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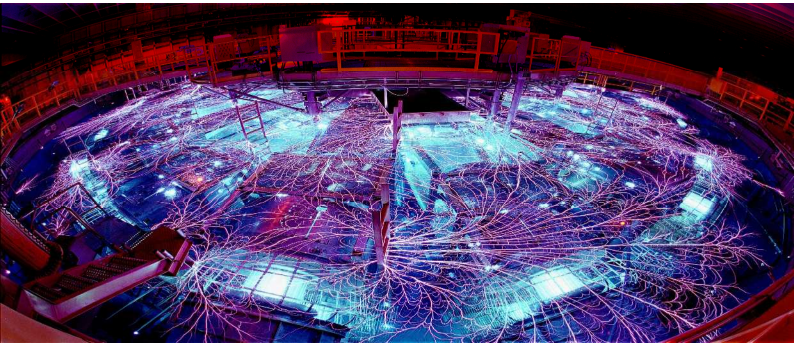

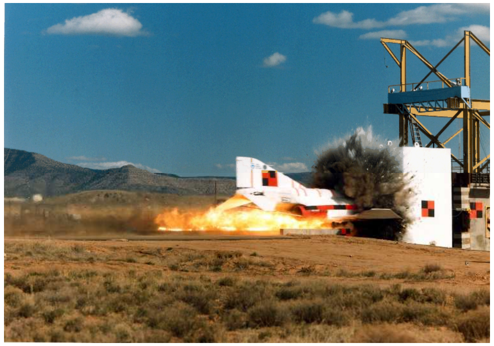

Viscoelasticity in Biomechanics and Aerospace Engineering

Kevin Troyer, PhD

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

My Background

- Orthopaedic Bioengineering Research Laboratory
- United States Food and Drug Administration
- Honeywell Federal Manufacturing and Technologies
- Sandia National Labs

Introduction to Viscoelasticity

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- Viscoelasticity: The relationship between stress and strain (stiffness) **depends on time**.

- Examples of viscoelastic materials
  - Biological tissue: bone, ligament, skin, cardiac tissue
  - Foam and foam composites
  - Epoxy: electronic encapsulates, bonding materials
  - Rubber: tires, o-rings
  - Metals (especially at high temperature)

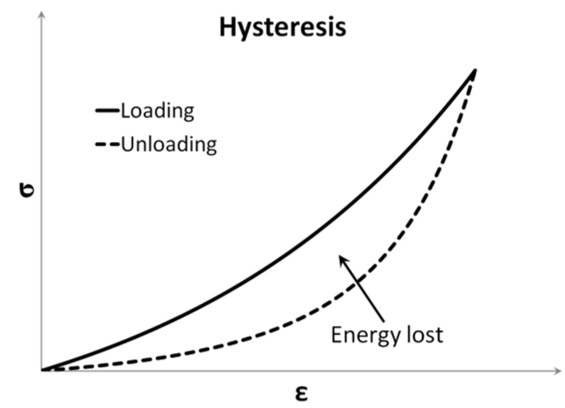
- Why study viscoelasticity?
  - Structural applications
  - Probe
  - Causal links between phenomena and microstructure

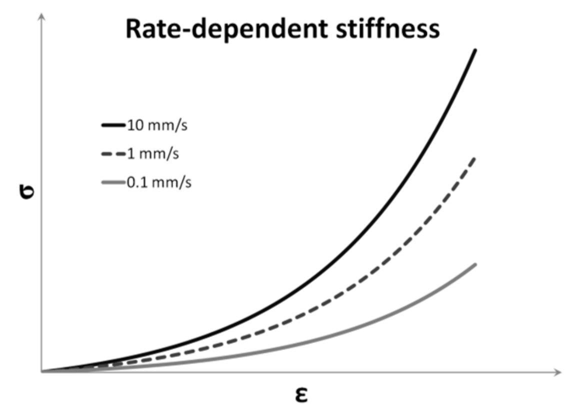
Viscoelastic Phenomena

Stress Relaxation

Creep

- Some important viscoelastic phenomena
  - **Creep**
    - Progressive deformation under constant stress
  - **Stress relaxation**
    - Stress decay under constant strain
  - **Hysteresis**
    - Energy dissipation during a loading cycle
  - **Rate-dependent stiffness**
    - Relaxation during loading events
- Origins of tissue viscoelasticity:
  - Intermolecular viscoelasticity of collagen fibrils[1]
  - Interactions between solid-phase constituents (fibers and matrix)[2,3]
  - Fluid movement through the tissue[4]
- Origins of polymer viscoelasticity:
  - Distortion of chemical bonds (length and angle)
  - Molecular rearrangements

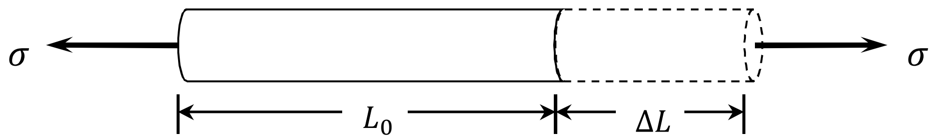




[1]Mosler, 1985; [2]Ciarletta, 2006; [3]Puxkandl, 2002; [4]Henninger, 2010

Viscoelastic Equations

Consider a 1-D rod subjected to a small instantaneous strain $\varepsilon_0 = \frac{\Delta L}{L_0}$

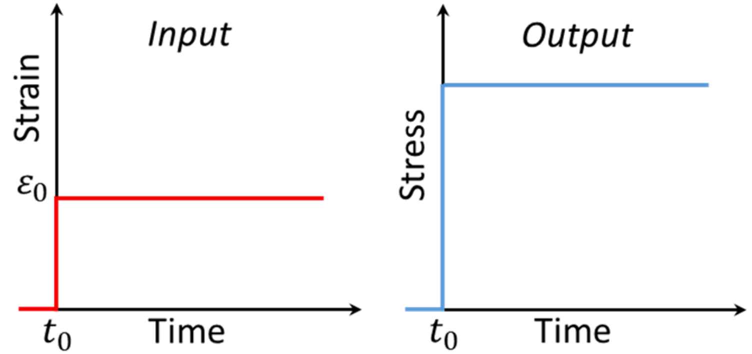


Linear elastic material

$$\sigma = \mathrm{E}\varepsilon_0 \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

Young's modulus:

- Resistance to deformation
- Independent of time

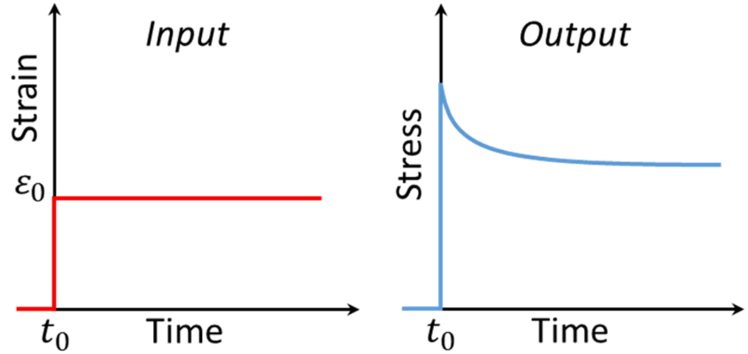


Linear viscoelastic material

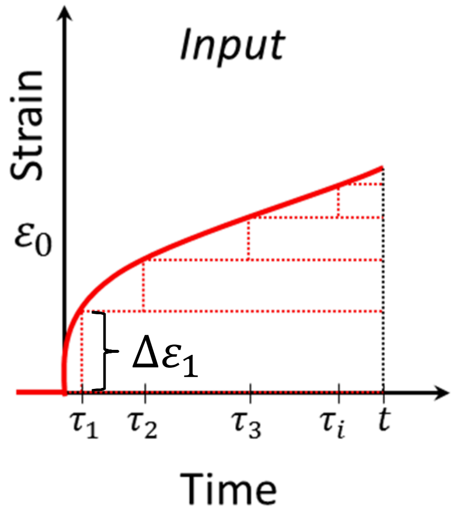
$$\sigma(t) = \mathrm{E}(t)\varepsilon_0 \Rightarrow E(t) = \frac{\sigma(t)}{\varepsilon_0}$$

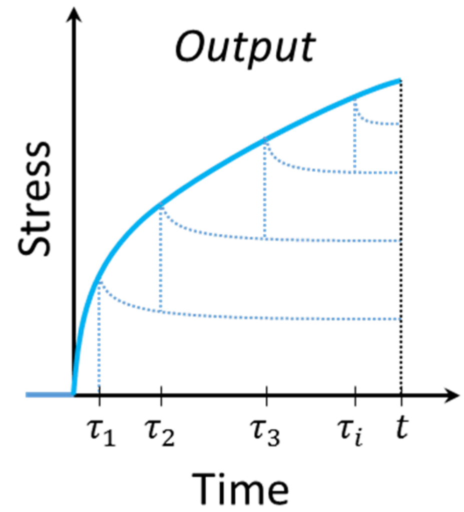
Relaxation modulus:

- Resistance to deformation
- **Dependent upon time**



Viscoelastic Equations





Heaviside step function

$$H(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 1 \end{cases}$$

We can approximate the strain history using a discrete number of steps: $\varepsilon(t) = \sum_{i=1}^{r} \Delta\varepsilon_i H(t - \tau_i)$

The resulting stress is: $\sigma(t) = \sum_{i=1}^{r} \Delta\varepsilon_i E(t - \tau_i) H(t - \tau_i)$

As the number is steps increases to infinity, and imposing $H(t - \tau_i)$ = 1 since $t \geq$ 1, we converge on the hereditary integral:

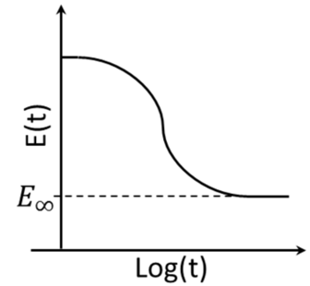
$$\sigma(t) = \int_0^t E(t - \tau) \frac{d\varepsilon(\tau)}{d\tau} d\tau$$

$$\sigma = E \times \varepsilon$$

Viscoelastic Equations

- The Prony Series

$$E(t) = \sum_{i=1}^{N} E_i e^{-\frac{t}{\tau_i}} + E_\infty \longrightarrow$$



- Example:

$$\sigma(t) = \int_0^t \left[E_1 e^{-\frac{t-\tau}{0.1}} + E_2 e^{-\frac{t-\tau}{1}} + E_3 e^{-\frac{t-\tau}{10}} + E_4 e^{-\frac{t-\tau}{100}} \right] \frac{d\varepsilon(\tau)}{d\tau} d\tau + E_\infty \varepsilon(t)$$

