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Numerical Investigation of Probability Measures Utilized in a Maximum Entropy Approach

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Outline

- Maximum Entropy Approach (MEA) Background
- Required Engineering Choices
  - Sampling Methods
  - Reduced Order Models (ROMs)
  - Empirical Likelihood (EL)
- Example System for Demonstration
- Results
  - Fixed-Interface
  - Free-Interface
- Conclusions/Remarks

Maximum Entropy Approach

- Combines random matrices, maximum entropy to select distribution, and maximum likelihood estimate for the distribution
- Introduced by Soize in 2000
- Assumes semi-positive definite matrices in current implementation
- Apply to system EOM
- $[M]\ddot{X} + [K]X = Fcos(\omega t)$
- Other EOM can be used, but require more attention

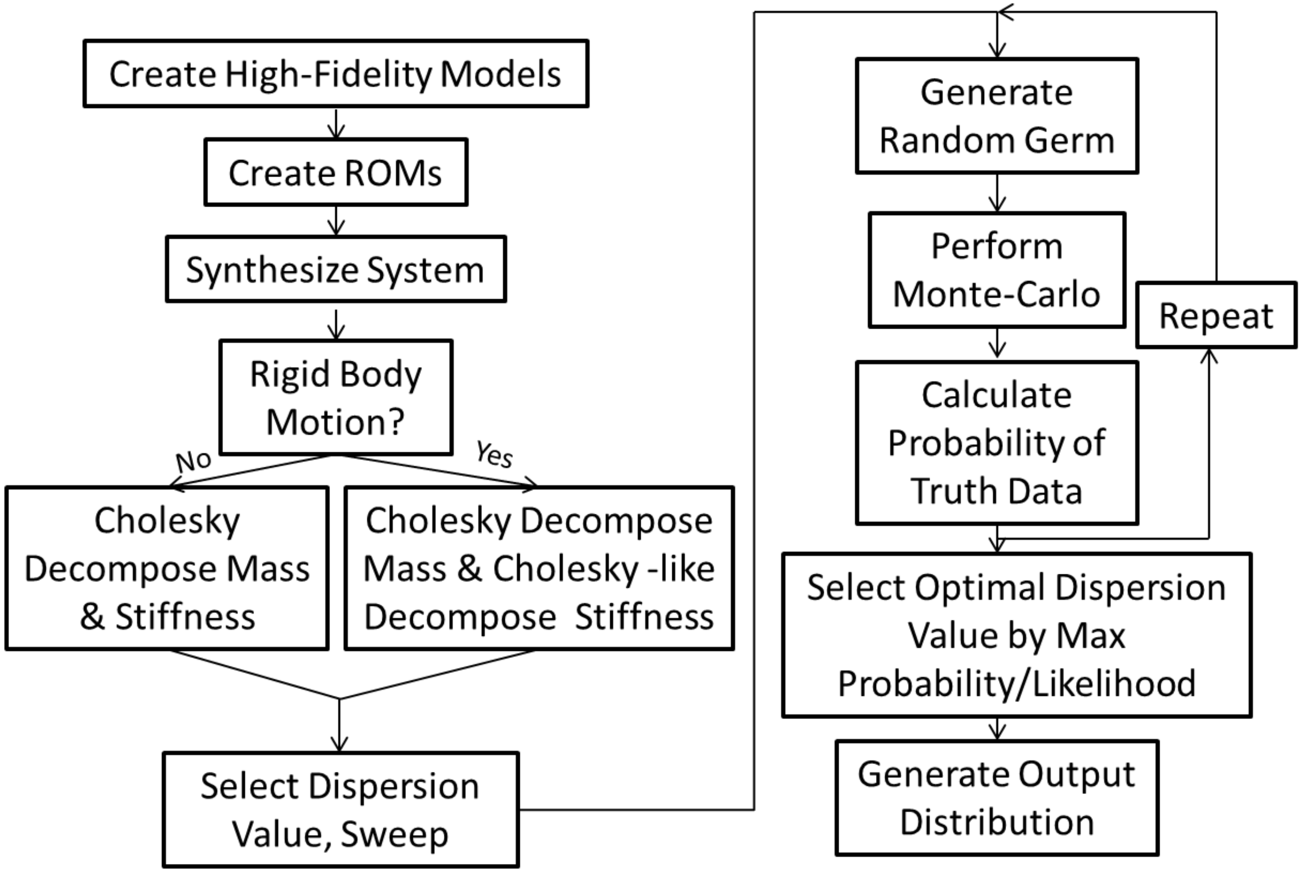
MEA Cont.

