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Title: High-Order Thermal Radiative Transfer

Author(s): Woods, Douglas Nelson
Cleveland, Mathew Allen
Wollaeger, Ryan Thomas
Warsa, James S.

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High-Order Thermal Radiative Transfer

Douglas Woods, Mathew Cleveland, Ryan Wollaeger, James Warsa

Los Alamos National Laboratory

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Research objectives

- Assess the sensitivity of the linearized thermal radiation transport (TRT) equations to finite element order on unstructured meshes.
- Investigate the sensitivity of the nonlinear TRT equations due to evaluating the opacities and heat capacity at nodal temperatures in 2-D using high-order finite elements.

Background

- Linear transport spatially converges with $O(h^{p+1})$ on unstructured meshes.
  - More degrees of freedom in fewer mesh zones
  - Increased accuracy per CPU time
- Limited amount of research available on high-order methods for TRT (Maginot, 2016).
  - Linearized equations
  - 1-D

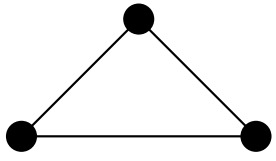
High-order finite elements

Approximate temperature as a polynomial expansion

$$T(\boldsymbol{x}) \approx \sum_j T_j b_j(\boldsymbol{x})$$

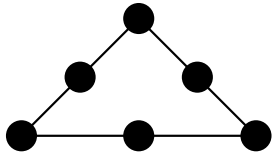
- Low-order (1^{st}-order) basis functions assume flat (linear) shape:

$$b_j(\boldsymbol{x}) = ax + by + c$$



- High-order (2^{nd}-order) basis function allow for solution curvature:

$$b_j(\boldsymbol{x}) = ax^2 + by^2 + cxy + dx + ey + f$$



Thermal radiation transport

The nonlinear thermal radiative transport (TRT) equations:

$$\frac{1}{c}\frac{I^{n+1}-I^{n}}{\Delta t}+\mathbf{\Omega}\cdot\boldsymbol{\nabla}I^{n+1}+\sigma_t I^{n+1}=\frac{\sigma_s}{4\pi}E^{n+1}+\sigma_a B^{n+1}+S_0^{n+1},$$

$$\rho C_v\frac{T^{n+1}-T^{n}}{\Delta t}=\int\int c\sigma_a\left(I^{n+1}-B^{n+1}\right)d\nu d\Omega',$$

$$B^{n+1}=\frac{2h\nu^3}{c^2}\left(e^{h\nu/kT_e^{n+1}}-1\right)^{-1}$$

Thermal radiation transport

The linearized thermal radiative transport (TRT) equations:

$$B^{n+1} \approx B^n + \left.\frac{\partial B}{\partial T}\right|_{T^n} \delta T$$

$$\rho C_v \frac{\delta T}{\Delta t} = \int\int c\sigma_a \left(I^{n+1} - B^{n+1}\right) d\nu d\Omega',$$

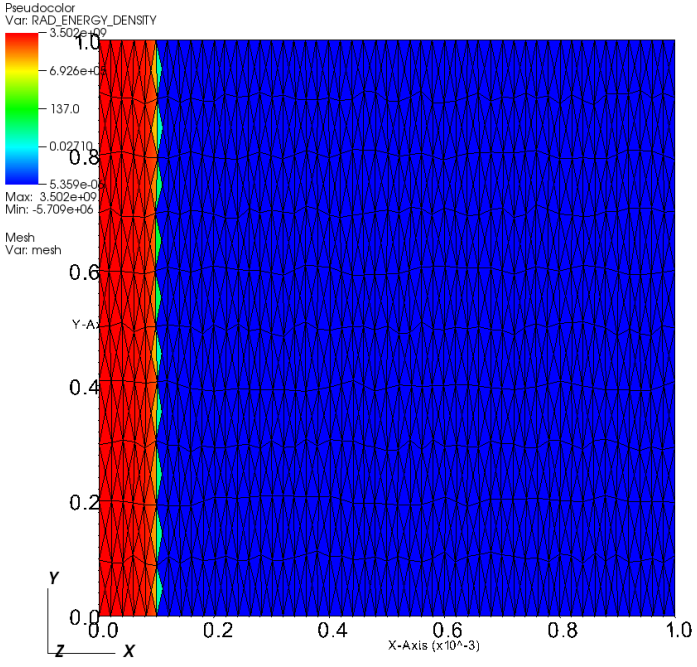
$$B^{n+1} \approx B^n + \left.\frac{\partial B}{\partial T}\right|_{T^n} \delta T$$

$$\frac{1}{c}\frac{I^{n+1} - I^n}{\Delta t} + \mathbf{\Omega}\cdot\boldsymbol{\nabla} I^{n+1} + \sigma_t I^{n+1} = \frac{\sigma_s}{4\pi} E^{n+1} + \sigma_a B^{n+1} + S_0^{n+1},$$

Linearized equations susceptible to maximum principle violations.

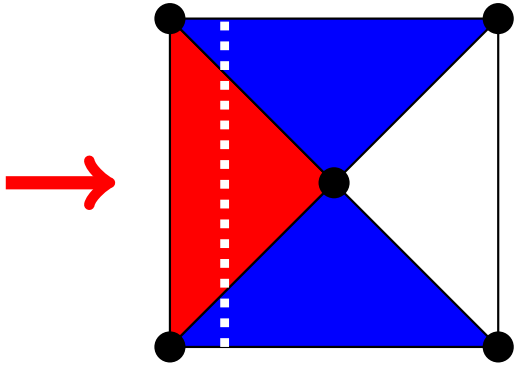
Test problem: Marshak wave

- 1-D, time dependent, material coupled, gray
- $\sigma_t = 10^6 \left(\frac{0.001}{T}\right)^3$
- $C_v = 1.3784 \times 10^{14}$ erg/keV/g
- Gray boundary source 0.150 keV
- As thermal wave propogates, incident radiation penetrates futher into the material.

