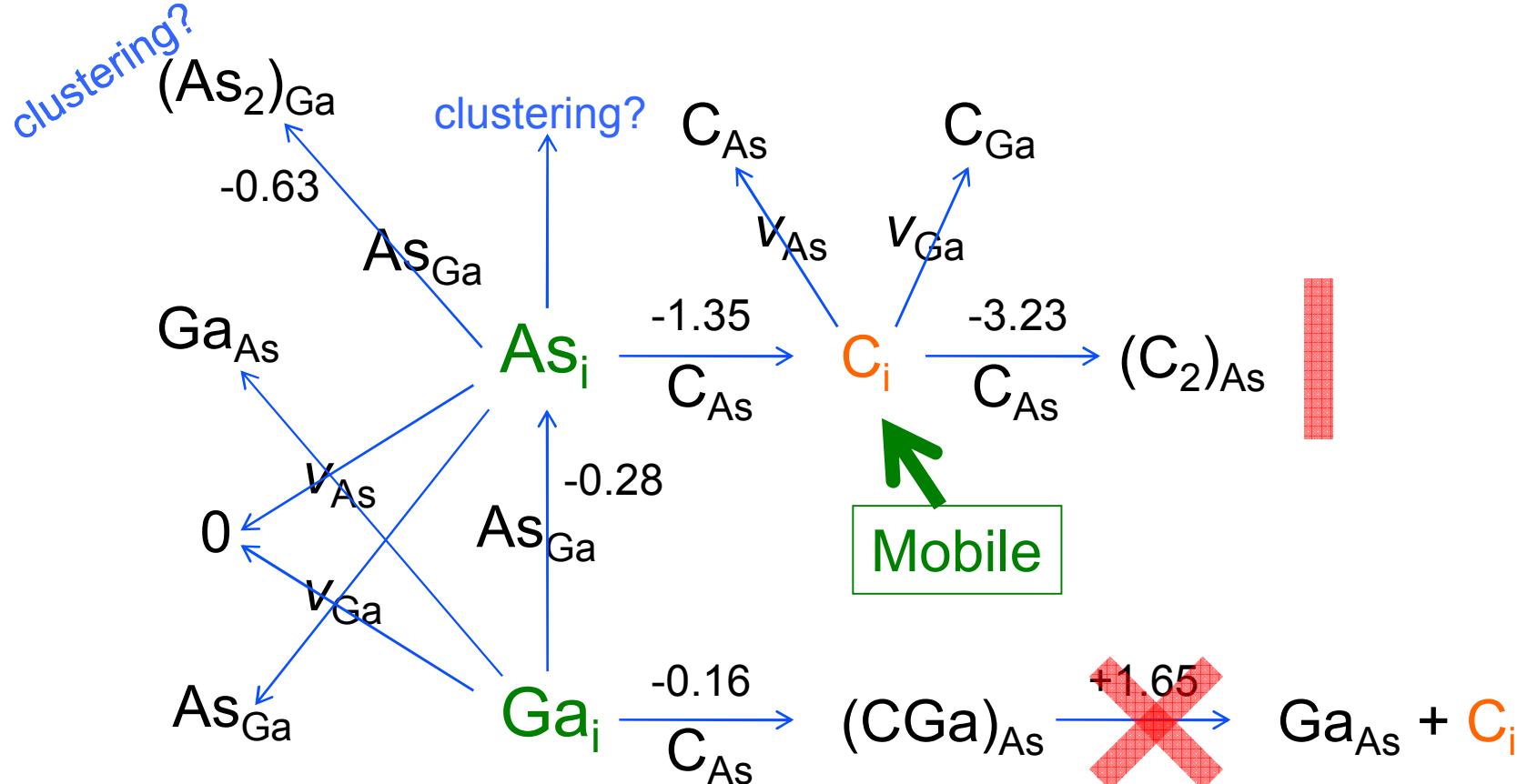
## Energy level scale expanded compared to KS eigenstates

- “band gap problem” is ... not a problem

# GaAs: C-doped reaction network

P.A. Schultz, J. Res. Eng: Rad. Effects **30**, 257 (2012).

SeqQuest, LDA, 216-site, thermodynamic energy with  $E_f = VBE$  (p-type)  
Reaction networks initiated by identified mobile species:  $As_i$ ,  $Ga_i$

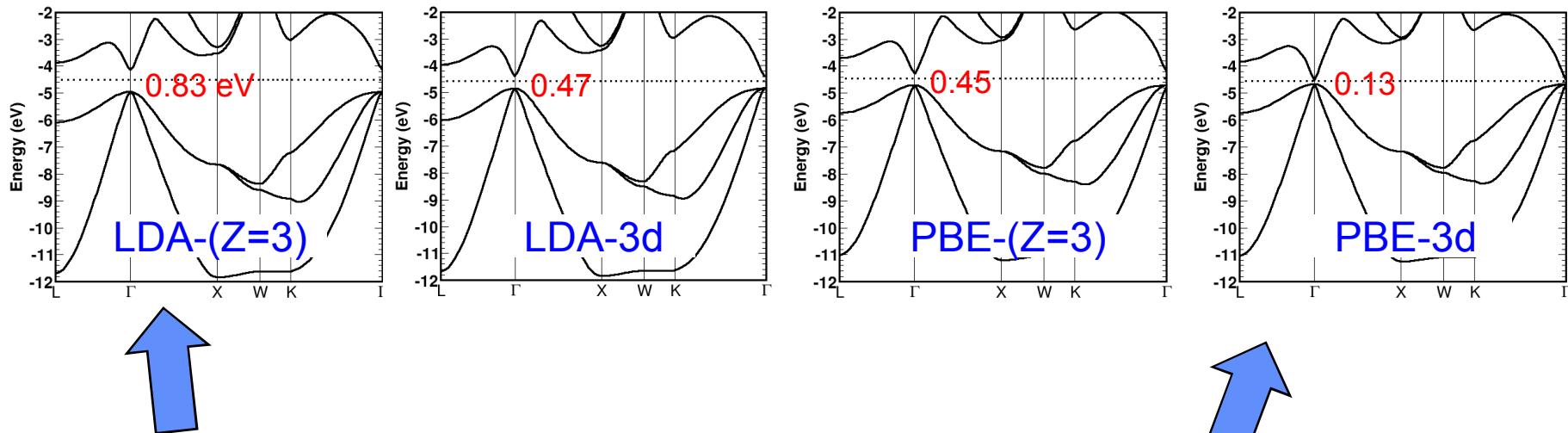


Reliable defect levels means reliable chemistry

# GaAs: A theoretical laboratory

P.A. Schultz and A.H. Edwards, NIMB **327**, 2-8 (2014).

Change the Ga pseudopotential and the functional, and the **KS band gap shrinks...**



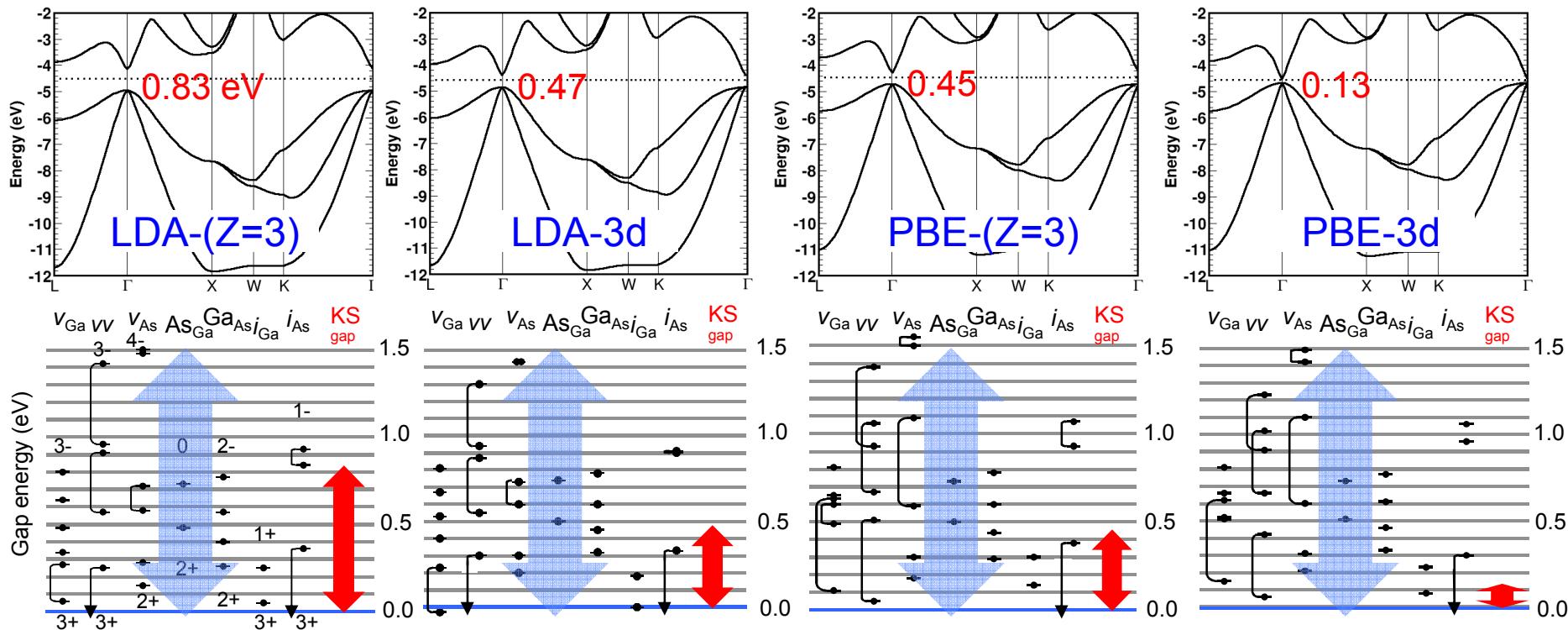
from 0.83 eV, LDA, Ga( $Z=3$ ) pseudopotential (PP) ...