- How to compute numerically:

$$\sigma(t+\Delta t) = \sum_{i=1}^{N} \left[e^{-\frac{\Delta t}{\tau_i}} h_i(t) + \frac{E_i \left(1 - e^{-\frac{\Delta t}{\tau_i}}\right)}{\Delta t / \tau_i} \Delta\varepsilon \right] + E_\infty \varepsilon(t+\Delta t)$$

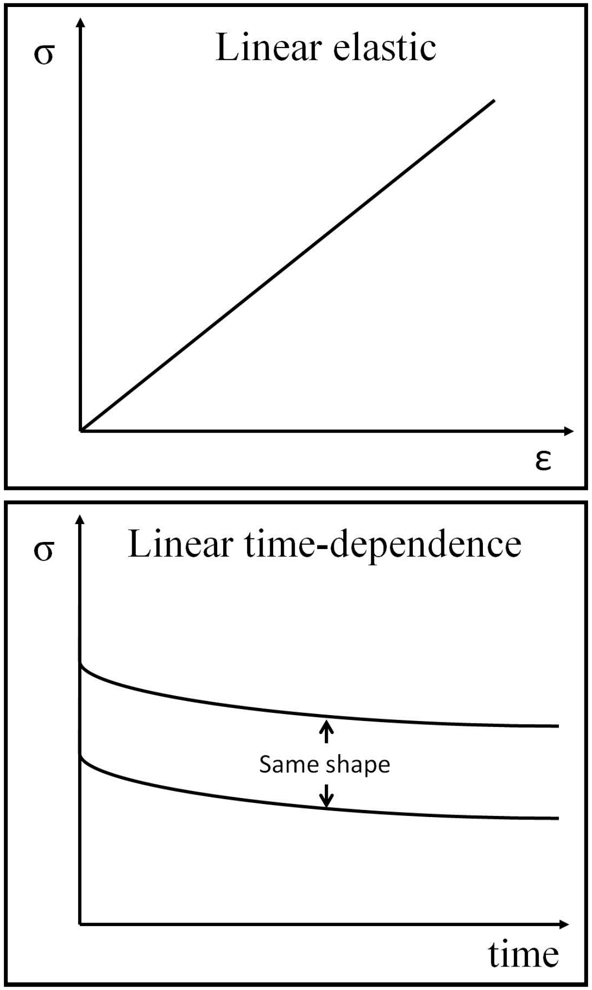
- All forms of $E(t)$ must satisfy thermodynamic restrictions
  - Monotonically decreasing function

Types of Viscoelasticity

- Linear viscoelasticity

$$\sigma(t) = \int_0^t \underbrace{E(t-\tau)}_{\text{Relaxation modulus}} \frac{d\varepsilon}{d\tau} d\tau$$

- Assumptions:
  1. Linear relationship between stress and strain
  2. Relaxation modulus is independent of strain level

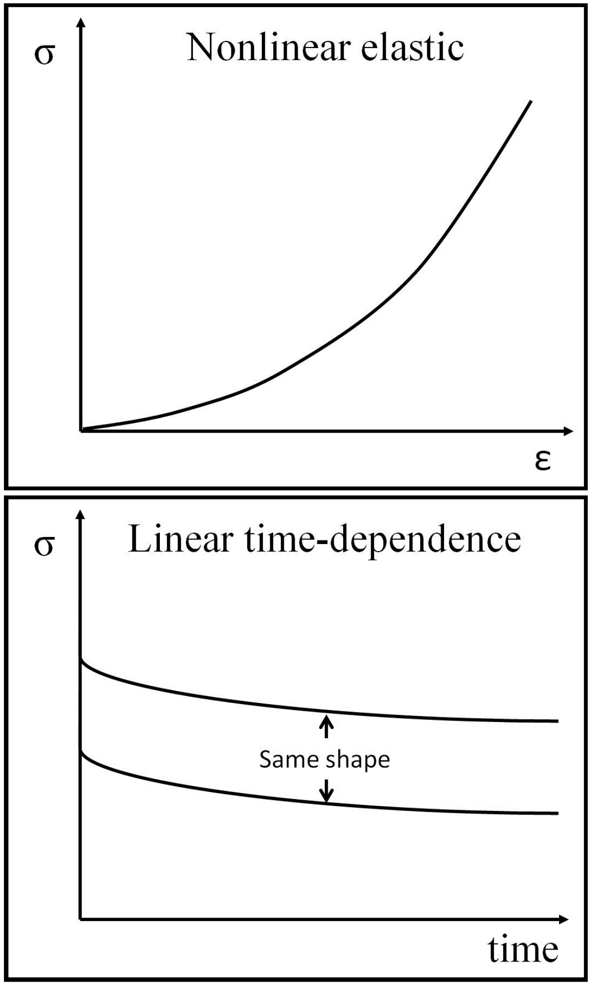


Types of Viscoelasticity

- Quasi-linear viscoelasticity

$$\sigma(t,\varepsilon) = \int_0^t \underbrace{G(t-\tau)}_{\text{Viscous component}} \underbrace{\frac{\partial \sigma^e(\varepsilon)}{\partial \varepsilon}}_{\text{Hyperelastic component}} \frac{d\varepsilon}{d\tau} d\tau$$

- Generalization of linear viscoelasticity
- Widespread use
  - Relatively simple mathematical interpretation
  - Easily incorporated into finite element software
- Assumptions:
  - Relaxation modulus is independent of the applied strain level (**linear viscous assumption**)



Types of Viscoelasticity

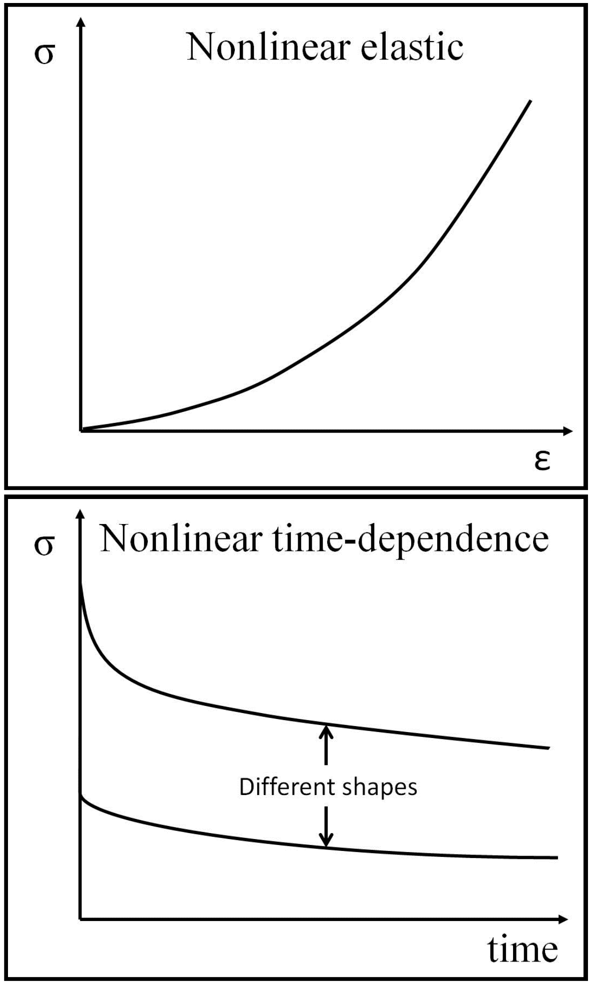
- Nonlinear viscoelasticity

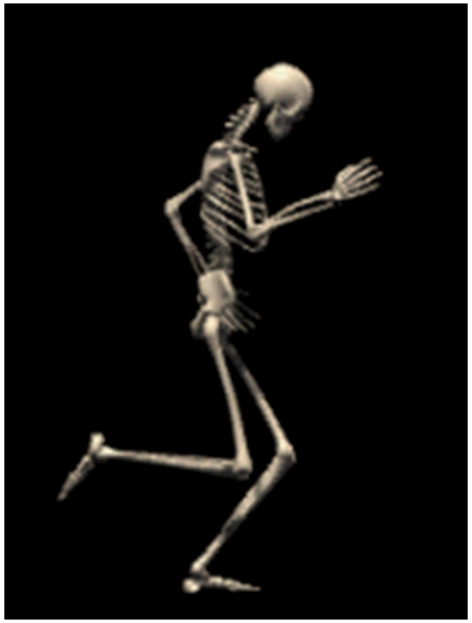
$$\sigma(t,\varepsilon) = \int_0^t E(\varepsilon, t-\tau)\frac{d\varepsilon}{d\tau}d\tau$$

- No assumptions with regard to material linearity

- Non-separable relaxation modulus
  - Simultaneously describe **elastic** and **viscous** nonlinearities

- Overall objectives of this research:
  - Develop a robust viscoelastic characterization technique to capture nonlinear viscoelasticity
  - Implement this behavior into FE software





