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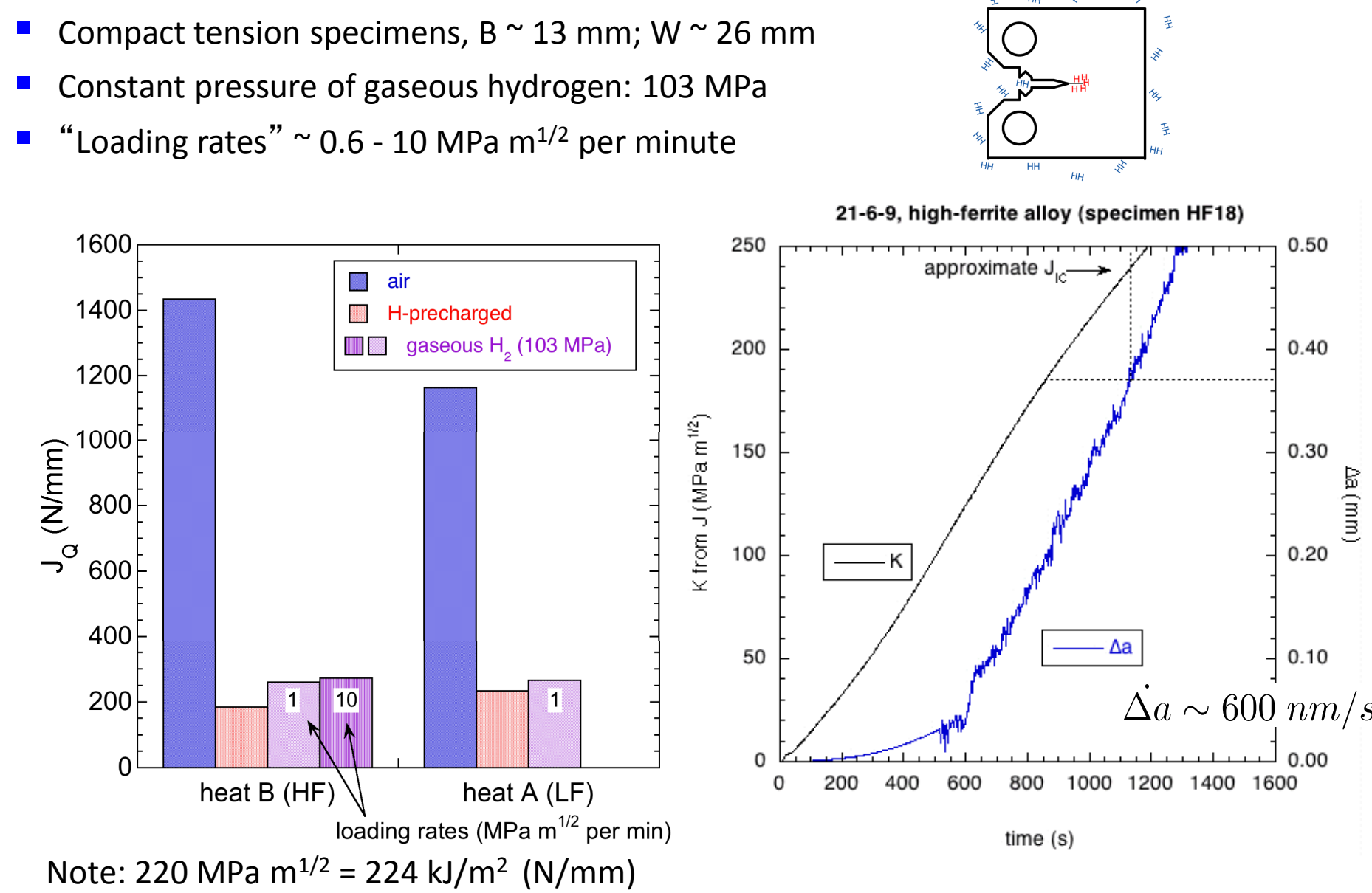
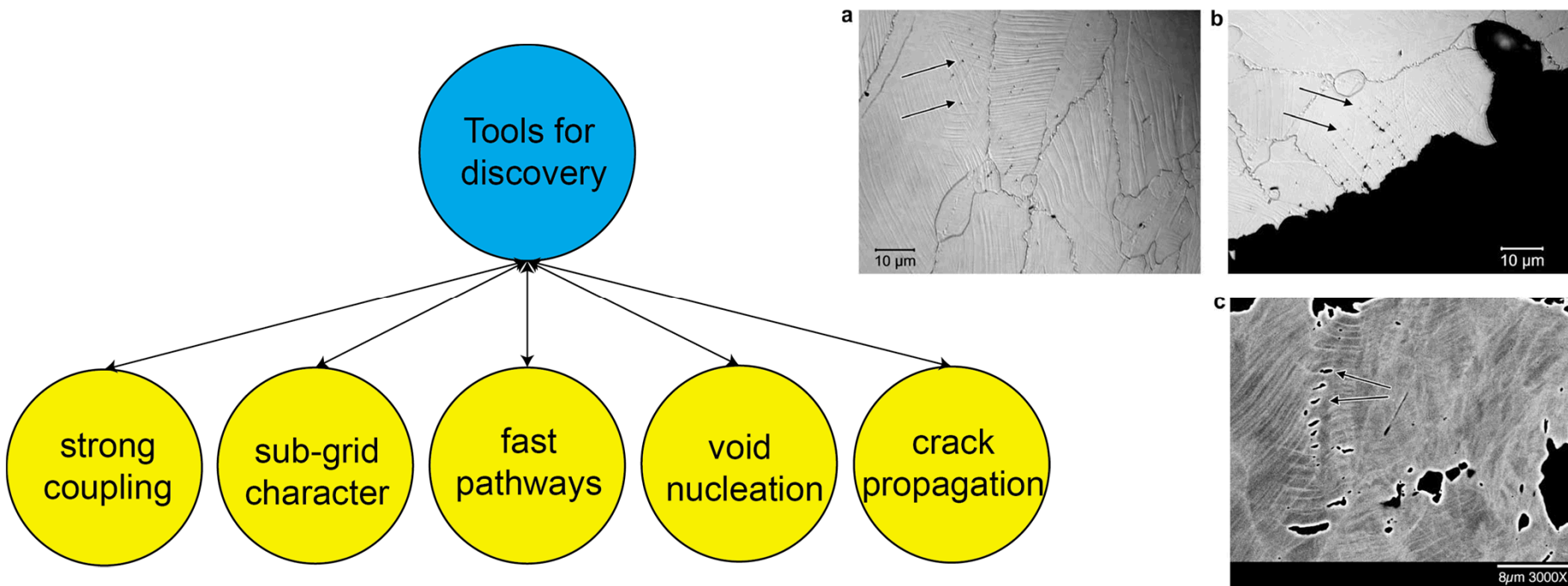


