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A New Particle Method Designed to Augment an Existing Shock Physics Code, and Thoughts on a GIMP Approach to the Same Problem

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Introduction

- Shocks and the MPM (GIMP/CPDI) are problematic
  - Very noisy particles
  - MPM codes, CFDLIB, Uintah, Kodiak
- Solutions
  - Artificial viscosity
  - Removing the null space
  - Coupling to other methods?
    - Mostly trouble so far, data at many locations/interpolation.
- The ELPM (Eulerian/Lagrangian Particle Method, Schumacher)
  - Augment existing technology with strength particles
  - Add Lagrangian capability to Eulerian hydrocode
- Can we derive similar methods using variational calculus/GIMP ideas?
  - Consistency

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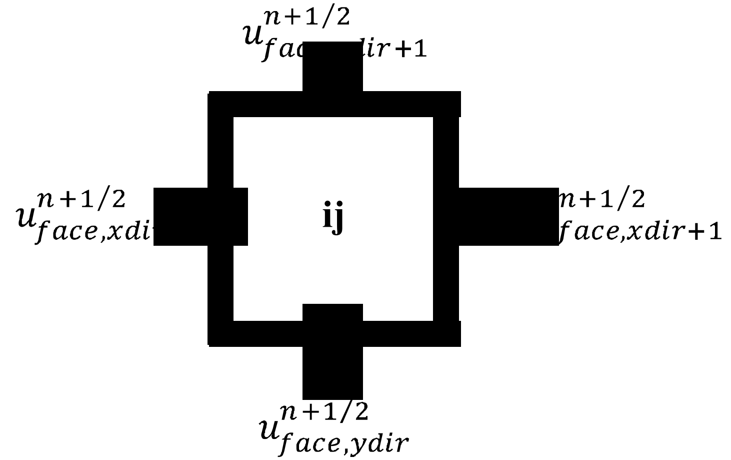
ELPM

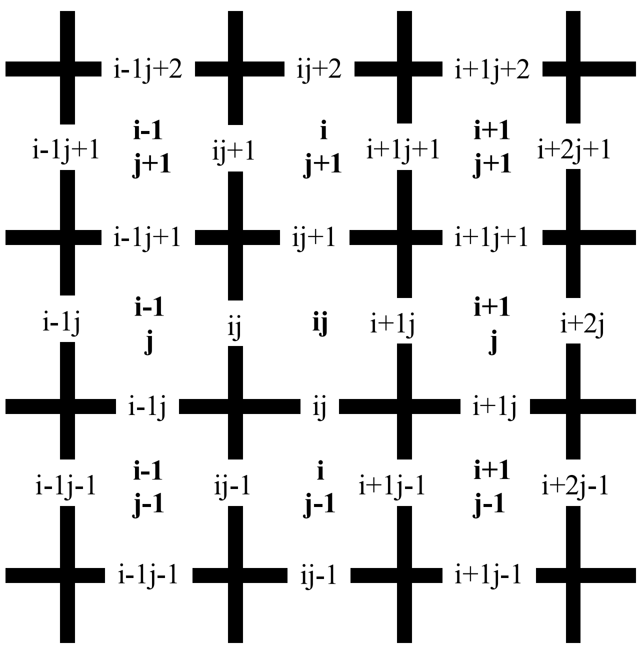
A “Hybrid Method” (term is not universal)

- Dual representations of material at all times
- Particles/Finite Elements
  - Avoid mesh tangling
  - Particles: contact, compressive response
  - FEM: tension/shear
  - Johnson, G.R., Beissel, S. R., Gerlach, C.A., Another Approach to a Hybrid Particle- Finite Element Algorithm for High-Velocity Impact, *Int J Impact Engineering*, 38, 397-405 (2011)
- Particles/Eulerian (MAC) Grid
  - Track interfaces using particles
  - Zheng, W., Zhu, B., Kim, B., Fedkiw, R., A New Incompressibility Discretization for a Hybrid Particle MAC Grid Representation with Surface Tension, *JCP*, 280, 96-142 (2015)
- ELPM
  - Particles: Strength, interfaces
  - MAC Grid: Pressure

