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RELAP-7 Progress Report: A Mathematical Model for 1-D Compressible, Single-Phase Flow Through a Branching Junction

Prepared by
Idaho National Laboratory
Idaho Falls, Idaho 83415

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RELAP-7 Progress Report: A Mathematical Model for 1-D Compressible, Single-Phase Flow Through a Branching Junction

R. A. Berry[1,3], L. Zou[1], R.C. Johns[2], R. C. Martineau[1],
D. Andrs[1], H. Zhang[1], J. E. Hansel[1]

[1]Idaho National Laboratory
[2]Los Alamos National Laboratory
[3]Principal Author

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Summary

This report presents a mathematical model for *compressible*, single-phase flow through a general 1-input, 1-output (2-port) pipe junction which could have an active component between the two pipes, as well as a general branching junction of an arbitrary number of 1-dimensional pipes.

1 Introduction

In the literature, the abundance of pipe network junction models, as well as inclusion of dissipative losses between connected pipes with loss coefficients, has been treated using the *incompressible flow* assumption of constant density. This approach is fundamentally, physically wrong for compressible flow with density change. This report introduces a mathematical modeling approach for general junctions in piping network systems for which the transient flows are *compressible* and single-phase. The junction could be as simple as a 1-pipe input and 1-pipe output with differing pipe cross-sectional areas for which a dissipative loss is necessary, or it could include an active component, between an inlet pipe and an outlet pipe, such as a pump or turbine. In this report, discussion will be limited to the former. A more general branching junction connecting an arbitrary number of pipes with transient, 1-D compressible single-phase flows is also presented.

These models will be developed in a manner consistent with the use of a general equation of state like, for example, the recent Spline-Based Table Look-up method [1] for incorporating the IAPWS-95 formulation [2] to give accurate and efficient calculations for properties for water and steam with RELAP-7 [3].

2 Simple, Single Input and Single Ouput Pipe Junction

Consider the simple connection of two pipes having different cross-sectional areas, as shown in Figure 1. For both pipes we take the positive flow direction to be from left to right.

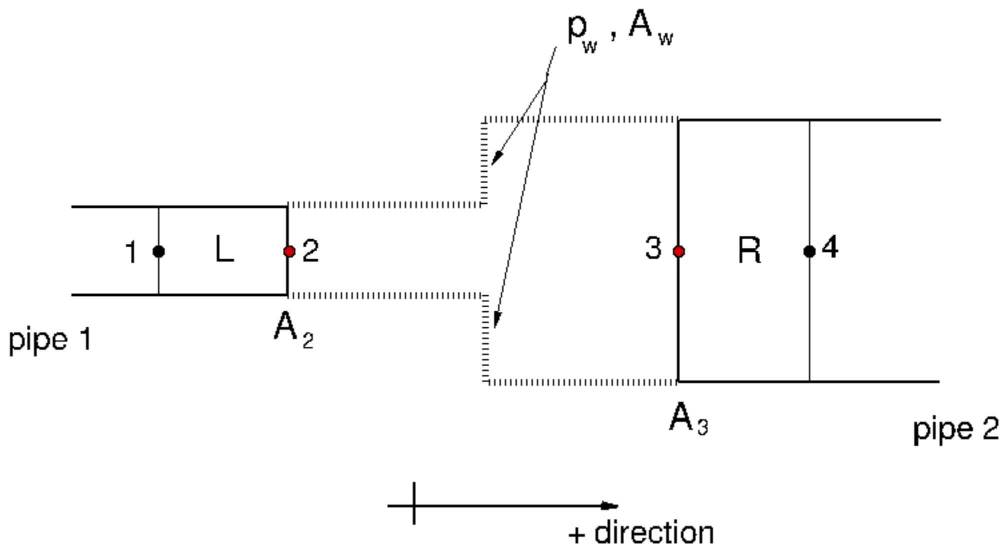

Figure 1. Simple, pipe junction schematic with one inlet and one outlet pipe and a cross-sectional area change.

For subsequent use, the pressure on the *washer shaped* area A_w is denoted p_w. Let us assume that the area available for flow is $A_f \equiv min(A_2, A_3)$. The area of this *washer shaped* face can then be written as $A_w = max(A_2, A_3) - A_f$.

For compressible flow, to connect pipe 1 and pipe 2 we need *six equations* (closures) to relate the *six unknowns* $\rho_2, p_2, u_2, \rho_3, p_3, u_3$, i.e. the fluid density, pressure, and velocity, at nodes 2 and 3 respectively, at new time level $n+1$. Because the pipe flows are compressible, characteristic information must be included from each pipe for the flow immediately adjacent to the junction. There are generally four ways in which to get this characteristic information from the immediate interior of the pipe: (1) Use Riemann invariants from characteristics in the space-time domain as in the classical *method of characteristics* [4], (2) Rewrite the governing partial differential equations into characteristic form and get the Riemann invariants at the boundary using *characteristic-biased differencing* [5,6], (3) *Extrapolate* appropriate information from the interior [7], and (4) Utilize *partial Riemann solvers* to obtain pipe interior influence [8,9]. Here we will utilize the last method. For a single compressible fluid, using the approach of [8] and [9] we get the first two equations by writing two approximate *half-Riemann problems*, for a left-running acoustic wave in pipe 1 and for a right-running acoustic wave in pipe 2:

$$p_2^{n+1} - p_L + \rho_L(c_L - u_L)(u_2^{n+1} - u_L) = 0 \tag{1}$$
$$p_3^{n+1} - p_R - \rho_R(c_R - u_R)(u_2^{n+1} - u_R) = 0 \tag{2}$$

and entropy wave(s)

$$\boldsymbol{If}\ u_2^{n+1} \geqslant 0\ \boldsymbol{then}$$
$$\rho_2^{n+1} = \rho_L + \frac{p_2^{n+1} - p_L}{c_L^2} \tag{3a}$$
$$\boldsymbol{If}\ u_3^{n+1} < 0\ \boldsymbol{then}$$
$$\rho_3^{n+1} = \rho_R + \frac{p_3^{n+1} - p_R}{c_R^2} \tag{3b}$$

where the old-time information available from the flow in each pipe is given by

$$(\cdot)_L = \frac{(\cdot)_1^n + (\cdot)_2^n}{2}$$
$$(\cdot)_R = \frac{(\cdot)_3^n + (\cdot)_4^n}{2}\,.$$

Usually, either Eqn. (3a) or Eqn. (3b) will hold, but usually not both simultaneously. Thus we have so far three equations, Eqns. (1), (2), and either (3a) or (3b). We need at least three additional equations for mathematical closure.

By assuming negligible fluid mass in the junction (i.e. between the two pipe boundary nodes adjacent to the junction, points 2 and 3) quasi-steady state relationships for mass and total energy balance can be written. For mass balance between the points 2 and 3

$$\rho_2^{n+1} u_2^{n+1} A_2 = \rho_3^{n+1} u_3^{n+1} A_3 \tag{4}$$

and for total energy balance between the points 2 and 3

$$\rho_2^{n+1} u_2^{n+1} A_2 \left[e(\rho_2^{n+1}, p_2^{n+1}) + \frac{p_2^{n+1}}{\rho_2^{n+1}} + \frac{1}{2}(u_2^{n+1})^2 \right] = \rho_3^{n+1} u_3^{n+1} A_3 \left[e(\rho_3^{n+1}, p_3^{n+1}) + \frac{p_3^{n+1}}{\rho_3^{n+1}} + \frac{1}{2}(u_3^{n+1})^2 \right].$$

By using Eqn. (4), this equation reduces to

$$e(\rho_2^{n+1}, p_2^{n+1}) + \frac{p_2^{n+1}}{\rho_2^{n+1}} + \frac{1}{2}(u_2^{n+1})^2 = e(\rho_3^{n+1}, p_3^{n+1}) + \frac{p_3^{n+1}}{\rho_3^{n+1}} + \frac{1}{2}(u_3^{n+1})^2 . \tag{5}$$

We still require at least one more equation to gain closure.

A couple of ways to get this last equation will be examined next. They will be referred to as *Technique 1* and *Technique 2*. From a fundamental viewpoint, both techniques are austensibly equivalent (at least in 1-D), though from a practical viewpoint, they will *not* match easily.

Technique 1

Technique 1 is related to the *Second Law of Thermodynamics* which considers dissipative flow processes which lead to an entropy *increase* (or entropy production), e.g. for flow between point 2 and point 3. Here, however, we make the key physical assumption that the entropy increase from point 2 to point 3 is isomorphic to the *decrease* of stagnation pressure of the flow between points 2 and 3.

For *no losses*, $p_{02} = p_{03}$, that is, the stagnation pressure at point 2 is identical to the stagnation pressure at point 3 [10]. Puting this into equation form $p(h, s)$

$$p(h_2^{n+1} + \frac{1}{2}(u_2^{n+1})^2, s(\rho_2^{n+1}, e_2^{n+1})) = p(h_3^{n+1} + \frac{1}{2}(u_3^{n+1})^2, s(\rho_3^{n+1}, e_3^{n+1})) . \tag{6a}$$

The system of equations is now closed (for the case of no losses between points 2 and 3).