... to 0.13 eV, PBE-3d, Ga( $Z=13$ ) pseudopotential (PP)

# GaAs: A theoretical laboratory

P.A. Schultz and A.H. Edwards, NIMB 327, 2-8 (2014).

Change the Ga pseudopotential and the functional, and the **KS band gap** changes ...



... but span of (total-energy-based) defect levels, the “defect band gap”, does not

Defect levels/gap **insensitive** to size of Kohn-Sham gap!

# GaAs Computational model lessons

---

- KS band gap not a problem for Si and GaAs defects
- Defect levels insensitive to size of Kohn-Sham gap!
  - total-energy differences vs. eigenvalue-referenced
  - GaAs is ideal theoretical laboratory for testing methods
- Detailed control of boundary conditions crucial: FDSM works
- Is this unique to Si and GaAs?

# Simple intrinsic defects in AlAs: Energy levels

MRS Symposia Proceedings 1370, (MRS Spring 2011); SAND2012-2938 (April 2012)

Verified cell-convergence

Calibrated:  $v_{\text{Al}}^{(\text{u})}$

Checked:  $\text{As}_{\text{Al}}$

Verified:  $vv$

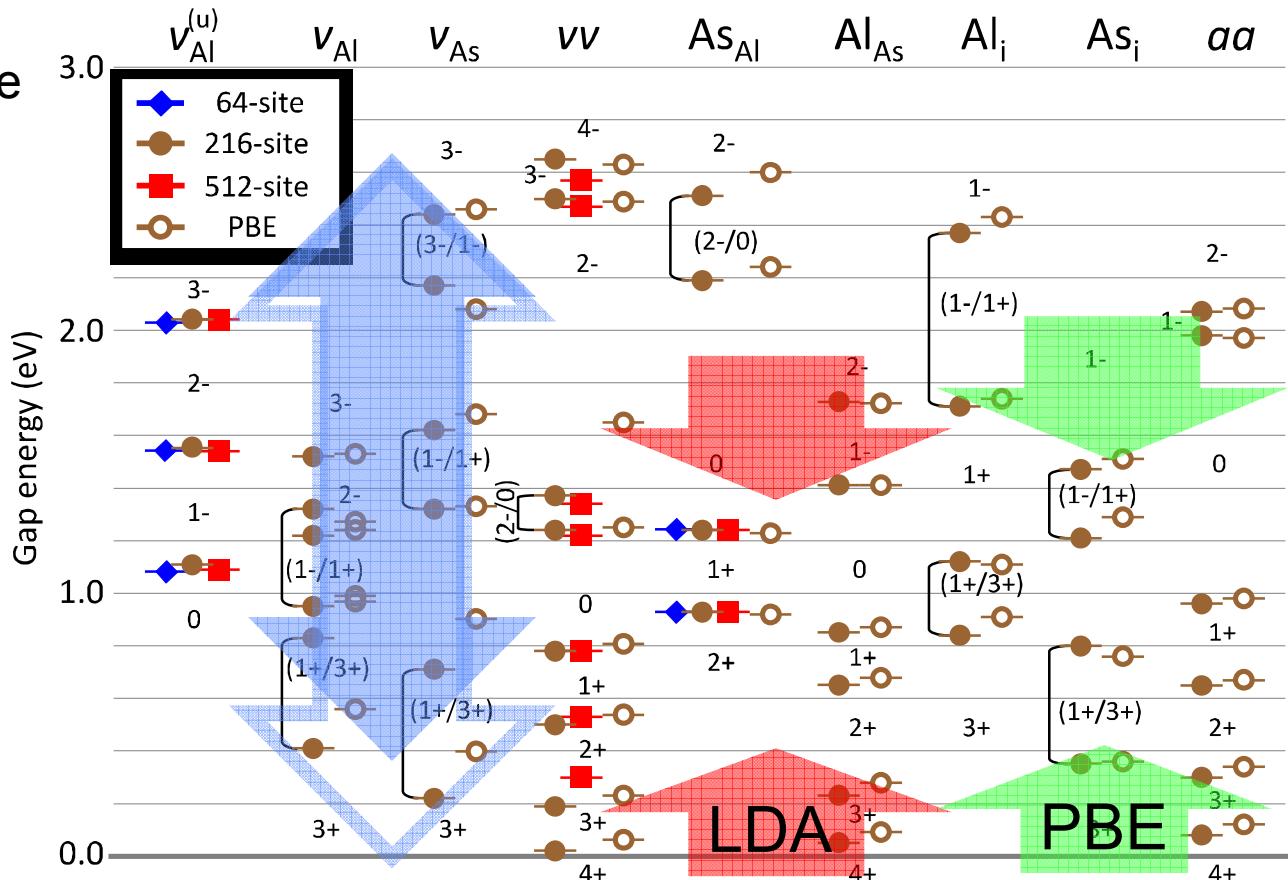
AlAs band gap

KS-LDA: 1.37 eV

KS-PBE: 1.53 eV

Defect span: 2.3 eV

Experiment: 2.16<sup>i</sup> eV



Very similar to GaAs defects, with some new features  
A reverse band gap problem?

# GaP intrinsic defects

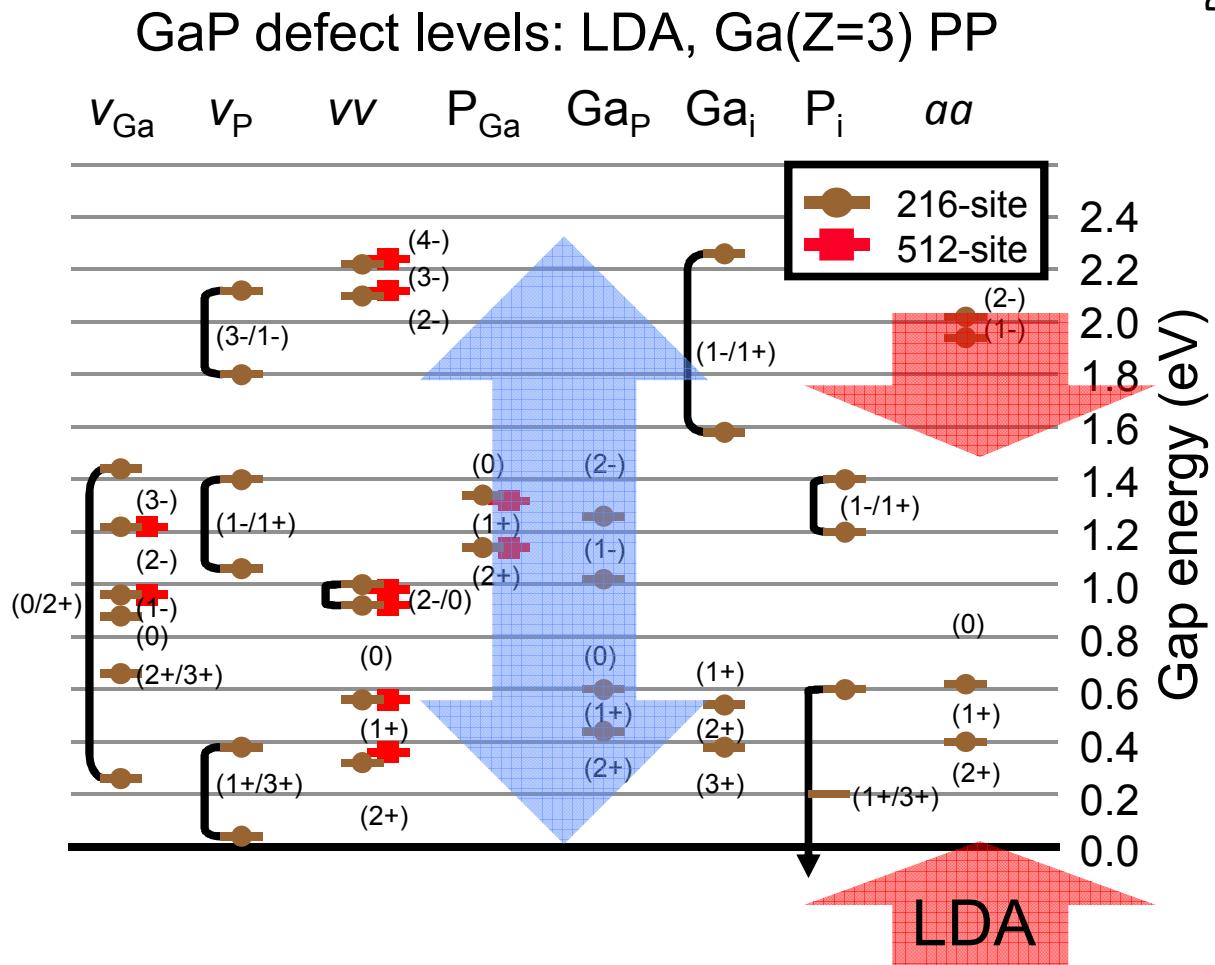
216-site results = 512-site  
Verification: cell-converged