Nonlinear Viscoelasticity of Tendon

APPLICATIONS IN BIOMECHANICS

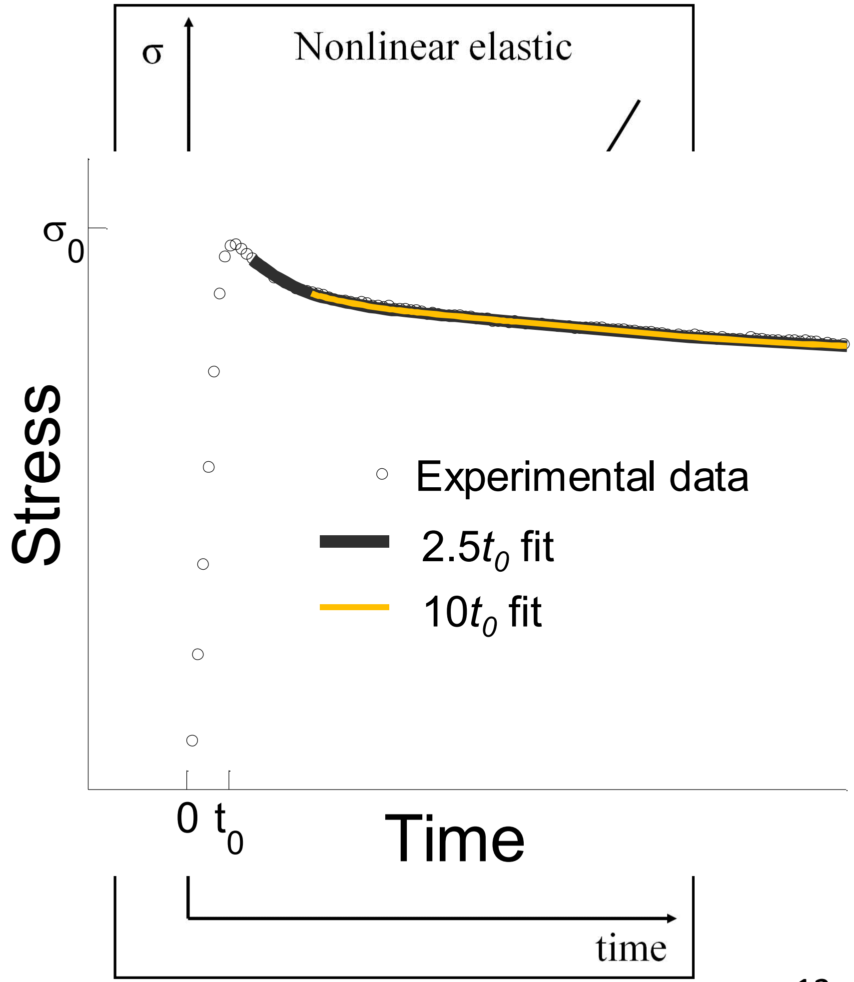
Motivation

- QLV popularity
  - Relatively straight-forward mathematical interpretation
  - Easily incorporated into FE software packages

- Limitations of QLV
  - *In vivo* tissues are subjected to varying loading conditions
  - Cannot capture deformation-dependent properties[1-7]

- Current nonlinear viscoelastic characterization techniques
  - May affect the predictive accuracy of the FE model

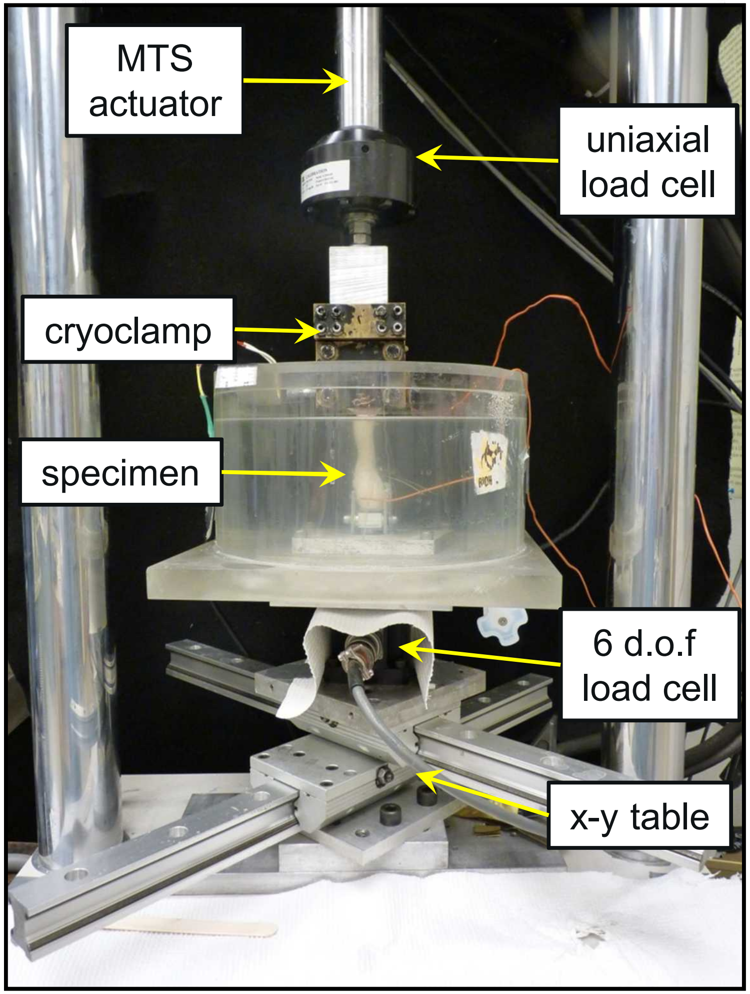
- Goals for Experiment 2:
  1. Develop a FE nonlinear viscoelastic formulation
  2. Validate predictive accuracy of the FE model



[1]Duenwald+, 2009; [2]Hingorani+, 2004; [3]Provanzano+, 2001; [4]Troyer and Puttlitz, 2011; [5]Ambrosetti-Giudici+, 2010; [6]Thornton+, 1997; [7]Bonifasi-Lista, 2005

Methods: Experimental Setup

- Ovine Achilles tendon (n=7)
  - Selected for: relatively <u>large size</u> and <u>constant cross section</u>
- Dissection and potting:
  - Carefully removed muscle belly
  - Potted calcaneous in PMMA bone cement
  - Frozen (-20 ° C)
- Testing protocol
  - **<u>Preconditioning</u>**: 7% strain, 0.5 Hz (50 cycles), 1 Hz (50 cycles)
  - **<u>Stress relaxation</u>** (ramp rate: 10 mm/s):
    - Strain magnitudes 1% to 6%
    - Hold: 100 s
  - **<u>Dynamic</u>**:
    - Strain amplitudes: 3% and 6%
    - Frequencies: 1 Hz and 10 Hz



Methods: Finite Element Formulation

- Total stress:

$$\sigma^t(\varepsilon^t,t)=E_\infty(\varepsilon^t)\cdot\varepsilon^t+\sum_{i=1}^{4}\frac{1}{1+\Delta t/\tau_i+\Delta t/2\tau_i}\left[E_i(\varepsilon^t)\cdot\Delta\varepsilon+\left(1+\frac{\Delta t}{2\tau_i}\right)\sigma^{i-1}\right]+\sigma_0$$

- Tangent stiffness:

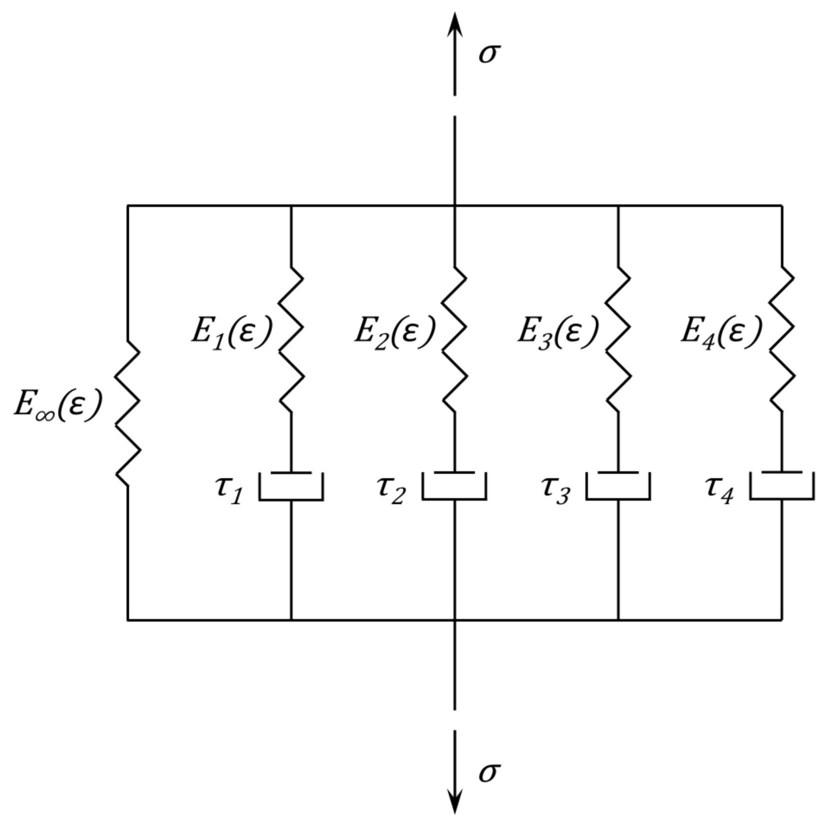
$$C^t=E_\infty(\varepsilon^t)+\sum_{i=1}^{4}\left[\frac{1}{1+\Delta t/2\tau_i}\right]E_i(\varepsilon^t)$$

- Tension-only FE model
  - Linear truss element (T3D2)
  - Model geometry: **<u>initial length</u>** and **<u>area</u>** definitions obtained experimentally