If there are *losses* between points 2 and 3, then $p_{02} \neq p_{03}$ and loss coefficients ξ_{2-3} and ξ_{3-2} are introduced for flow from points 2 to 3 or, respectively, for flow between point 3 and 2. For the first case, $u_2^{n+1} \geqslant 0$, and

$$\begin{aligned} p(h_2^{n+1} + \frac{1}{2}(u_2^{n+1})^2, s(\rho_2^{n+1}, e_2^{n+1}))\,(1 - \xi_{2-3}) \,+\, p(\rho_2^{n+1}, e_2^{n+1})\,\xi_{2-3} \\ = p(h_3^{n+1} + \frac{1}{2}(u_3^{n+1})^2, s(\rho_3^{n+1}, e_3^{n+1})) \end{aligned} \tag{6b}$$

or for the second case, $u_3^{n+1} \leqslant 0$, and

$$\begin{aligned} p\big(h_3^{n+1} + \frac{1}{2}(u_3^{n+1})^2, s(\rho_3^{n+1}, e_3^{n+1})\big)\,(1-\xi_{3-2}) \;+\; p(\rho_3^{n+1}, e_3^{n+1})\,\xi_{3-2} \\ = p\big(h_2^{n+1} + \frac{1}{2}(u_2^{n+1})^2, s(\rho_2^{n+1}, e_2^{n+1})\big) \end{aligned} \tag{6c}$$

Remark: In the limit of very low-speed, incompressible flow, these loss coefficient forms become identical to classically used loss coefficients.

Sometimes it may be necessary to use an alternative loss coefficient to the one used above. For example, if a large tank boundary condition is considered, no flow exists in the tank and the static pressure is equal to the stagnation pressure. Instead it makes sense to use an alternative form of loss coefficient, denoted $\hat{\xi}_{2-3}$ or $\hat{\xi}_{3-2}$. With use of these alternative loss coefficients, for the first case, with $u_2^{n+1} \geqslant 0$

$$p\big(h_2^{n+1} + \frac{1}{2}(u_2^{n+1})^2, s(\rho_2^{n+1}, e_2^{n+1})\big)\,(1-\hat{\xi}_{2-3}) = p\big(h_3^{n+1} + \frac{1}{2}(u_3^{n+1})^2, s(\rho_3^{n+1}, e_3^{n+1})\big) \tag{6d}$$

while for the second case, with $u_3^{n+1} \leqslant 0$

$$p\big(h_3^{n+1} + \frac{1}{2}(u_3^{n+1})^2, s(\rho_3^{n+1}, e_3^{n+1})\big)\,(1-\hat{\xi}_{3-2}) = p\big(h_2^{n+1} + \frac{1}{2}(u_2^{n+1})^2, s(\rho_2^{n+1}, e_2^{n+1})\big)\,. \tag{6e}$$

Technique 2

An alternative to the use of Eqns. (6a), (6b), or (6c) for closure of the governing equation system is to employ a quasi-steady momentum flux balance between points 2 and 3 [11–14]. This again assumes negligble mass between these two points.

The quasi-steady momentum balance between the two respective pipe boundary points, points 2 and 3, is written

$$\rho_2^{n+1}(u_2^{n+1})^2 A_2 + p_2^{n+1} A_2 + p_w(A_3 - A_2) = \rho_3^{n+1}(u_3^{n+1})^2 A_3 + p_3^{n+1} A_3\,. \tag{7}$$

The pressure p_w on the *washer shaped* area A_w must be specified to achieve correct physical characteristics, just as realistic k_{2-3} and k_{3-2} were needed with *Technique 1*. A somewhat realistic choice (though more sophisticated choices could be investigated) for p_w is

$$p_w = \begin{cases} p_2 & A_2 > A_3 \\ p_3 & A_2 < A_3\,. \end{cases}$$

3 General, Single Input and Single Ouput Pipe Junction

In the previous section, a simple two-pipe junction was considered with a single direction orientation. In general, each pipe may have its own orientation (such as in RELAP-7) which must be accounted for. In this section the procedure for connecting two pipes via a *general* junction is given using *Technique 1* from the previous section. The nomenclature is also generalized to a more algorithmic form. Again consider two pipes of differing cross-section, connected through an active component junction though which transient, compressible single-phase fluid flow occurs. The nodalization is shown schematically in Figure 2.

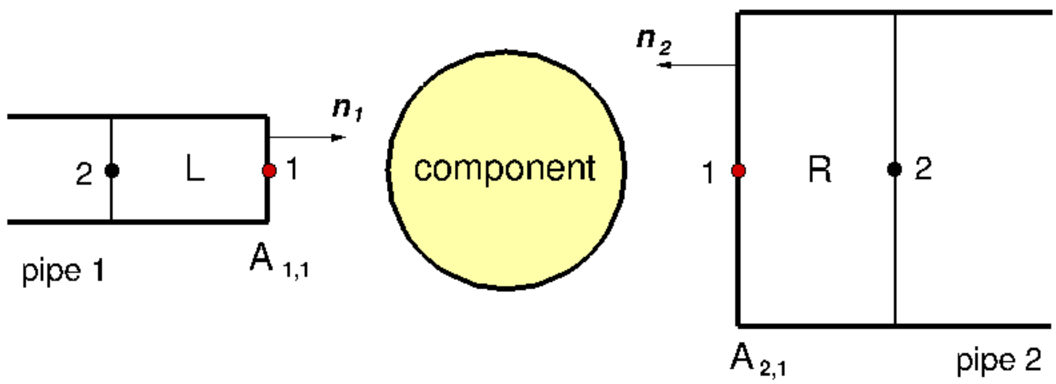

Figure 2. Two pipes of differing cross-section, connected through an active component junction through which transient, compressible single-phase fluid flows.