Simulating hydrogen embrittlement and fast pathways for diffusion

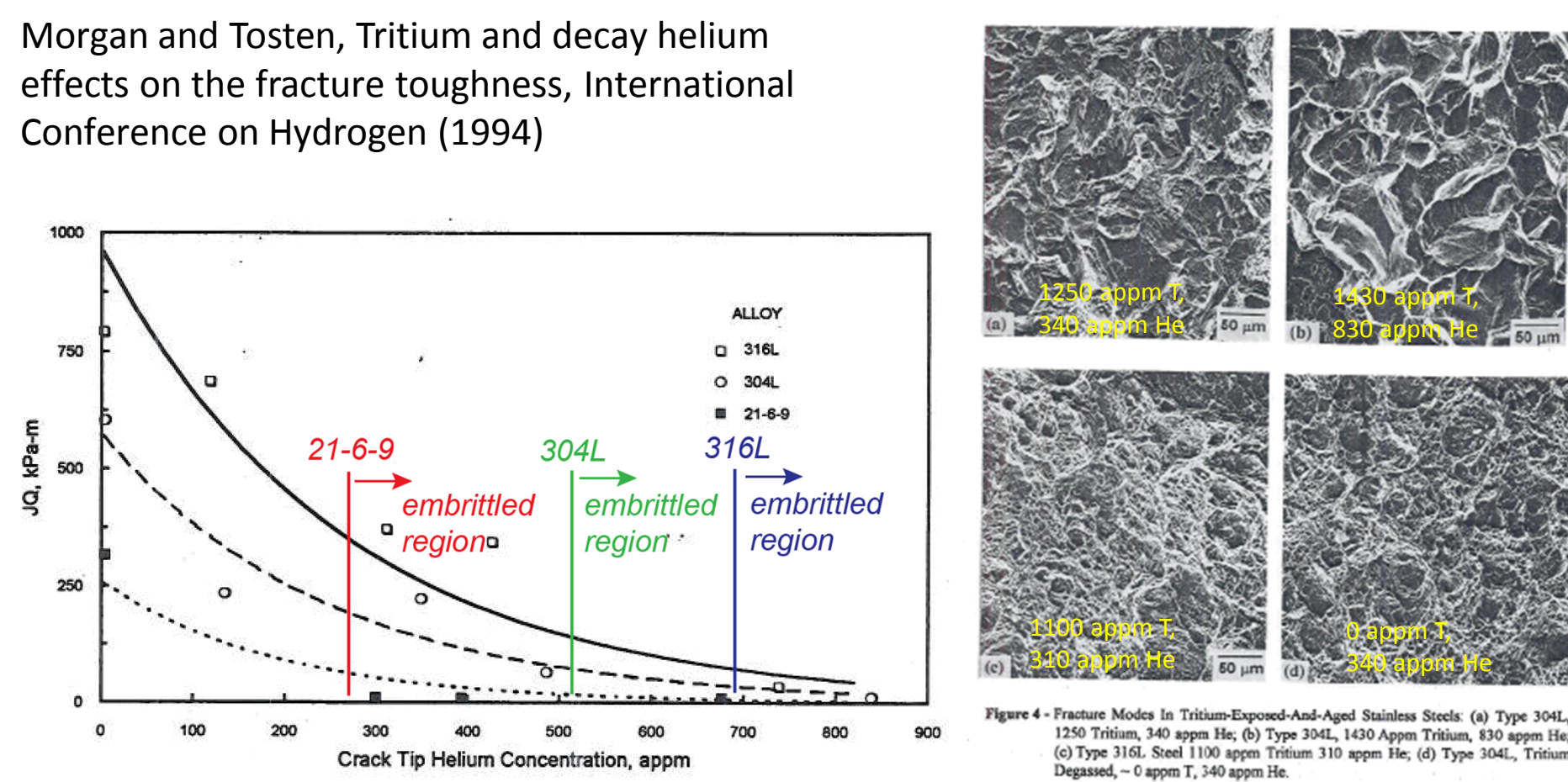
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- Motivated through observation
- Strong chemo-mechanical coupling
- Capture sub-grid processes through a surface approach
- Explore fast pathways for diffusion at structural and microstructural scales
- Develop models for H/T/He embrittlement w/focus on void nucleation



- Fracture toughness degrades with increasing helium concentrations
- Both tritium and helium are requisite for degradation
- Transition from void evolution to fracture along twin and grain boundaries



This path heavily leverages Sofronis/McMeeking (1989)* and Krom (1998). Recent work by Leo and Anand (2013).

Transport of hydrogen in the current configuration

$$D^* \dot{c}_l + C^* \text{div} v - \nabla_x \cdot d_l \nabla_x c_l - \nabla_x \cdot \frac{c_l V_H}{J} d_l \nabla_x \tau_h + \frac{\theta_l}{J} \frac{\partial N_T}{\partial \epsilon_p} \dot{\epsilon}_p - \frac{\theta_l N_T}{J^2} \dot{J} = 0$$