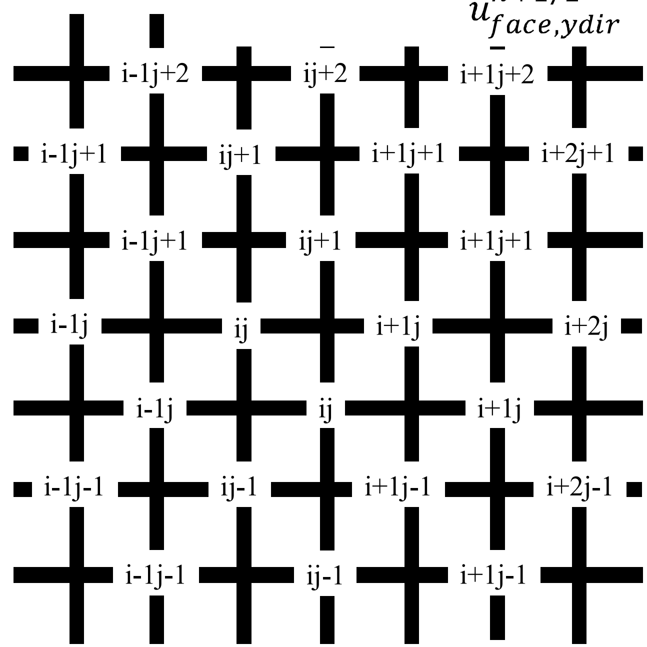
Kodiak (the existing hydrocode)

- **Lagrangian step followed by remap to original (MAC) grid**
  - **Finite Volume discretization of the Lagrangian step**
  - **Remap (interface tracking and mixing)**