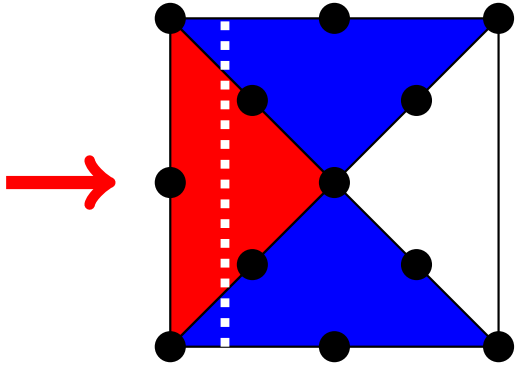


High-order finite elements - conceptual mass averaging

- Higher-order elements have better resolution within each mesh zone.
- Cell average temperature is better mass weighted.



(a) 1st-order FEM.



(b) 2nd-order FEM.

Linearized thermal radiation transport test definitions

1. Linearized system, structured mesh, 1^{st}-order elements
2. Linearized system, structured mesh 2^{nd}-order elements
3. Linearized system, unstructured mesh, 1^{st}-order elements
4. Linearized system, unstructured mesh 2^{nd}-order elements
5. Nonlinear system, structured mesh, 1^{st}-order elements
6. Nonlinear system, structured mesh 2^{nd}-order elements
7. Nonlinear system, unstructured mesh, 1^{st}-order elements
8. Nonlinear system, unstructured mesh 2^{nd}-order elements

Remember: odds are odd, evens are even.

Linearized TRT case matrix - Δt and Δx are part of sufficient conditions to adhere to maximum principle

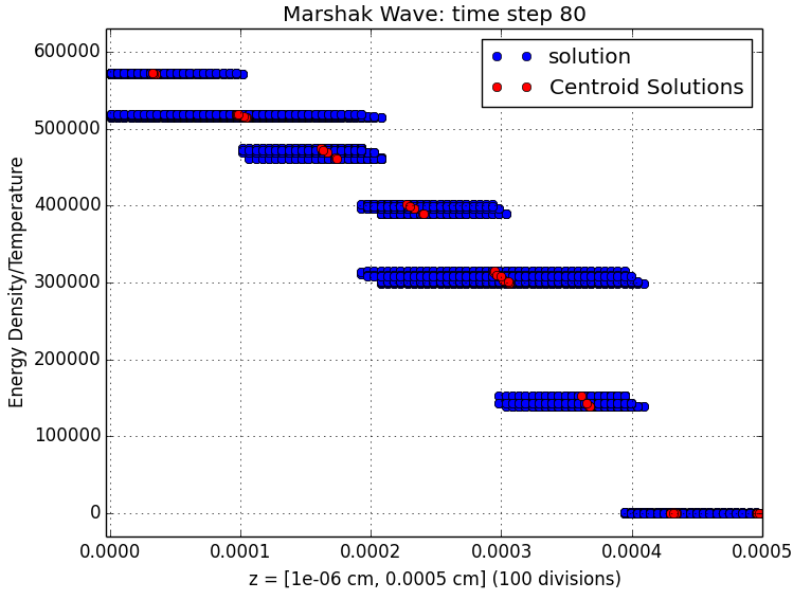
Case matrix for refining the time step and mesh

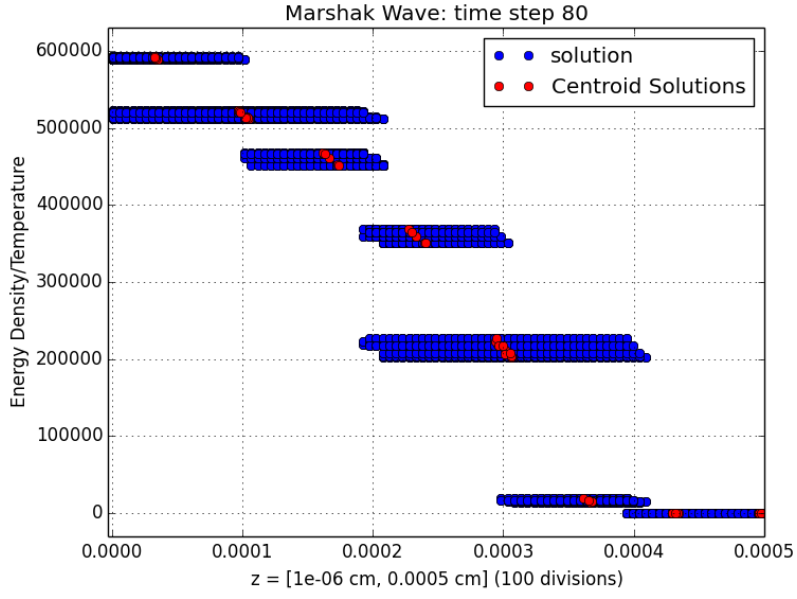
- Populated with test numbers that adhere to the maximum principle, i.e., $T(0 < x' < x, y, t) > T(x, y, t)$

Δt (sec.) \ no. of mesh zones	100	400	1600	6400
8×10^{-10}				
8×10^{-11}	1, 3			
8×10^{-12}	1, 2, 3, 4	1, 2, 3, 4	2	
8×10^{-13}	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4
8×10^{-14}	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4

Linearized TRT coarse unstructured mesh - HO FEM does not propagate energy as far as LO

Electron temperatures from linearized system on meshes with 100 zones for $\Delta t = 8 \times 10^{-12}$ seconds at time step 80.