GaP band gap

KS-LDA: 1.51<sup>i</sup> eV

Defect span: 2.35 eV

Experiment: 2.35<sup>i</sup> eV



# InP intrinsic defects

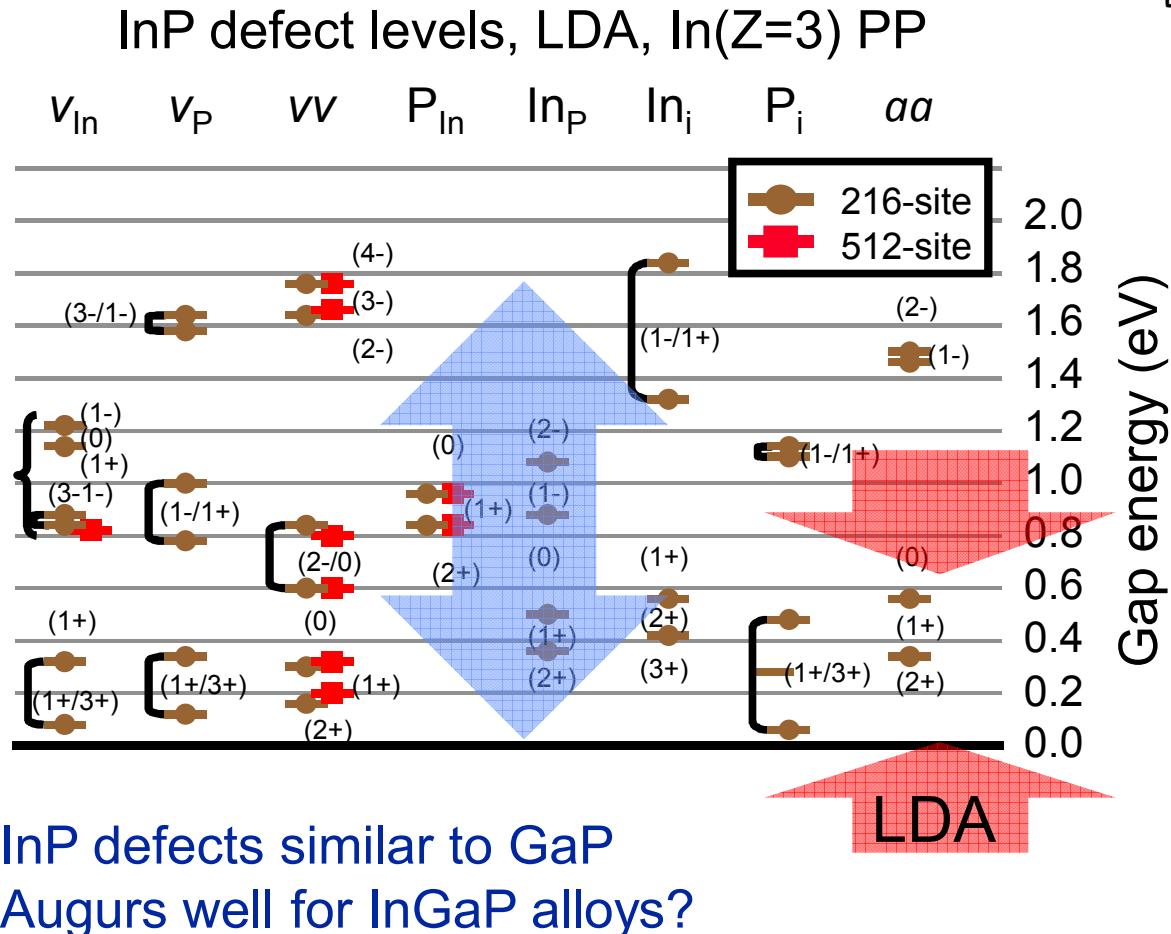
216-site results = 512-site  
Verification: cell-converged

InP band gap

KS-LDA: 0.67 eV

Defect span: 1.7 eV

Experiment: 1.42 eV



# AIP intrinsic defects

A.H. Edwards, H. Barnaby, A.C. Pineda, P.A. Schultz, IEEE-Trans. Nucl. Sci. **60**, 4109 (2013)

216-site results = 512-site  
Verification: cell-converged

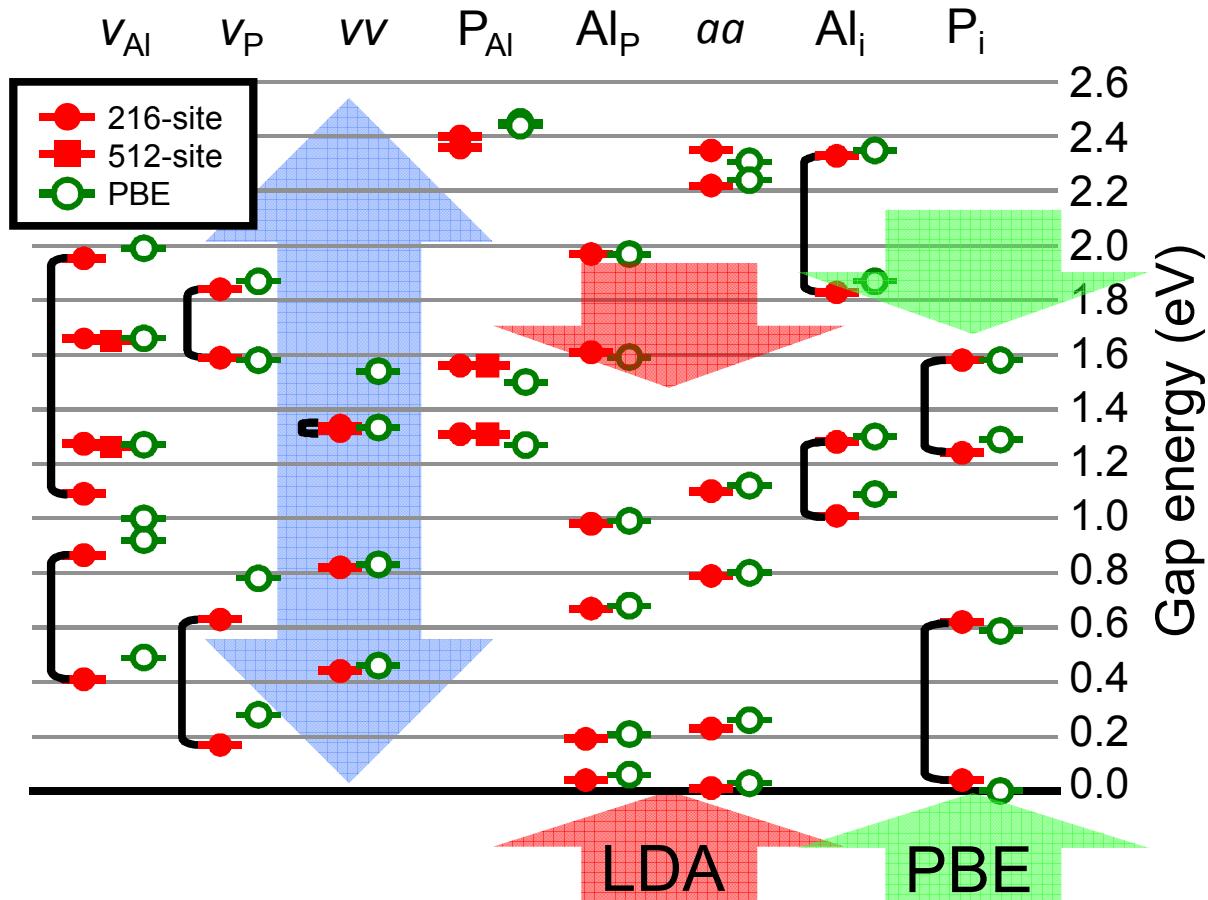
AIP band gap

KS-LDA: 1.48 eV

KS-PBE: 1.67 eV

**Defect span: 2.55 eV**

Experiment: 2.51 eV



# The DFT “Defect band gap”

- Kohn-Sham gap: **outside** bounds of VB to CB *band eigenvalues*
- Defect band gap: **inside** bounds of transition **energies** for defect levels

Band gaps: **experiment**, Kohn-Sham, **DFT defect gap**

Si	1.17 eV		AlAs	2.16 <sup>i</sup> eV		AlP	2.51 eV	
	KS	Defect		KS	Defect		KS	Defect
lda	0.49	1.2	lda	1.37	>2.3	lda	1.48	2.55
pbe	0.62	1.2	pbe	1.53	>2.3	pbe	1.67	2.55
GaAs		1.52 eV		GaP		2.35 <sup>i</sup> eV		InP
	KS	Defect		KS	Defect		KS	Defect
lda	0.83	1.54	lda	1.51	2.35	lda	0.67	1.7
lda-3d	0.47	1.52	lda-3d	1.47	2.35	lda-3d	0.66	1.7
pbe	0.45	1.50	pbe	1.74	2.35	pbe	0.47	1.7
pbe-3d	0.13	1.50	pbe-3d	1.52	n/c	pbe-3d	0.46	n/c

**Total energy** defect gap insensitive to Kohn-Sham gap  
**Defect band gap** matches (overshoots?) experiment

# Other examples

---

IV-IV: 3C-SiC (cubic)

GGA/PBE KS Gap: 1.38 eV

**Defect Gap (PBE):** ~2.4

**Experimental Gap:** 2.40

II-VI: CdTe (3d-valence)

LDA KS Gap: 0.81 eV

PBE KS Gap: 0.69 eV

**Defect Gap (LDA&PBE):** ~1.6

**Experimental band gap:** 1.60

Close correlation of the **defect gap** with **experiment band gap**

What about a crystal that is not tetrahedral, and a large gap?