Primary Grid



Staggered Grids

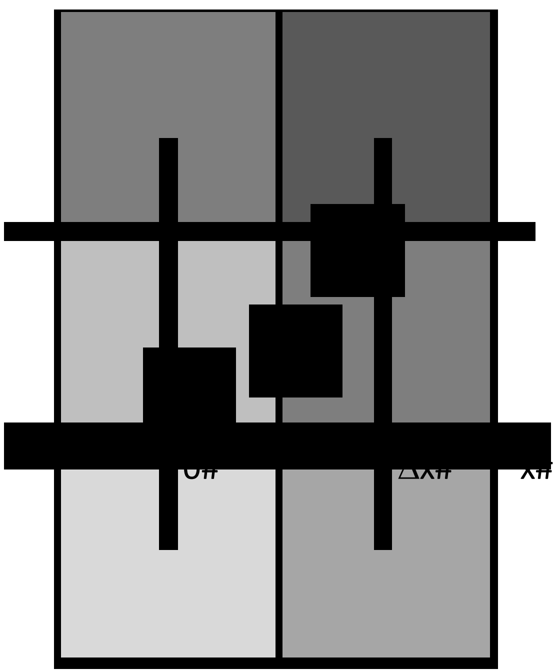
Kodiak

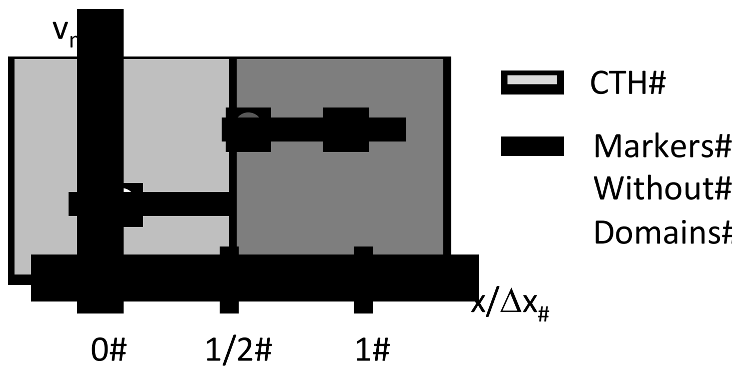
Kinematics

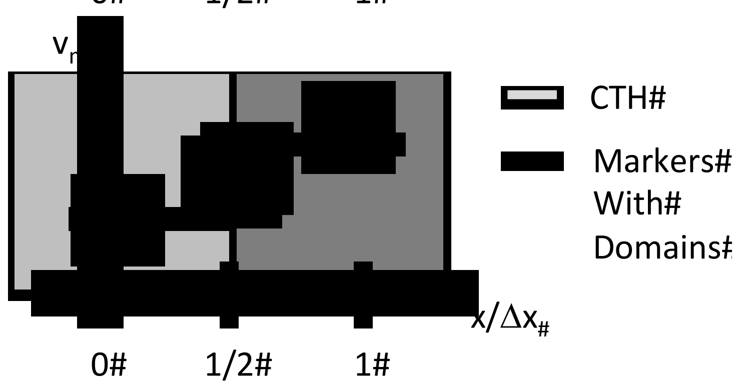
- **Piecewise constant velocity (staggered cells)**
- **Piecewise constant velocity gradient (primary cells)**

Use "MAC interpolation" for particles/markers (marker/cell overlaps)

- Welch, J., Harlow, F., Shannon, J., and Daly, B., The MAC Method, A Computing Technique for Solving Viscous, Incompressible, Transient Fluid-Flow Problems Involving Free Surfaces, LA-3425, March 1966







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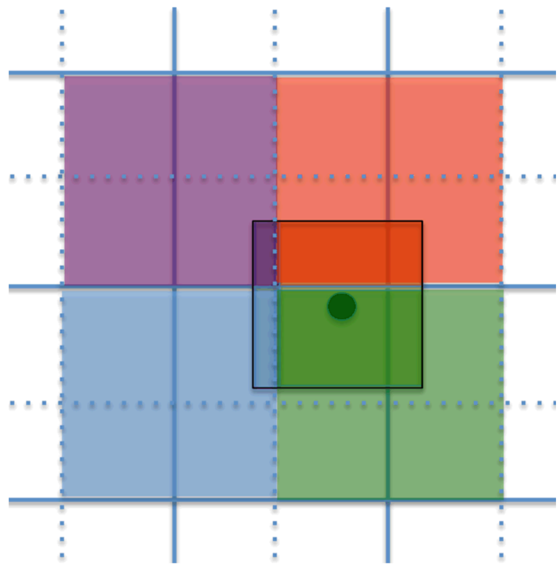
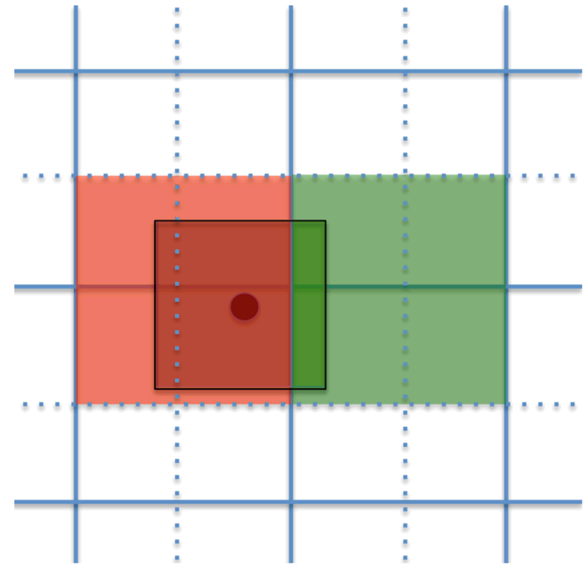
Kinematics and MAC Interpolation

- **Different data in different cells**

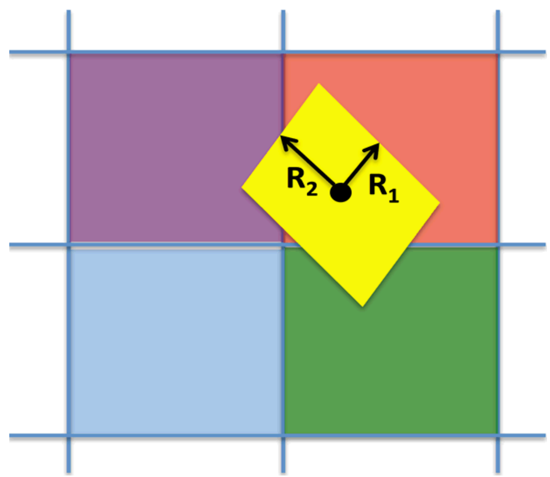


X-direction Velocities



Y-direction Velocities

- **Deforming domains**



Sutherland, I.E., Hodgman G.W., Reentrant polygon clipping, *Communications of the ACM*, **17**, 32-42 (1974)