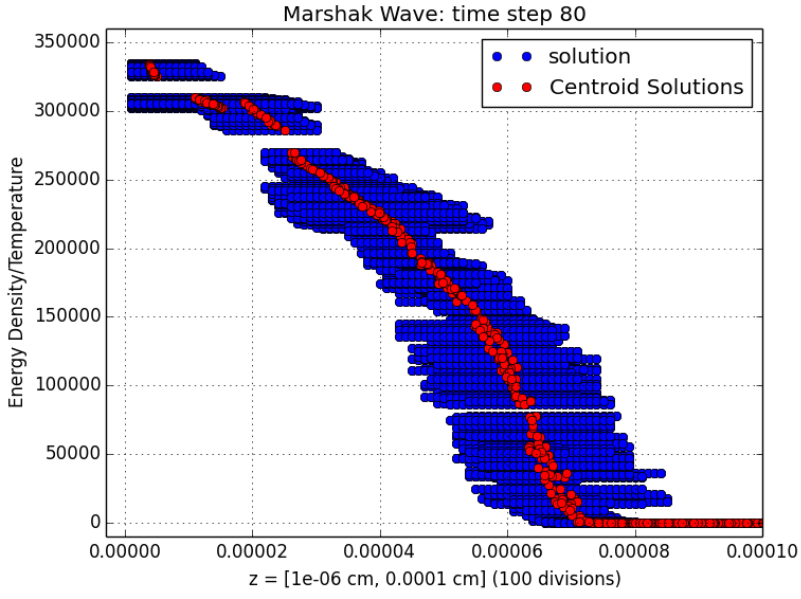


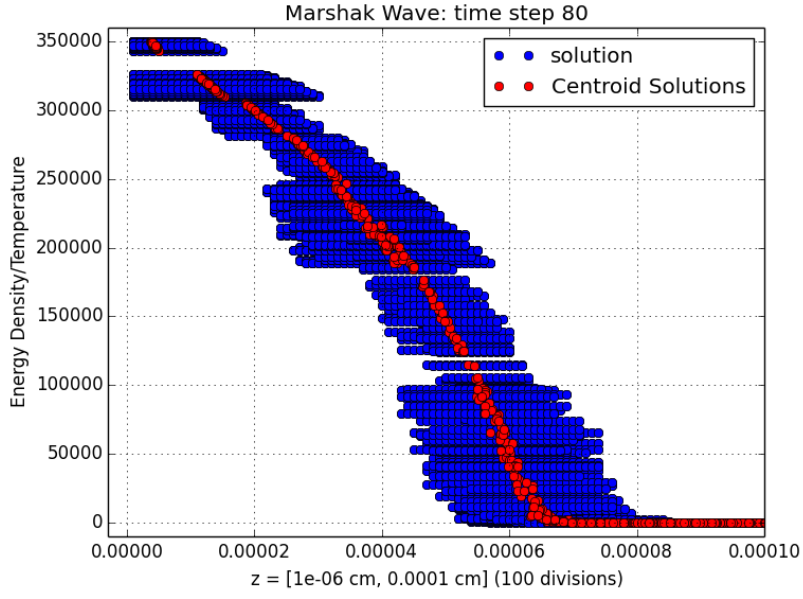


(a) Test 3 (1^{st}-order, unstructured). (b) Test 4 (2^{nd}-order, unstructured).

Linearized TRT refined unstructured mesh - mesh resolution affects continuity of temperature profile

Electron temperatures from linearized system on structured meshes with 6400 zones for $\Delta t = 8 \times 10^{-13}$ seconds at time step 80.





(a) Test 3 (1^{st}-order, unstructured). (b) Test 4 (2^{nd}-order, unstructured).

Nonlinear TRT test definitions

1. Linearized system, structured mesh, 1^{st}-order elements
2. Linearized system, structured mesh 2^{nd}-order elements
3. Linearized system, unstructured mesh, 1^{st}-order elements
4. Linearized system, unstructured mesh 2^{nd}-order elements
5. Nonlinear system, structured mesh, 1^{st}-order elements
6. Nonlinear system, structured mesh 2^{nd}-order elements
7. Nonlinear system, unstructured mesh, 1^{st}-order elements
8. Nonlinear system, unstructured mesh 2^{nd}-order elements

Remember: odds are odd, evens are even.

Nonlinear TRT case matrix - nonlinear solver convergence restricted by time step size

Case matrix for refining the time step and mesh

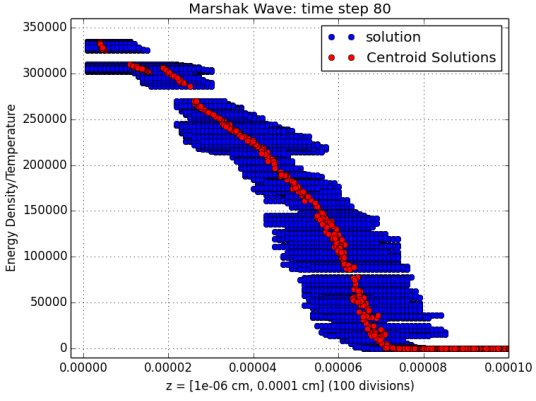
- Populated with test numbers that provide "reasonable" solutions

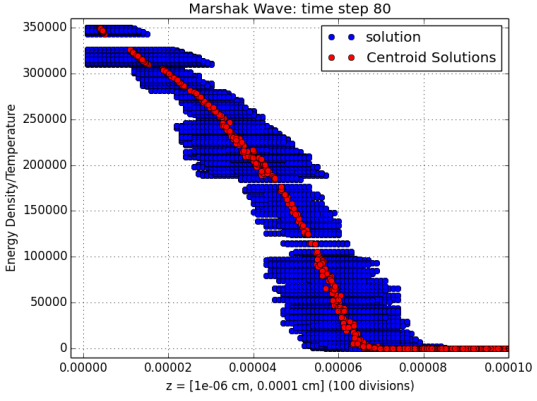
Δt (sec.) \ no. of mesh zones	100	400	1600	6400
8×10^{-10}				
8×10^{-11}	5, 7			
8×10^{-12}	5, 6, 7, 8	5, 6, 7, 8	5, 7	5, 7
8×10^{-13}	5, 6, 7, 8	5, 6, 7, 8	5, 6, 7, 8	5, 6, 7, 8
8×10^{-14}	5, 6, 7, 8	5, 6, 7, 8	5, 6, 7, 8	5, 6, 7, 8

Nonlinear TRT refined unstructured mesh - LO FEM is affected by linearization

Electron temperatures from linearized system on structured meshes with 6400 zones for $\Delta t = 8 \times 10^{-13}$ seconds at time step 80.

