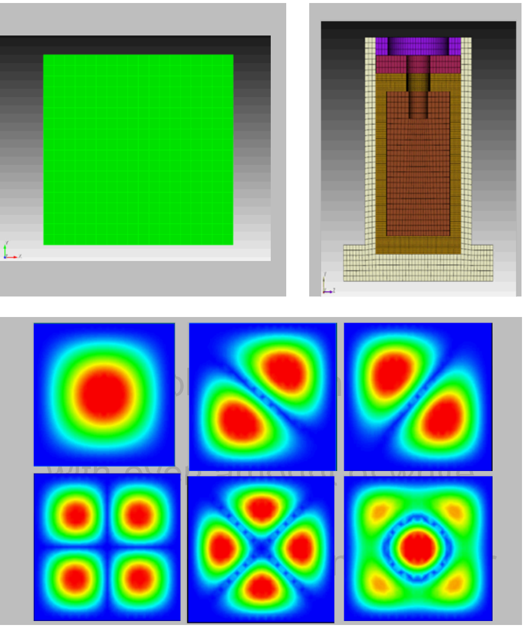
SAND2016-8028C

Craig-Bampton Substructuring of Linear Viscoelastic Finite Element Models



R.J. Kuether[1] & K.L. Troyer[1]

[1]Sandia National Laboratories

Presented at 2016 ASME IDETC on August 22, 2016.

Exceptional service in the national interest

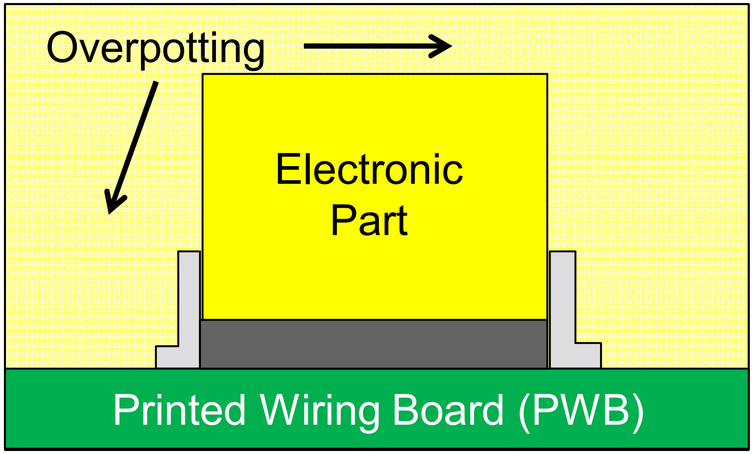
Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

What is our goal?

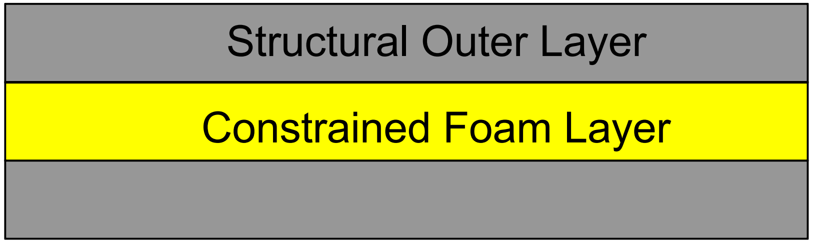
- Develop a Craig-Bampton type reduced order model (ROM) for subcomponent finite element models with **linear viscoelastic material behavior**
- Reduce computational burden of repetitive numerical solutions for large scale assemblies while preserving accuracy
- Incorporate **non-viscous damping** into subcomponent models via material property data