The six unknown variables at the two pipes' boundaries are $\rho_{i,1}^{n+1}$, $p_{i,1}^{n+1}$, and $u_{i,1}^{n+1}$ for $i = 1, 2$ corresponding to the solution of six independent equations, which must be supplied.

A half Riemann problem is used to obtain requisite flow information from each pipe consistent with the characteristics. A simple Godunov-type solver is employed. First the old-time pipe flow information for each pipe is defined:

$$\begin{aligned}\bar{\rho}_i &= \frac{\rho^n_{i,1} + \rho^n_{i,2}}{2} \\ \bar{p}_i &= \frac{p^n_{i,1} + p^n_{i,2}}{2} \\ \bar{u}_i &= \frac{u^n_{i,1} + u^n_{i,2}}{2} \\ \bar{c}_i &= \frac{c^n_{i,1} + c^n_{i,2}}{2} \qquad i = 1, 2.\end{aligned}$$

Here the first subscript index indicates the pipe, while the second subscript index indicates the node number on that pipe. In addition, for algorithmic convenience an *implicit switch function* is defined for each pipe end at the junction

$$S_i = \frac{u^{n+1}_{i,1} + |u^{n+1}_{i,1}|}{2|u^{n+1}_{i,1}| + \varepsilon} \qquad i = 1, 2$$

where ε is a small parameter to avoid potential division by too small of a number in the numerical procedures.

When various components are placed between pipes 1 and 2, the boundary conditions at the two pipe end nodes are coupled with the particular component. To determine the time-updated solution for these two pipe boundary nodes the following algorithm was developed:

If $u^{n+1}_{1,1} \cdot \hat{n}_1 \geqslant 0$ (outflow from pipe 1)

$$\rho^{n+1}_{1,1} = \bar{\rho}_1 + \frac{p^{n+1}_{1,1} - \bar{p}_1}{\bar{c}^2_1} \tag{8}$$

$$p^{n+1}_{1,1} = \bar{p}_1 - \bar{\rho}_1(\bar{c}_1 - \bar{u}_1)(u^{n+1}_{1,1} - \bar{u}_1)[2S_1 - 1] \tag{9}$$

else $u^{n+1}_{1,1} \cdot \hat{n}_1 < 0$ (flow into pipe 1 from junction)

$$p^{n+1}_{1,1} = \bar{p}_1 + \bar{\rho}_1(\bar{c}_1 - \bar{u}_1)(u^{n+1}_{1,1} - \bar{u}_1)[2S_1 - 1] \tag{10}$$

end if.

If $u_{2,1}^{n+1} \cdot \hat{n}_2 \geqslant 0$ (outflow from pipe 2)

$$\rho_{2,1}^{n+1} = \bar{\rho}_2 + \frac{p_{2,1}^{n+1} - \bar{p}_2}{\bar{c}_2^2} \tag{11}$$

$$p_{2,1}^{n+1} = \bar{p}_2 - \bar{\rho}_2(\bar{c}_2 - \bar{u}_2)(u_{2,1}^{n+1} - \bar{u}_2)[2S_2 - 1] \tag{12}$$

else $u_{2,1}^{n+1} \cdot \hat{n}_2 < 0$ (flow into pipe 2 from junction)

$$p_{2,1}^{n+1} = \bar{p}_2 + \bar{\rho}_2(\bar{c}_2 - \bar{u}_2)(u_{2,1}^{n+1} - \bar{u}_2)[2S_2 - 1] \tag{13}$$

end if.

Because it usually occurs that either $u_{1,1}^{n+1} \cdot \hat{n}_1 \geqslant 0$ or $u_{2,1}^{n+1} \cdot \hat{n}_2 \geqslant 0$ we have three equations with six unkowns. Consequently, three more equations (at least) are required to obtain mathematical closure.

By assuming negligible fluid mass between pipes 1 and 2, quasi-steady-state mass and total energy balances may be written between these two pipe boundary nodes. Using the quasi-steady mass balance

$$\rho_{1,1}^{n+1} u_{1,1}^{n+1} \cdot \hat{n}_1 A_{1,1} + \rho_{2,1}^{n+1} u_{2,1}^{n+1} \cdot \hat{n}_2 A_{2,1} = 0 \tag{14}$$

and from the quasi-steady total energy balance

$$\begin{aligned} \rho_{1,1}^{n+1} u_{1,1}^{n+1} \cdot \hat{n}_1 A_{1,1} \left[e(\rho_{1,1}^{n+1}, p_{1,1}^{n+1}) + \frac{p_{1,1}^{n+1}}{\rho_{1,1}^{n+1}} + \frac{1}{2}(u_{1,1}^{n+1})^2 \right] & \\ + Q_{1-2} + Q_{2-1} - W_{1-2} - W_{2-1} & \\ - \rho_{2,1}^{n+1} u_{2,1}^{n+1} \cdot \hat{n}_2 A_{2,1} \left[e(\rho_{2,1}^{n+1}, p_{2,1}^{n+1}) + \frac{p_{2,1}^{n+1}}{\rho_{2,1}^{n+1}} + \frac{1}{2}(u_{2,1}^{n+1})^2 \right] &= 0 \end{aligned}$$