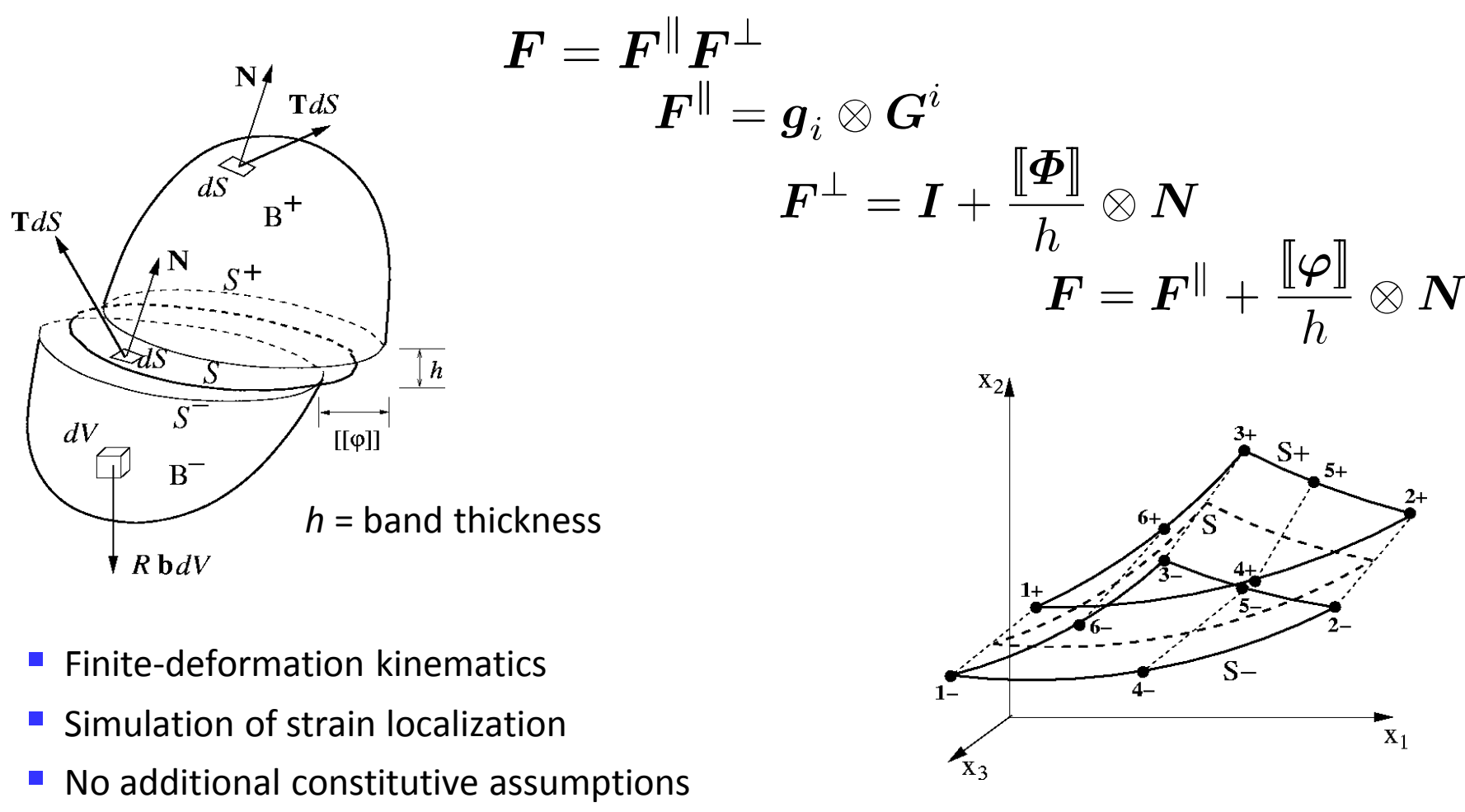
Transport of hydrogen in the reference configuration (push back)

$$D^* \dot{C}_L - \nabla_X \cdot d_l C^{-1} \nabla_X C_L + \nabla_X \cdot \frac{d_l V_H}{RT} C^{-1} \nabla_X \tau_h C_L + \theta_T \frac{d N_T}{d \epsilon_p} \dot{\epsilon}_p$$

transient term diffusion term advection term from hydrostatic stress source term from trapping

$$D_L = F^{-1} d_l F^{-T} = d_l C^{-1}$$

Goal: Capture sub-grid processes through methods that regularize the jump



- Finite-deformation kinematics
- Simulation of strain localization
- No additional constitutive assumptions

Yang, Mota and Ortiz (IJNME, 2005), Armero and Garikipati (IJSS, 1996)

Fox and Simo (1990), Callari, Armero, Abati (2010)

$$\text{redefine space } X = \Phi(\xi^1, \xi^2, \xi^3) = \bar{\Phi}(\xi^1, \xi^2) + N(\xi^1, \xi^2) \xi^3 \quad G_i = \Phi_{,i} = \frac{\partial X}{\partial \xi^i}$$

$$\text{include jump in } C \quad C(X) = \bar{C}(\bar{\Phi}[\xi^1, \xi^2]) + \frac{[C](\Phi[\xi^1, \xi^2])}{h} \xi^3 \quad \nabla_X C = (\nabla \bar{\Phi})^{-T} \frac{\partial C}{\partial \xi^i}$$

Finite element implementation is straightforward

$$\nabla_X C|_{\xi^3=0} = [B] \begin{bmatrix} \{C\}^+ \\ \{C\}^- \end{bmatrix} = [[B]^+ \quad [B]^-] \begin{bmatrix} \{C\}^+ \\ \{C\}^- \end{bmatrix}$$

$$B_{ia}^\pm = [G_i^1 \quad G_i^2 \quad G_i^3] \cdot \left[\frac{1}{2} \frac{\partial N_a}{\partial \xi^1} \quad \frac{1}{2} \frac{\partial N_a}{\partial \xi^2} \quad \pm \frac{1}{h} N_a \right]$$

$i = \#$ dimensions, $a = \#$ nodes

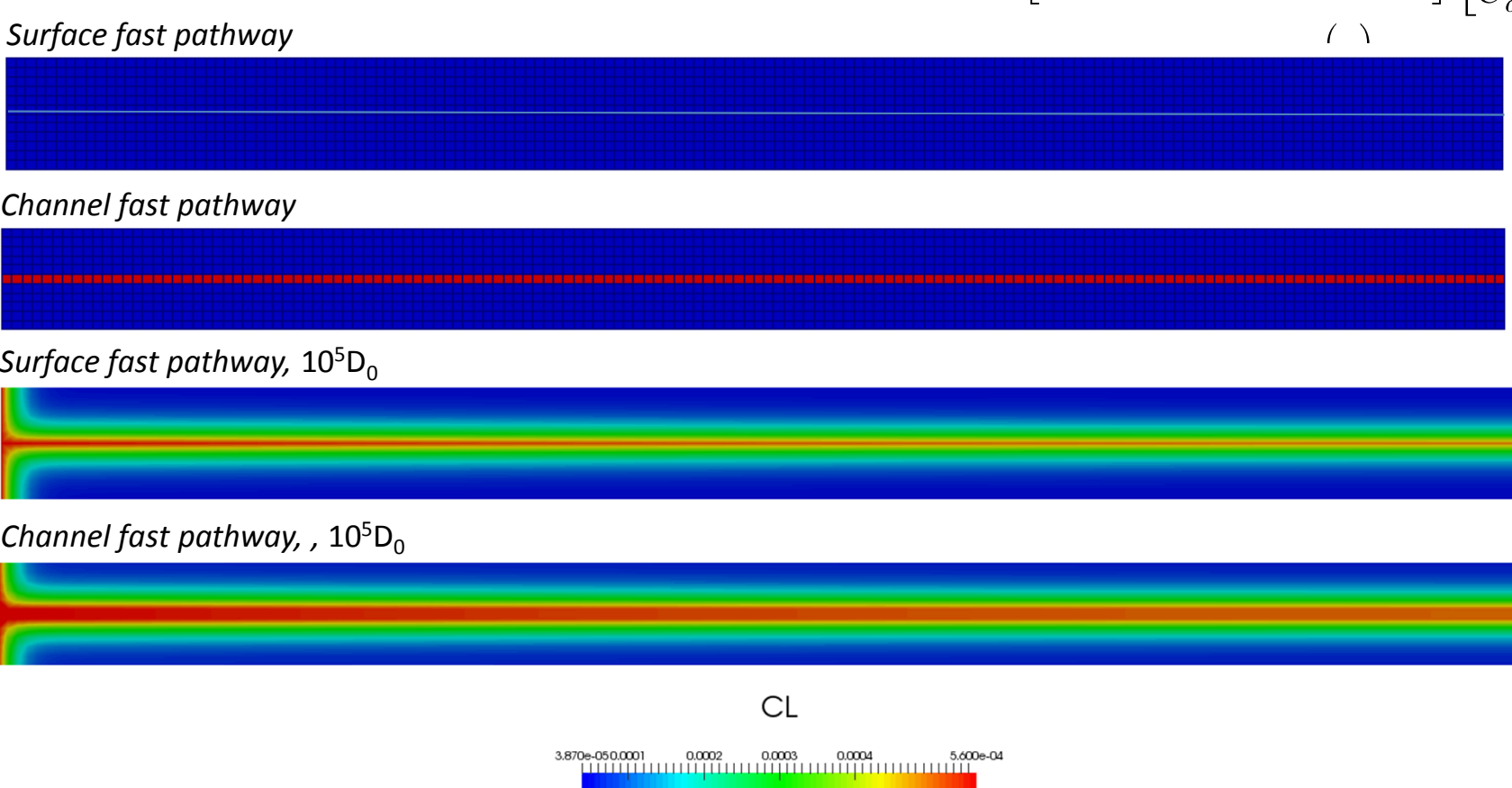
Given this gradient operator, we can use the same PDE for finite-deformation diffusion