Applications with Linear Viscoelastic Behavior

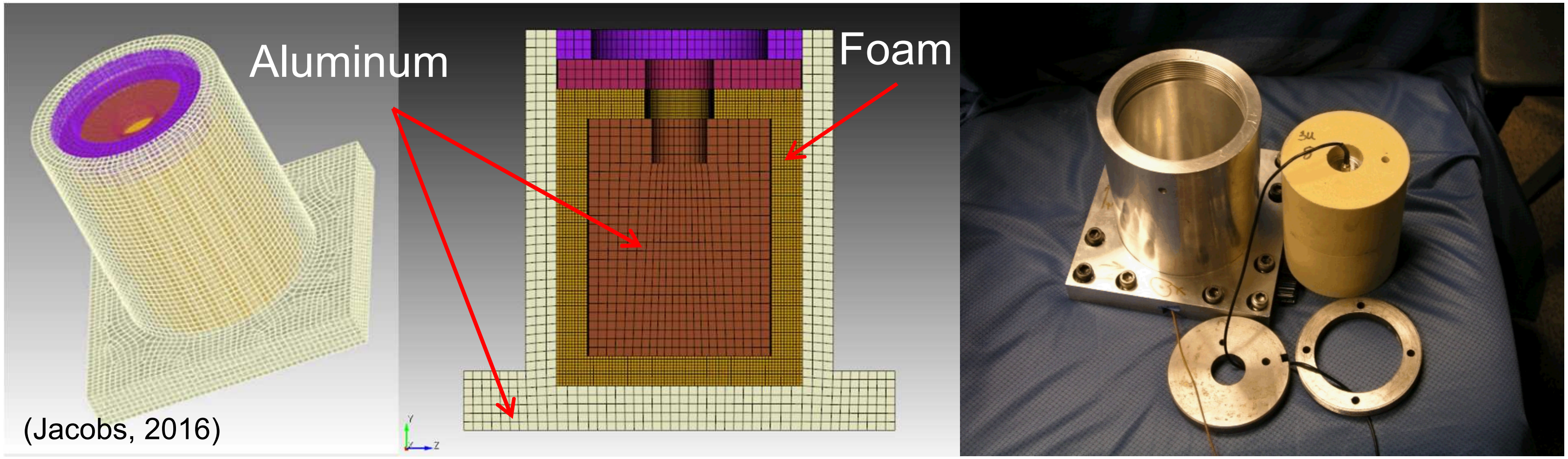
Encapsulation



Sandwich Structured Composites



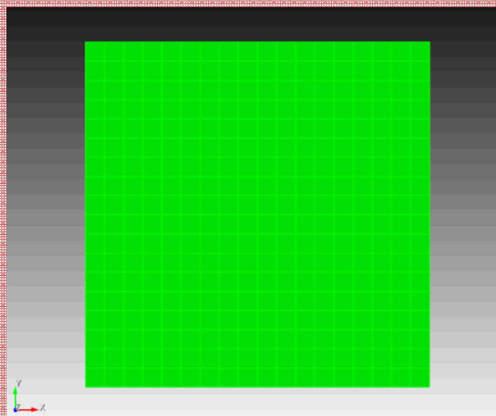
Ministack



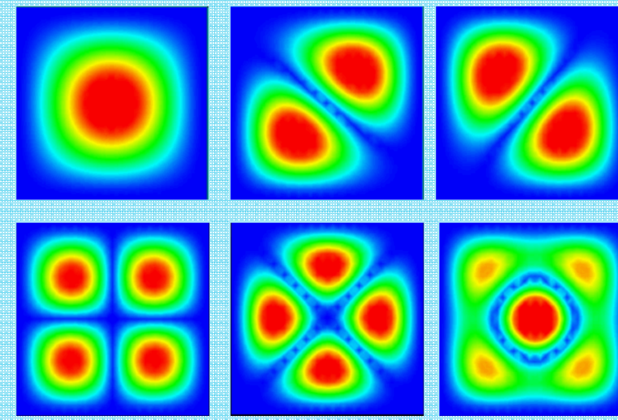
Substructuring Approach

FE mesh in physical coordinates

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{f}_{matl}\left(\mathbf{x},\dot{\mathbf{x}}\right) = \mathbf{f}\left(t\right)$$



Determine appropriate subcomponent basis based on the physical equations of motion