# CsI defect level spectrum - DFT

R. M. Van Ginhoven and P.A. Schultz, J.Phys.: Cond. Matter **25**, 495504 (2013)

250-site results = 432-, 686-site

Verification: cell-converged

$v_I$  levels match experiment

Validation of accuracy

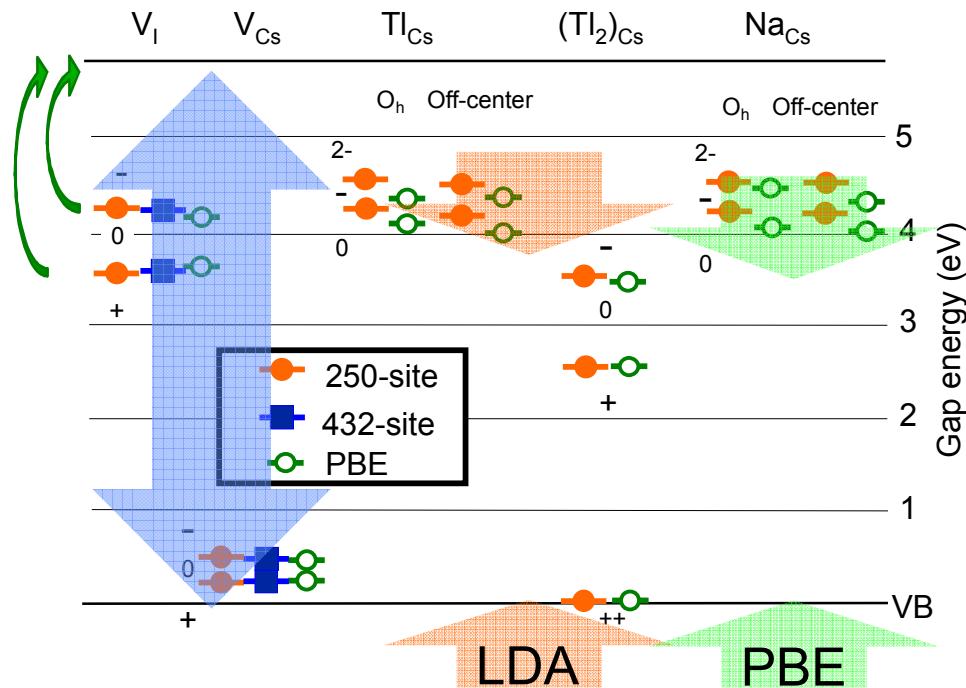
CsI band gap

KS-LDA: 3.80 eV

KS-PBE: 3.58

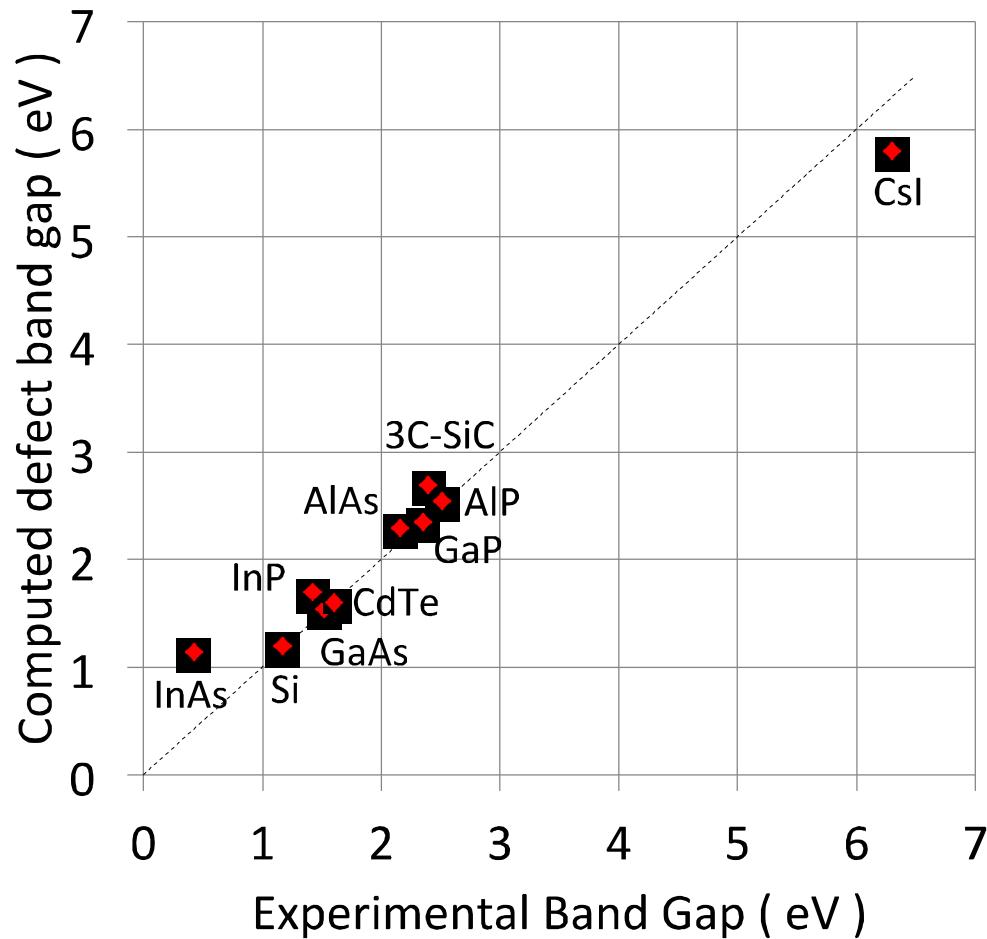
Defect span: >5.8 eV

Experiment: 6.3 eV



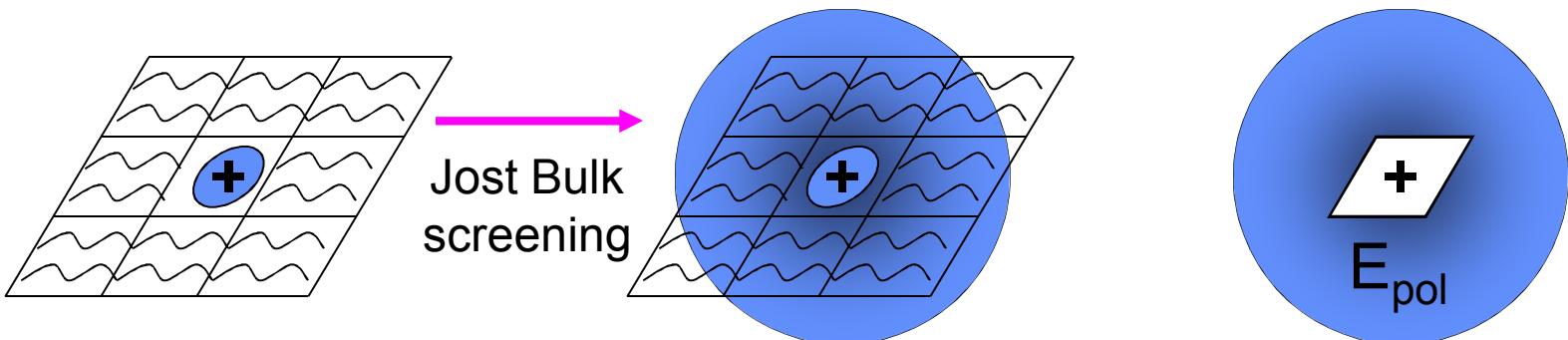
Not a band **gap** problem, a **band edge problem**—  
where are they cf. total energy defect levels?

# The Defect Gap vs. the Band Gap



Defect gap = experiment, despite a band gap problem

# The polarization model



For extrapolation to bulk, need energy of screening outside of supercell:  $E_{pol}$

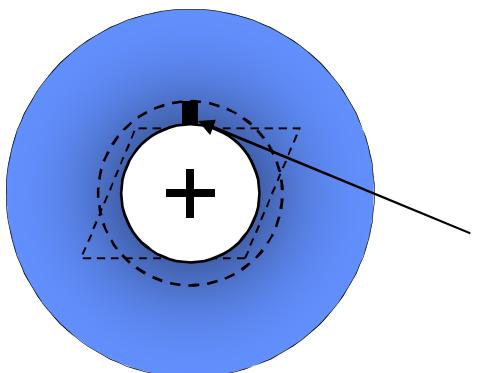
Jost model:  $E_{pol} = \frac{(1 - 1/\epsilon_0) q^2}{R_{jost}}$

$$R_{jost} = R_{vol} - R_{skin}$$

$q$  = charge on defect

$$R_{jost} = R_{vol} - R_{skin}$$

$R_{vol}$  = radius of volume sphere

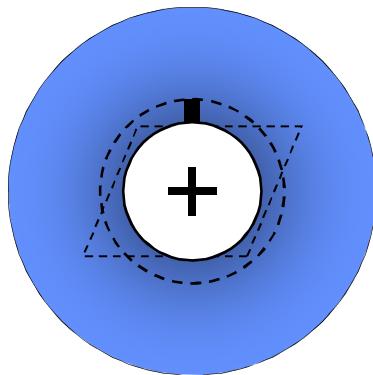


Two parameters for any material

$R_{skin}$  = unscreened volume **inside** cell.  
fit: = 1.3-1.7 Bohr

$\epsilon_0$  = static dielectric constant - expt  
Si    GaAs    InP    GaP    AlAs    InAs  
11.8    13    12.5    11.2    10.1    15.15

# How big is bulk screening?



$$E_{\text{pol}} = \frac{(1 - 1/\epsilon_0) q^2}{R_{\text{jost}}}$$

Defects mostly converged at 64-site cells

$E_{\text{pol}}$  mostly insensitive to  $\epsilon_0$  at 10-15, use GaAs (LDA)

Charge  $q = +1, -1 \quad +2, -2 \quad +3, -3 \quad +4, -4$

Screening: 1.09 eV 4.36 eV 9.81 eV 17.43 eV

*This is lower bound on classical screening energy*

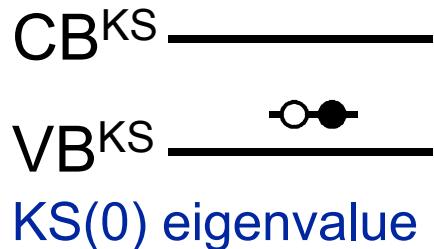
Bulk classical screening outside defect is **huge**  
 Key insight to understanding KS gap vs. defect gap

# How is a good defect band gap possible?

---

Conventional picture:

Defect state depicted as eigenvalue inside KS eigenvalue gap

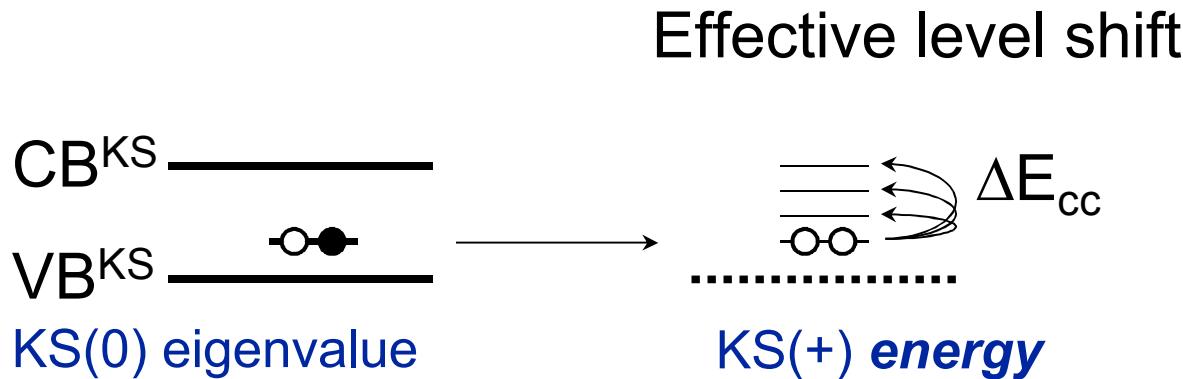


**Sham and Kohn** [Phys. Rev. 145, 561 (1966)]

the KS eigenfunctions and eigenenergies are auxiliary functions of the KS equations, and “must *not* be interpreted as corresponding to elementary excitations.”