$$D^* \dot{C}_L - \nabla_X \cdot d_l C^{-1} \nabla_X C_L + \nabla_X \cdot \frac{d_l V_H}{RT} C^{-1} \nabla_X \tau_h C_L + \theta_T \frac{d N_T}{d \epsilon_p} \dot{\epsilon}_p$$

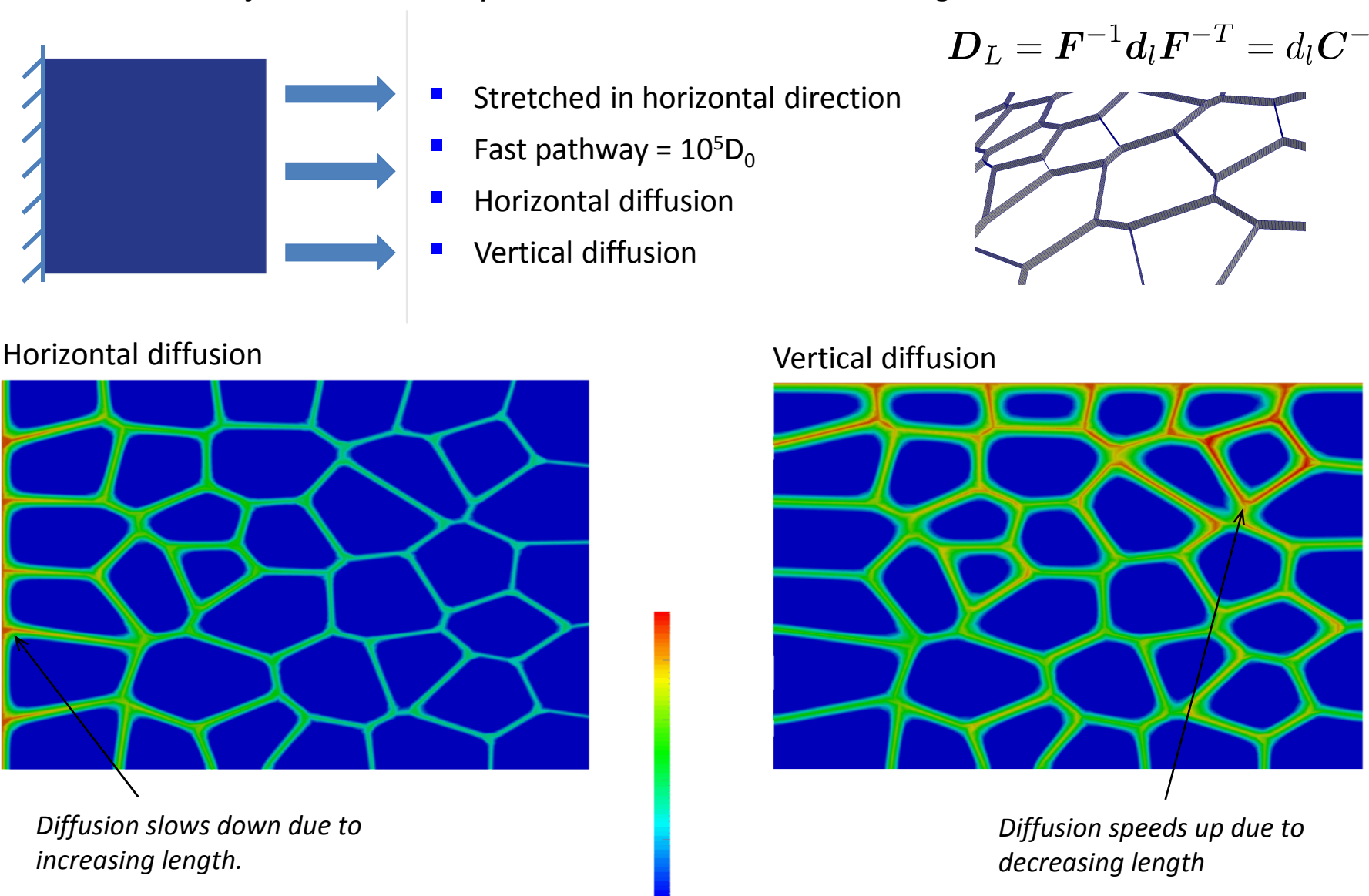
Implementation of fast pathway (no deformation):

$$\int_{B^+} \nabla_X v D_L(X) \nabla_X C \, dV \approx h \sum_{j=1}^{N_M} \bar{W}(\xi_j^1, \xi_j^2) [v_a^+(\xi_j^1, \xi_j^2) \quad v_a^-(\xi_j^1, \xi_j^2)] \begin{bmatrix} \bar{B}^+(\xi_j^1, \xi_j^2) \\ \bar{B}^-(\xi_j^1, \xi_j^2) \end{bmatrix} D_L(X)$$

