

# SECANT QKD Grand Challenge

## Sandia Enabled Communications and Authentication Network using Quantum Key Distribution



**Maturing continuous variable QKD  
and New Ideas**

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# Talk outline

1. On-chip realization of CV-QKD hardware
2. Quantum seal development
3. New Ideas and application of the quantum silicon photonics toolbox
4. Objectives, milestones, deliverables

# Quantum seal development

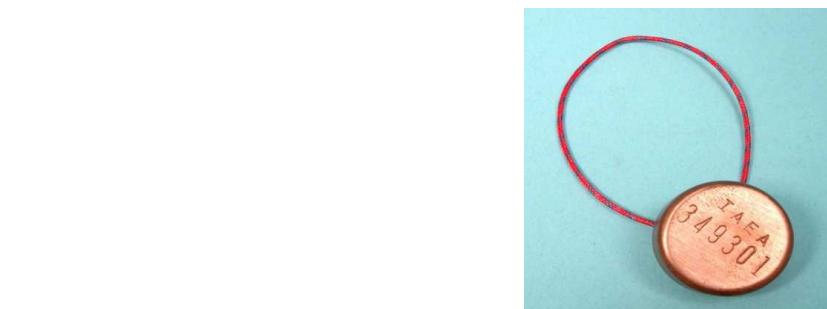
# Secure seals



Seals are an important tool for nuclear non-proliferation safeguards (IAEA)



Physical seal (made from plastic, steel etc.). [Picture from Acme seals]



Metal seal with insulating wire (Cobra seal)