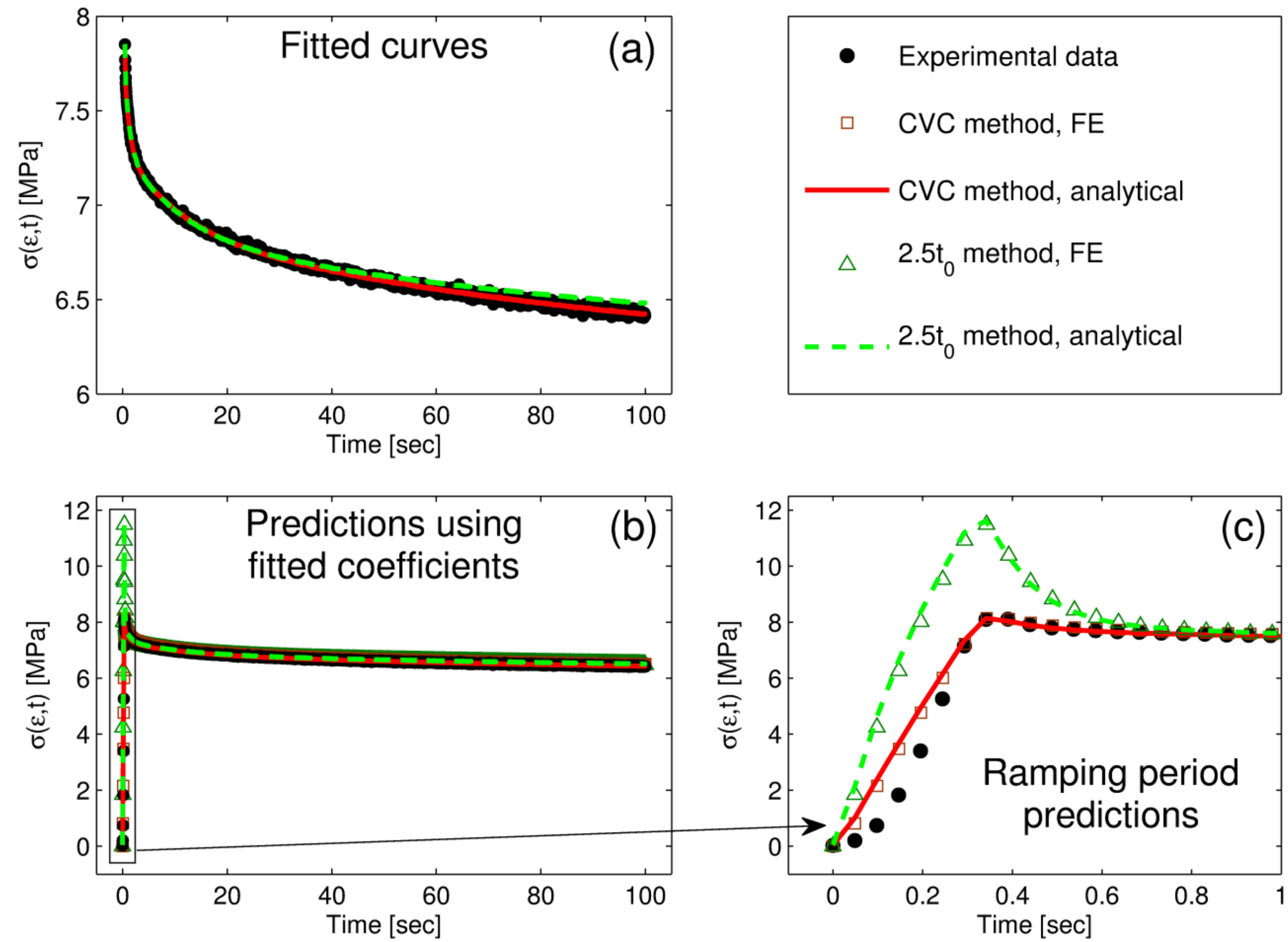
- FE model used to predict stress relaxation and dynamic behavior

- Comparisons
  - Non-weighted RMSE
  - Percent error (Kruskal-Wallis, *post hoc*: Wilcoxon rank-sum test with Bonferroni adjustment ($p<0.005$)

Five –component mechanical model



Results: Stress Relaxation



Results: Relaxation Moduli

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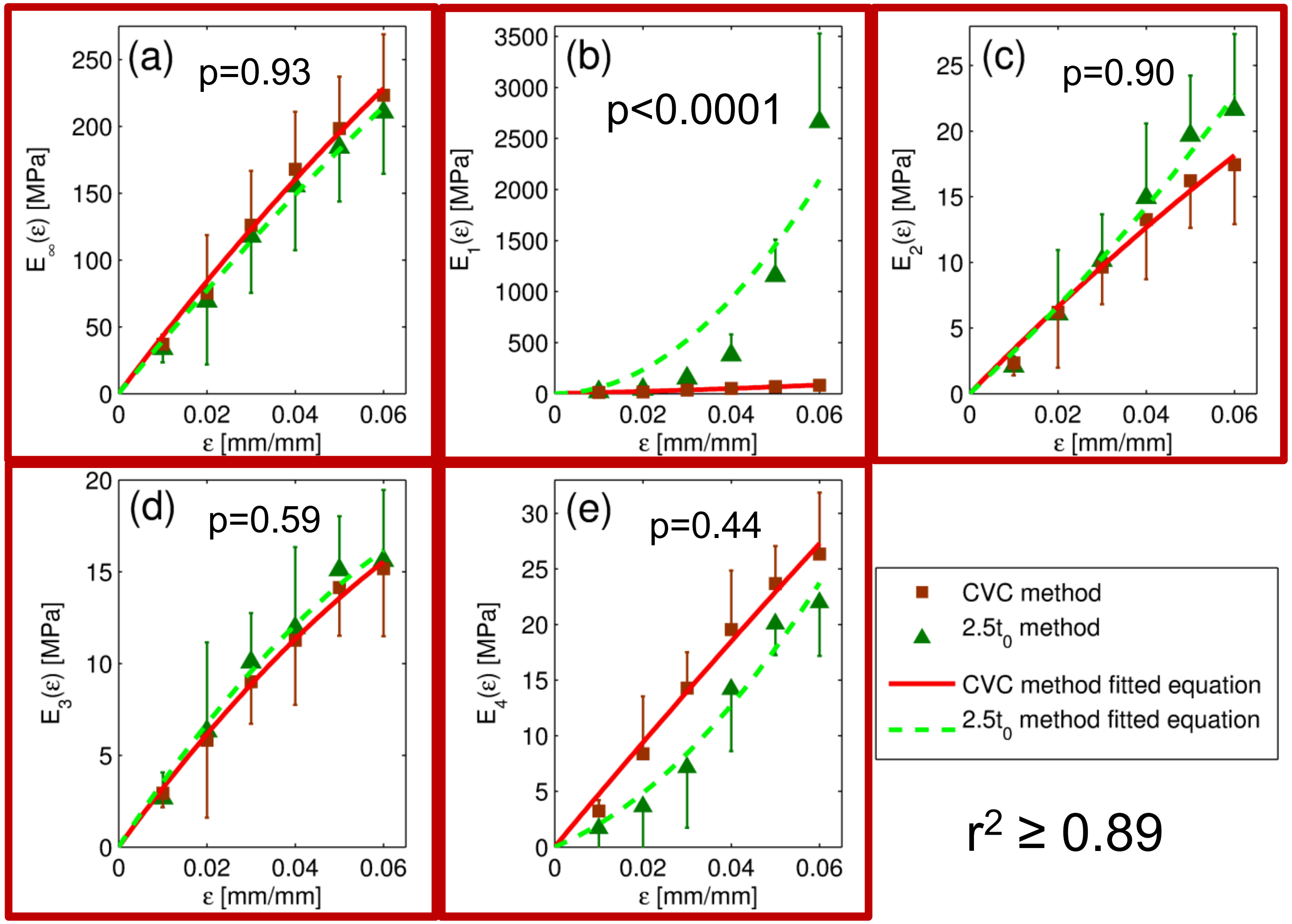
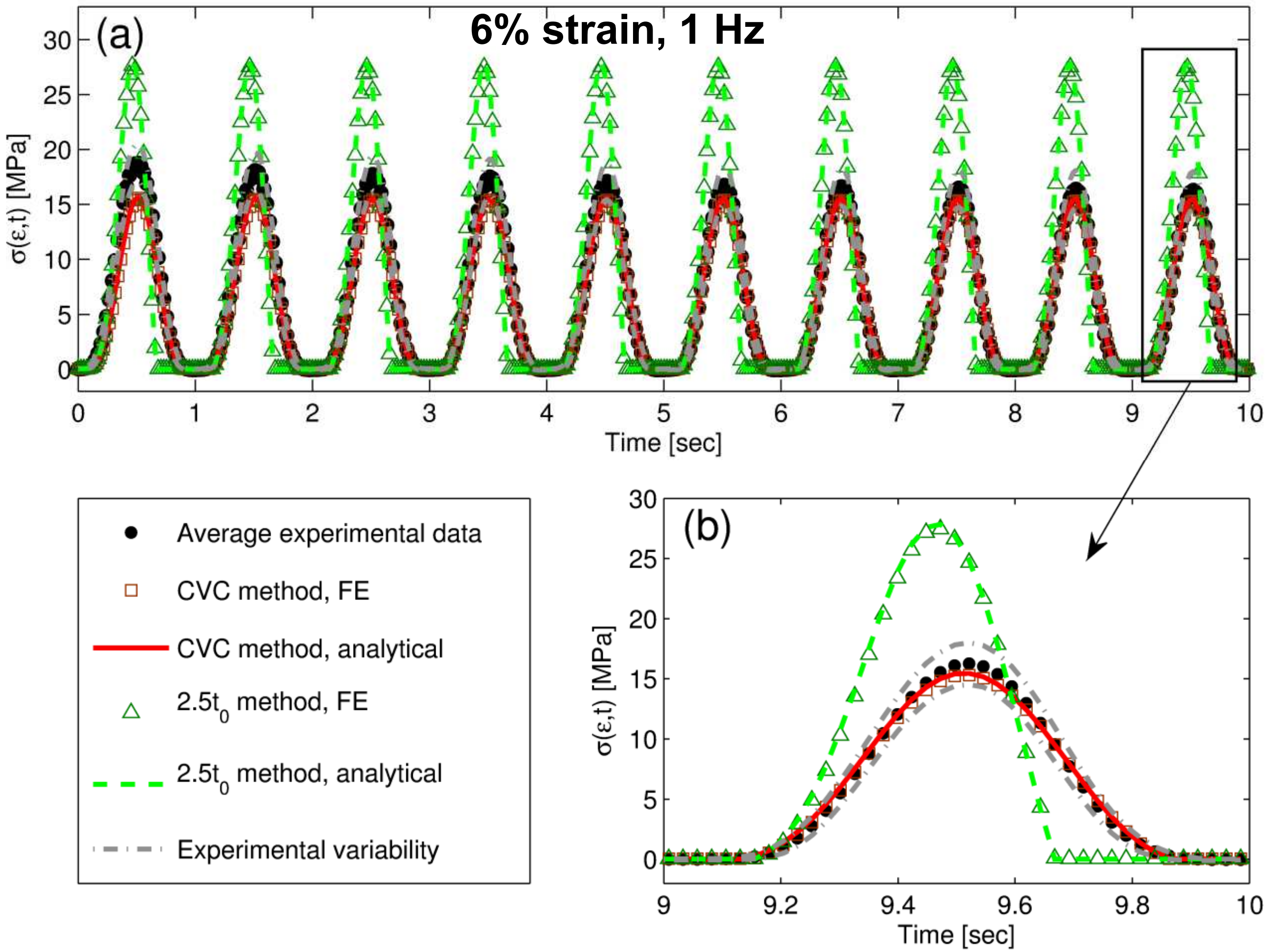
$$E(\varepsilon,t)=E_\infty(\varepsilon)+E_1(\varepsilon)e^{-t/0.1}+E_2(\varepsilon)e^{-t/1}+E_3(\varepsilon)e^{-t/10}+E_4(\varepsilon)e^{-t/100}$$