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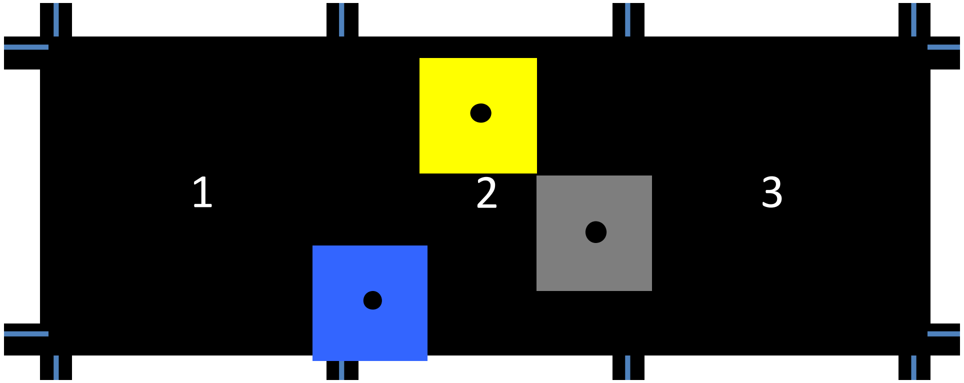
ELPM

Final Step

– Update particle strength and interpolate to the grid

$$\boldsymbol{\sigma} = \boldsymbol{S} - P\boldsymbol{I}$$



Advantages:

- Diffusion free advection of internal variables.
- Interface tracking.

The ELPM implementation in Kodiak is called CLAWS (Coupled Lagrangian Approach With Strength)

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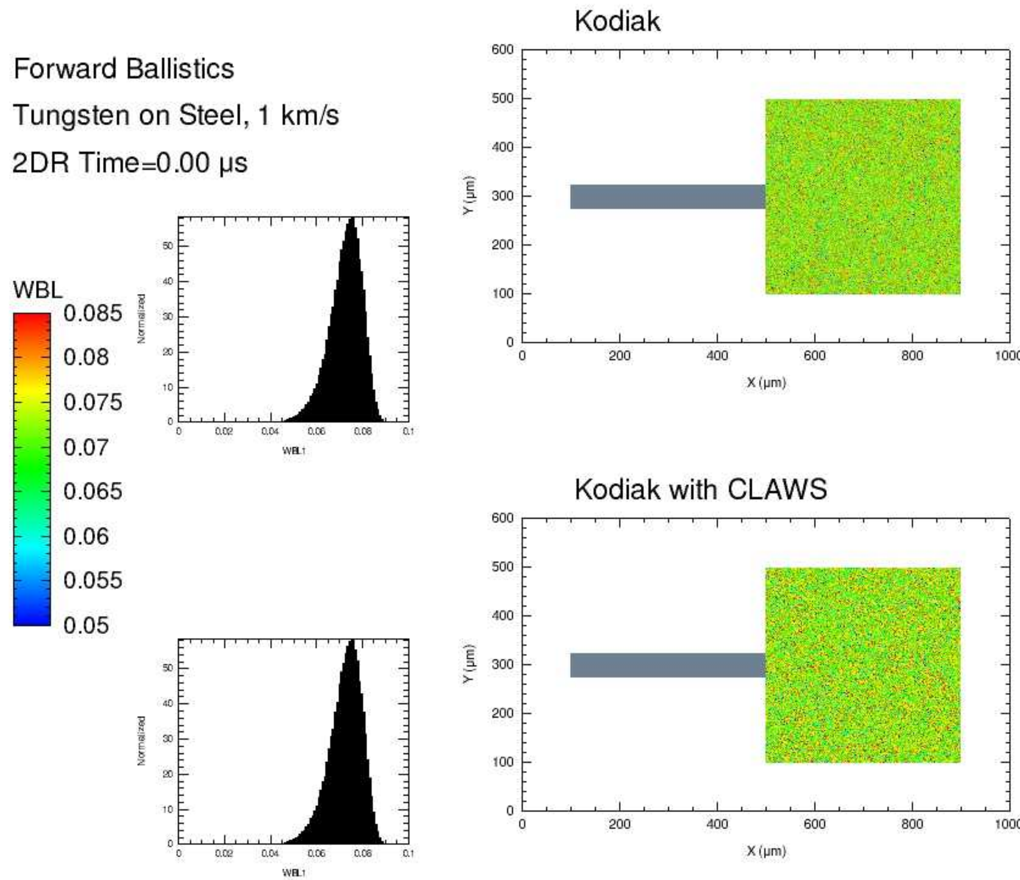
Ballistics Calculations