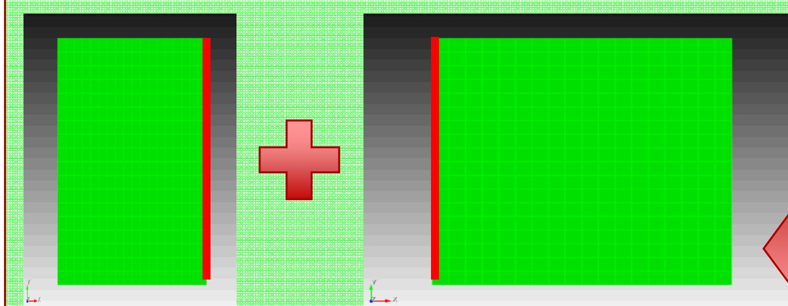
Project full equations of motion onto a small set of basis vectors

Related by transformation:

$\mathbf{x}(\mathrm{t}) = \mathbf{T}_{CB}\mathbf{q}(t)$ with $\mathbf{q}(\mathrm{t}) \ll \mathbf{x}(t)$

$$\therefore\ \hat{\mathbf{M}}\ddot{\mathbf{q}} + \hat{\mathbf{K}}\mathbf{q} + \mathbf{f}_{matl}\left(\mathbf{q},\dot{\mathbf{q}}\right) = \hat{\mathbf{f}}\left(t\right)$$

Assemble each subcomponent equations along shared interface DOF



Linear Viscoelasticity with Prony Series

- **Stress dependent upon time**

$$\boldsymbol{\sigma}(t) = \int_0^t E(t-\tau)\frac{d\boldsymbol{\varepsilon}}{d\tau}d\tau$$

- **Prony series**

$$E(t) = E_\infty + (E_g - E_\infty)\zeta(t)$$

$$\zeta(t) = \sum_{i=1}^{N} E_i e^{-t/\tau_i}$$

Relaxation modulus: describes time- and history-dependent behavior!

⋮ (skipping detailed mathematics)

- **Subcomponent FEA equations of motion**

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}_{v,K}\int_0^t \zeta_K(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}_{v,G}\int_0^t \zeta_G(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}_e\mathbf{x} = \mathbf{f}(t)$$

*Typically have many degrees-of-freedom!

Reduction of Subcomponent Equations of Motion

- **The transformation basis of a Craig-Bampton reduction employs:**
  - Fixed interface modes
  - Static constraint modes
- **Partition the subcomponent equations of motion into interior DOF and boundary DOF as**

$$\begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{ib} \\ \mathbf{M}_{bi} & \mathbf{M}_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}}_i \\ \ddot{\mathbf{x}}_b \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{K,ii} & \mathbf{K}_{K,ib} \\ \mathbf{K}_{K,bi} & \mathbf{K}_{K,bb} \end{bmatrix} \int_0^t \zeta_K(t-\tau) \begin{Bmatrix} \dot{\mathbf{x}}_i \\ \dot{\mathbf{x}}_b \end{Bmatrix} d\tau + \begin{bmatrix} \mathbf{K}_{G,ii} & \mathbf{K}_{G,ib} \\ \mathbf{K}_{G,bi} & \mathbf{K}_{G,bb} \end{bmatrix} \int_0^t \zeta_G(t-\tau) \begin{Bmatrix} \dot{\mathbf{x}}_i \\ \dot{\mathbf{x}}_b \end{Bmatrix} d\tau + \begin{bmatrix} \mathbf{K}_{e,ii} & \mathbf{K}_{e,ib} \\ \mathbf{K}_{e,bi} & \mathbf{K}_{e,bb} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_i \\ \mathbf{x}_b \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_{ext}(t) \end{Bmatrix}$$

How to deal with viscoelastic terms?