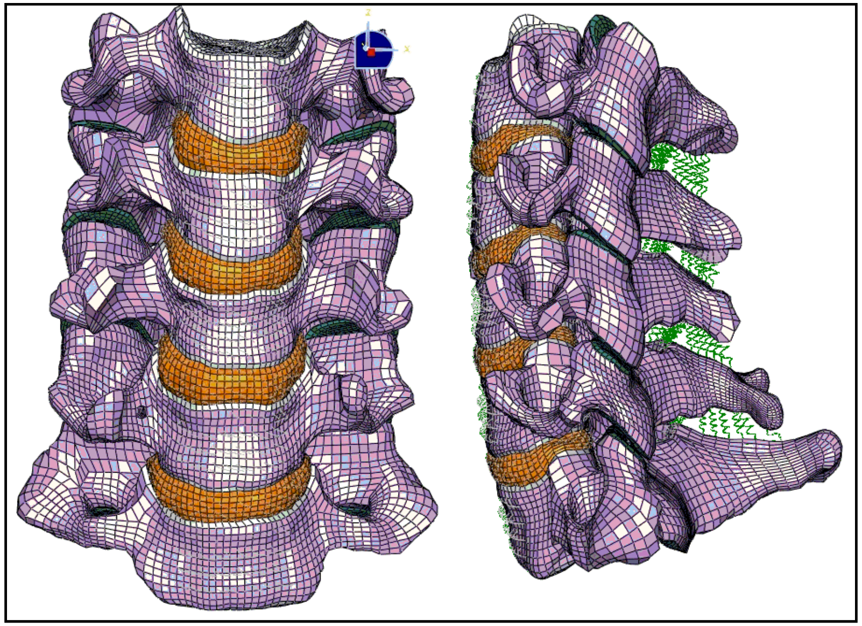


$r^2 \geq 0.89$

Results: Cyclic Predictions



Adapted from Womack, 2009

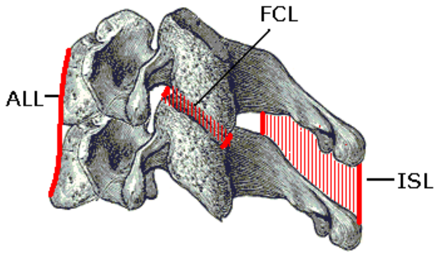


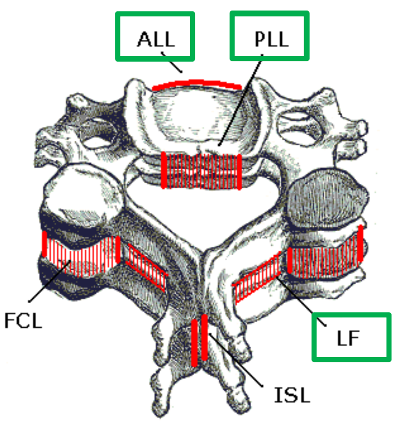
Nonlinear Viscoelasticity of Spinal Ligaments

APPLICATIONS IN BIOMECHANICS

Motivation

- Role of spinal ligaments:
  - Facilitate 3D physiologic motion patterns
  - Maintain static vertebral postures
  - Limit excessive motion
  - Absorb additional energy during traumatic loading events
- Requires consideration of viscoelastic behavior
- Few studies have explicitly characterized the viscoelastic behavior
  - QLV theory[1,2]
  - Over-simplified nonlinear models[3-5]
  - Affect the predictive accuracy of the model
- Goals of experiment:
  1. Characterize the nonlinear viscoelastic constitutive behavior of ALL, PLL, LF
  2. Validate this constitutive relationship via cyclic predictions



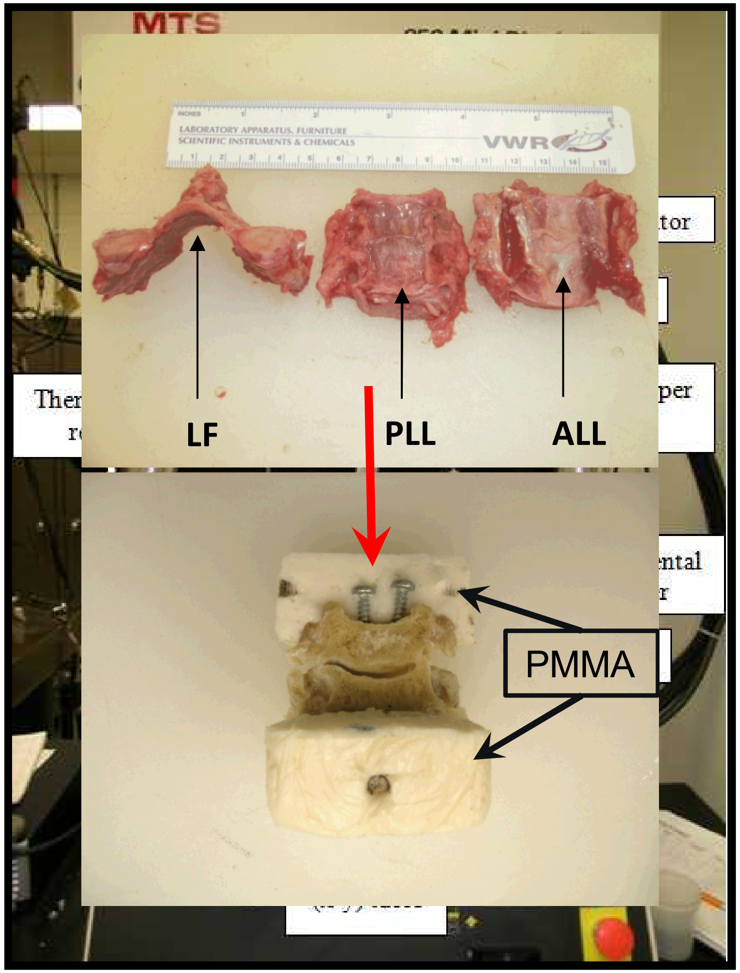


Adapted from Womack, 2009

[1]Lucas+, 2008; [2]Little and Khalsa, 2005; [3]Troyer and Puttlitz, 2011; [4]Yahia+, 1991; [5]Ambrosetti-Giudici+, 2010

Experimental Methods

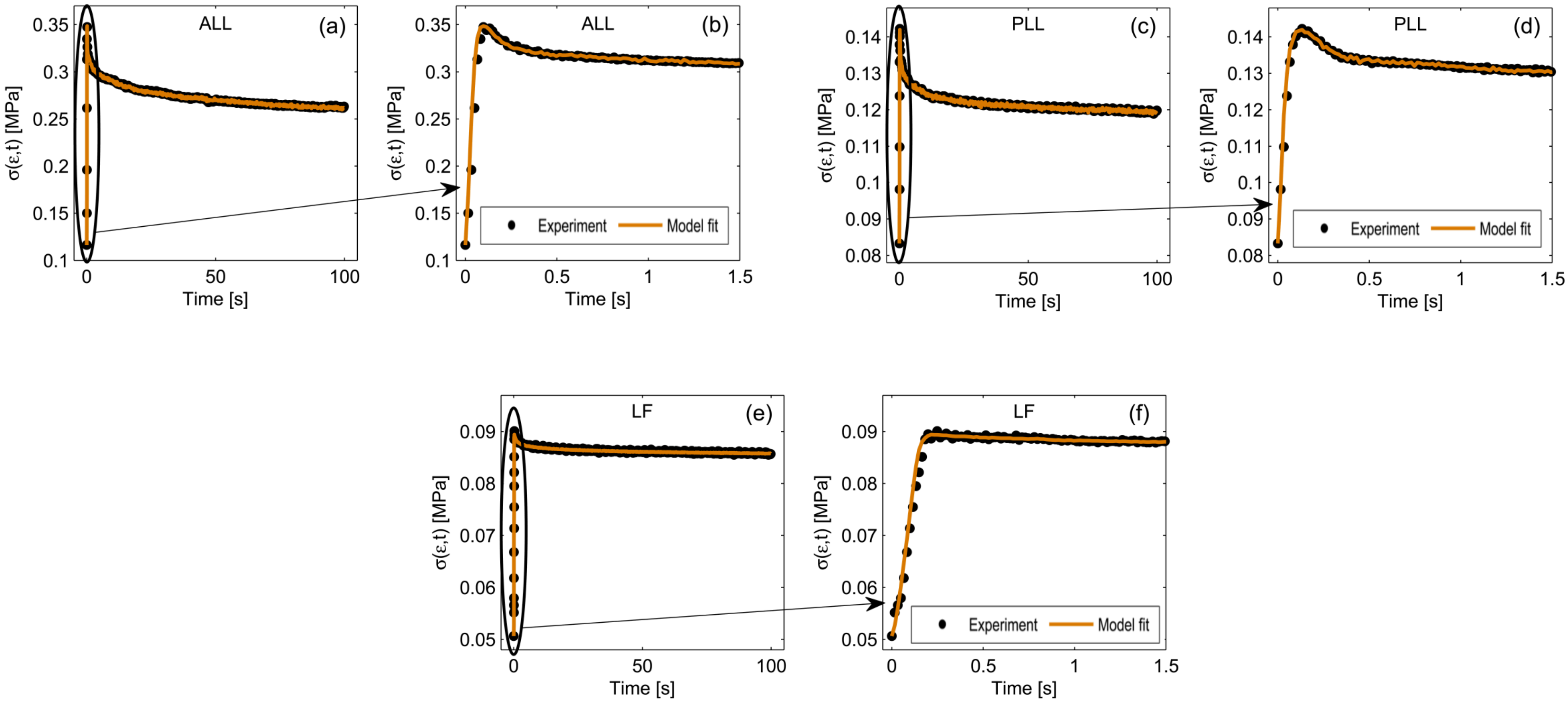
- Experimental data subset of a larger study[1]
  - Three cervical spinal ligaments:
    1. Anterior longitudinal ligament (ALL; n=8)
    2. Posterior longitudinal ligament (PLL; n=8)
    3. Ligamentum flavum (LF; n=6)
- Dissection and potting
  - Isolated via removal of non-osteoligamentous tissue
  - Potted in polymethlmethacrylate for attachment to the testing device (858 Mini Bionix II; MTS)
- Experimental setup
  - Environmental chamber (isotonic saline, 37 C)
  - Translation (x-y) table
- <u>Validation data acquisition</u>: Cyclic behavior
  - Cyclic frequency sweep: 0.001 Hz to 1 Hz
  - Strain amplitudes: 10% and 15% (peak-to-peak)
- <u>Fitted data acquisition</u>: Relaxation behavior
  - Incremental strain magnitudes: 4%-25% strain[2]
  - Ramp: <0.3 s, hold: 100 s, recover: 600 s