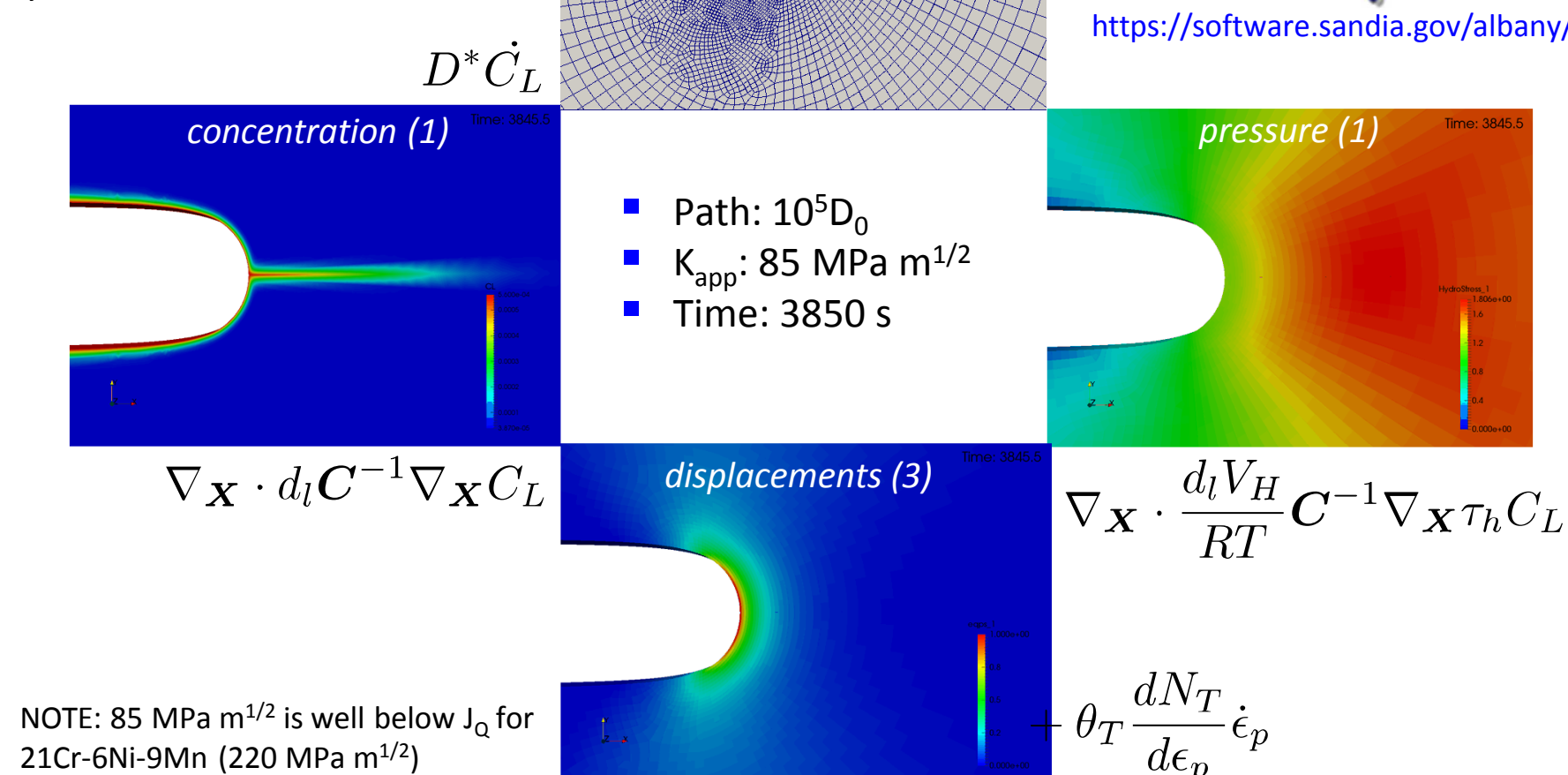
$$\begin{bmatrix} \bar{B}^+(\xi_j^1, \xi_j^2) & \bar{B}^-(\xi_j^1, \xi_j^2) \end{bmatrix} \begin{bmatrix} C_a^+ \\ C_a^- \end{bmatrix}$$



Grain boundary diffusion coupled with mechanical loading:



Units are scaled in the balance of linear momentum, conservation of concentration, and L_2 projection to improve condition number of the system



NOTE: 85 MPa $m^{1/2}$ is well below J_Q for 21Cr-6Ni-9Mn (220 MPa $m^{1/2}$)

We seek to find descriptors of helium bubble formation that result from the radioactive decay of tritium (T) to He.

- Schaldach and Wolfer (2004) focus on the total number of clusters (total bubble density)

- 1000s of ODEs for helium clusters are condensed into 3 coupled ODEs written in
 - Single He (monomers) N_p , total bubble density, N_b , and bubble volume fraction S_b
 - ODEs are nonlinear. We can integrate them implicitly with Newton's method
 - Chemo-mechanical solution (T, u) . Solve ODEs (dependent on T) at integration points.

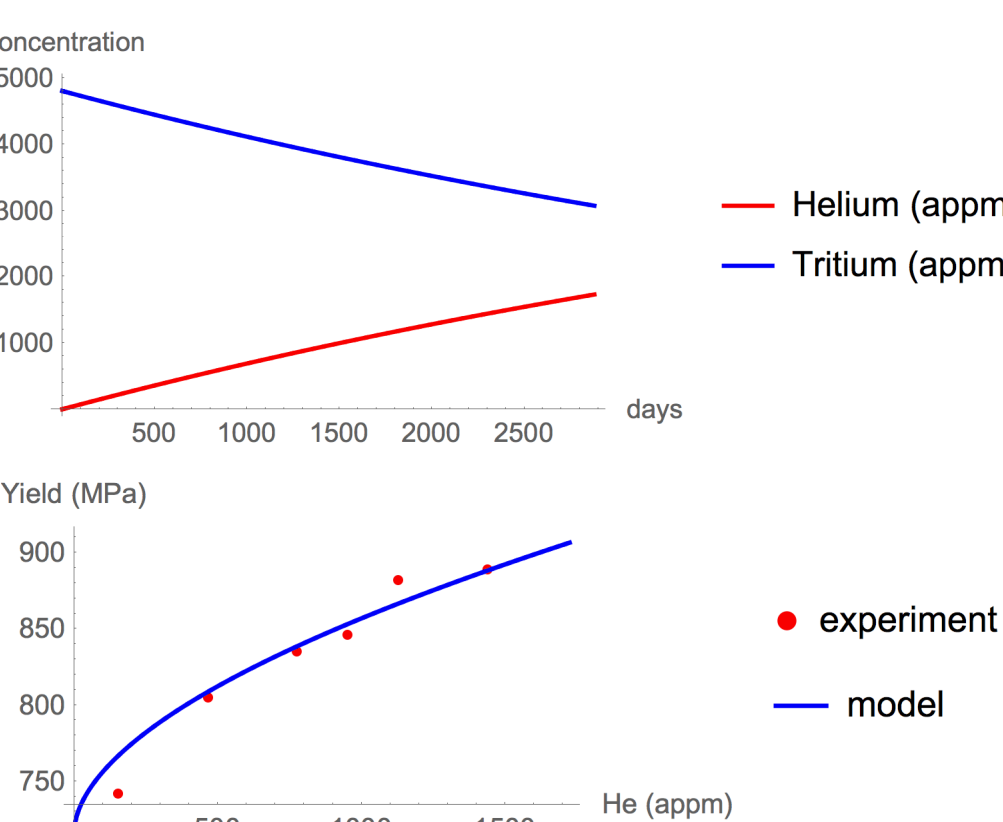
$$\frac{dN_1}{dt} = G(t) - 16\pi(r_1 + r_1)DN_1^2 - 4\pi DN_1 \left(\frac{3}{4\pi} \right)^{\frac{1}{3}} S_b^{\frac{1}{3}} N_b^{\frac{2}{3}}$$