Linearized Complex Fixed Interface Modes

- **Restrain all the boundary DOF and retain the interior portion of the EOM**

$$\mathbf{M}_{ii}\ddot{\mathbf{x}}_i + \mathbf{K}_{K,ii}\int_0^t \zeta_K(t-\tau)\dot{\mathbf{x}}_i d\tau + \mathbf{K}_{G,ii}\int_0^t \zeta_G(t-\tau)\dot{\mathbf{x}}_i d\tau + \mathbf{K}_{e,ii}\mathbf{x}_i = \mathbf{0}$$

- **Compute "linearized complex fixed interface eigenmodes"**
  - Iterative approach that uses linearized quadratic eigensolver in Sierra/SD

Linearized quadratic eigenvalue problem: for each mode, iterate until $\mathbf{Im}(\boldsymbol{\lambda_r}) = \boldsymbol{\omega_0}$

$$\left(\lambda_r^2\mathbf{M}_{ii} + \lambda_r\mathbf{K}_{K,ii}\sum_{j=1}^{N_K}\frac{K_{coeff,j}}{\lambda_0 + 1/\tau_{K,j}} + \lambda_r\mathbf{K}_{G,ii}\sum_{j=1}^{N_K}\frac{G_{coeff,j}}{\lambda_0 + 1/\tau_{G,j}} + \mathbf{K}_{e,ii}\right)\boldsymbol{\varphi}_r = \mathbf{0}$$

with $\lambda_0 = i\omega_0$

$$\boldsymbol{\Phi}_{LCFI} = \begin{bmatrix} \mathrm{Re}(\boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \cdots & \boldsymbol{\varphi}_m) & \mathrm{Im}(\boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \cdots & \boldsymbol{\varphi}_m) \\ & \mathbf{0} & & & & \mathbf{0} & & \end{bmatrix}$$

Pseudo-Static Constraint Modes

- **Ignore inertia term; subcomponent EOM in the frequency domain becomes**

$$\left(i\omega \begin{bmatrix} \mathbf{K}_{K,ii} & \mathbf{K}_{K,ib} \\ \mathbf{K}_{K,bi} & \mathbf{K}_{K,bb} \end{bmatrix} \sum_{j=1}^{N_K} \frac{K_{coeff,j}}{i\omega + 1/\tau_{K,j}} + i\omega \begin{bmatrix} \mathbf{K}_{G,ii} & \mathbf{K}_{G,ib} \\ \mathbf{K}_{G,bi} & \mathbf{K}_{G,bb} \end{bmatrix} \sum_{j=1}^{N_G} \frac{G_{coeff,j}}{i\omega + 1/\tau_{G,j}} + \begin{bmatrix} \mathbf{K}_{e,ii} & \mathbf{K}_{e,ib} \\ \mathbf{K}_{e,bi} & \mathbf{K}_{e,bb} \end{bmatrix} \right) \begin{Bmatrix} \mathbf{X}_i \\ \mathbf{X}_b \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{F}_{ext} \end{Bmatrix}$$

- **Set X_b = I, ω>>0, and solve top equation for X_i**

$$\left(i\omega \mathbf{K}_{K,ii} \sum_{j=1}^{N_K} \frac{K_{coeff,j}}{i\omega + 1/\tau_{K,j}} + i\omega \mathbf{K}_{G,ii} \sum_{j=1}^{N_G} \frac{G_{coeff,j}}{i\omega + 1/\tau_{G,j}} + \mathbf{K}_{e,ii} \right) \mathbf{X}_i = -\left(i\omega \mathbf{K}_{K,ib} \sum_{j=1}^{N_K} \frac{K_{coeff,j}}{i\omega + 1/\tau_{K,j}} + i\omega \mathbf{K}_{G,ib} \sum_{j=1}^{N_G} \frac{G_{coeff,j}}{i\omega + 1/\tau_{G,j}} + \mathbf{K}_{e,ib} \right)$$

- **Resulting complex, pseudo-static modes split into real and imaginary parts**

$$\mathbf{\Psi}_{PSCM} = \left[\mathrm{Re} \begin{Bmatrix} \mathbf{X}_i \\ \mathbf{I}_{bb} \end{Bmatrix} \quad \mathrm{Im} \begin{Bmatrix} \mathbf{X}_i \\ \mathbf{I}_{bb} \end{Bmatrix} \right]$$