[1]Troyer and Puttlitz, 2011; [2]Bass+, 2007

Results

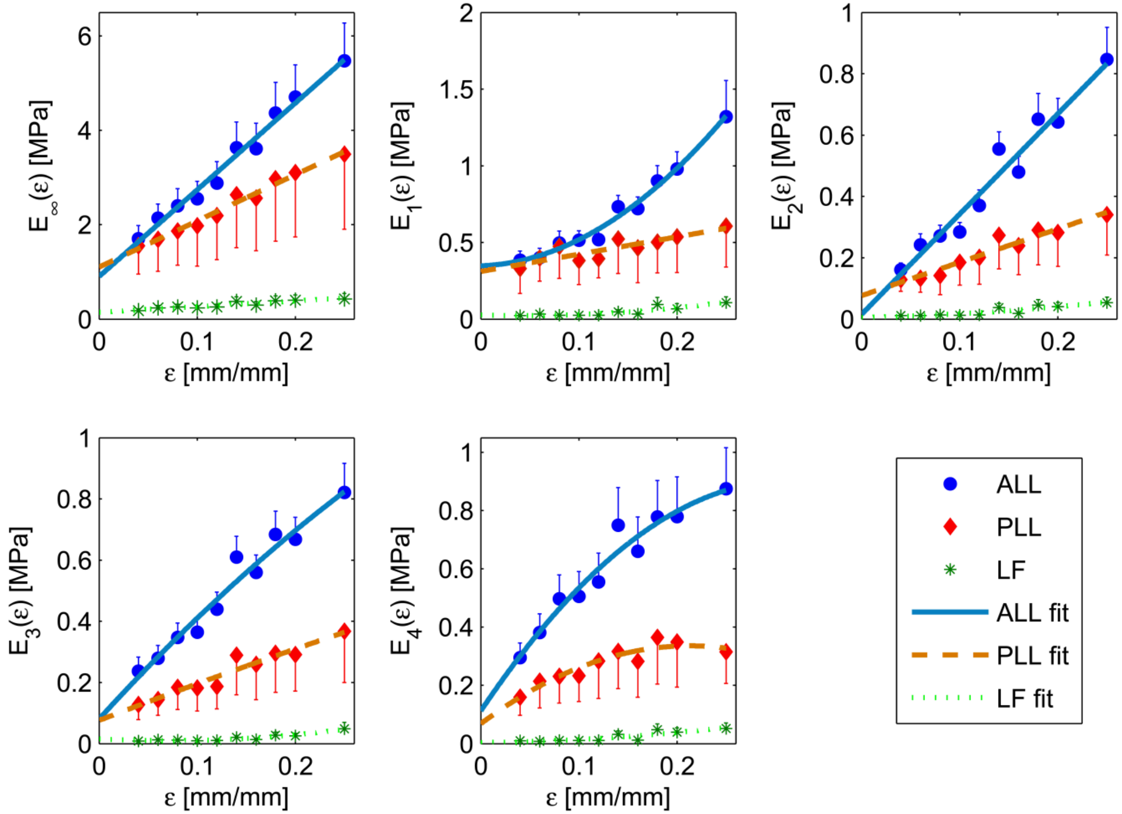
- Constitutive equation fit both the long-term and short-term relaxation data well`



Results

- The strain-dependent moduli for each ligament type was unique ($p \leq 0.0376$ for all comparisons)
  - ***ALL and PLL*** → dominated by **long-term** and **short-term** moduli
  - **LF** → **consistent** across temporal decades
  - **LF** → **reduced** with respect to the longitudinal ligaments

$$E(\varepsilon,t) = E_{\infty}(\varepsilon) + E_1(\varepsilon)e^{-t/0.1} + E_2(\varepsilon)e^{-t/1} + E_3(\varepsilon)e^{-t/10} + E_4(\varepsilon)e^{-t/100}$$



Reduced Order Models of Structures with Linear Viscoelastic Materials

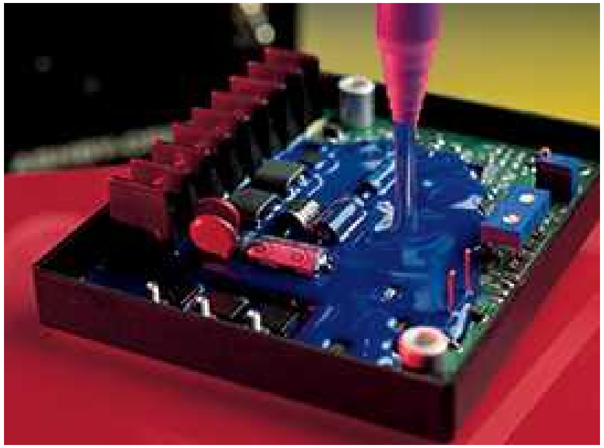
APPLICATIONS IN AEROSPACE ENGINEERING

What is our goal?

- Develop reduced order models (ROMs) of finite element models with linear viscoelastic material behavior for **time domain** structural dynamic simulations
- Reduce computational burden of repetitive numerical solutions while preserving the accuracy of the full order model
- Incorporate non-viscous damping into ROMs via material property data

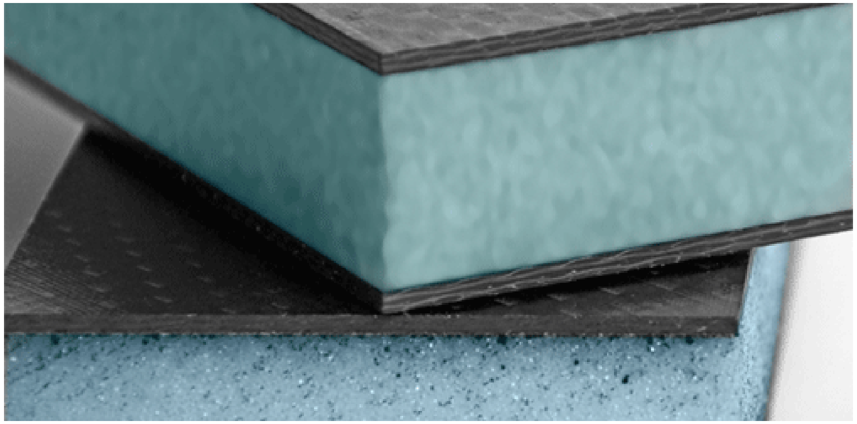
Applications with Linear Viscoelastic Behavior

Encapsulation



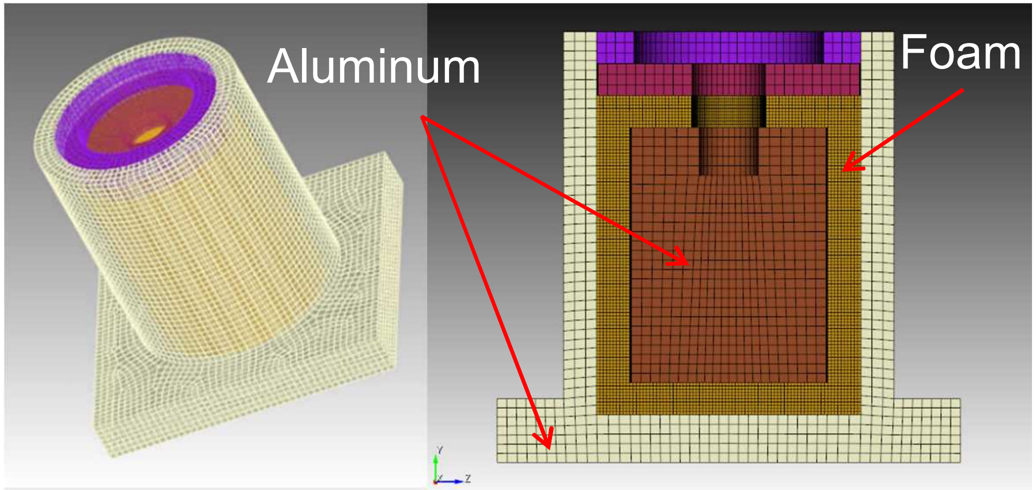
(http://www.epoxies.com/blog/try-our-new-flame-resistant-potting-compounds/)

Sandwich Structured Composites



(http://altairenlighten.com/2012/07/sandwich-structures/)

Ministack

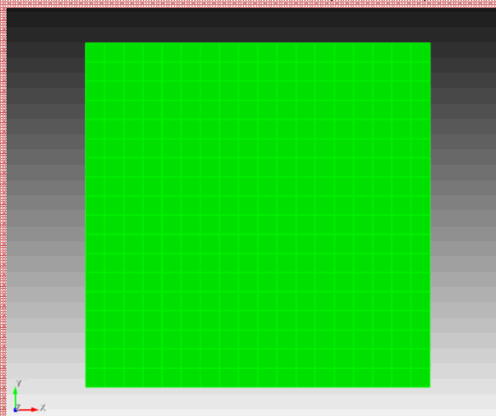


(Jacobs, 2016)

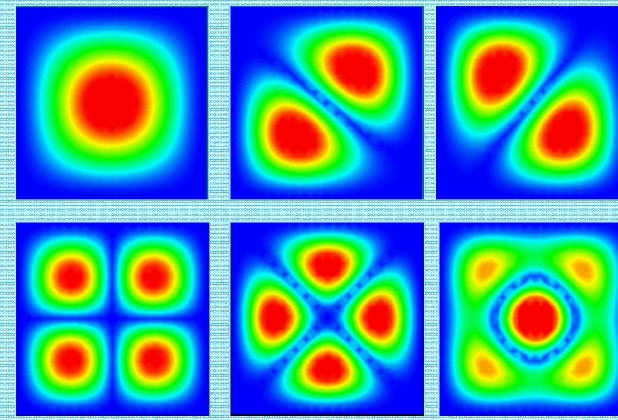
Reduced Order Modeling

FE mesh in physical coordinates

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{f}_{matl}\left(\mathbf{x}, \dot{\mathbf{x}}\right) = \mathbf{f}\left(t\right)$$



Determine appropriate basis, or shape vectors based on the physical equations of motion



Project full equations of motion onto a small set of basis vectors

Related by transformation:

$$\mathbf{x}(\mathrm{t}) = \mathbf{T}\mathbf{q}(t)$$

$$^{*}\mathbf{q}(\mathrm{t}) \ll \mathbf{x}(t)$$

Solve reduced equations

$$\hat{\mathbf{M}}\ddot{\mathbf{q}} + \hat{\mathbf{K}}\mathbf{q} + \mathbf{f}_{matl}\left(\mathbf{q}, \dot{\mathbf{q}}\right) = \hat{\mathbf{f}}\left(t\right)$$

Linear Viscoelasticity with Prony Series

- **Stress dependent upon time**

$$\boldsymbol{\sigma}(t) = \int_0^t E(t-\tau)\frac{d\boldsymbol{\varepsilon}}{d\tau}d\tau$$

- **Prony series**

$$E(t) = E_\infty + (E_g - E_\infty)\zeta(t)$$

$$\zeta(t) = \sum_{i=1}^{N} E_i e^{-t/\tau_i}$$

Relaxation modulus: describes time- and history-dependent behavior!