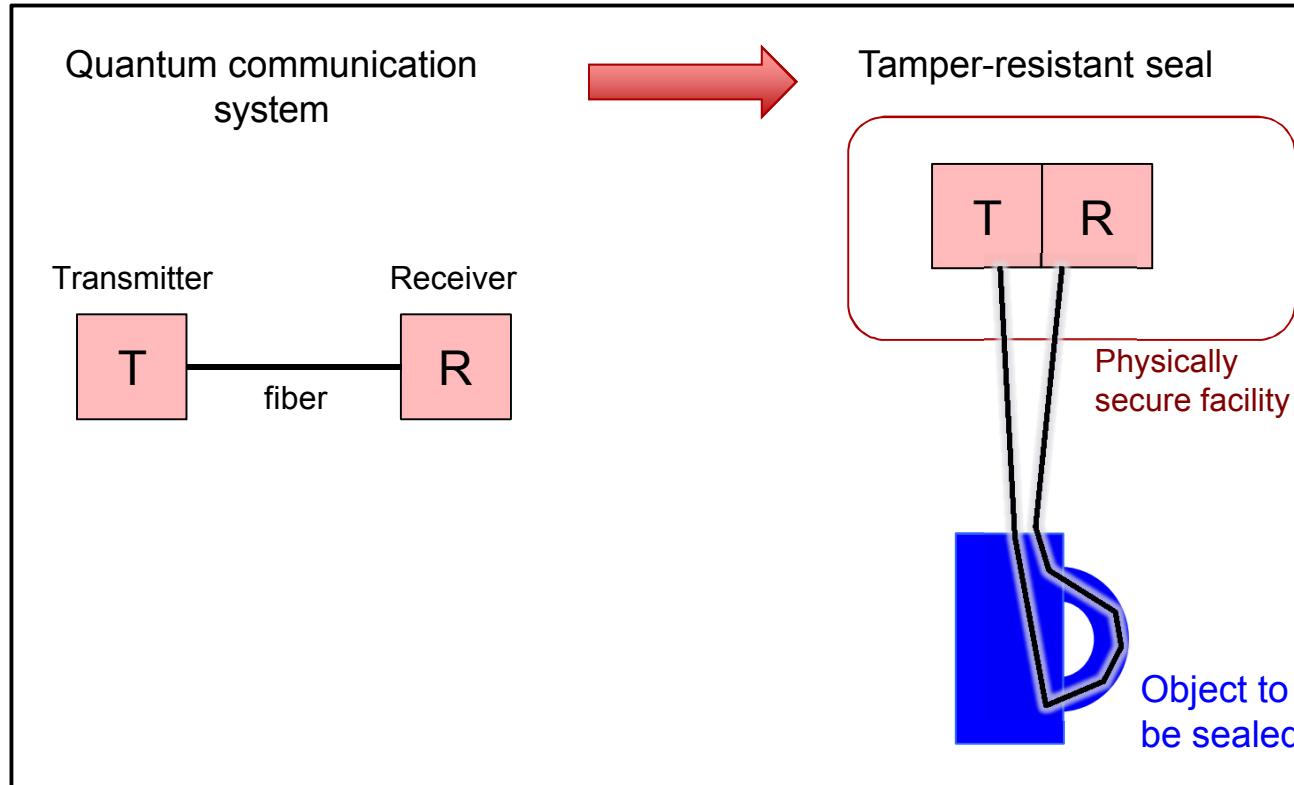


Fiber optic seals  
[Picture from Canberra Industries]

But these seals simply measure light intensity and compare against a fiducial value.

Vulnerable to duplication of laser source and adiabatic tapering in of counterfeit source.

# QKD-enabled optical seals



- Calibrate seal (loss, noise, covariance matrix) upon installation.
- QKD performed up to channel estimation. No error correction or production of key necessary.
- Alice and Bob right next to each other, so no classical “communication” necessary

QKD provides:

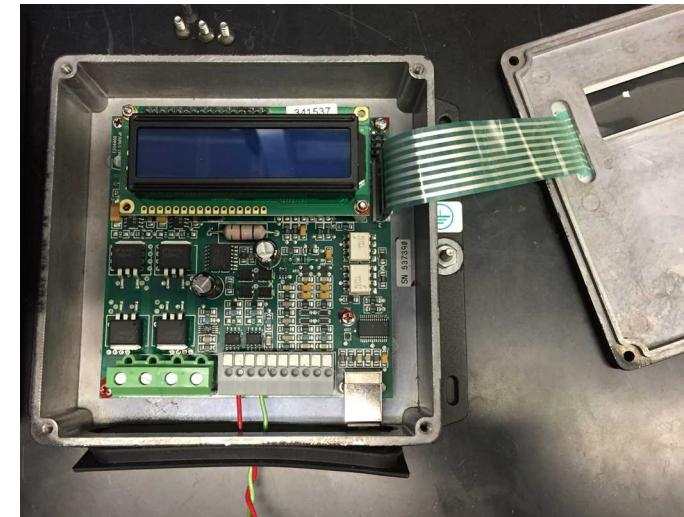
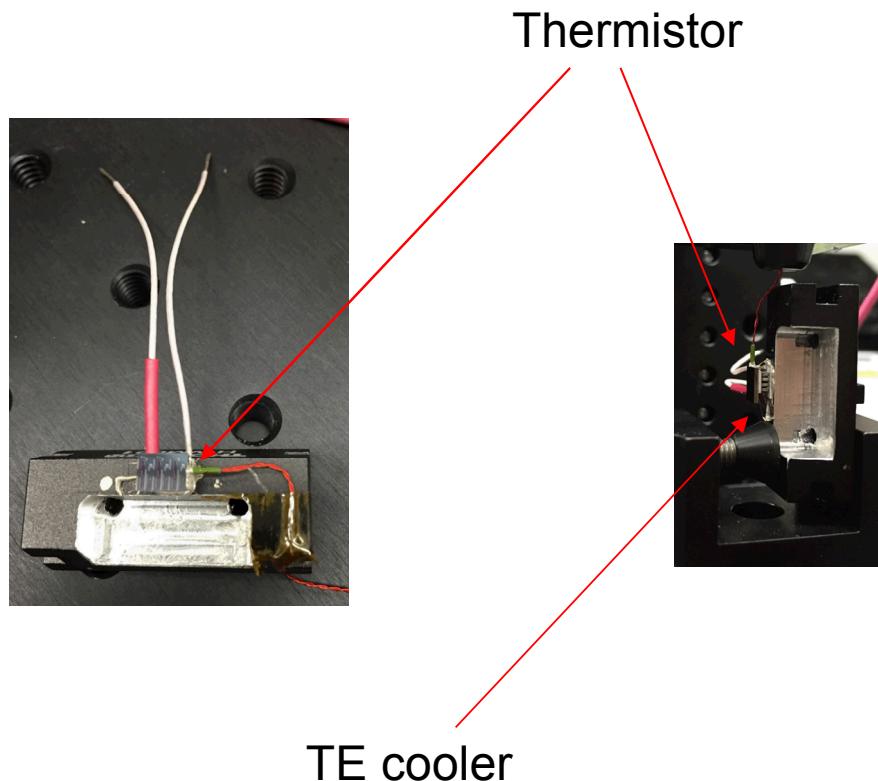
1. Source authentication
2. Channel integrity testing