Assembly of Viscoelastic CB-ROMs

- **Subcomponent transformation matrix defined as**

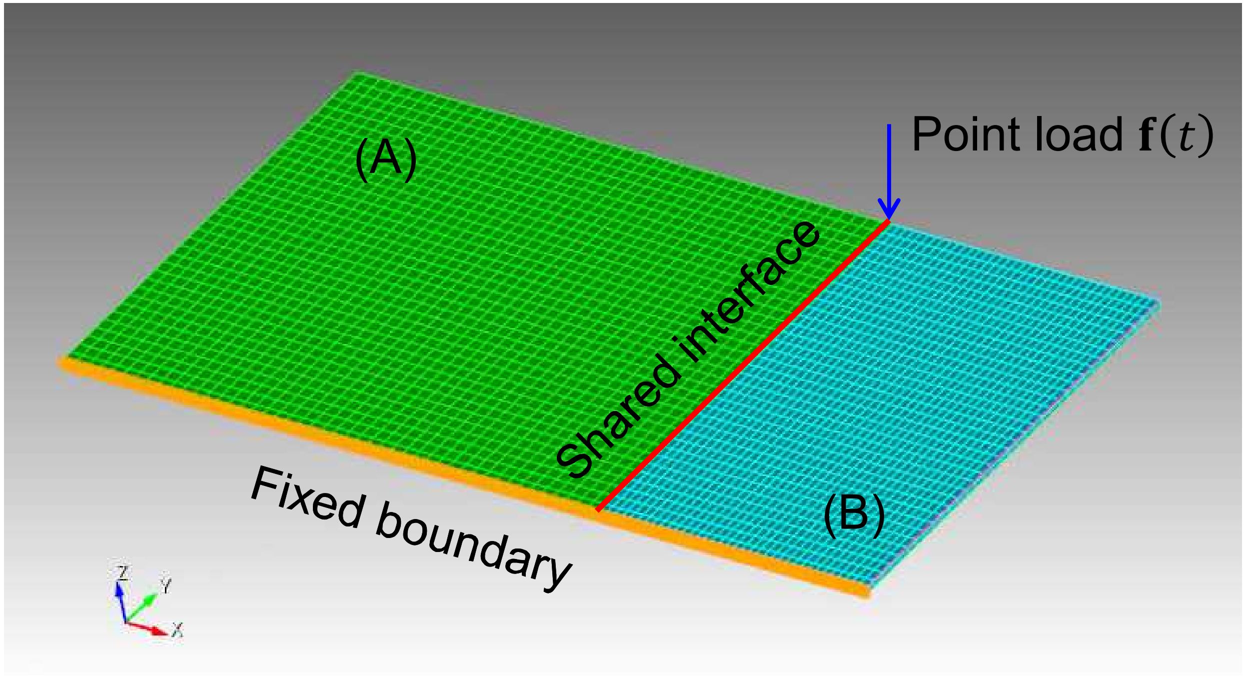
$$\begin{Bmatrix} \mathbf{x}_i \\ \mathbf{x}_b \end{Bmatrix} = \begin{bmatrix} \mathrm{Re}(\mathbf{\Phi}) & \mathrm{Im}(\mathbf{\Phi}) & \mathrm{Im}(\mathbf{X}_i) & \mathrm{Re}(\mathbf{X}_i) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_k^r \\ \mathbf{q}_k^i \\ \mathbf{x}_b^i \\ \mathbf{x}_b^r \end{Bmatrix} = \mathbf{T}_{CB}\mathbf{q}$$

$$\hat{\mathbf{M}}\ddot{\mathbf{q}} + \hat{\mathbf{K}}_K \int_0^t \zeta_K(t-\tau)\dot{\mathbf{q}}(\tau)d\tau + \hat{\mathbf{K}}_G \int_0^t \zeta_G(t-\tau)\dot{\mathbf{q}}(\tau)d\tau + \hat{\mathbf{K}}_e\mathbf{q} = \mathbf{T}_{CB}^{\mathrm{T}}\mathbf{f}_{ext}(t)$$