# How is a good defect band gap possible?

What about:  
final state effects?

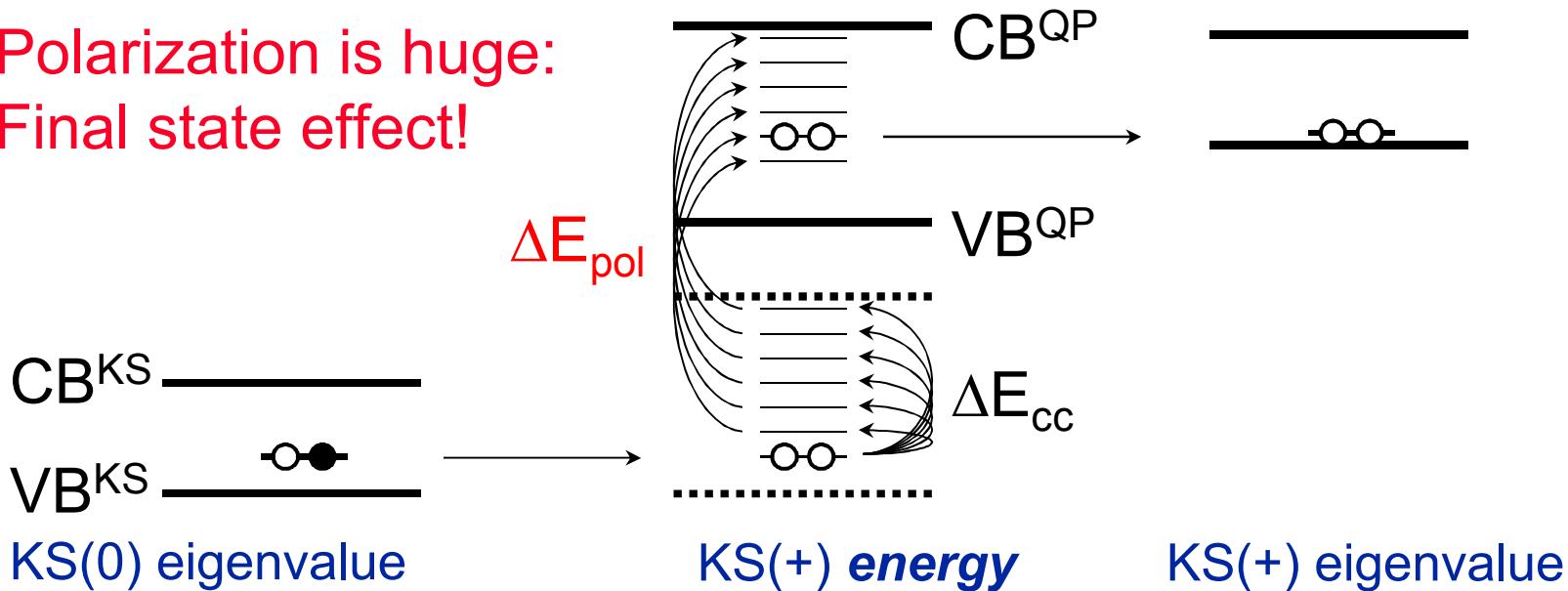


Central cell electronic relaxation (quantum):  $\Delta E_{cc}$

Sham and Kohn [Phys. Rev. 145, 561 (1966)]  
the KS eigenfunctions and eigenenergies are auxiliary functions of the KS equations,  
and “must *not* be interpreted as corresponding to elementary excitations.”

# How is a good defect band gap possible?

Polarization is huge:  
Final state effect!



Central cell relaxation (quantum):  $\Delta E_{\text{cc}}$

Long range screening (classical):  $\Delta E_{\text{pol}} > E_g$

Defect levels bounded by (screened) quasiparticle gap, not eigenvalue gap

Sham and Kohn [Phys. Rev. 145, 561 (1966)]

the KS eigenfunctions and eigenenergies are auxiliary functions of the KS equations, and “must *not* be interpreted as corresponding to elementary excitations.”

Not only eigenvalues but eigenstates are meaningless

# Conclusions

---

- **Total energy** DFT defect levels not constrained by KS band gap problem
- Semilocal DFT+FDSM - quantitative (~0.1 eV) for defect levels in semiconductors
- **Defect band gap** is good predictor of experimental band gap
- KS interpretation of band gap is not-even-wrong for defect levels
  - Sham and Kohn's ignored warning about misinterpreting KS eigenvalues
- **Rigorous charge boundary conditions** more crucial than KS band gap
  - **band edge problem** — where are they? — is the more serious question
- Path to better functionals: “fixing” using KS gap as primary metric is misguided
- **Diligence in credibility — verification/validation/UQ — crucial to predictive DFT**

Thanks to Arthur H. Edwards (AFRL) and also Renee M. Van Ginhoven (AFRL/RDHEC)

# ----- Supporting slides -----

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# A supercell theory of defect energies

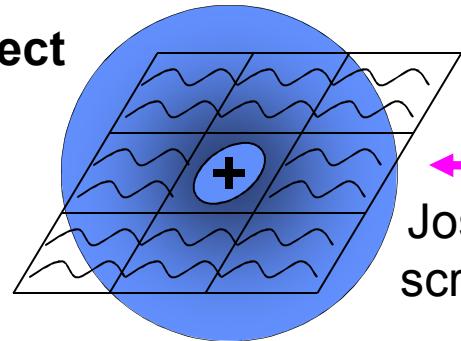
Peter A. Schultz, Phys. Rev. Lett. **96**, 246401 (2006).

**Target system:**  
isolated defect

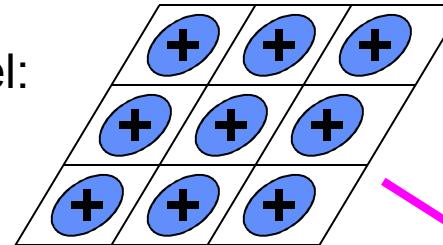
=

**Computational  
model for  
isolated defect**

( + DDO  
for defect  
banding)

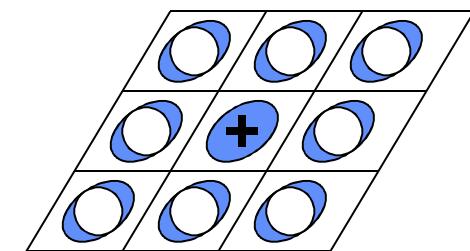


Standard  
DFT model:  
Supercell

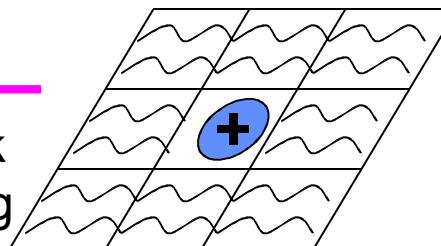


LMCC to fix  
boundary  
conditions

## Finite Defect Supercell Model



Jost Bulk  
screening

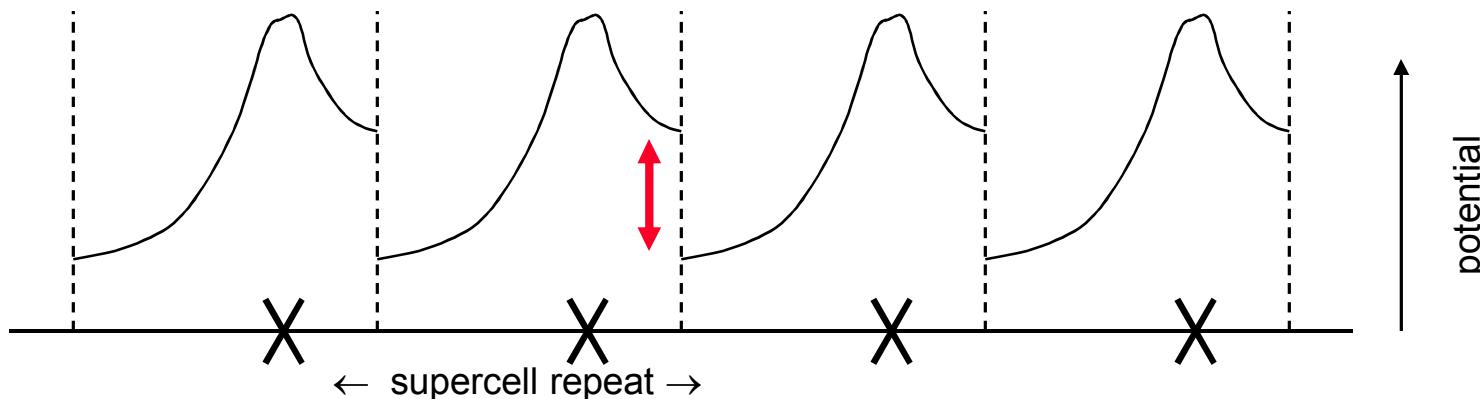


Crystal embedding  
to fix  $\mu_e$

**FDSM: *Ab initio* computational model** – connect model to physics  
Calculations with rigorous control of charge boundary conditions