- Since semi-positive definite, can Cholesky decomposition
  - $[K] = [L_K]^T[L_K]$
- Insert random germ
  - $[\boldsymbol{K}] = [L_K]^T[\boldsymbol{G}(\delta_K)][L_K]$
  - $\mathbb{E}[\boldsymbol{G}] = I$
  - $Var[\boldsymbol{G}] \propto \delta_K$
- 2 possible uses
  - Calibrate dispersion to known data
  - Apply dispersion value to accommodate unknown error

Calibration of Dispersion Parameter

- Like any calibration, can be computationally expensive
- Truth data can be any selected output
  - Used natural frequencies in this analysis
- Can be thought of in two ways
  - Use mean and variance of truth data: unbiased estimate
  - Use instances from the distribution, assume model matches mean output: bias estimate
  - Doesn't affect analysis, just explanation
- $\delta^{opt} = \arg\max \Pr\,(Truth|\delta) = \arg\max \mathcal{L}(\text{Truth}, \delta)$

Process Flow Chart



Sampling Methods

Monte-Carlo (MC)

- Simplest sampling method
- Pick values at random from distribution
  - All values are independent
- Easy to program
  - Adjusted quasi-random numbers
- Is easy to scale
- Easy to parallelize
- Computationally expensive
  - ~10,000 for convergence

Latin Hypercube (LHC)

- Stratification of MC
- Split distribution into equal probable sections
  - Value chosen randomly from section
  - Or midpoint for repeatability
- Massive reduction in points
  - ~100 for convergence
- Must pre-generate evaluation points
  - Memory issues for large sample numbers or large matrices
- Can also be parallelized
- Some numerical issues for unbounded distributions

Reduced Order Models

Fixed-Interface (CB)

- Uses fixed-interface modal DOF and physical interface displacement DOF
- Primal formulation of Craig-Bampton
- Ensure linear independence of DOF
- When synthesized, maintain interface DOF
- Good for flexible bodies with rigid connections

Free-Interface (CC)

- Uses free-interface modal DOF and interfacial force DOF
- Doesn't automatically ensure linear independence if rigid body information
- Similar to Dual CB, but no Lagrange multipliers
- When synthesized, no interface information is maintained
- Good for large interfaces

Likelihood Function

Log-likelihood

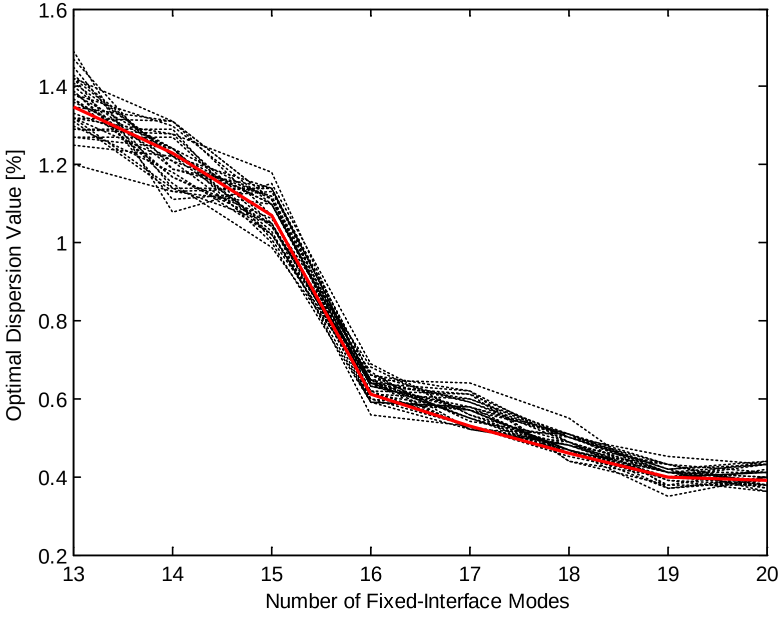
- Simplifies joint distribution to summation since independence of modes
  - Each frequency can be determined independently
- Negative log-likelihood used for future research in optimization routine