- **Primal assembly through constraint applied to real part of pseudo-static constraint mode**

$$\begin{Bmatrix} \mathbf{q}^{(\mathrm{A})} \\ \mathbf{q}^{(\mathrm{B})} \end{Bmatrix} = \begin{Bmatrix} \mathbf{q}_k^{r,(\mathrm{A})} \\ \mathbf{q}_k^{i,(\mathrm{A})} \\ \mathbf{x}_b^{i,(\mathrm{A})} \\ \mathbf{x}_b^{r,(\mathrm{A})} \\ \mathbf{q}_k^{r,(\mathrm{B})} \\ \mathbf{q}_k^{i,(\mathrm{B})} \\ \mathbf{x}_b^{i,(\mathrm{B})} \\ \mathbf{x}_b^{r,(\mathrm{B})} \end{Bmatrix} = \mathbf{L} \begin{Bmatrix} \mathbf{q}_k^{r,(\mathrm{A})} \\ \mathbf{q}_k^{i,(\mathrm{A})} \\ \mathbf{x}_b^{i,(\mathrm{A})} \\ \mathbf{q}_k^{r,(\mathrm{B})} \\ \mathbf{q}_k^{i,(\mathrm{B})} \\ \mathbf{x}_b^{i,(\mathrm{B})} \\ \mathbf{x}_b^{r,(\mathrm{B})} \end{Bmatrix} = \mathbf{L}\ddot{\mathbf{q}}_u$$

Application to Plate Models in Sierra/SD



- **Viscoelastic Sandwich Plates**
  - Aluminum 6061-T6 (linear elastic)
  - PMDI 22 foam (linear viscoelastic)