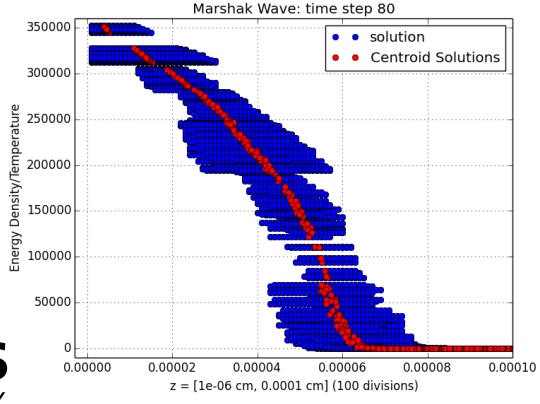
Test 3 (linearized, 1st-order, unstructured).

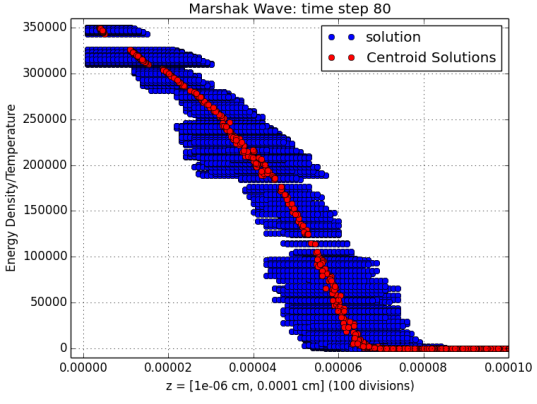




Test 4 (linearized, 2nd-order, unstructured).

Test 7 (nonlinear, 1st-order, unstructured).

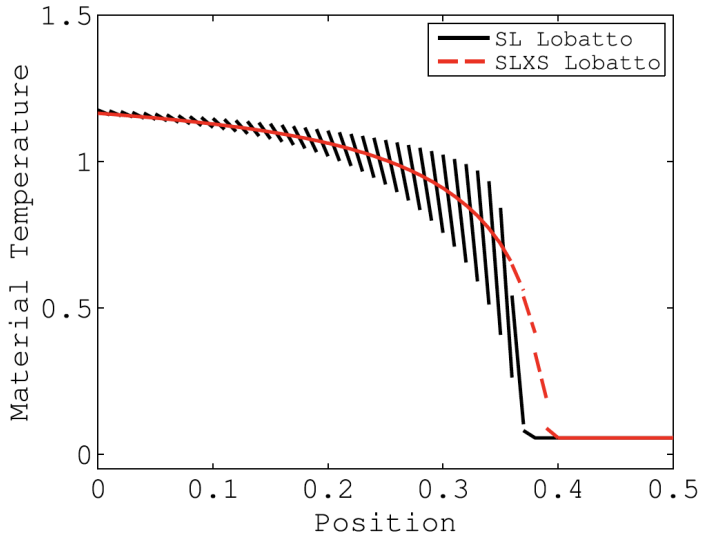




Test 8 (nonlinear, 2nd-order, unstructured).

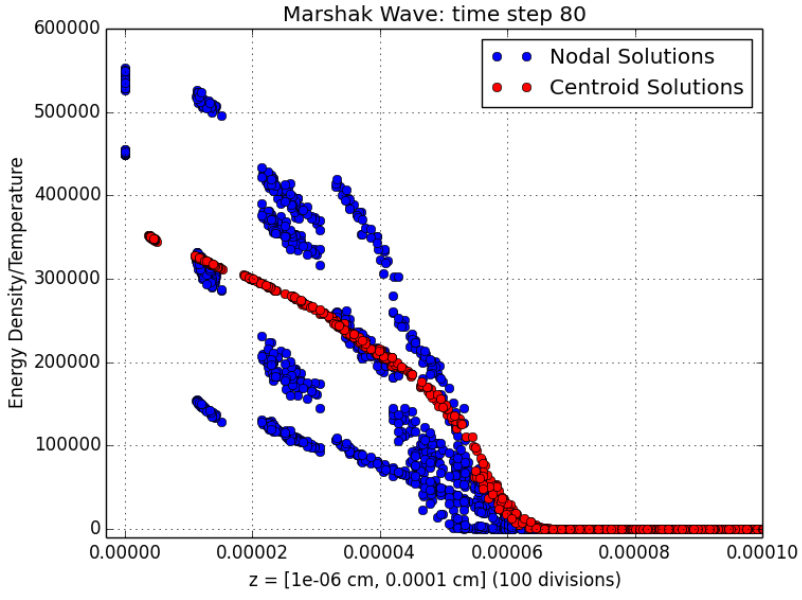
Serrano vs. Hidalgo: opacities and heat capacities evaluated at nodal temperatures

- Serrano solves using cell-wise average opacities and heat capacities.
- Maginot (2016) and Luke Cornejo showed the nodal solutions are discontinuous.
- Evaluating material properties at nodal temperatures greatly smooths out nodal solutions.