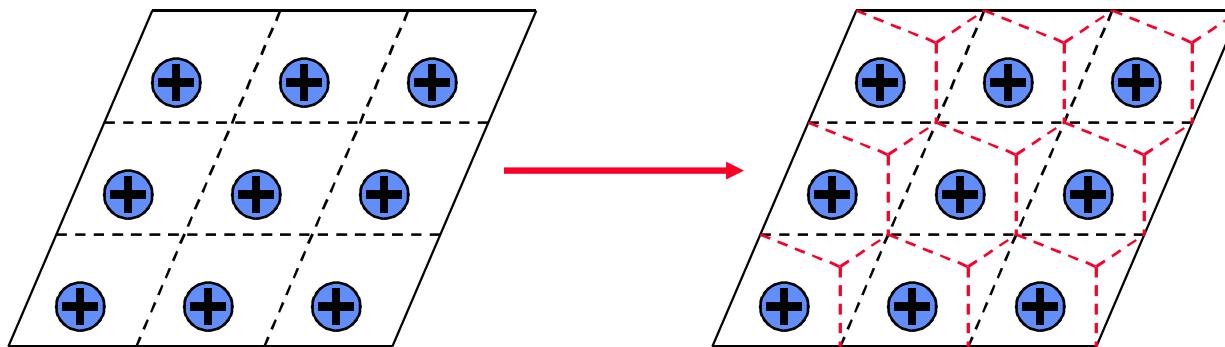
# LMCC potential in bulk systems

A complication in bulk systems ...



Discontinuity in potential from LMCC at supercell boundary!

... is solved by using Wigner-Seitz cells around LMCC positions



With Wigner-Seitz local volume, LMCC potential is continuous

# The electron chemical potential $\mu_e$

- Standard  $E_{\text{form}}$  of charged defects needs electron reservoir:

$$E_{\text{form}}(q) = E_{\text{defect}}(q) - E_{\text{xtal}}(0) - \sum N_i \mu_i + q \mu_e$$

linked

- Supercells with charge:  $\phi_{\text{def}}(r) = \phi_{\text{pbc}}(r) + C_{\text{def}}$

Periodic potential  $\phi_{\text{def}}(r)$  only known to within a constant  $C_{\text{def}}$

$C_{\text{def}} = \text{fcn}\{\text{defect type, configuration, cell shape, cell size, ...}\}$

$E_{\text{defect}}(q)$  has  $qC_{\text{def}}$  term in its internal energy

- Standard ad hoc workarounds unsatisfactory - unquantitative

- matching VB,CB edge, band structure features, average potentials ...
- Issue: renormalizing infinities, defect modified bands, band-bending, ...
- calibration uncertainty of “few tenths of eV” (Garcia & Northrup) - best case

Needed a more rigorous scheme to fix electron reservoir

# Defect energy and level calculation

---

## Finite Defect Supercell Model Formation Energy

$$E_{\text{form}}(q) = E_{\text{defect}}(q) - E_{\text{xtal}}(0) - \sum N_i \mu_i + E_{\mu_0}(q) + E_{\text{pol}}(q)$$

$E_{\text{defect}}(q)$ : DFT energy with LMCC potential

$- E_{\text{xtal}}(0) - \sum N_i \mu_i$  : match number of each type of atom

$E_{\mu_0}(q)$ : fix chemical potential  $\mu_e$  to common electron reservoir

$E_{\text{pol}}(q)$ : bulk polarization response

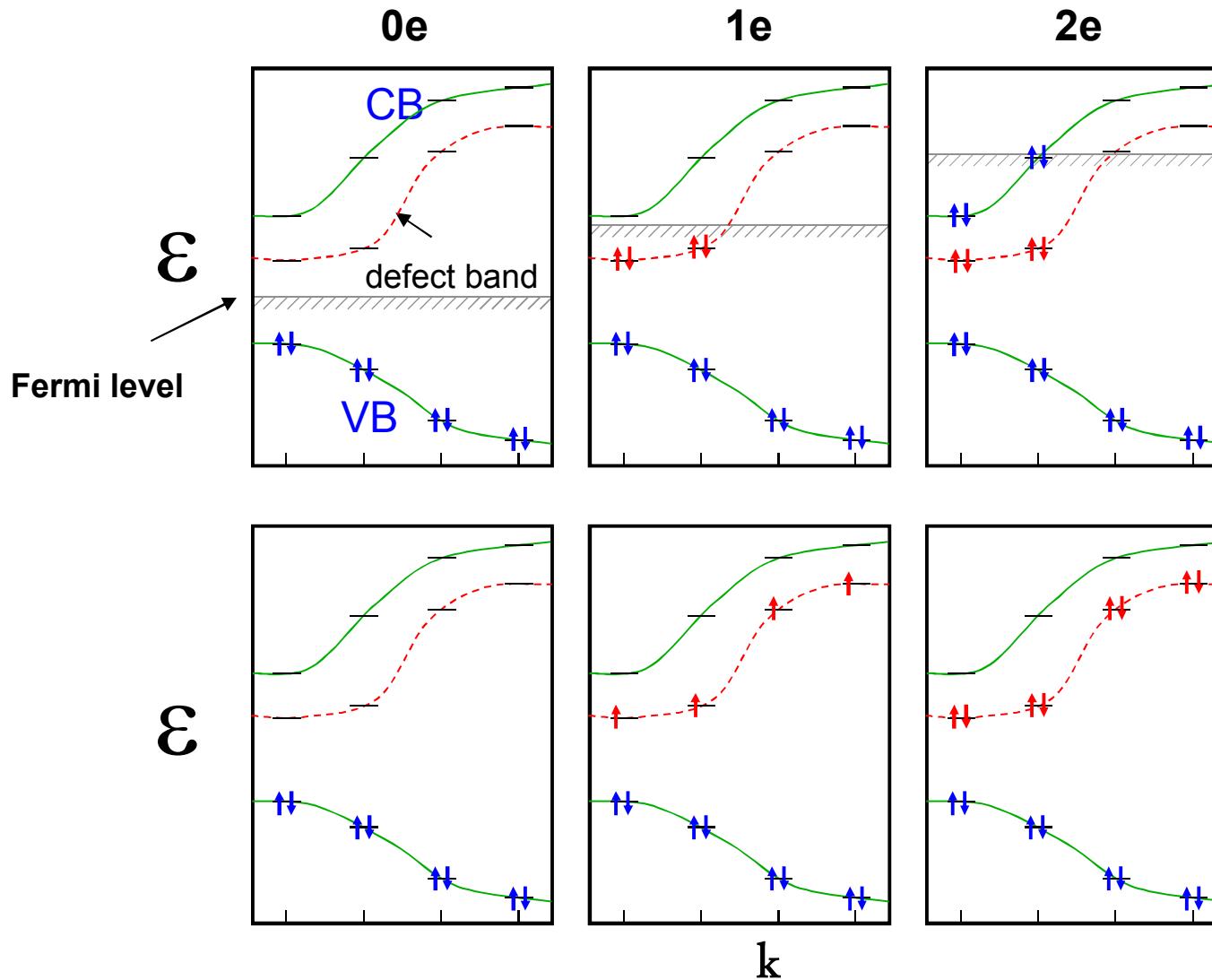
## Defect level calculation

$$\Delta E(q/q-1) = E_{\text{form}}(q) - E_{\text{form}}(q-1)$$

Need to set spectrum vs. VB/CB by single marker.

All defect levels for all defects then fixed by continuity.

# Defect banding: Discrete Defect Occupations



# Charged cell convergence - Jellium

PHYSICAL REVIEW B

VOLUME 51, NUMBER 7

15 FEBRUARY 1995-I

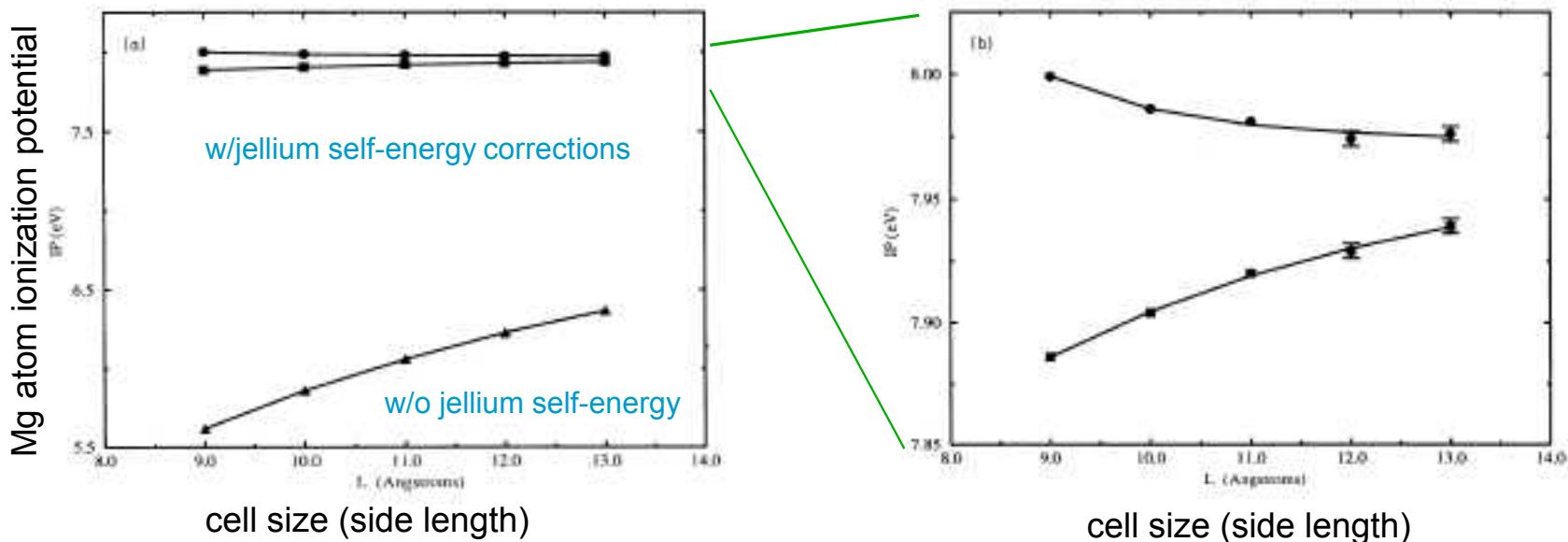
## Periodic boundary conditions in *ab initio* calculations