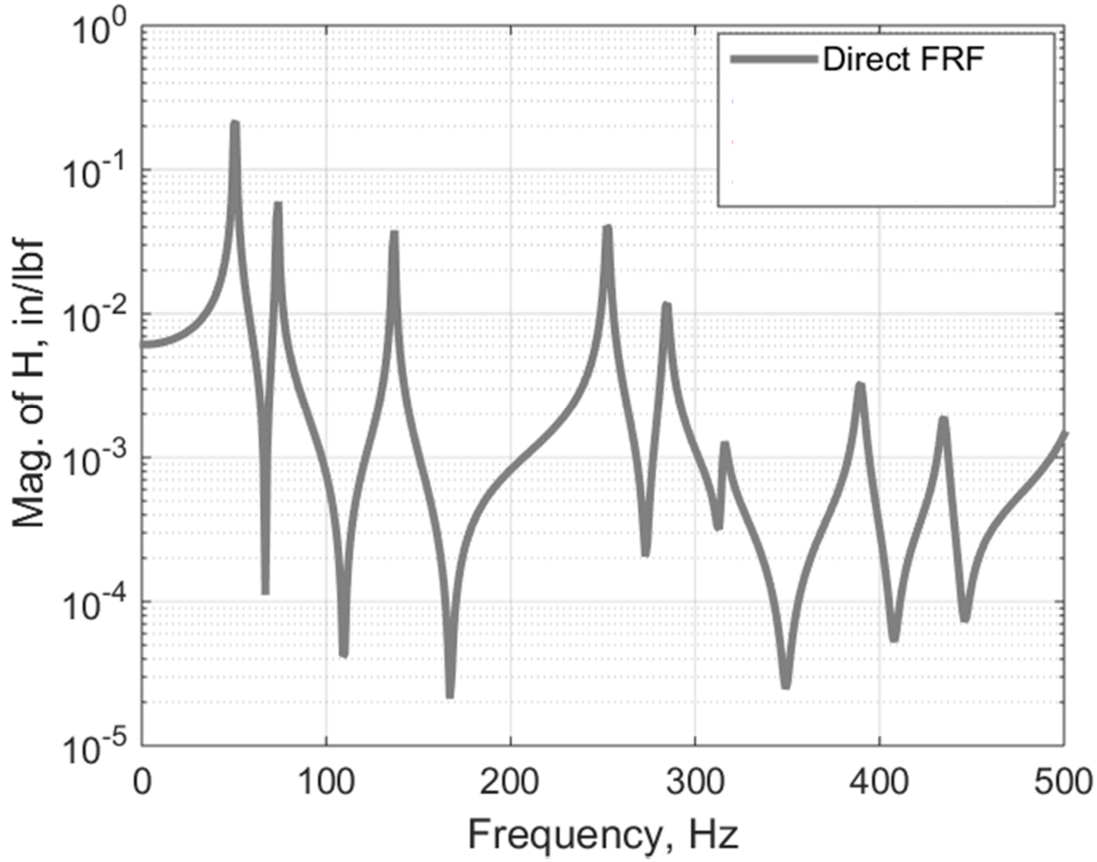
Modal Bases

- Goal: predict response up to 500 Hz
  - Rule of thumb: keep modes up to 2x frequency of interest
  - 1155 interface DOF