Empirical log-likelihood

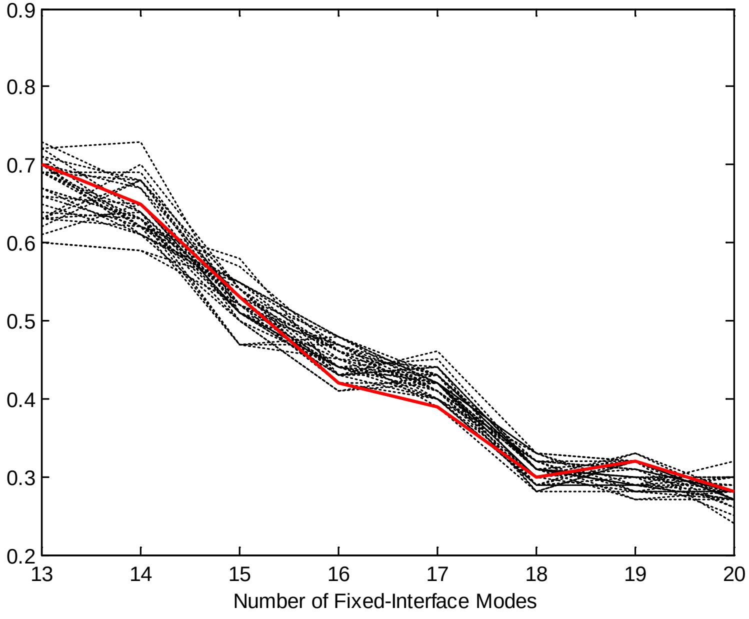
- Perform probability measure multiple times
  - Take highest probability of individual evaluations
  - Computational statistics tool
- More computationally expensive
  - Allows for higher repeatability
  - Less subject to MC randomness
- Can give some confidence intervals on results

Likelihood Cont.

System B



System D



Probability Measure

Histogram

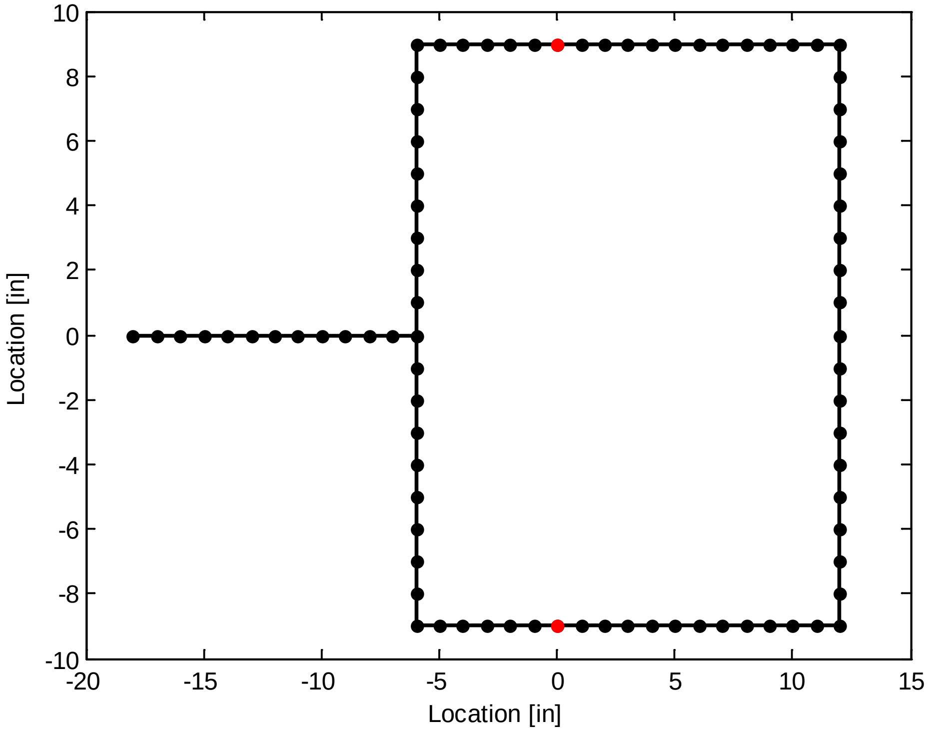
- Simple
- Requires engineering judgment
  - Specify bounds
    - Number of bins
    - Size of bins
  - Chosen as percentage of true value
- Count instances within tolerance as proportion to probability
- Optimal percentage changes with dispersion

Gaussian Kernel

- Applies a kernel estimator to generate density function
  - Evaluate exact probability at truth value
- Assuming Gaussian smoothing
- No additional input from engineer
- “ksdensity.m”
- Since large amount of samples, less assumptions than histogram
  - Used as truth data