⋮ (skipping detailed mathematics)

- **FEA equations of motion**

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}_{v,K}\int_0^t \zeta_K(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}_{v,G}\int_0^t \zeta_G(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}_e\mathbf{x} = \mathbf{f}(t)$$

*Typically have many degrees-of-freedom!

Linearized Complex Eigenmode Basis

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}_{v,K}\int_0^t \zeta_K(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}_{v,G}\int_0^t \zeta_G(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}_e\mathbf{x} = \mathbf{f}(t)$$

- **Linearized Complex Eigenmodes**
  - Iterative approach that uses linearized quadratic eigensolver in Sierra/SD

Linearized quadratic eigenvalue problem: for each mode, iterate until $\mathbf{Im}(\boldsymbol{\lambda_r}) = \boldsymbol{\omega_0}$

$$\left(\lambda_r^2\mathbf{M} + \lambda_r\mathbf{K}_{v,K}\sum_{i=1}^{N_K}\frac{K_{coeff,i}}{\lambda_0 + 1/\tau_{K,i}} + \lambda_r\mathbf{K}_{v,G}\sum_{i=1}^{N_G}\frac{G_{coeff,i}}{\lambda_0 + 1/\tau_{G,i}} + \mathbf{K}_e\right)\mathbf{u}_r(\lambda_0) = \mathbf{0} \quad \textbf{with} \quad \lambda_0 = i\omega_0$$

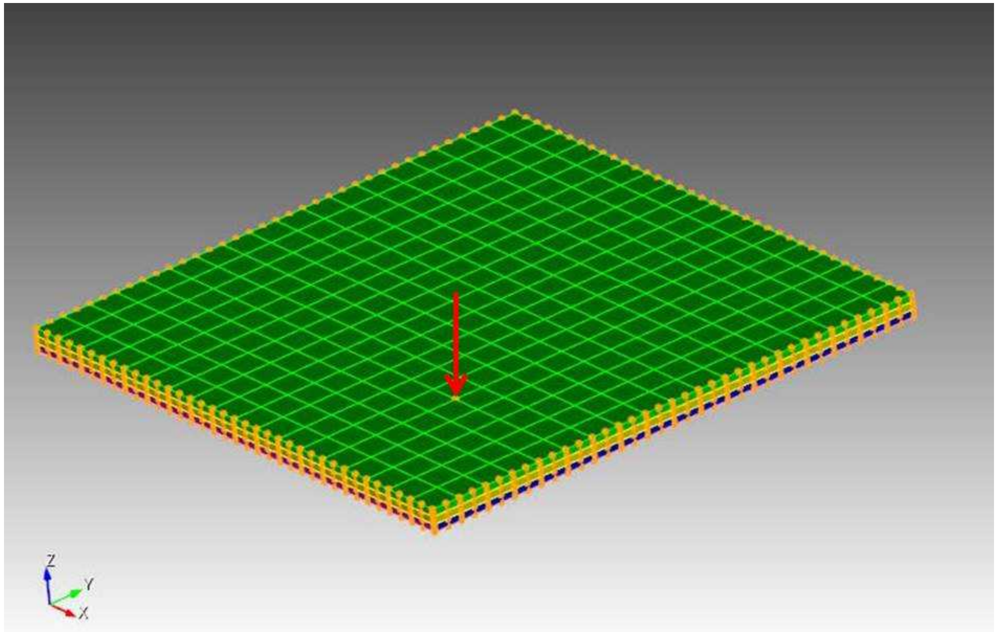
Quasi-static Correction

$$\mathbf{R}_{qs} = \left(i\omega\mathbf{K}_{v,K}\sum_{i=1}^{N_K}\frac{K_{coeff,i}}{i\omega + 1/\tau_{K,i}} + i\omega\mathbf{K}_{v,G}\sum_{i=1}^{N_G}\frac{G_{coeff,i}}{i\omega + 1/\tau_{G,i}} + \mathbf{K}_e\right)^{-1}\mathbf{b} \quad \textbf{with} \quad \omega >> 0$$

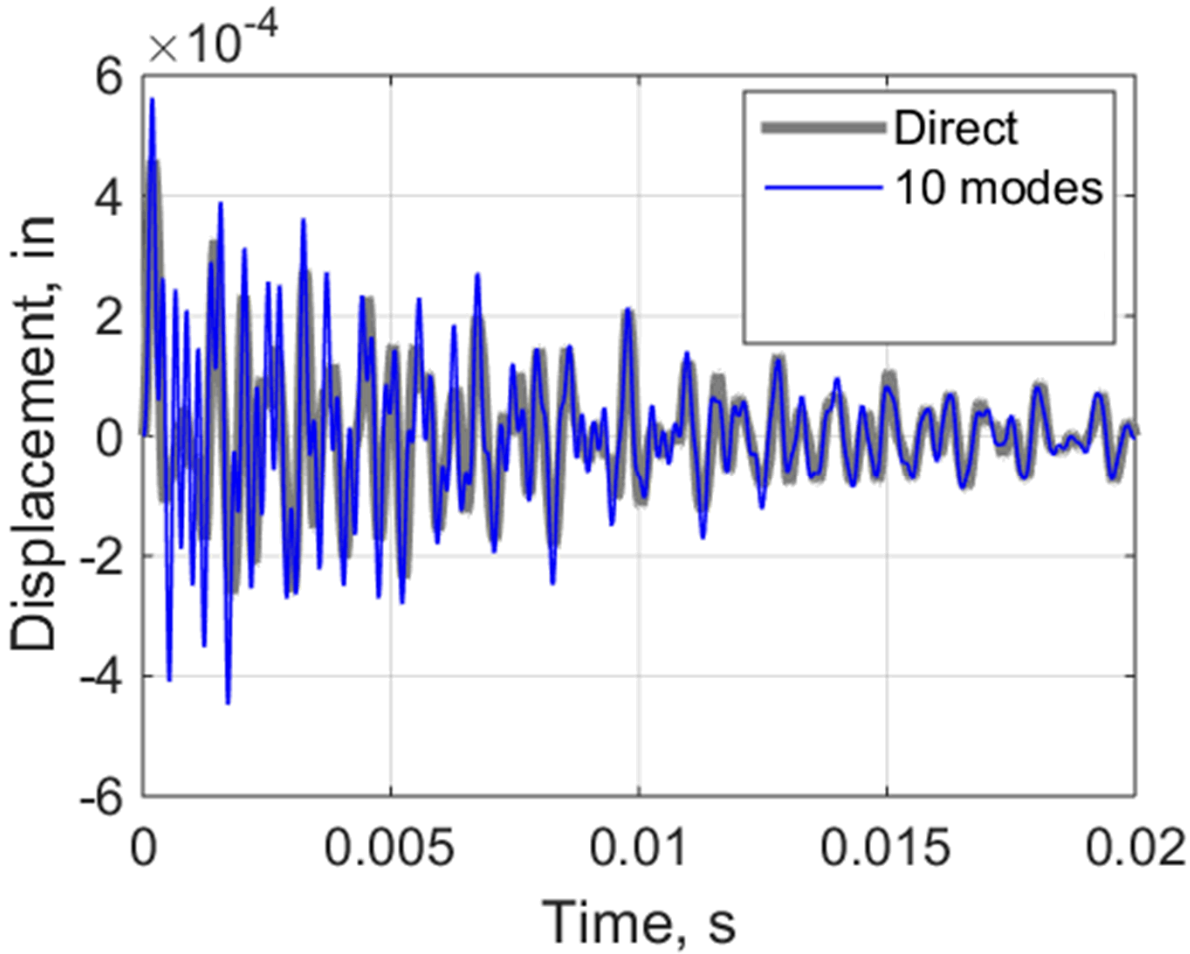
$$\Rightarrow \quad \mathbf{T} = [\mathrm{Re}(\mathbf{u}_1 \quad \mathbf{u}_2 \quad \dots \quad \mathbf{u}_{N_R} \quad \mathbf{R}_{qs}) \quad \mathrm{Im}(\mathbf{u}_1 \quad \mathbf{u}_2 \quad \dots \quad \mathbf{u}_{N_R} \quad \mathbf{R}_{qs})]$$