Provides security against a wider range of attacks/tamper modes

Will focus on CV-QKD-based implementations

# Hardware: towards long-term stability

New mount for feedback stabilized temperature control



# Sensitivity analysis

A completely general framework for understanding what information about the channel parameters we can obtain from measurement is given by the Cramer-Rao lower bound:

Given data  $X_1, \dots, X_N$ , i.i.d. drawn according to a parameterized distribution  $p(x|\theta)$ ,

$$\text{cov}T_\theta(X_1, \dots, X_N) \geq \frac{F(\theta)^{-1}}{N}$$

Covariance matrix for any estimator of  $\theta$        $\xrightarrow{\hspace{10em}}$  Fisher information matrix

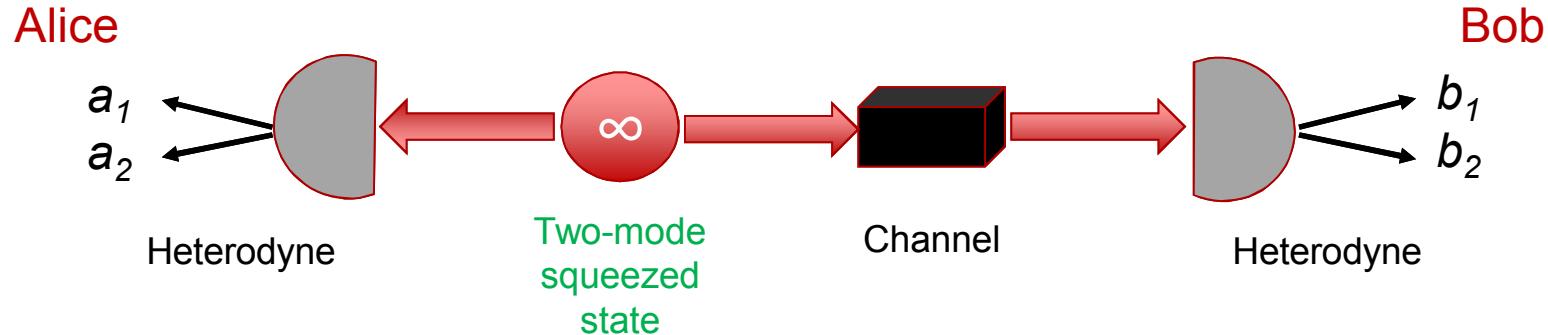
$$F(\theta)_{ij} = \int dx p(x|\theta) \frac{\partial \ln p(x|\theta)}{\partial \theta_i} \frac{\partial \ln p(x|\theta)}{\partial \theta_j}$$

Quantifies the influence each of the parameters have on the measurement outcomes. This in turn dictates how estimable the parameters are from the measurements.

What is  $p(x|\theta)$  for the seal concept?

# Sensitivity analysis

In the entanglement-based picture,



Assume the channel is a passive Gaussian channel characterized by  $T$  and  $\varepsilon$ . Then:

$$p(a_1, a_2, b_1, b_2) = \frac{1}{(2\pi)^2 \sqrt{\det \Lambda}} \exp \left[ -\frac{1}{2} A^\top \Lambda^{-1} A \right]$$