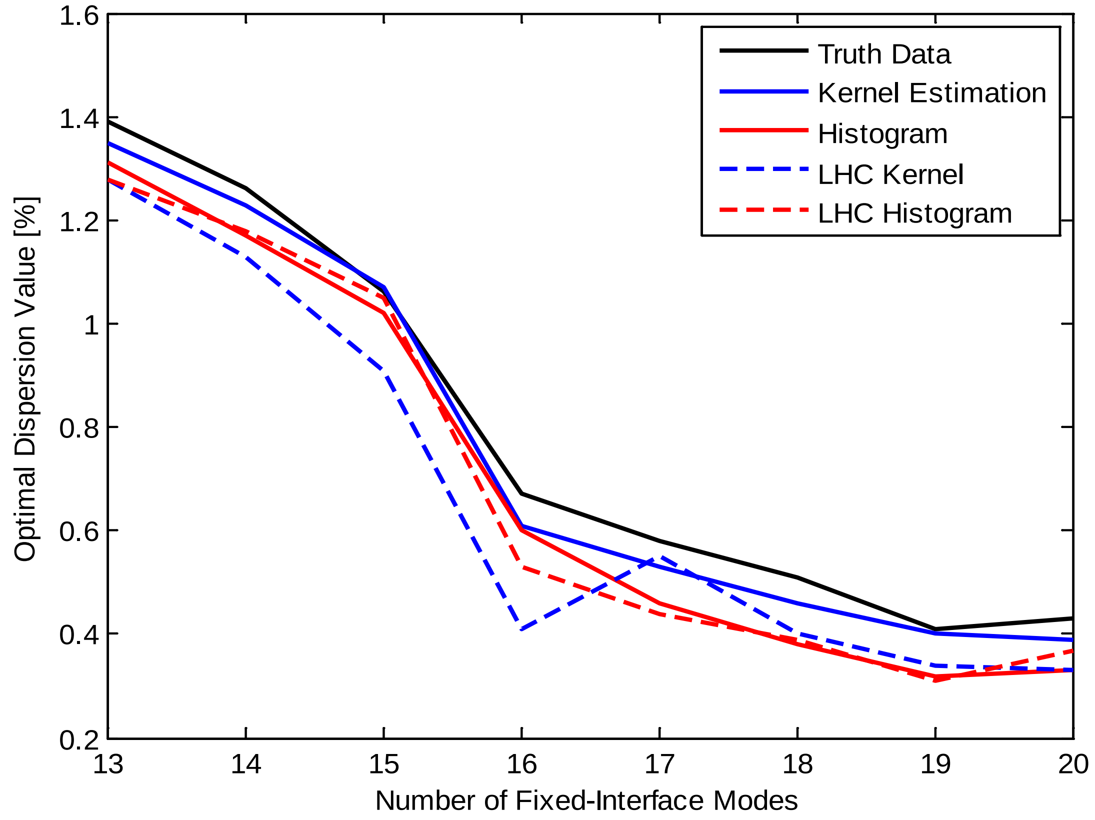
Demonstration System

- 2 Systems- “B” Left, “D” Right
- Use first 11 elastic Free-Free natural frequencies