$$\frac{dN_b}{dt} = 8\pi(r_1 + r_1)DN_1^2$$

$$\frac{\eta}{\Omega} \frac{dS_b}{dt} = 16\pi(r_1 + r_1)DN_1^2 + 4\pi DN_1 \left(\frac{3}{4\pi} \right)^{\frac{1}{3}} S_b^{\frac{1}{3}} N_b^{\frac{2}{3}}$$

$$C_{He} = N_1 + \left(\frac{\eta}{\Omega} \right) S_b$$

$G(t)$ - helium source term
 r_1 - initial bubble radius
 η - number of He atoms/vacant site
 Ω - partial molar volume



Comparison of current model to experimental data by Robinson and Thomas (1991). Data from hydrogen yields α_1 (hydrogen = tritium). Nonlinearity in fit stems from average bubble radius, not fitting parameter α_2 . Both parameters are needed to accurately fit Robinson's data.

From helium bubble ODEs, we have:

N_b total bubble density
 S_b bubble volume fraction

Calculate average bubble radius:

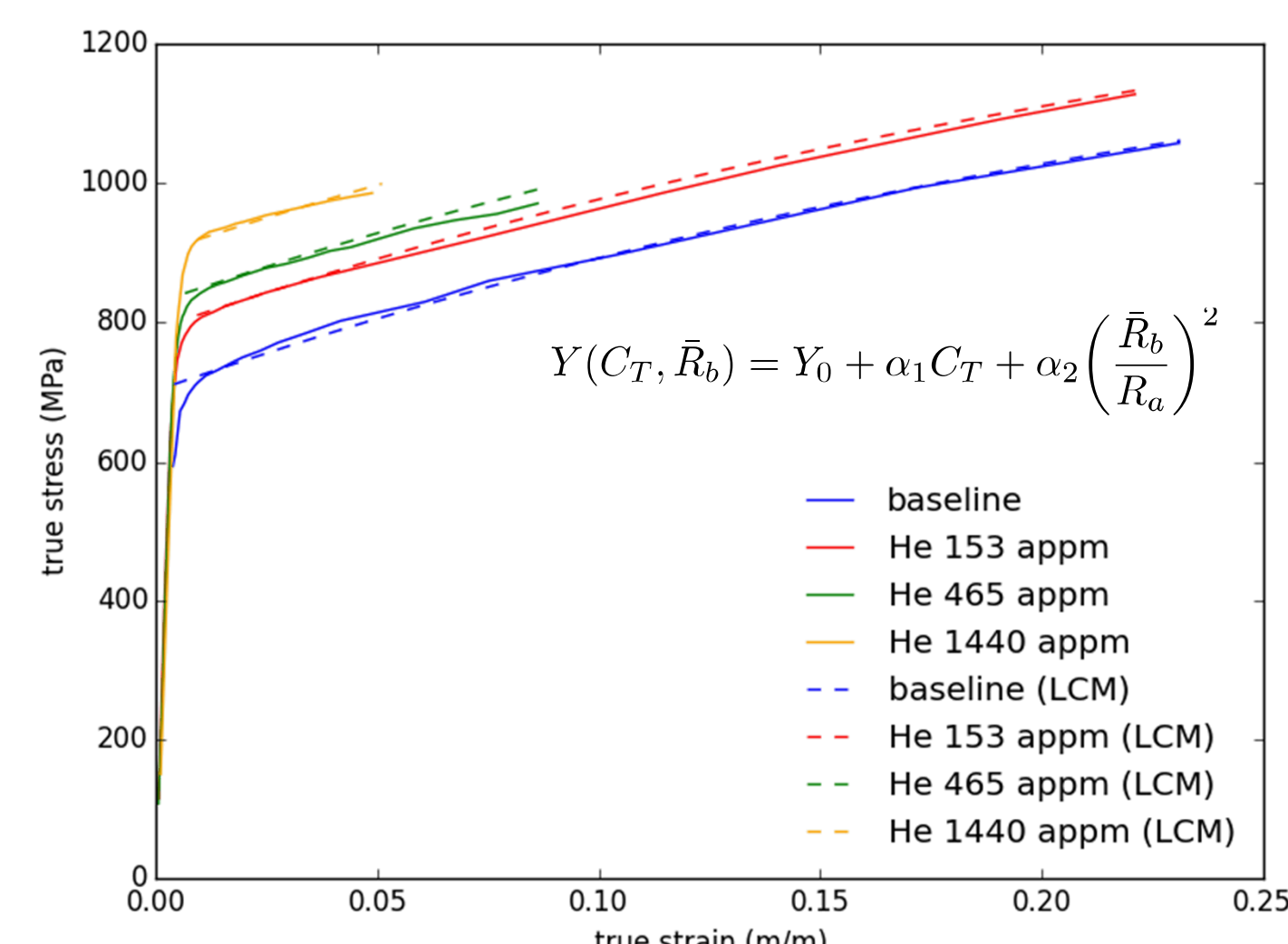
$$\bar{R}_b = \left(\frac{3S_b}{4\pi N_b} \right)^{\frac{1}{3}}$$

Assumptions for yield stress σ_y :

- Rate independent yield stress σ_0
- Proportional to T concentration, C_T
- Misfit strengthening from He is dominant (Arsenlis, Wolfer, JNM)
- Misfit strengthening varies w/R_b^2
- Normalize w/He atomic radius R_s

$$Y(C_T, \bar{R}_b) = Y_0 + \alpha_1 C_T + \alpha_2 \left(\frac{\bar{R}_b}{R_a} \right)^2$$