$$A = (a_1, a_2, b_1, b_2)^\top$$

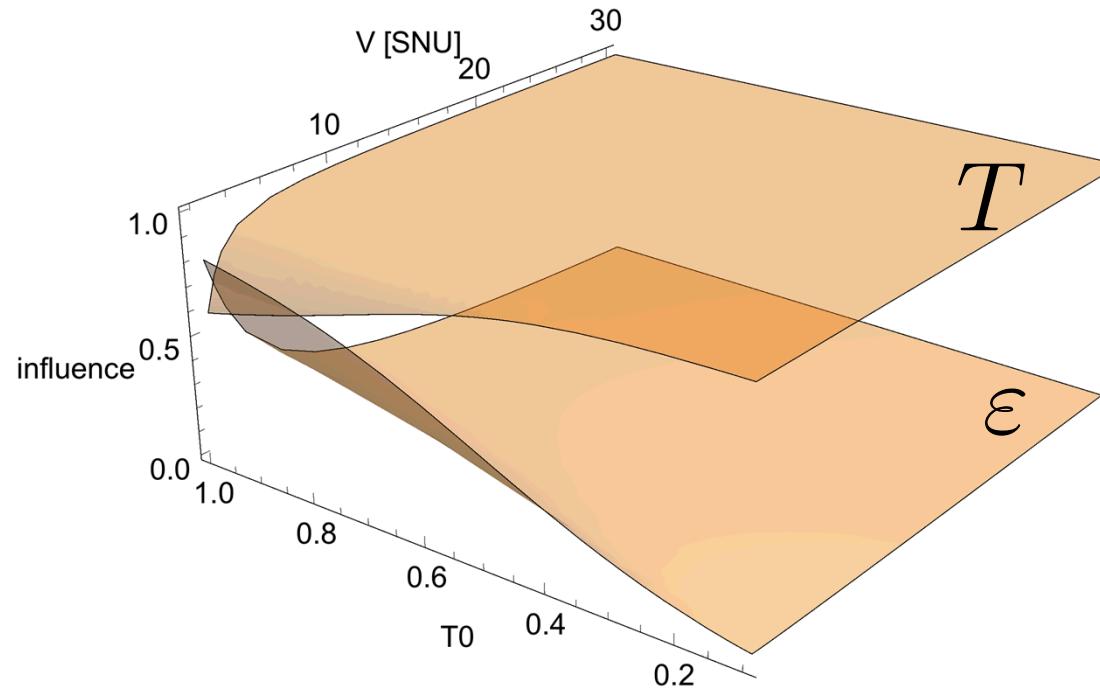
$$\Lambda = \begin{bmatrix} V + 1 + \sigma^2 & 0 & \frac{C}{2} & 0 \\ 0 & V + 1 + \sigma^2 & 0 & -\frac{C}{2} \\ \frac{C}{2} & 0 & T(V + \chi) + 1 + \sigma^2 & 0 \\ 0 & -\frac{C}{2} & 0 & T(V + \chi) + 1 + \sigma^2 \end{bmatrix}$$

$$V = V_A + 1$$

$$C = \sqrt{T(V^2 - 1)}$$

$$\chi = \frac{1 - T}{T} + \varepsilon$$

# Sensitivity analysis



- Excess noise has a large influence only when the modulation variance is small and the channel transmission is high
- Channel transmission has the most influence in the opposite regime
- Fundamental tradeoff between estimating these two parameters

# Hypothesis testing framework

Tamper detection formalized within a hypothesis testing framework

$\mathcal{H}_0$  : Data is **consistent** with calibration run

$\mathcal{H}_1$  : Data is **inconsistent** with calibration run

Require data processing protocol to perform this hypothesis test :

1. With as few assumptions on the data generating distribution as possible
2. In real-time, using simple arithmetic operations (ideally, FPGA implementable)
3. With as little data as possible

# Hypothesis testing framework

What is the data in CV-QKD (with homodyne measurements)?

$$(\mathbf{d}_1, \dots, \mathbf{d}_N) = \left( \begin{pmatrix} d_A \\ d_B \end{pmatrix}_1, \dots, \begin{pmatrix} d_A \\ d_B \end{pmatrix}_N \right)$$

Two continuous random variables per pulse

Simple statistics we can calculate about this data:

$$\bar{d}_i = \frac{1}{N} \sum_{k=1}^N d_i$$

Sample means

$$S = \frac{1}{N-1} \sum_{k=1}^N (\mathbf{d} - \bar{\mathbf{d}})(\mathbf{d} - \bar{\mathbf{d}})^\top$$

Sample covariance matrix

$$= \begin{bmatrix} \sigma_A^2 & \sigma_{AB} \\ \sigma_{AB} & \sigma_B^2 \end{bmatrix}$$