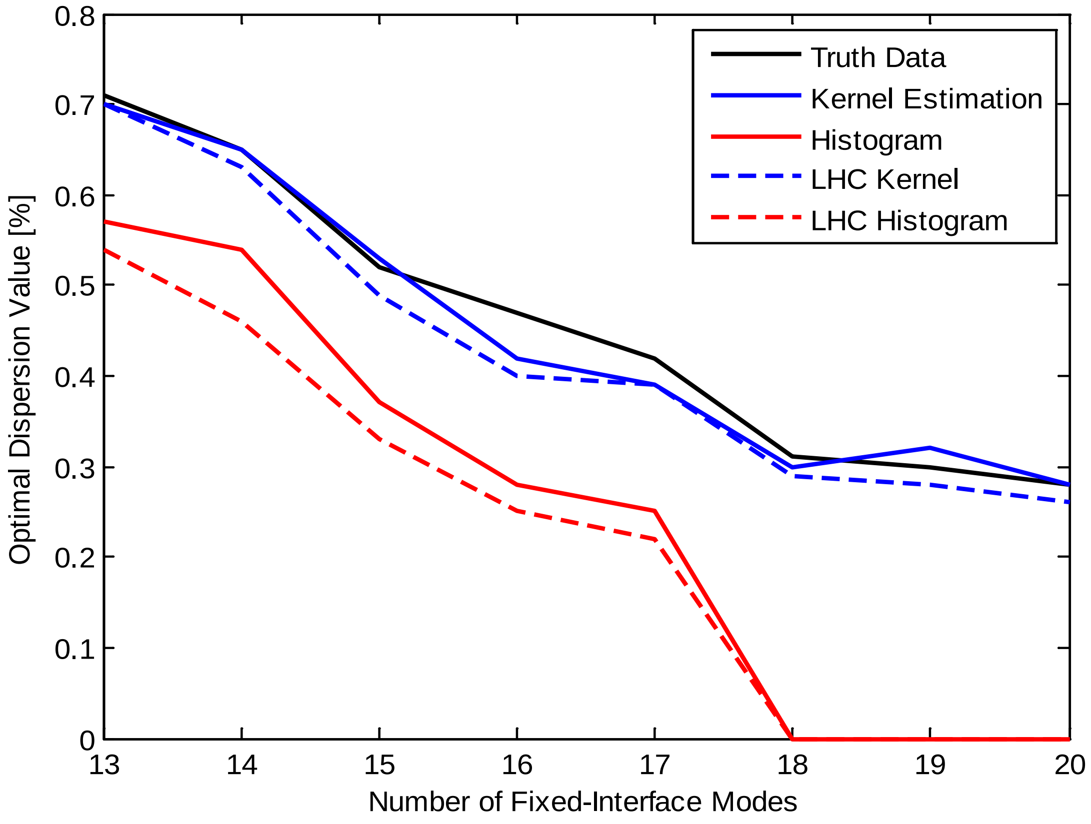
Fixed-Interface Reduction Results

System B



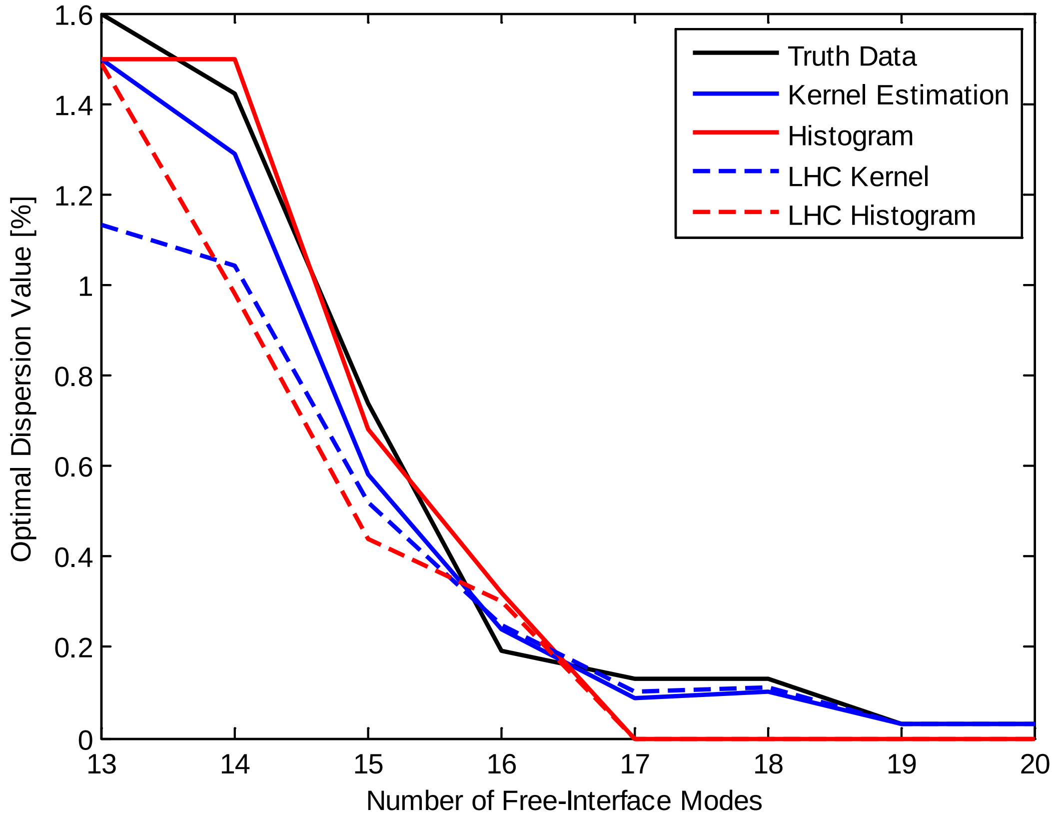
Fixed-Interface Reduction Results

System D



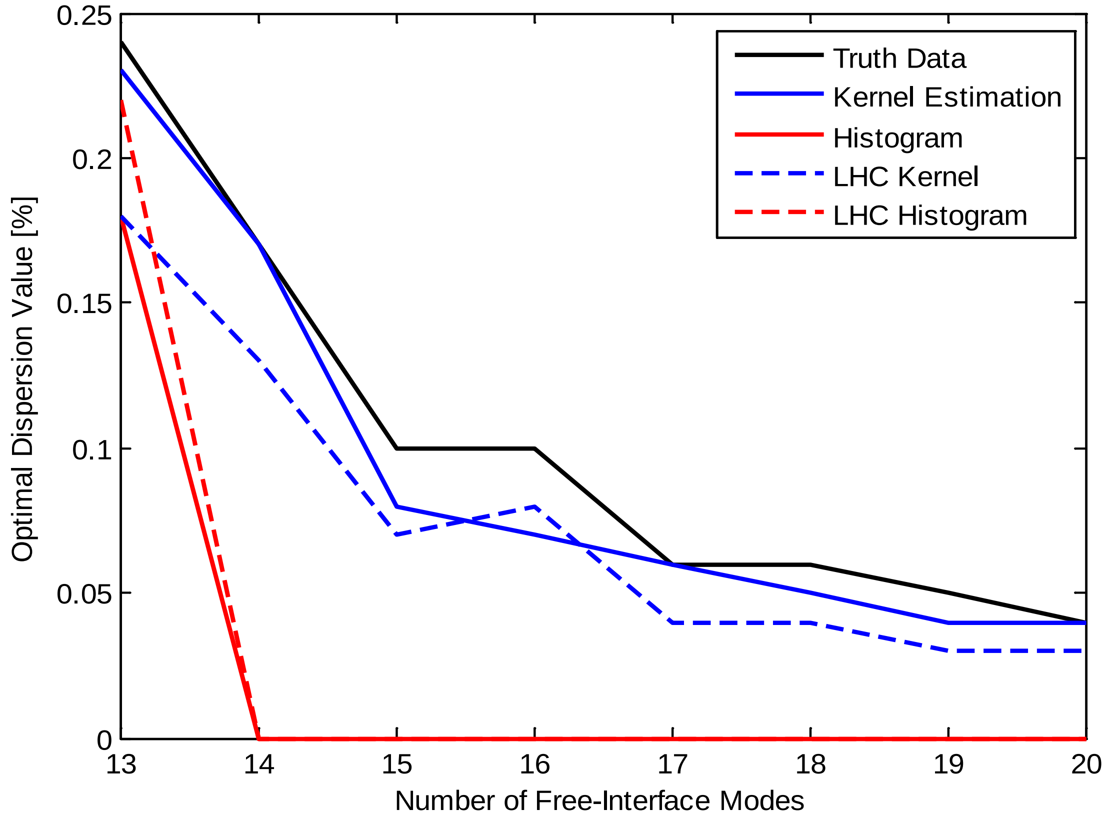
Free-Interface Reduction Results

System B



Free-Interface Reduction Results

System D



Remarks/ Future Research

- Using MC sampling is typically more conservative
- Using a histogram has discontinuity and requires more engineering decisions
- System B shows less variability
- At 10K/200, no observed conservative trend

- Uncertainty bounds on dispersion parameter
- Optimization routine for optimal parameter

Conclusions

- Use MEA to quatify model-form error for ROM
- Implemented MEA to two systems
- Numerical investigation of engineering choices
  - Sampling methods
  - ROMs
  - Likelihood function
  - Probability measure
- With no optimization methods, recommend
  - Gaussian kernel estimator
  - Monte-Carlo sampling
  - Empirical likelihood
- Choice of ROM is problem dependent

Special Thanks

- Advisor: Dan Kammer (Wisconsin)
- Matt Brake (Rice/Sandia*)
- Marc Mignolet (ASU)
- Audience

- Questions?

*Sandia is a multi-program laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under contract DE-AC04-94AL85000.