G. Makov and M. C. Payne

Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, United Kingdom

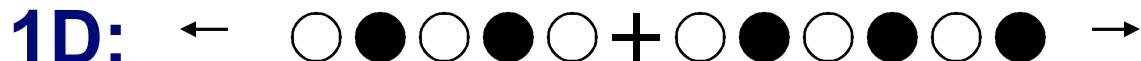
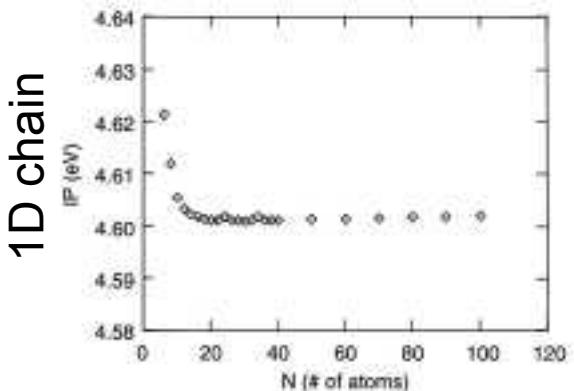
(Received 19 July 1994)

Figure 3

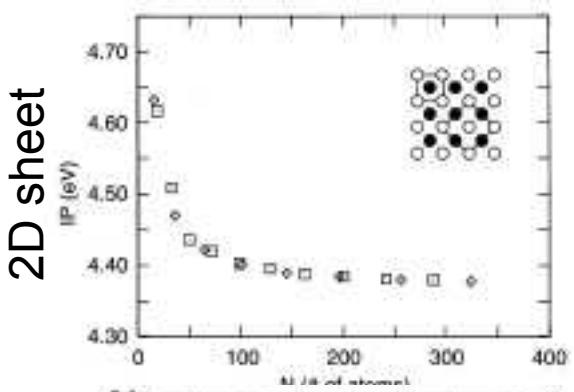


Variation in computed total energy due to incorrect charge potential

# LMCC: NaCl - Cl vacancy ionization



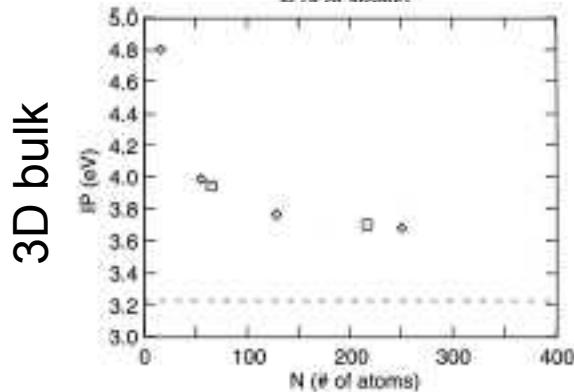
Supercell size dependence due to polarization.  
Larger supercell  $\rightarrow$  more polarization  
Apparent  $L^{-3}$  scaling = 1D classical dielectric screening



**2D:** single-layer 2D square sheet (polar&non-polar)

Apparent  $L^{-2}$  scaling = 2D classical dielectric screening

Insensitive to cell type, polar vs. non-polar



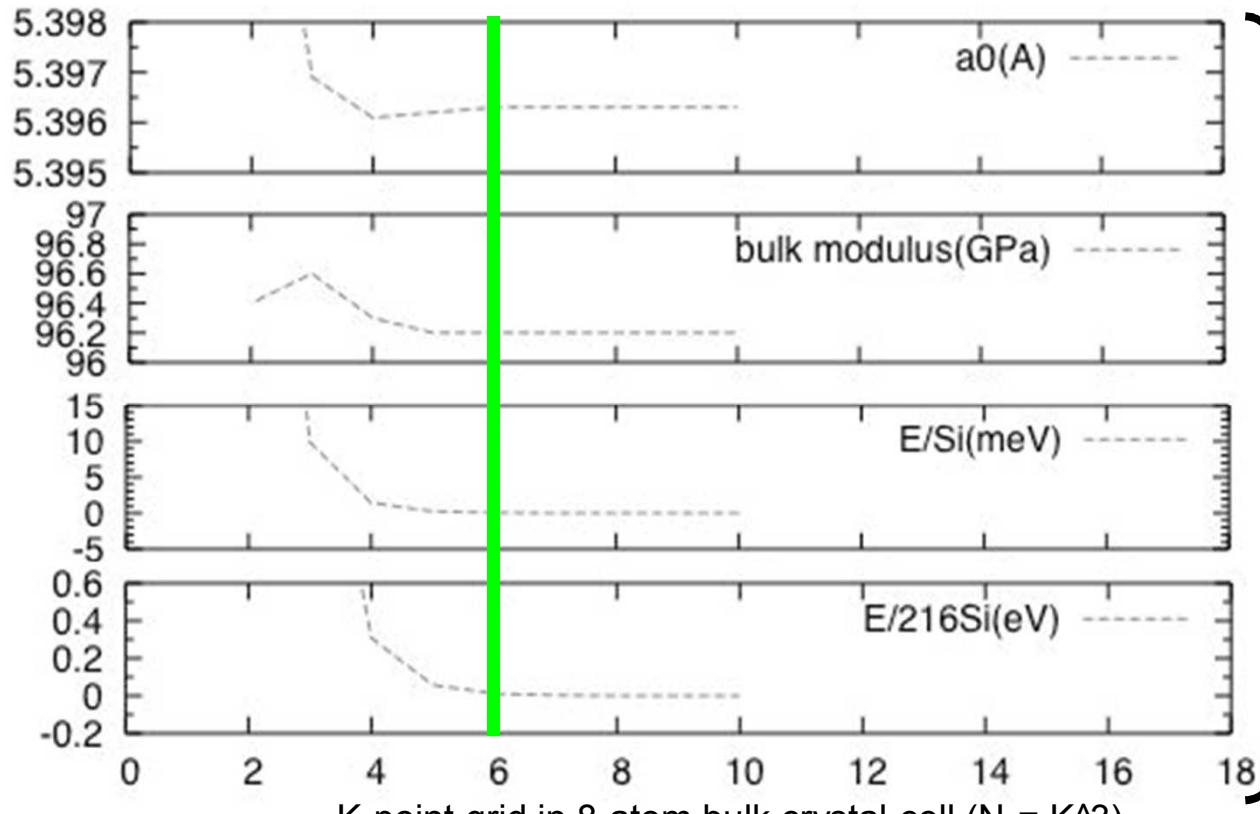
**3D:** bulk-layer 3D square sheet (fcc&sc cells)