Compare Kodiak and Kodiak with CLAWS

- **Tungsten penetrator, 1 km/s.**
- **Stationary steel target block with Weibull distributed failure strains**
- **For 1 p/c CLAWS spatial assignment of failure strains is identical to Kodiak.**



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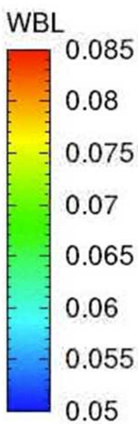
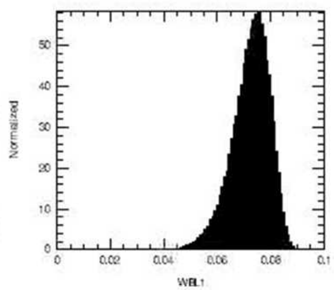
Ballistics Calculations

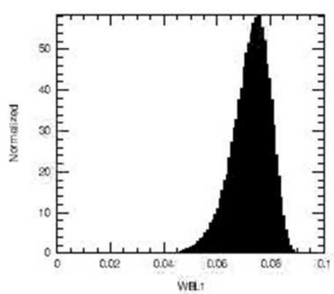
Forward Ballistics

Tungsten on Steel, 1 km/s

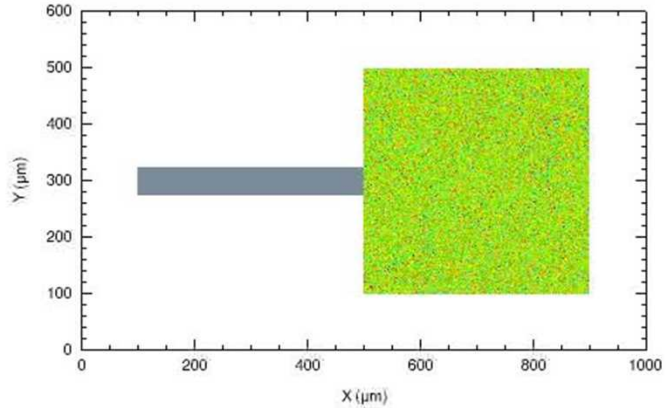
2DR Time=0.00 μs



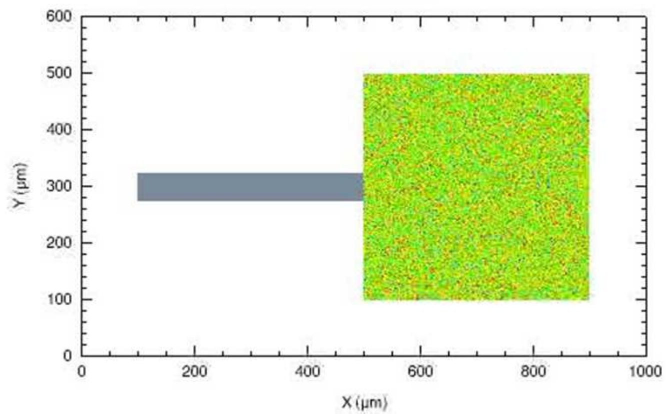




Kodiak



Kodiak with CLAWS



Ballistics Calculations

Forward Ballistics

Tungsten on Steel, 1 km/s

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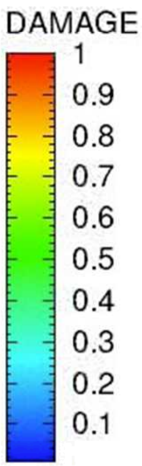