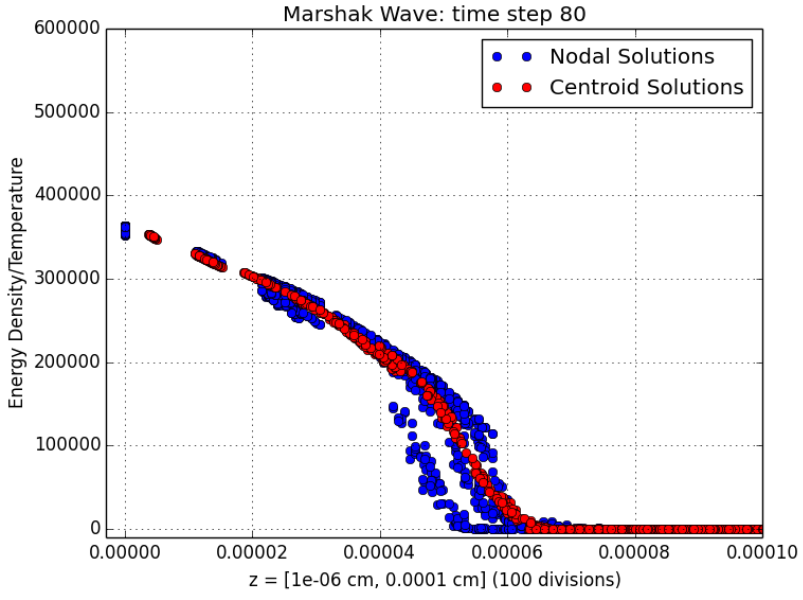


Maginot et al. (2016).

Serrano vs. Hidalgo - nodal solutions are substantially closer to cell average solutions

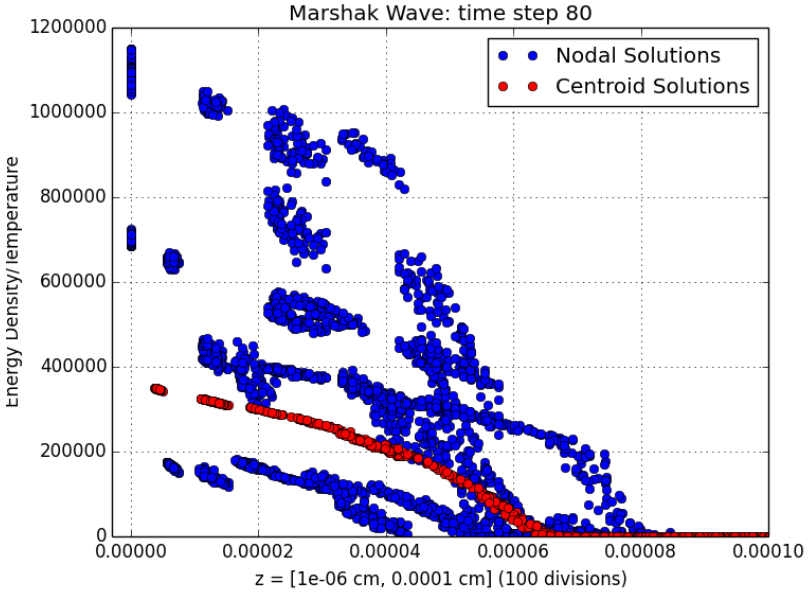


(a) Test 7 (nonlinear, 1st-order, unstructured) with Serrano.

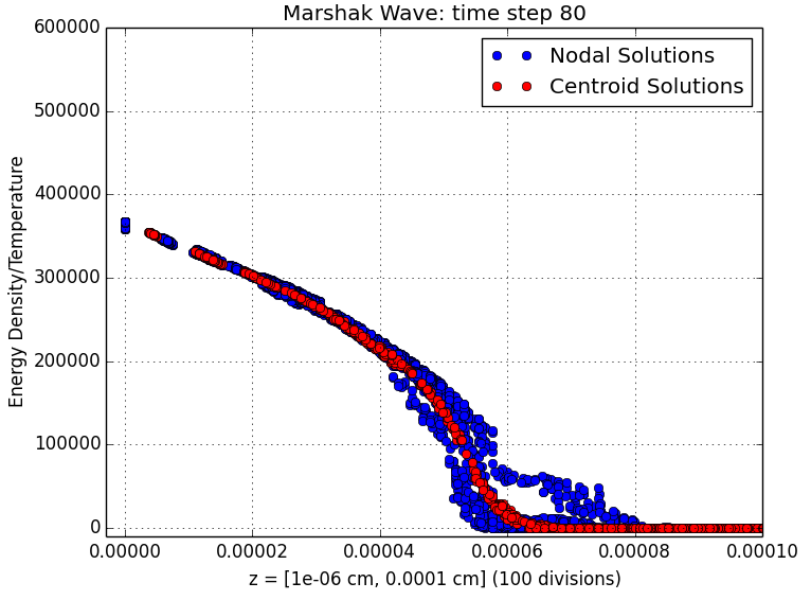


(b) Test 7 (nonlinear, 1st-order, unstructured) with Hidalgo.

Serrano vs. Hidalgo - nodal solutions are substantially closer to cell average solutions



(a) Test 8 (nonlinear, 2nd-order, unstructured) with Serrano.



(b) Test 8 (nonlinear, 2nd-order, unstructured) with Hidalgo.

Conclusions

- Linearization affects the solution
  - Low-order elements are much more sensitive to the linearization
  - Higher-order elements are far less sensitive to the linearization
- Nonlinear low-order elements tend to agree with nonlinear high-order elements
  - Nonlinear low-order may propagate less energy
- Evaluating material properties at the nodal temperatures reduces blading

Future Work

- Finite element analysis for maximum principle
  - Determine the time step size to tolerate the maximum nodal energy deposition before violating the maximum principle
- Refined mesh at boundary may reduce "gaps"
- Time step ramp-up may allow solutions to nonlinear system
- Compare to analytic local thermodynamic equilibrium diffusion solution

QUESTIONS?

Thank you!