If there is a pump between pipe 1 and pipe 2 then clearly there would be a work input term in the total energy equation above. Or, if there was a turbine between pipes 1 and 2 there would be an appropriate work output term. A similar statement can be made for devices between pipe 1 and 2 which involve heat input or output. For our simple junction case the heat addition (or removal) and work output (or input) terms will be taken equal to zero. In addition, because there are only two pipes connected to the device (junction) the mass flux balance above can be used to simplify and rewrite the total energy balance as

$$\left[e(\rho_{1,1}^{n+1}, p_{1,1}^{n+1}) + \frac{p_{1,1}^{n+1}}{\rho_{1,1}^{n+1}} + \frac{1}{2}(u_{1,1}^{n+1})^2\right] + \frac{Q_{1-2}}{\dot{m}} + \frac{Q_{2-1}}{\dot{m}} - \frac{W_{1-2}}{\dot{m}} - \frac{W_{2-1}}{\dot{m}} - \left[e(\rho_{2,1}^{n+1}, p_{2,1}^{n+1}) + \frac{p_{2,1}^{n+1}}{\rho_{2,1}^{n+1}} + \frac{1}{2}(u_{2,1}^{n+1})^2\right] = 0$$

where

$$\dot{m} = \rho_{1,1}^{n+1} u_{1,1}^{n+1} \cdot \hat{n}_1 A_{1,1} = \rho_{2,1}^{n+1} u_{2,1}^{n+1} \cdot \hat{n}_2 A_{2,1}\,.$$

Or,

$$e(\rho_{1,1}^{n+1}, p_{1,1}^{n+1}) + \frac{p_{1,1}^{n+1}}{\rho_{1,1}^{n+1}} + \frac{1}{2}(u_{1,1}^{n+1})^2 - [e(\rho_{2,1}^{n+1}, p_{2,1}^{n+1}) + \frac{p_{2,1}^{n+1}}{\rho_{2,1}^{n+1}} + \frac{1}{2}(u_{2,1}^{n+1})^2] = 0\,. \qquad (15)$$

At least one more equation is still needed to provide mathematical closure.

For a junction the final closure is related to the Second Law of Thermodynamics wherein dissipative processes which undergo an entropy *increase* (or entropy production). An equivalent idea, isomorphic to the principle of entropy increase, is employed here, in which the stagnation pressure will *decrease* for a dissipative process. To use this idea we first state, or define, stagnation pressure for a *compressible fluid* in terms of the other flow variables (at the two pipe boundary nodes) as:

$$\begin{aligned} p_{0i,1}^{n+1} &= p(h_{0i,1}^{n+1}, s_{i,1}^{n+1}) \\ &= p(h_{i,1}^{n+1} + \frac{1}{2}(u_{i,1})^{n+1}, s(\rho_{i,1}^{n+1}, e_{i,1}^{n+1})) \\ &= p(e_{i,1}^{n+1} + \frac{p_{i,1}^{n+1}}{\rho_{i,1}^{n+1}} + \frac{1}{2}(u_{i,1})^{n+1}, s(\rho_{i,1}^{n+1}, e_{i,1}^{n+1})) \\ &= p(e(\rho_{i,1}^{n+1}, p_{i,1}^{n+1}) + \frac{p_{i,1}^{n+1}}{\rho_{i,1}^{n+1}} + \frac{1}{2}(u_{i,1})^{n+1}, s(\rho_{i,1}^{n+1}, e(\rho_{i,1}^{n+1}, p_{i,1}^{n+1}))) \quad i = 1, 2 \end{aligned}$$

For *no losses* [10]:

$$p_{01,1}^{n+1} = p_{02,1}^{n+1} \qquad (16)$$

Remark 1: If the device between pipes 1 and 2 was a pump, not only would there be a work input term in the total energy equation, but also an increase in stagnation pressure, i.e.

$$p_{02,1}^{n+1} \geqslant p_{01,1}^{n+1} \quad (for\, flow\, from\, pipe\, 1\, to\, pipe\, 2)$$

or

$$p_{01,1}^{n+1} \geqslant p_{02,1}^{n+1} \quad (for\, flow\, from\, pipe\, 2\, to\, pipe\, 1)\,.$$

In either case, an additional relationship (could be coupled) relating this change would need to be supplied. Similarly may be said of other potential devices such as a turbine, etc.