Apparent  $L^{-1}$  scaling = 3D classical dielectric screening

Strictly screening due to large supercell volume

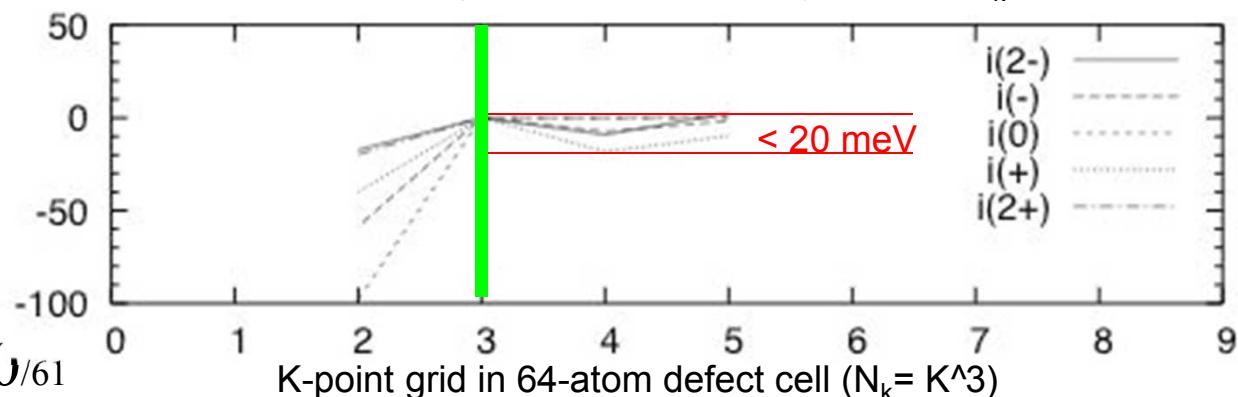
Insensitive to cell shape

# BZ convergence: Si self-interstitial



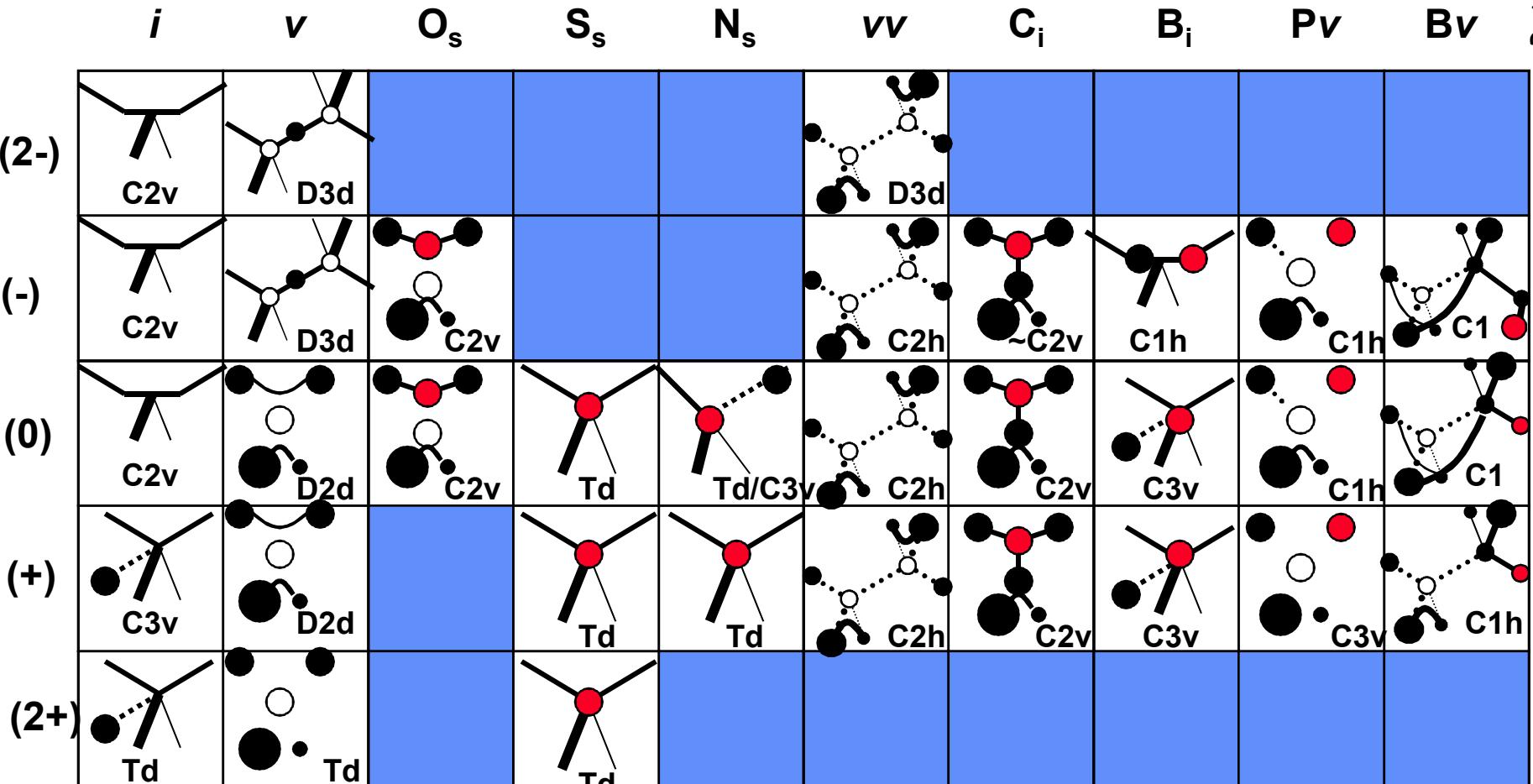
Bulk properties ( $a_0, B, E/\text{Si}, E_{\text{tot}}$ ) converge quickly:  
at  $6^3$  k w/8-cell  
=  $3^2$  k w/64-cell  
=  $2^3$  k w/216-cell

Defect energies should not vary faster than bulk, IF computational model is valid.



Interstitial formation energies in 64-site cell vary  $< 20$  meV  
{10 meV w/o  $i(+)$ } beyond equivalent of  $6^3$  k-grid in 8-site.

# Silicon defect structures



GGA:  $E(C2v) < E(D3d)$  for  $v(-)$

# Si: new P-v and B-v charge states

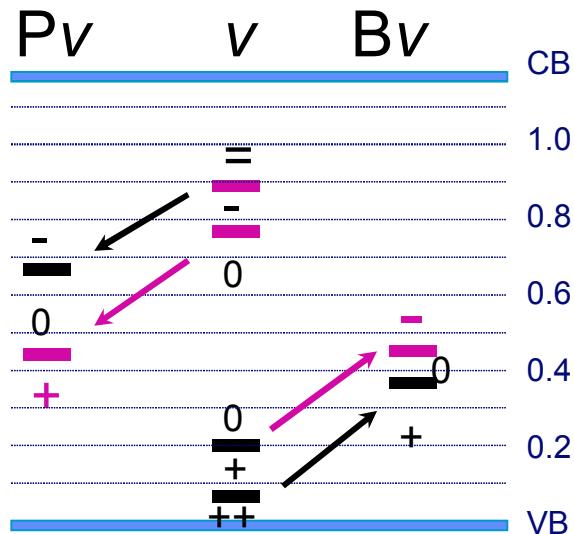
- Silicon level calculations - over 15 defects with levels

$i(=/-/0/+/++), v(=/-/0/+/++), vv(=/-/0/+), C_i(-/0/+), B_i(-/0/+), Pv, Bv$

$O_s$ (A-center),  $O_i$ ,  $N_s$ ,  $S_s$ ,  $v_2O$ ,  $v_2O_2$ ,  $H_i$ ,  $vP_2$ ,  $v_2P$ , ...

DFT “defect band gap” matches experiment (1.2 eV)

DFT: mean  $|error| = 0.10$  eV, max error~0.2 eV



Task: Theory quantified  $v(=/-)$ ,  $v(-/0)$

Discovery: Theory predicted  $Pv(+)$  and  $Bv(-)$

“Absolute prediction”

new levels >0.4 eV from band edge

validation error: 0.2

$Pv(0/+)$  subsequently confirmed in experiment  
[Larsen, et al PRL 97, 106402 (2006)]

VALIDATION is key to quantitative DISCOVERY - GaAs is ALL discovery

# Calibrating the polarization model: $v_{\text{Ga}}$

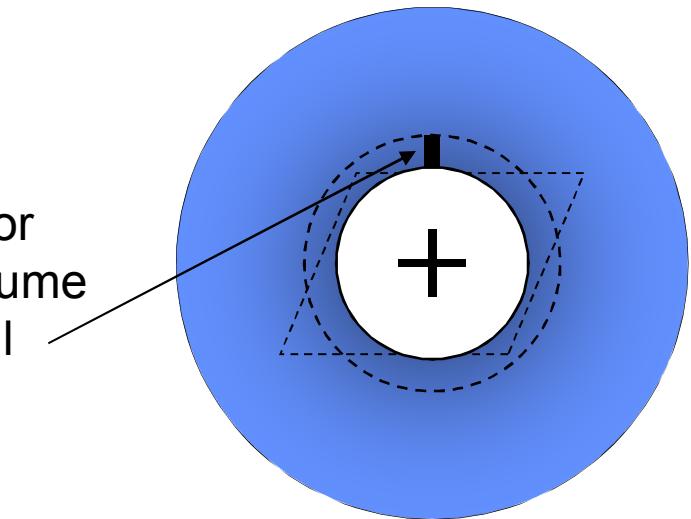
Jost model:

$$E_{\text{pol}} = \frac{(1 - 1/\epsilon_0) q^2}{R_j}$$

$$R_j = R_{\text{vol}} - R_{\text{skin}}$$

$R_{\text{skin}}$  accounts for unscreened volume **inside** supercell

Need  $\epsilon_0$  (use 13), and  $R_{\text{skin}}$  (fit)



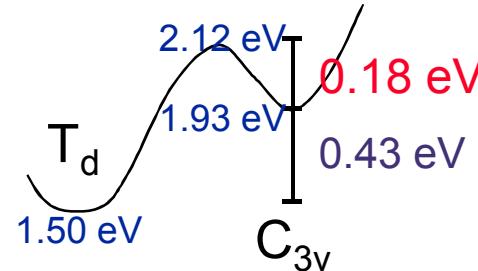
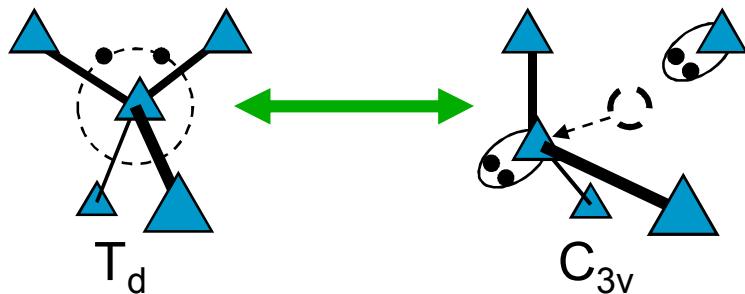
Why use  $v_{\text{Ga}}$ ?

Need higher charge states (0 to -3), best if not strongly distorted (near  $T_d$ )

Energy(eV)	$v_{\text{Ga}}(0)$	$E(2/-1)-E(1/0)$	$E(3/-2)-E(2/-1)$	$a\text{As}: E(0/+)-E(+/2+)$
64-site	2.81	0.167	0.174	0.231
216-site	2.69	0.168	0.152	0.246
512-site	2.75	0.162	0.141	0.252

# GaAs EL2 and the As antisite

EL2 = antisite  $\text{As}_{\text{Ga}}(0)$



216-site =  
512-site  
(~ 64-site)

	Experiment -EL2	SeqQuest/FDSM - $\text{As}_{\text{Ga}}$
EL2(0/1+)	$E_c$ -0.74 eV	$E_c$ -0.81 eV
EL2(1+/2+)	$E_v$ +0.54 eV	$E_v$ +0.48 eV
Splitting:	0.24 eV ( $E_g = 1.52$ )	0.25 eV
EL2*	no donor states	no donor states
Reorientation:	~0.3 eV	~0.2 eV

Verification: 64-216-512-1000-site supercell results match

Validation: DFT matches experiment for EL2 w/in 0.1eV