Matt Cleveland
Ryan Wollaeger
Jim Warsa

Oregon State University

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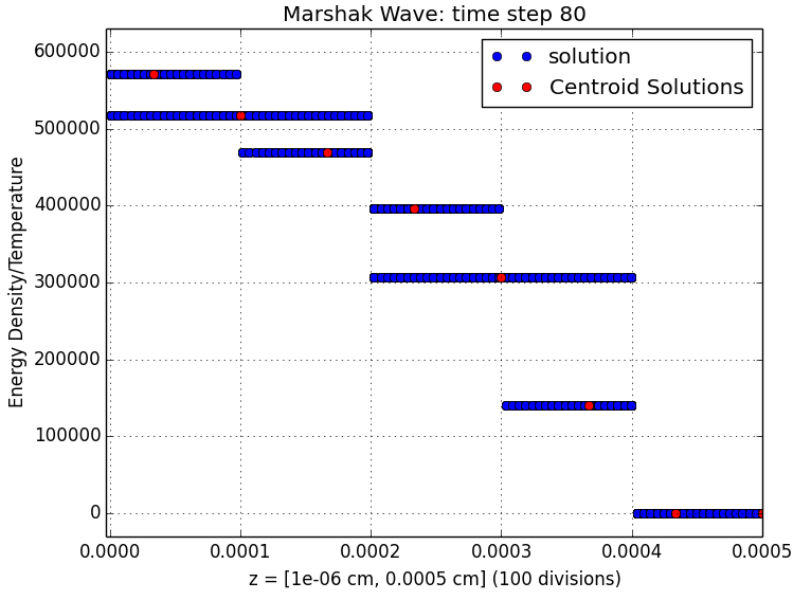
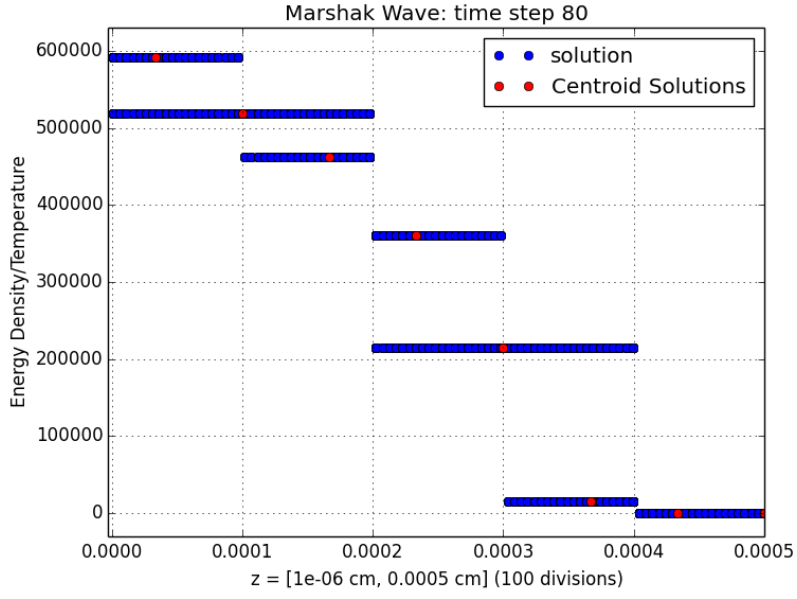
Linearized TRT coarse structured mesh - HO FEM does not propagate energy as far as LO

Electron temperatures from linearized system on meshes with 100 zones for $\Delta t = 8 \times 10^{-12}$ seconds at time step 80.



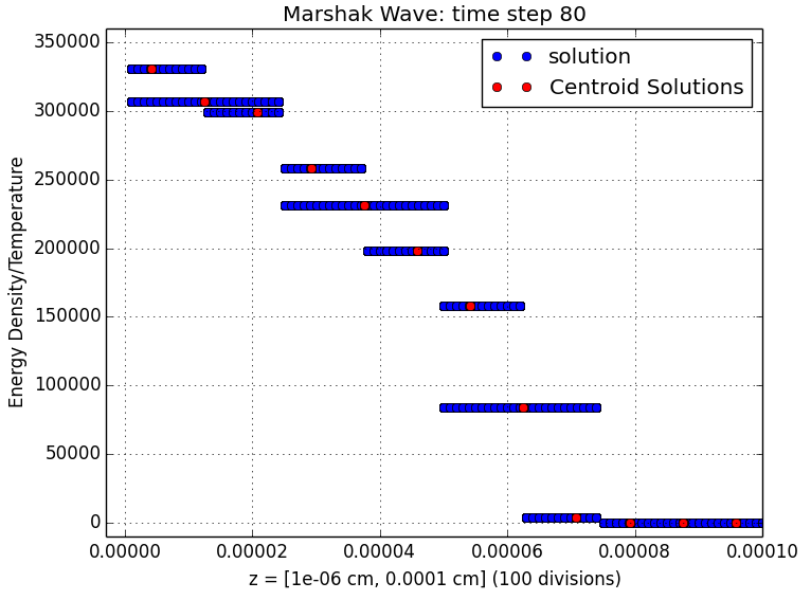
(a) Test 1 (1^{st}-order, structured).