For the case of a simple junction *with losses*:

If $u_{1,1}^{n+1} \cdot \hat{n}_1 \geqslant 0$ (outflow from pipe 1)

$$(1 - \xi_{1-2}) p_{01,1}^{n+1} + \xi_{1-2} p_{1,1}^{n+1} - p_{02,1}^{n+1} = 0\,. \tag{17}$$

If $u_{2,1}^{n+1} \cdot \hat{n}_2 \geqslant 0$ (outflow from pipe 2)

$$(1 - \xi_{2-1}) p_{02,1}^{n+1} + \xi_{2-1} p_{2,1}^{n+1} - p_{01,1}^{n+1} = 0\,. \tag{18}$$

Here, ξ_{1-2} and ξ_{2-1} are loss coefficients for compressible flow from pipe 1 to pipe 2, or from pipe 2 to pipe 1, respectively.

Remark 2: In the limit of low speed or incompressible flow the form used for these loss coefficients becomes identical to those used classically.

4 Junction Connecting Arbitrary Number of Pipes

A junction connecting an arbitrary number pipes is now examined. Consider a fixed number k of 1-D pipes connecting to this junction. At a given instant in time there are r pipes with flow *into* this junction and s pipes with flow *from* this junction. Both r and s can (and generally will) change with time during a transient, but their sum is time-invariant, so

$$r + s = k$$

and at each instant in time (and iteration) the number of pipes r flowing into the junction and pipes s flowing from the junction must be determined or identified. The nodal labeling for a representative five-pipe junction, junction J, is shown schematically in Figure 3.

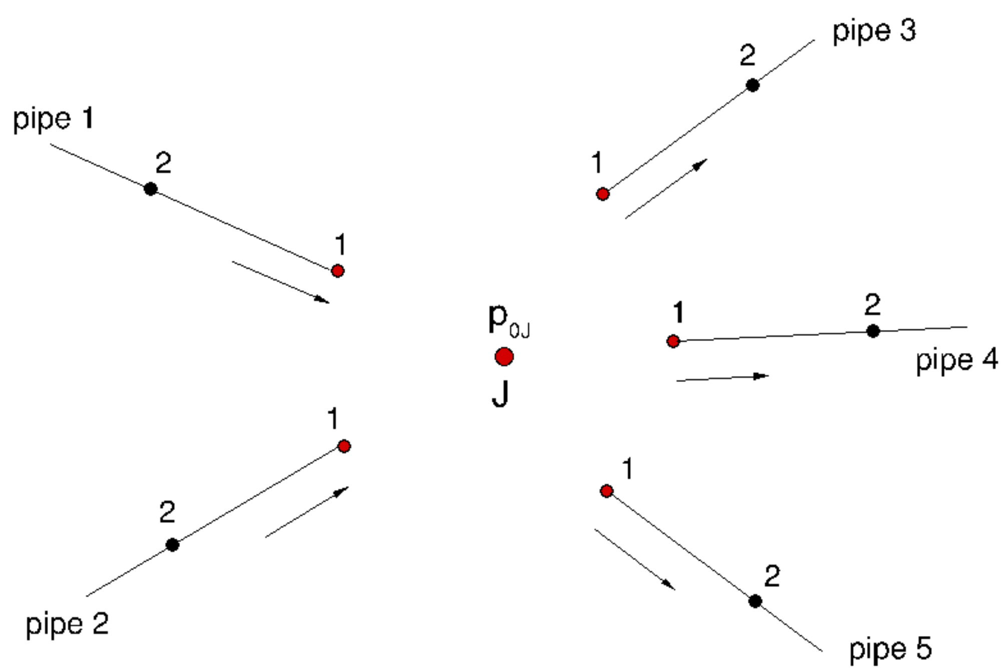

Figure 3. Instantaneous flow state for a representative five-pipe, compressible flow junction J.

So, obviously, $k = 5$ for the example illustrated in this Figure. At the instant shown there are $r = 2$ pipes flowing into junction J (pipes 1 and 2) and there are $s = 3$ pipes flowing from junction J (pipes 3, 4, and 5). For this model, a ficticious junction point, J, is placed at the junction flow centroid where we imagine a (time dependent) stagnation pressure $p_{0J}(t)$ exists. This is merely a useful mathematical artifice, employed for ease of conception and mathematical codification of equations. This junction stagnation pressure p_{0J} is added to the list of unknown variables to be solved for, so an additional equation must also be supplied to facilitate this solution.

The unknowns are now the boundary node values at each pipe end connecting to the junction, plus p_{0J}, i.e.

$$\rho_{i,1}^{n+1},\ u_{i,1}^{n+1},\ p_{i,1}^{n+1},\ p_{0J}^{n+1} \quad i = 1, \cdots, k \ \ (where\, subscript\ i\ denotes\, pipe\, \#)$$

For this five-pipe junction example there are $(3 \times 5) + 1 = 16$ unknowns, so there must be 16 independent equations specified for the system to allow for their solution.