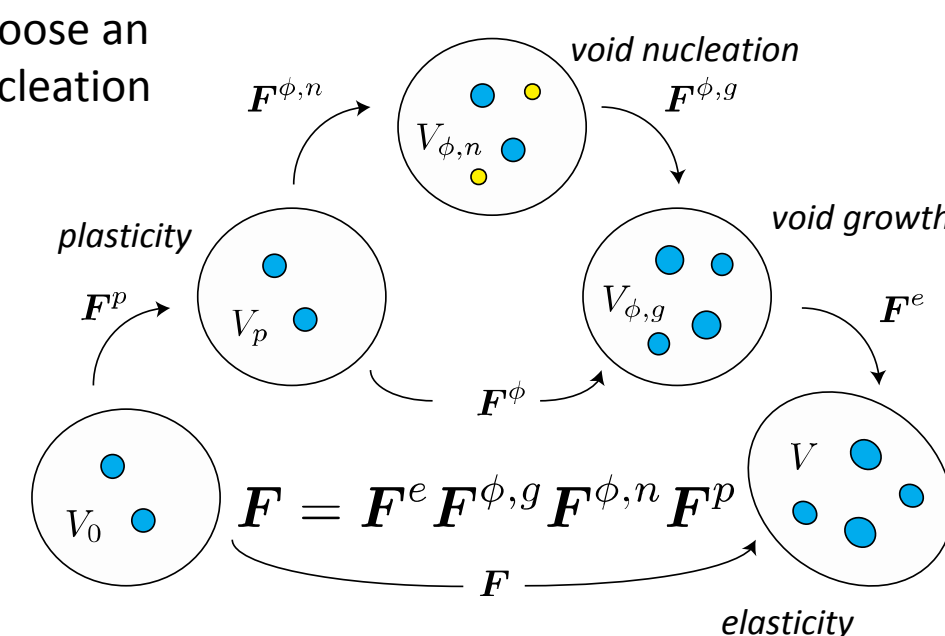
α_1 and α_2 are constants fit to experiments



From Robinson and Thomas, "Accelerated Fracture due to Tritium and Helium in 21-6-9 Stainless Steel" (1991). We cannot locate the source data. Missing He concentrations of 774, 950, and 1128 appm.

In spirit of Chu and Needleman (1980) we choose an appropriate state variable to capture void nucleation through elevated stresses at pile-ups.

- Void nucleation driven by deformation (dislocations, twins)
 - Tritium C_T hastens process
 - Helium amplifies process
- Void growth through Cocks-Ashby
- Void coalescence governed by S_b
 - Nucleated voids connected by smaller helium bubbles



$$\sigma_y(C_T, \bar{R}_b) = Y(C_T, \bar{R}_b) + \kappa \quad \kappa = 2\mu\epsilon_{ss}$$

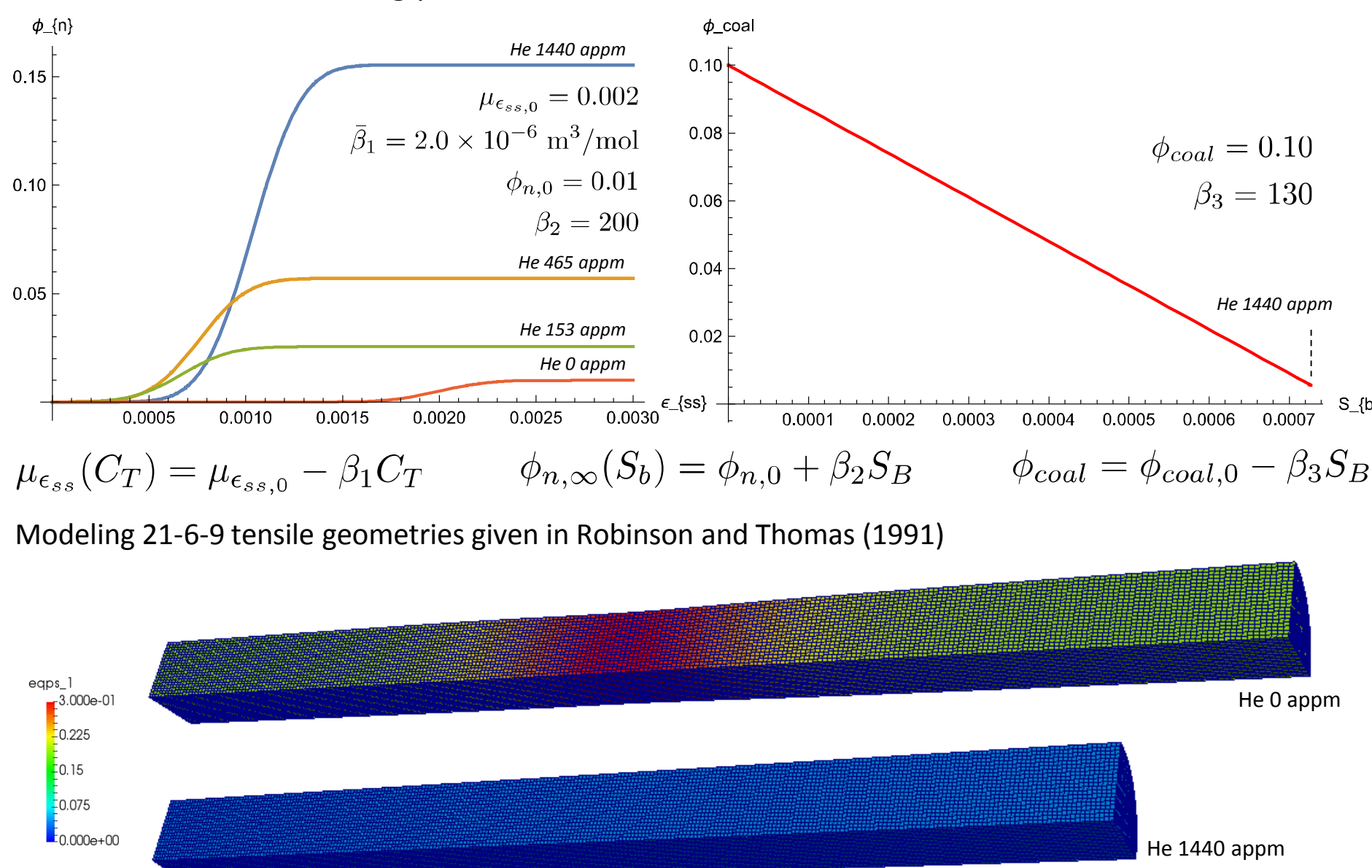
$$f_n(\epsilon_{ss}, C_T, S_b) = \frac{\phi_{n,\infty}(S_b)}{\sqrt{2\pi\epsilon_{ss}}} \exp \left[-\frac{1}{2} \left(\frac{\epsilon_{ss} - \mu\epsilon_{ss}(C_T)}{\sigma_{\epsilon_{ss}}} \right)^2 \right] \quad \text{void nucleation}$$

$$\phi_n(\epsilon_{ss}, C_T, S_b) = \frac{1}{2} \phi_{n,\infty}(S_b) \left[1 + \text{erf} \left(\frac{\epsilon_{ss} - \mu\epsilon_{ss}(C_T)}{\sqrt{2\sigma_{\epsilon_{ss}}}} \right) \right] \quad \mu_{\epsilon_{ss}}(C_T) = \mu_{\epsilon_{ss},0} - \beta_1 C_T$$