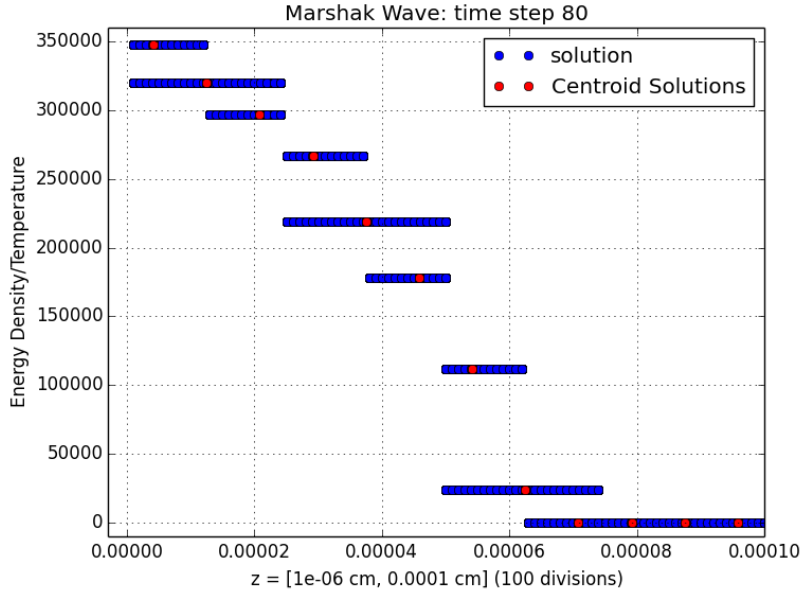
(b) Test 2 (2^{nd}-order, structured).

Linearized TRT refined structured mesh - HO FEM has higher temperature near boundary

Electron temperatures from linearized system on meshes with 6400 zones for $\Delta t = 8 \times 10^{-13}$ seconds at time step 80.



(a) Test 1 (1^{st}-order, structured).

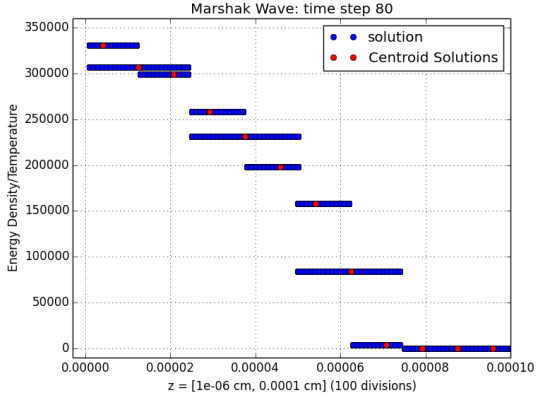


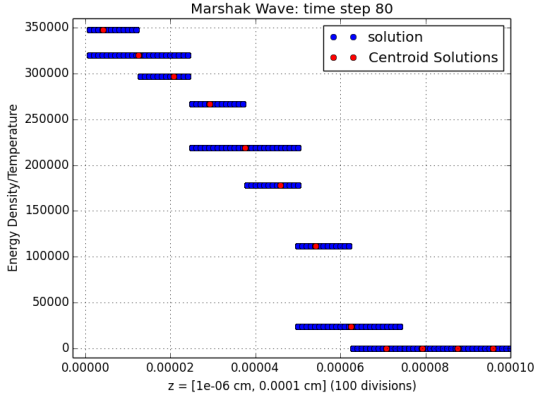
(b) Test 2 (2^{nd}-order, structured).

Nonlinear TRT refined structured mesh - LO FEM is affected by linearization

Electron temperatures from linearized system on meshes with 6400 zones for $\Delta t = 8 \times 10^{-13}$ seconds at time step 80.

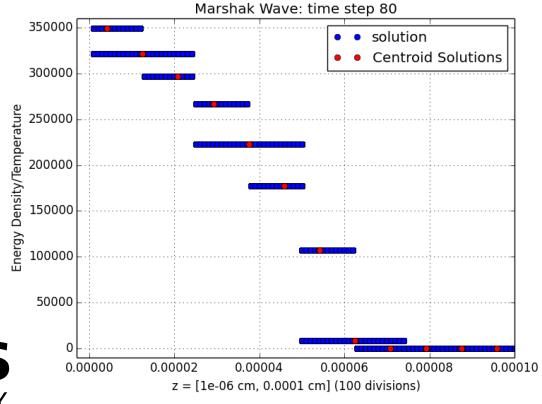
Test 1 (linearized, 1st-order, structured).

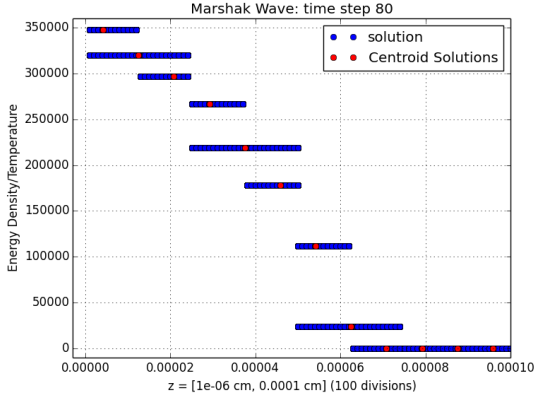




Test 2 (linearized, 2nd-order, structured).

Test 5 (nonlinear, 1st-order, structured).



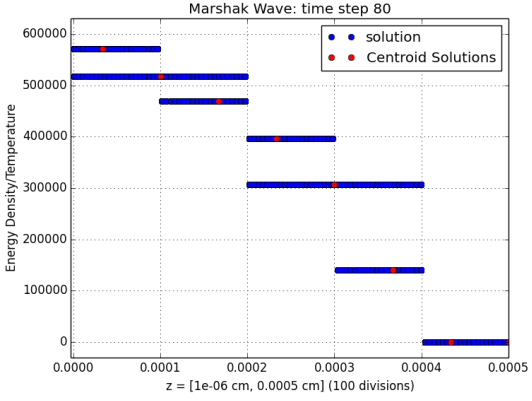


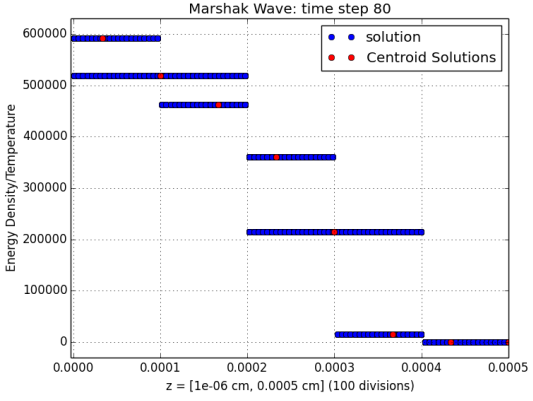
Test 6 (nonlinear, 2nd-order, structured).