We begin by defining the time-level n information available from the flow variables in each pipe (these variables carry the flow characteristic information consistent with the half Riemann solution for each pipe):

$$\bar{\rho}_i = \frac{\rho_{i,1}^n + \rho_{i,2}^n}{2}$$
$$\bar{p}_i = \frac{p_{i,1}^n + p_{i,2}^n}{2}$$
$$\bar{u}_i = \frac{u_{i,1}^n + u_{i,2}^n}{2}$$
$$\bar{c}_i = \frac{c_{i,1}^n + c_{i,2}^n}{2} \qquad i = 1, \cdots, k$$

An *implicit switch function* is also defined for each pipe boundary node at the junction

$$S_i = \frac{u_{i,1}^{n+1} + |u_{i,1}^{n+1}|}{2|u_{i,1}^{n+1}| + \varepsilon} \qquad i = 1, k$$

where ε is a small parameter used, for numerical purposes, to avoid division by zero.

We next begin building the requisite closure equations by first building the half Riemann problem equations from the available flow variables:

For each pipe i $(i = 1, \cdots, k)$

if $u_{i,1}^{n+1} \cdot \hat{n}_i \geqslant 0$ (outflow from pipe i into junction)

$$\rho_{i,1}^{n+1} = \bar{\rho}_i + \frac{p_{i,1}^{n+1} - \bar{p}_i}{\bar{c}_i^2} \tag{19}$$

$$p_{i,j}^{n+1} = \bar{p}_i - \bar{\rho}_i(\bar{c}_i - \bar{u}_i)(u_{i,1}^{n+1} - \bar{u}_i)[2S_i - 1] \tag{20}$$

else $u_{i,1}^{n+1} \cdot \hat{n}_i < 0$ (flow into pipe i from junction)

$$p_{i,j}^{n+1} = \bar{p}_i + \bar{\rho}_i(\bar{c}_i - \bar{u}_i)(u_{i,1}^{n+1} - \bar{u}_i)[2S_i - 1] \tag{21}$$

end if

End for

By assuming negligible mass in the junction, quasi-steady mass and total energy balance relations can be used at any instant to obtain two more closure relations. For quasi-steady mass balance

$$\sum_{i=1}^{k} \rho_{i,1}^{n+1} u_{i,1}^{n+1} \cdot \hat{n}_i A_{i,1} = 0 \tag{22}$$

and for quasi-steady total energy balance

$$\sum_{i=1}^{k} \rho_{i,1}^{n+1} u_{i,1}^{n+1} \cdot \hat{n}_i A_{i,1} \left[e(\rho_{i,1}^{n+1}, p_{i,1}^{n+1}) + \frac{p_{i,1}^{n+1})}{\rho_{i,1}^{n+1}} + \frac{1}{2}(u_{i,1}^{n+1})^2 \right] = 0\,. \tag{23}$$

For closure of the junction equations we also utilize the Second Law of Thermodynamics for dissipative processes. However, we conjecture an isomorphic alternative to that of entropy production (or *increase*) for dissipative processes, namely that dissipative processes result in a *decrease* of stagnation pressure for the flow.

For *no losses* [10]

$$p_{0i,1}^{n+1} = p_{0J}^{n+1} \qquad i = 1, \cdots, k$$

that is

$$\begin{aligned} p_{0J}^{n+1} &= p(h_{0i,1}^{n+1}, s_{i,1}^{n+1}) \\ &= p(h_{i,1}^{n+1} + \frac{1}{2}(u_{i,1}^{n+1})^2, s(\rho_{i,1}^{n+1}, e_{i,1}^{n+1})) \\ &= p(e(\rho_{i,1}^{n+1}, p_{i,1}^{n+1}) + \frac{p_{i,1}^{n+1}}{\rho_{i,1}^{n+1}} + \frac{1}{2}(u_{i,1}^{n+1})^2, s(\rho_{i,1}^{n+1}, e(\rho_{i,1}^{n+1}, p_{i,1}^{n+1}))) \quad i = 1, \cdots, k \end{aligned} \tag{24}$$

For flow *with losses*

For each pipe i $(i = 1, \cdots, k)$

if $u_{i,1}^{n+1} \cdot \hat{n}_i \geqslant 0$ (outflow from pipe 1)

$$(1 - \xi_{i-J}) p_{0i,1}^{n+1} + \xi_{i-J} p_{i,1}^{n+1} - p_{0J}^{n+1} = 0 \tag{25}$$

else $u_{i,1}^{n+1} \cdot \hat{n}_i < 0$ (flow into pipe i from junction)

$$p_{0i,1}^{n+1} + (\hat{\xi}_{J-i} - 1) p_{0J}^{n+1} = 0 \tag{26}$$

end if

End for

where

$$\begin{aligned} p_{0i,1}^{n+1} &= p(h_{0i,1}^{n+1}, s_{i,1}^{n+1}) \\ &= p(h_{i,1}^{n+1} + \frac{1}{2}(u_{i,1}^{n+1})^2, s(\rho_{i,1}^{n+1}, e_{i,1}^{n+1})) \\ &= p(e(\rho_{i,1}^{n+1}, p_{i,1}^{n+1}) + \frac{p_{i,1}^{n+1}}{\rho_{i,1}^{n+1}} + \frac{1}{2}(u_{i,1}^{n+1})^2, s(\rho_{i,1}^{n+1}, e(\rho_{i,1}^{n+1}, p_{i,1}^{n+1}))) \quad i = 1, \cdots, k \end{aligned}$$