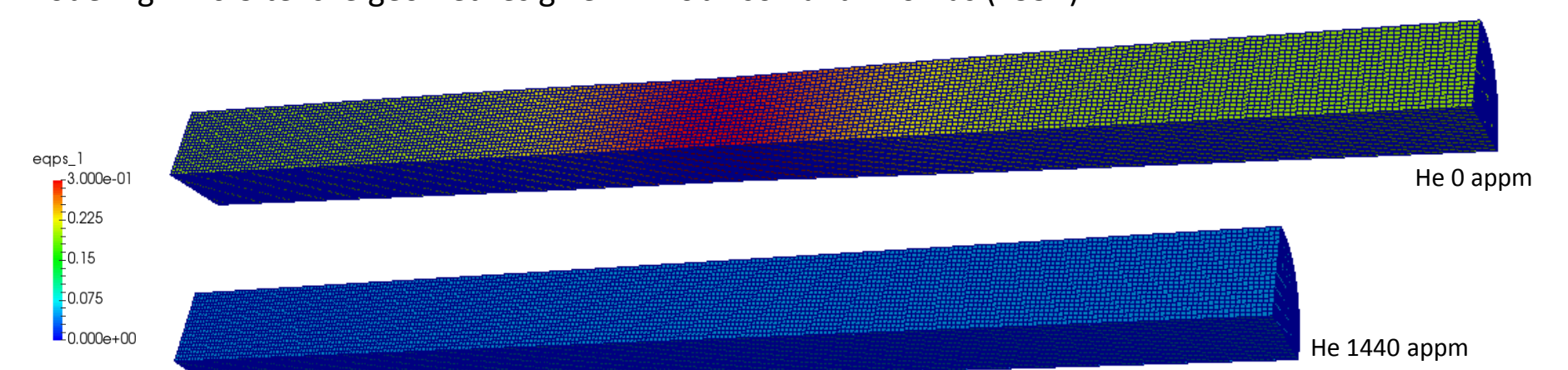
$$\dot{\phi} = \sqrt{\frac{2}{3}} \dot{\epsilon}_p \frac{1 - (1 - \phi)^{m+1}}{(1 - \phi)^m} \sinh \left[\frac{2(2m-1)\langle p \rangle}{2m+1} \frac{1}{\sigma_c} \right] \quad \text{void growth}$$

$$\phi_{coal} = \phi_{coal,0} - \beta_3 S_B \quad \text{void coalescence}$$

Initial constitutive modeling parameters for T/He embrittlement - more work needed

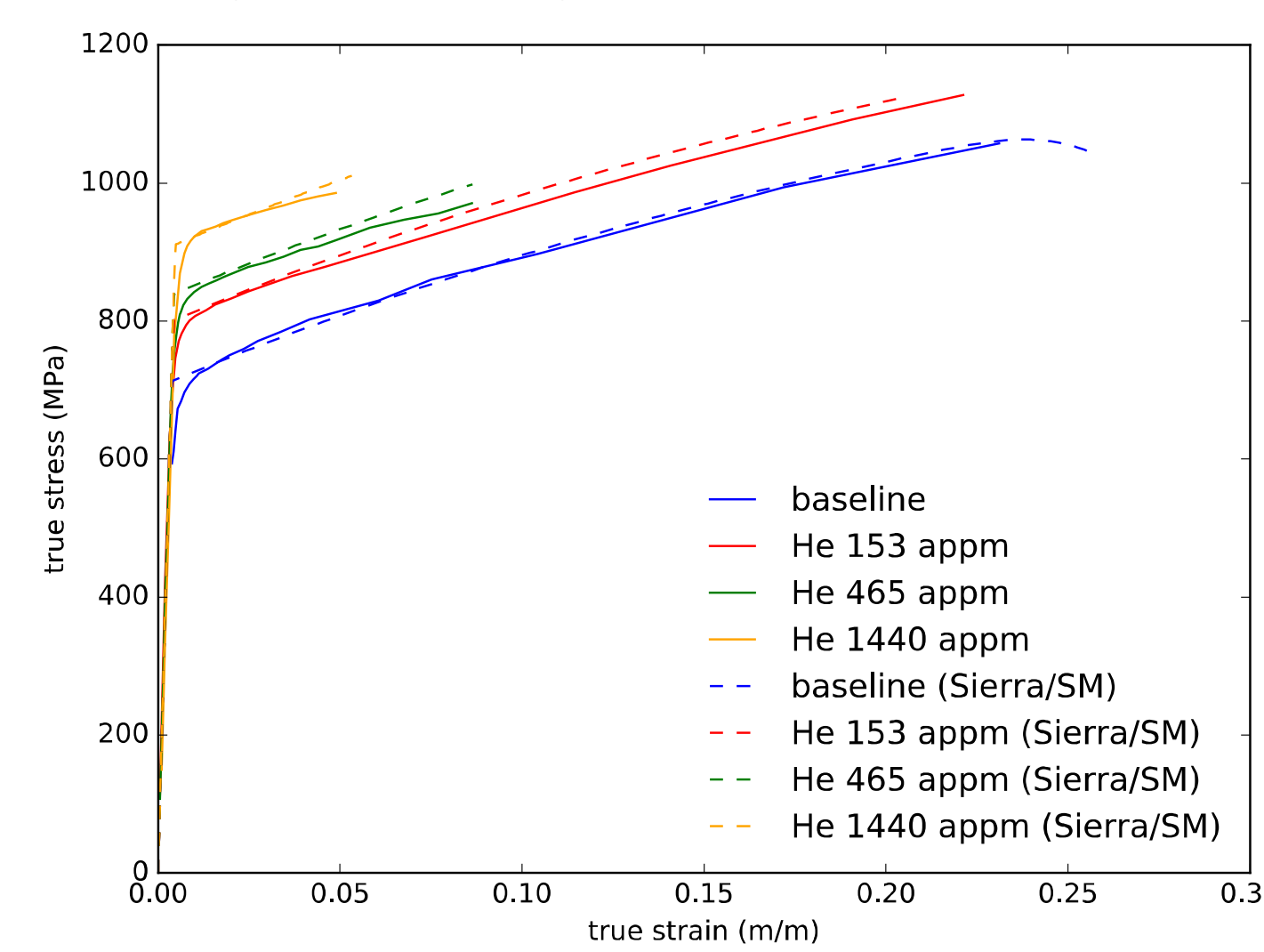


Modeling 21-6-9 tensile geometries given in Robinson and Thomas (1991)



T/He embrittled microstructures can move from nucleation to coalescence

Initial models for void evolution provide a pull for additional experimental data and enable a greater understanding of the crack resistance.



age 1.8 years in tritium

465 appm helium

total tritium concentration

621.4 mol/m³

helium bubble volume fraction

2.3e-4 m/m

strain metric for hardening

0.0013 m/m

void volume fraction

0.070 m/m³

surface elements

(1/4 symmetry)