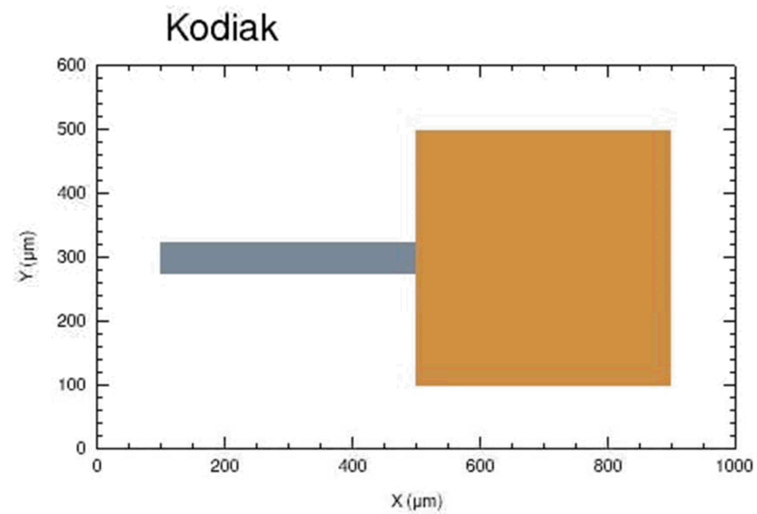

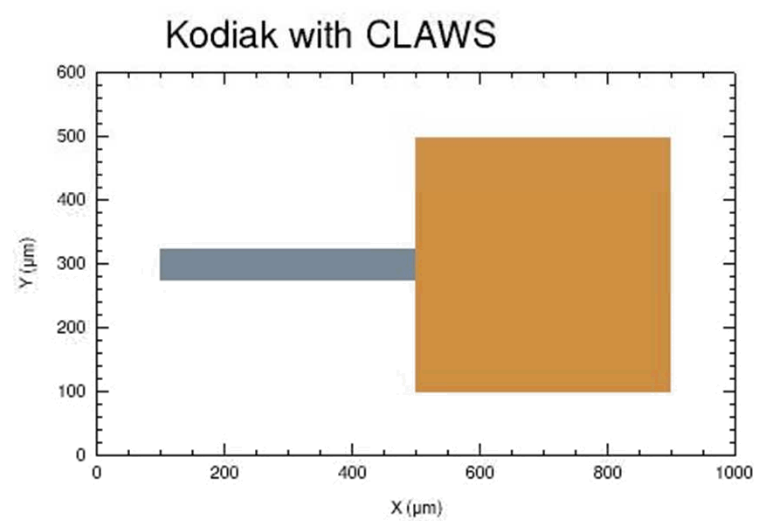

Can We Derive A Method Like ELPM From Variational Principles?

Why would you want to?

- **No ambiguity**
  - **Interpolation**
  - **Boundary Conditions**
- **Develop a family of methods**
  - **Smoother representation of solution fields (e.g. linear variation in velocity)**
  - **Larger data st**
  - **encils (1 neighbor OK, more probably not)**

Variational forms general enough?

Weak Form, Conservation of Momentum

$$\int_V \rho\boldsymbol{a}\cdot\delta\boldsymbol{v}dV + \int_V \boldsymbol{\sigma}:\nabla\delta\boldsymbol{v}dV = \int_V \rho\boldsymbol{b}\cdot\delta\boldsymbol{v}dV + \int_{\partial V}\boldsymbol{\tau}\cdot\delta\boldsymbol{v}dS$$

Where

- $\delta\boldsymbol{v}$ admissible (satisfies kinematic B.C.s)

Discretization

- $\delta\boldsymbol{v} = \sum_v \boldsymbol{v}_v S_v(\boldsymbol{x})$
- $\boldsymbol{\sigma} = \sum_p \boldsymbol{\sigma}_p \chi_p(\boldsymbol{x})$

FEM (Galerkin) $S = \chi$

GIMP (Petrov-Galerkin) $S \neq \chi$

ELPM/MAC grid ?