Application to Plate Model in Sierra/SD

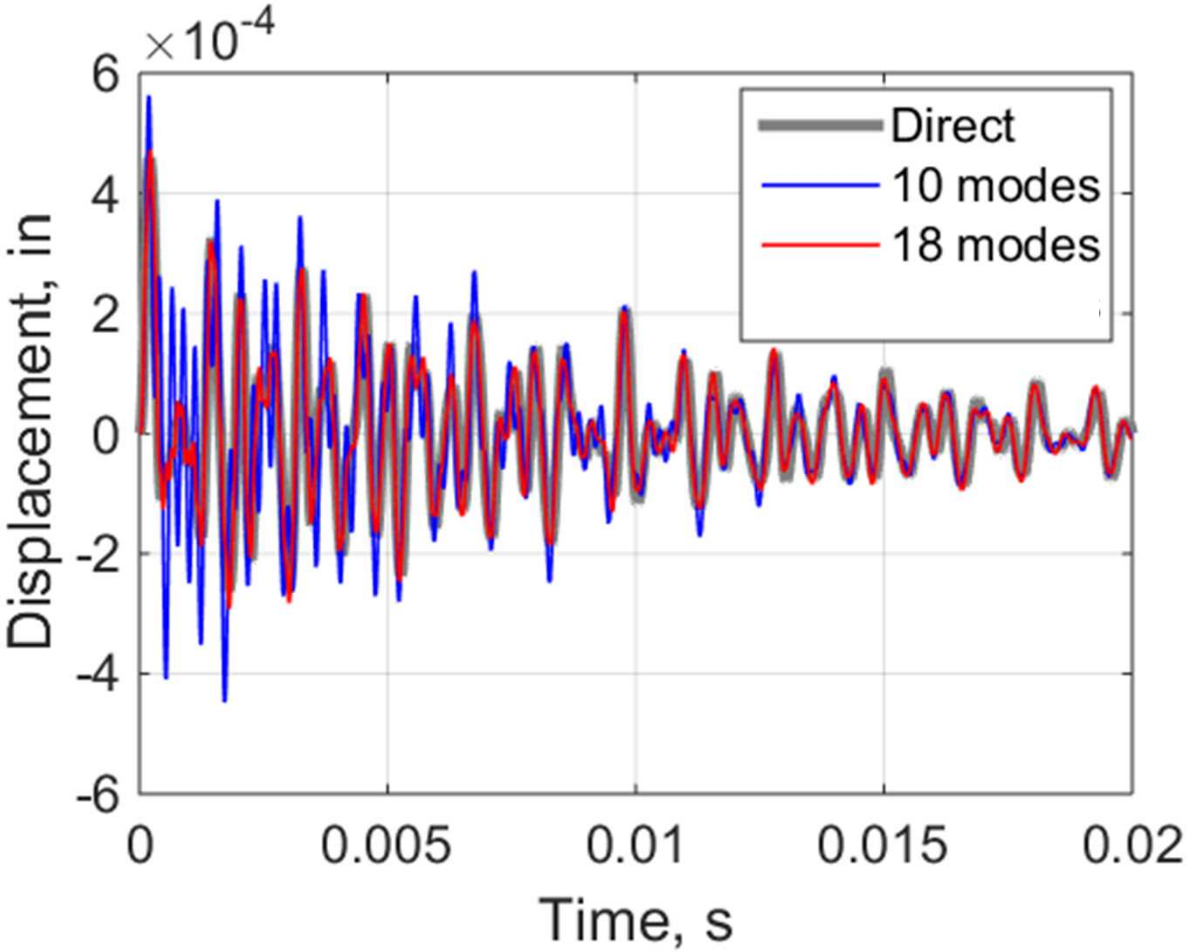
- **Viscoelastic Sandwich Plates**
  - Aluminum 6061-T6 (linear elastic)
  - PMDI 22 foam (linear viscoelastic)



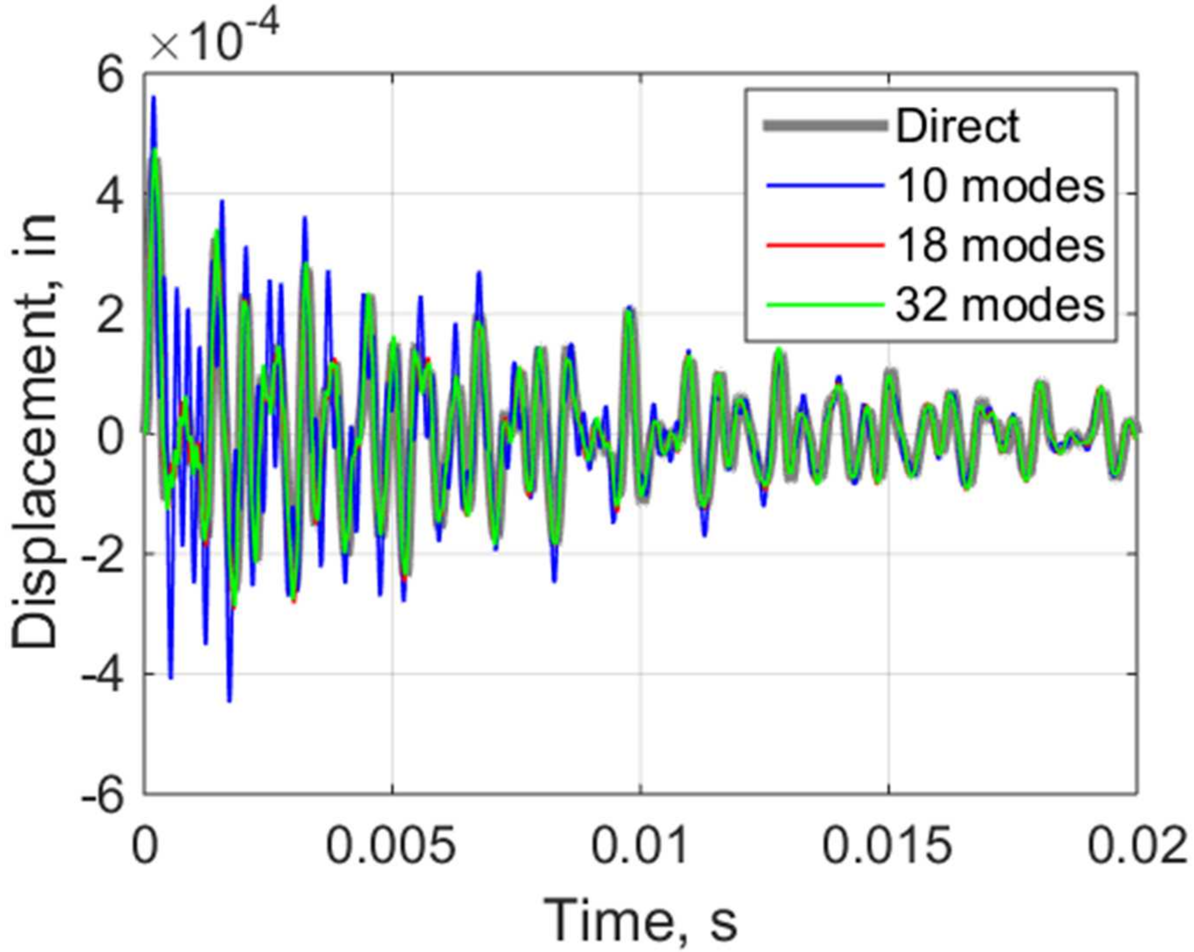
Viscoelastic Sandwich Plate with Linearized Complex Modes



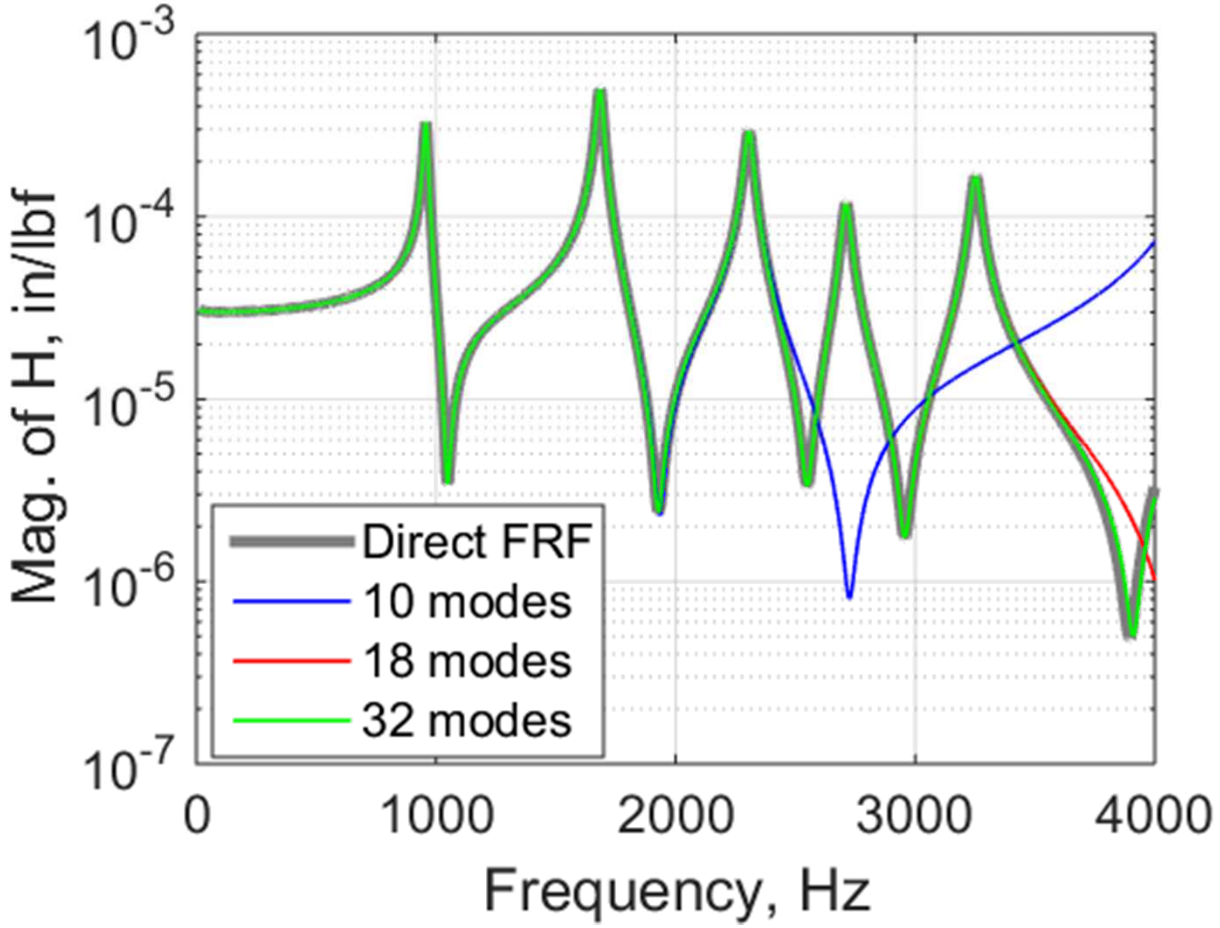
Viscoelastic Sandwich Plate with Linearized Complex Modes



Viscoelastic Sandwich Plate with Linearized Complex Modes



Viscoelastic Sandwich Plate with Linearized Complex Modes



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Transient Solutions of Sandwich Plate			
Model	**Eigensolution**	**Solution Time**	**Total Cost**
Full FEA model	n/a	~10 days* (dt = 1e-7)	~864,000 seconds (~14,400 min)
1501 mode ROM with real modes	229 seconds	2571 seconds (dt = 1e-6)	2800 seconds (~47 min)
32 mode ROM with linearized complex modes	4923 seconds	4.2 seconds (dt = 5e-6)	4927 seconds (~82 min)

*Estimate based on integration in Matlab on single processor

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Biomechanics work

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Reduced order modeling work

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