Collect these into the vector:

$$\theta = (\bar{d}_A, \bar{d}_B, \sigma_A, \rho_{AB}, \sigma_B)^\top$$

# Hypothesis testing framework

Now we can formalize the hypothesis test

$$\mathcal{H}_0 : \hat{\delta} = 0$$

$$\mathcal{H}_1 : \hat{\delta} \neq 0$$

$$\hat{\delta} = \theta - \theta_0$$

Test statistic:

$$\chi^2 = \hat{\delta}^T \Sigma_{\hat{\delta}}^{-1} \hat{\delta}$$

Sullivan *et al.*, J. Quality Tech.,  
39 66 (2007)

If we use maximum likelihood estimates for elements of  $\delta$ , this test statistic is **asymptotically chi-square distributed** with 5 degrees of freedom.

Therefore can use this test statistic to bound hypothesis testing error probabilities, e.g.

$$p(\chi^2) < 0.01 \implies \text{reject } \mathcal{H}_0$$

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Notes:

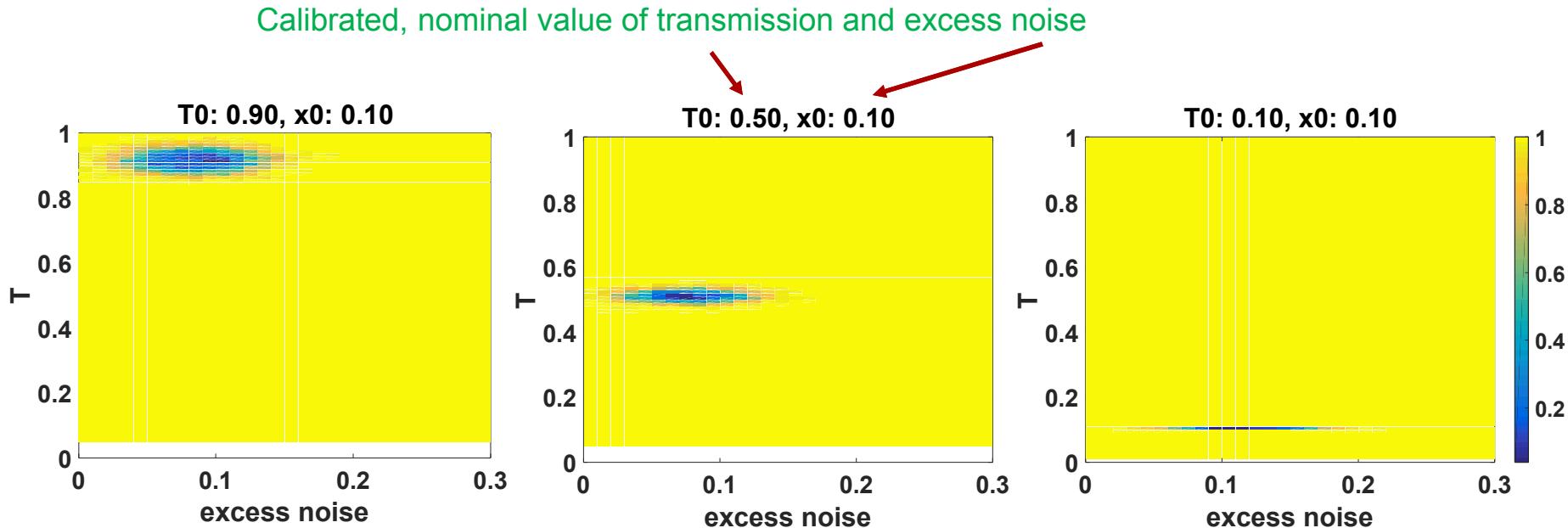
$\Sigma_{\hat{\delta}}$  is difficult to calculate directly, so we will use an approximation to it based on the Cramer-Rao bound.

No assumptions on data until now. This last approximation assumes the data collected under the calibration run is from a Gaussian channel.

# Hypothesis testing framework

Probability of rejecting null hypothesis (detecting tampering)

Simulated data ( $N = 10,000$ ,  $V_a = 30$  SNUs) under a passive Gaussian channel.



- Test is very sensitive to changes in channel transmission, especially at low transmission.
- Less sensitive to excess noise in this parameter regime (requires lower  $V_a$ ).
- Improve test to have greater power – achieve good rejection with smaller  $N$ .