Nonlinear TRT coarse structured mesh - LO FEM affected by linearization

Electron temperatures from linearized system on meshes with 100 zones for $\Delta t = 8 \times 10^{-12}$ seconds at time step 80.

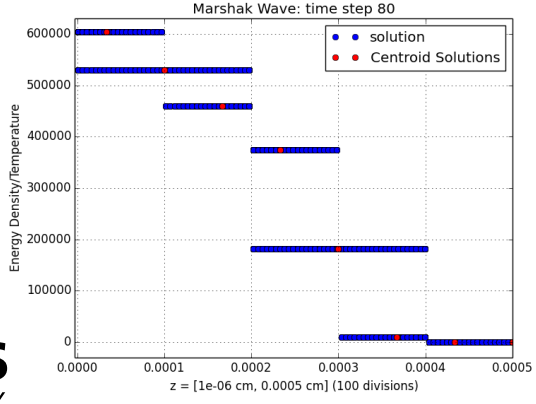
Test 1 (linearized, 1st-order, structured).

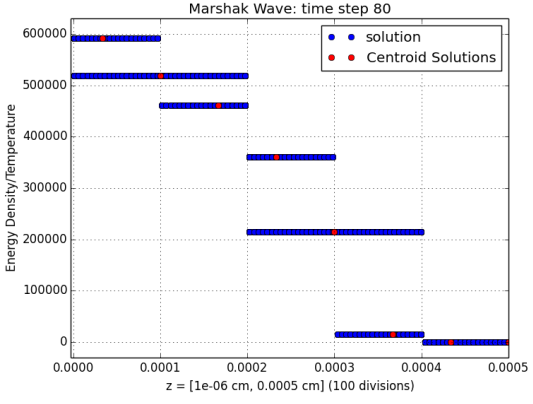




Test 2 (linearized, 2nd-order, structured).

Test 5 (nonlinear, 1st-order, structured).



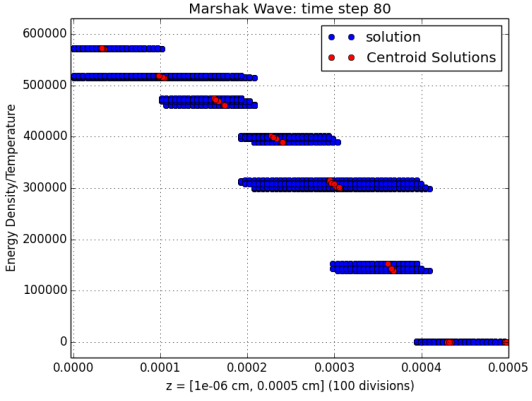


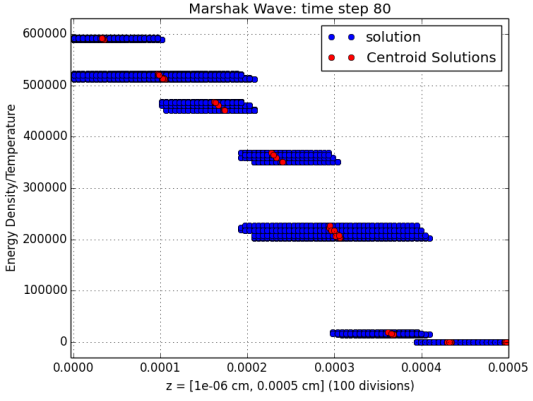
Test 6 (nonlinear, 2nd-order, structured).

Nonlinear TRT coarse unstructured mesh - nonlinear LO propagates less energy than HO

Electron temperatures from linearized system on meshes with 100 zones for $\Delta t = 8 \times 10^{-12}$ seconds at time step 80.

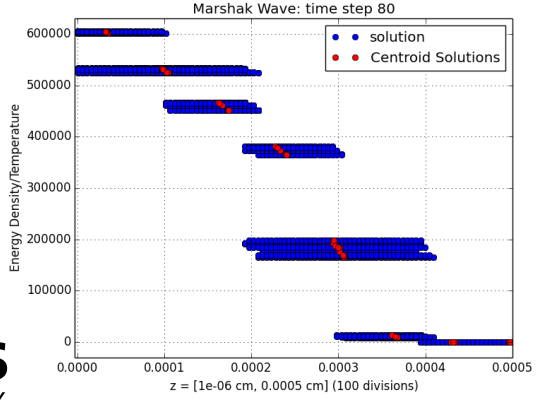
Test 3 (linearized, 1st-order, unstructured).

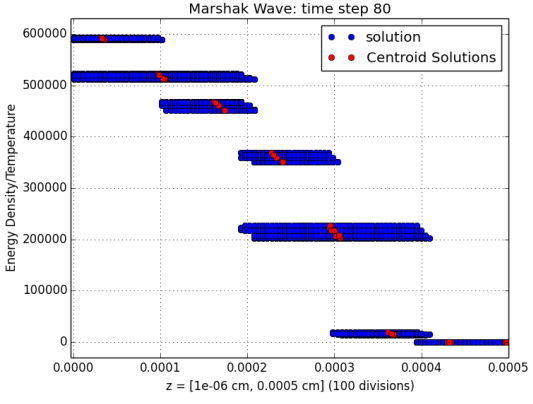




Test 4 (linearized, 2nd-order, unstructured).

Test 7 (nonlinear, 1st-order, unstructured).





Test 8 (nonlinear, 2nd-order, unstructured).