Remark 1: For $u_{i,1}^{n+1} \cdot \hat{n}_i \geqslant 0$ (flow from pipe i to junction) we have defined the loss coefficients by

$$\xi_{i-J} = \frac{p_{0i,1}^{n+1} - p_{0J}^{n+1}}{p_{0i,1}^{n+1} - p_{i,1}^{n+1}}$$

and for $u_{i,1}^{n+1} \cdot \hat{n}_i < 0$ (flow from junction to pipe i) we have defined the loss coefficients by

$$\hat{\xi}_{J-i} = \frac{p_{0J}^{n+1} - p_{0i,1}^{n+1}}{p_{0J}^{n+1}}$$

Remark 2: Alternatively, applying losses only on outflows from pipe i into the junction gives:

For each pipe i $(i = 1, \cdots, k)$

if $u_{i,1}^{n+1} \cdot \hat{n}_i \geqslant 0$ (outflow from pipe 1)

$$(1 - \xi_{i-J})p_{0i,1}^{n+1} + \xi_{i-J}p_{i,1}^{n+1} - p_{0J}^{n+1} = 0$$

else $u_{i,1}^{n+1} \cdot \hat{n}_i < 0$ (flow into pipe i from junction)

$$p_{0i,1}^{n+1} - p_{0J}^{n+1} = 0$$

end if

End for

And in a similar manner, losses could be applied only to the pipes i with inflow from the junction (using $\hat{k}_{J-i}$), i.e.

For each pipe i $(i = 1, \cdots, k)$

if $u_{i,1}^{n+1} \cdot \hat{n}_i \geqslant 0$ (outflow from pipe 1)

$$p_{0i,1}^{n+1} - p_{0J}^{n+1} = 0$$

else $u_{i,1}^{n+1} \cdot \hat{n}_i < 0$ (flow into pipe i from junction)

$$p_{0i,1}^{n+1} + (\hat{\xi}_{J-i} - 1)p_{0J}^{n+1} = 0$$

end if

End for

Alas, the mathematical system for the junction is not yet closed. We do not know how the flow energy partitions amongst the pipes with flow from the junction into the pipe. Because we do not know this, we *assume* the stagnation specific enthalpy is identical for each of the s pipes for which the flow is from the junction J into the pipe i [15]. For this example (at the instant shown) this means that pipes 3, 4, and 5 receiving flow from the juncion will be assumed to have the same stagnation specific enthalpy, i.e.

$$h_{03,1}^{n+1} = h_{04,1}^{n+1} = h_{05,1}^{n+1} \tag{27}$$

where

$$h_{0i,1}^{n+1} = e(\rho_{i,1}^{n+1}, p_{i,1}^{n+1}) + \frac{p_{i,1}^{n+1}}{\rho_{i,1}^{n+1}} + \frac{1}{2}(u_{i,1}^{n+1})^2$$

For this example this yielded two new equation; more generally, it yields $s-1$ new equations such that

$$h_{0i,1}^{n+1} = h_{0j,1}^{n+1} \quad \forall\, i,j \in \{s\} \,:\, i \neq j\,. \tag{28}$$

The system of equations is now mathematically closed: 16 equations for the 16 unknowns.

Remark 3: If the energy partitioning amongst the pipes receiving flow from the junction was known, based on some physical knowlege, additional relationships could be constructed to reflect this *additional knowledge*.

5 Final Note

The technique described in this report to represent the flow of single-phase compressible flow in a junction can be extended to the compressible two-phase, 7-equation model employed in RELAP-7. Roughly speaking, the number of implicit unknowns to be solved for at each junction (and, consequently, the number of equations necessary to relate those unknowns) will approximately double from that of the single-phase case. The five-pipe junction example of the previous section will have at least 32 implicit unknowns with an equal number of equations linking those values at the pipe boundaries adjacent to the junction. The extension of this methodology to the 7-equation two-phase model of RELAP-7 will be detailed at a later date.

In addition to the compressible junction method described in this report, it is noted that there is another technique available, possibly more efficient, to treat a pipe network junction. We refer to this technique here as the *finite junction volume method*. Using this approach, a junction is assigned a realistic volume; the multidimensional, time-dependent balance equations are then written for this junction volume. The transient equations for this volume are then solved to yield updated physical quantities that evolve with time. Though the pipes which connect to this junction are treated one-dimensionally, because of their three-dimensional orientation relative to the junction, their one-dimensional fluxes map to multidimensional fluxes for the junction volume. Similarly, the time-updated multidimensional momentum solution is projected appropriately to the direction of each pipe for interactive coupling. This technique is very conducive to solution by the numerical finite volume method, and is described in [11, 12, 16–18]. This method was considered early in the RELAP-7 development effort [11, 12], but was abandoned at that time, due to limitations of the underlying software-development framework capablilities. Because of recent enhancements to the underlying framework capabilites, and for other reasons which (for brevity) are not detail here, this finite junction volume method may warrant future revisitation.

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