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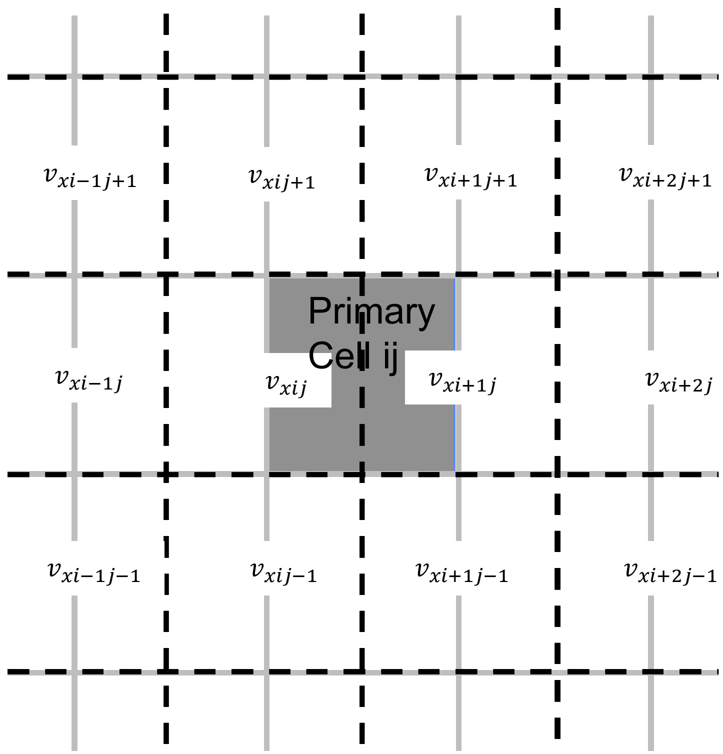
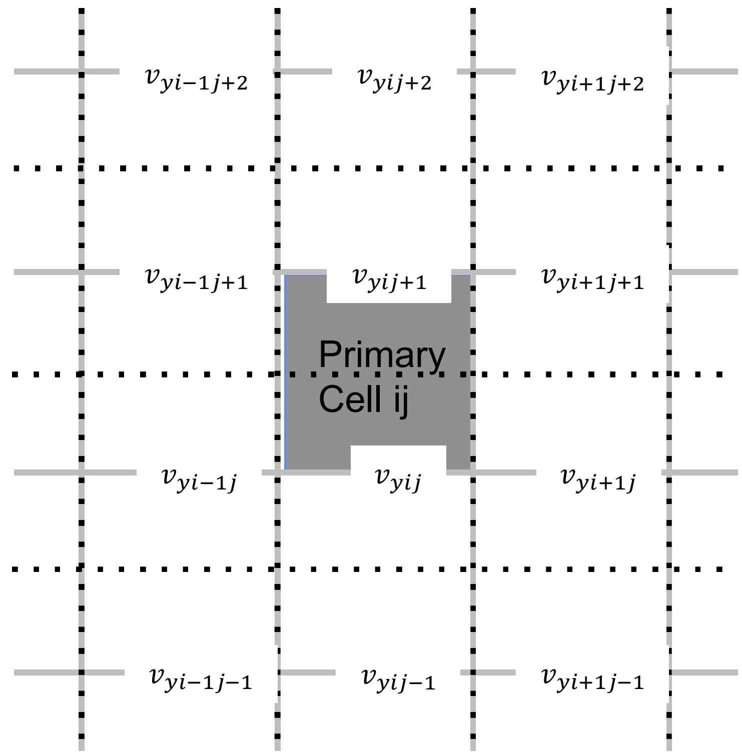
MAC Grid

This works:

- $\delta \boldsymbol{v} = \sum_{f_x} \delta v_{f_x} S_{f_x}(\boldsymbol{x}) \boldsymbol{e}_x + \sum_{f_y} \delta v_{f_y} S_{f_y}(\boldsymbol{x}) \boldsymbol{e}_y$
- $\boldsymbol{\sigma} = \sum_p \boldsymbol{\sigma}_p \chi_p(\boldsymbol{x})$





Simple Case $b = \tau = 0$

$$m_{f_i} = \sum_p m^p \overline{S_{f_i p}} V_p$$

$$\overline{S_{f_i p}} = \frac{1}{V_p} \int \chi_p S_{f_i} dV$$

$$\dot{p}_{f_i} = -\sum_p \sigma_{ij}^p \overline{\frac{\partial S_{f_i p}}{\partial x_j}} V_p$$

$$\overline{\frac{\partial S_{f_i p}}{\partial x_j}} = \frac{1}{V_p} \int \chi_p \frac{\partial S_{f_i}}{\partial x_j} dV$$

- Same as GIMP equations, except with faces (f_i) instead of vertices, and the shape functions are different in each coordinate direction (i).
- *Particle data updates face momenta directly*

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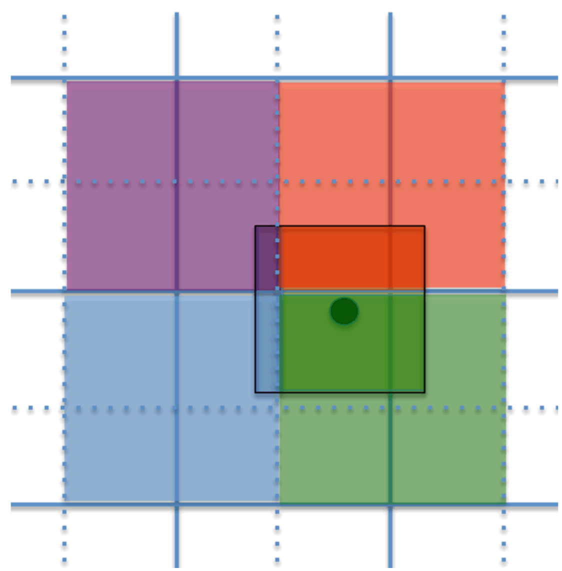
Weighting functions for various χ_p, S_{f_i}

Characteristic Functions:

- $\chi_p = 1$ on particle domain, 0 elsewhere.
- $S_{f_i} = 1$ on staggered cell f_i, 0 elsewhere.

$$\overline{S_{f_i p}} = \frac{1}{V_p} \int \chi_p S_{f_i} dV$$

= overlap volume fractions

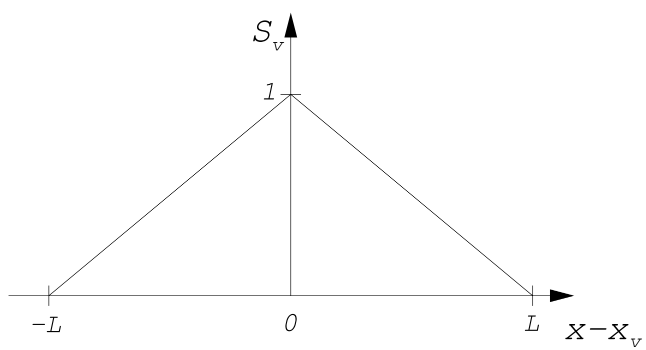


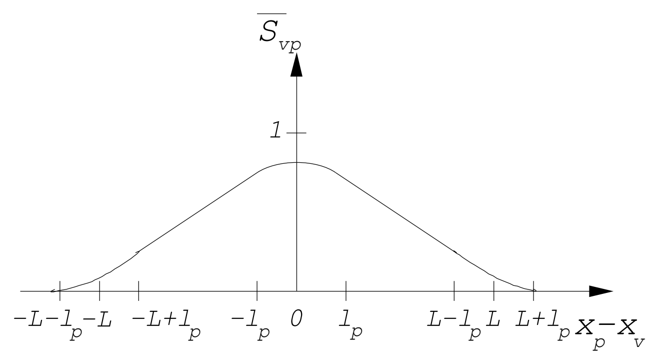
Same as ELPM

Weighting functions for various χ_p, S_{f_i}

Linear variation of face velocities:

- $\chi_p = 1$ on particle domain, 0 elsewhere.
- S_{f_i} linear on (f_{i-1}, f_i) and (f_i, f_{i+1}), 0 elsewhere.





Same as GIMP (as typically used)

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Conclusions

- ELPM improves strength modeling in Kodiak.
- Existing ELPM particle stress to grid interpolation functions may be derived using GIMP ideas on a MAC grid.
- A family of ELPM interpolation functions has been derived, allowing arbitrary smoothness.
  - Most useful (considering data stencil) is probably the "classic GIMP" shape function.
- A GIMP variant, "GIMP PtoF", could be implemented on a MAC grid.
  - Calculate face accelerations from particle stresses and gradients of the GIMP PtoF shape functions.
  - Avoids interpolation first to vertices, then to faces or cell centers.

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