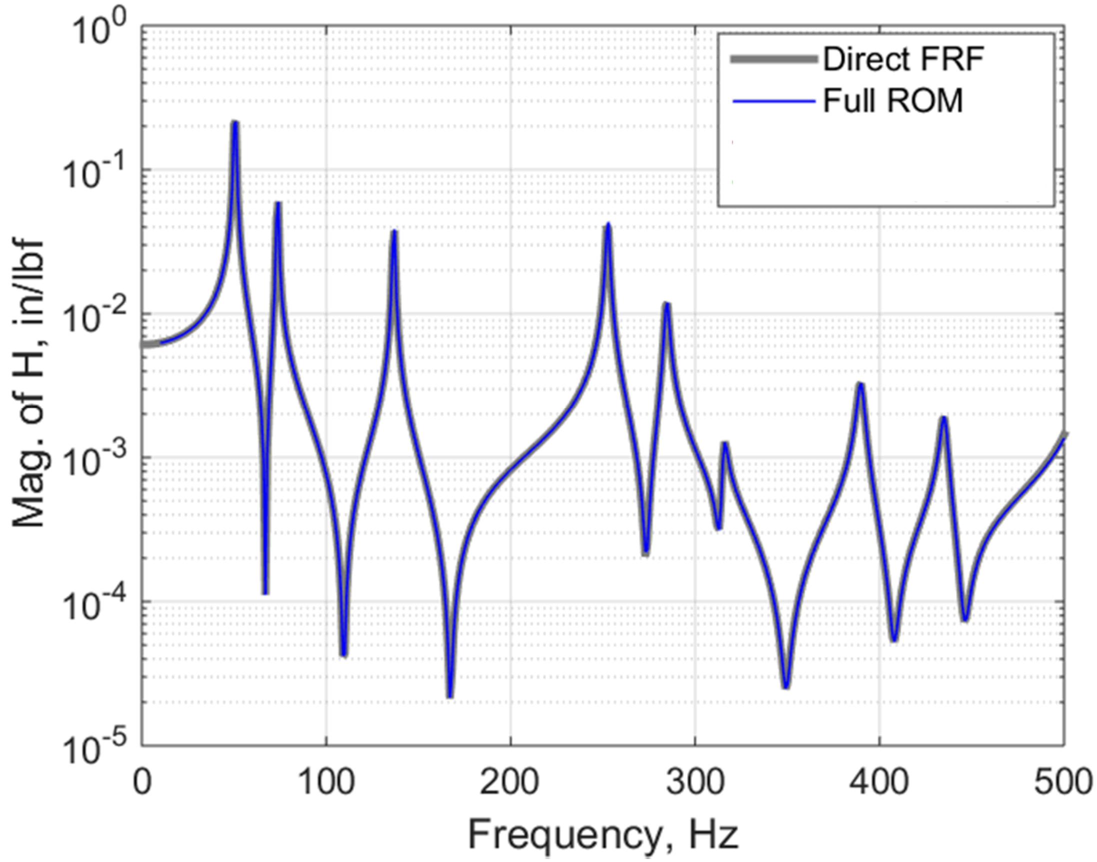
$$f_n = \frac{Im(\lambda)}{2\pi}$$

Fixed-Interface Mode	**Subcomponent (A)- 16”x12” plate**	**Subcomponent (B)- 8”x12” plate**
1	74.5 Hz	148.8 Hz
2	205.3 Hz	362.5 Hz
3	304.5 Hz	602.5 Hz
4	439.3 Hz	756.5 Hz
5	465.6 Hz	822.4 Hz
6	676.1 Hz	-
7	720.1 Hz	-
8	792.4 Hz	-
9	836.1 Hz	-
10	980.5 Hz	-

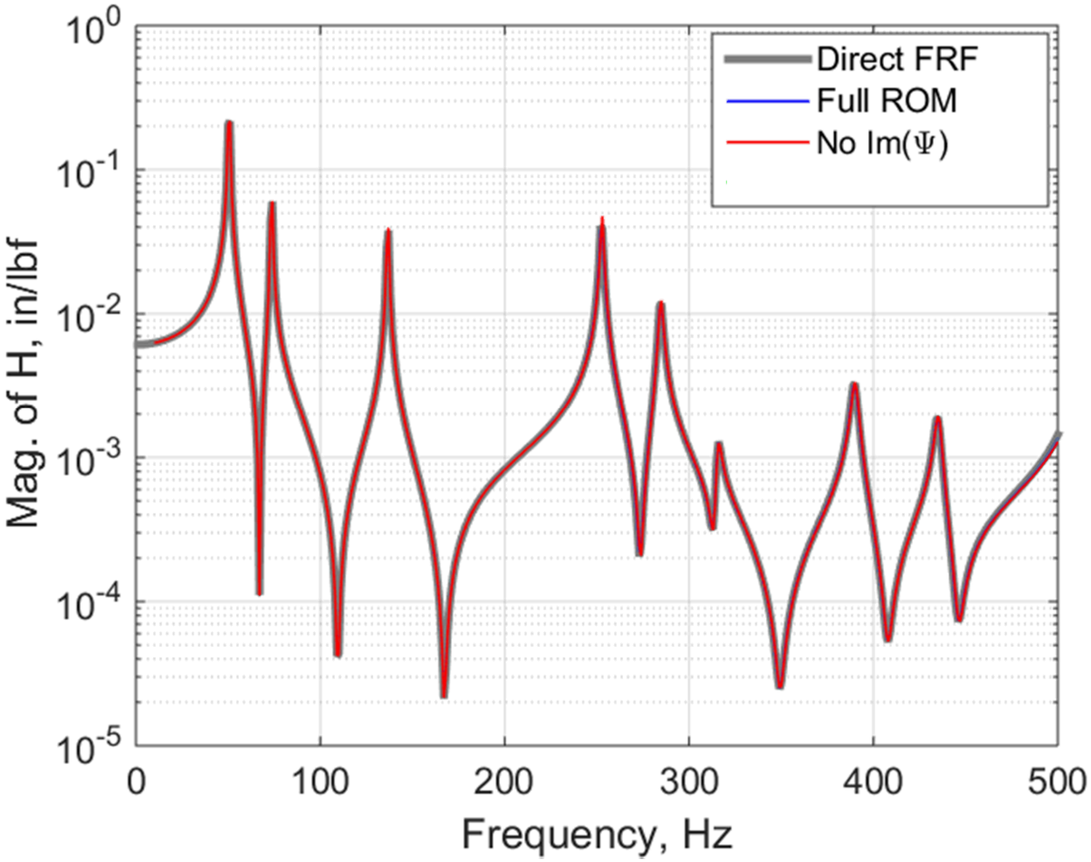
FRF Results Comparison



FRF Results Comparison

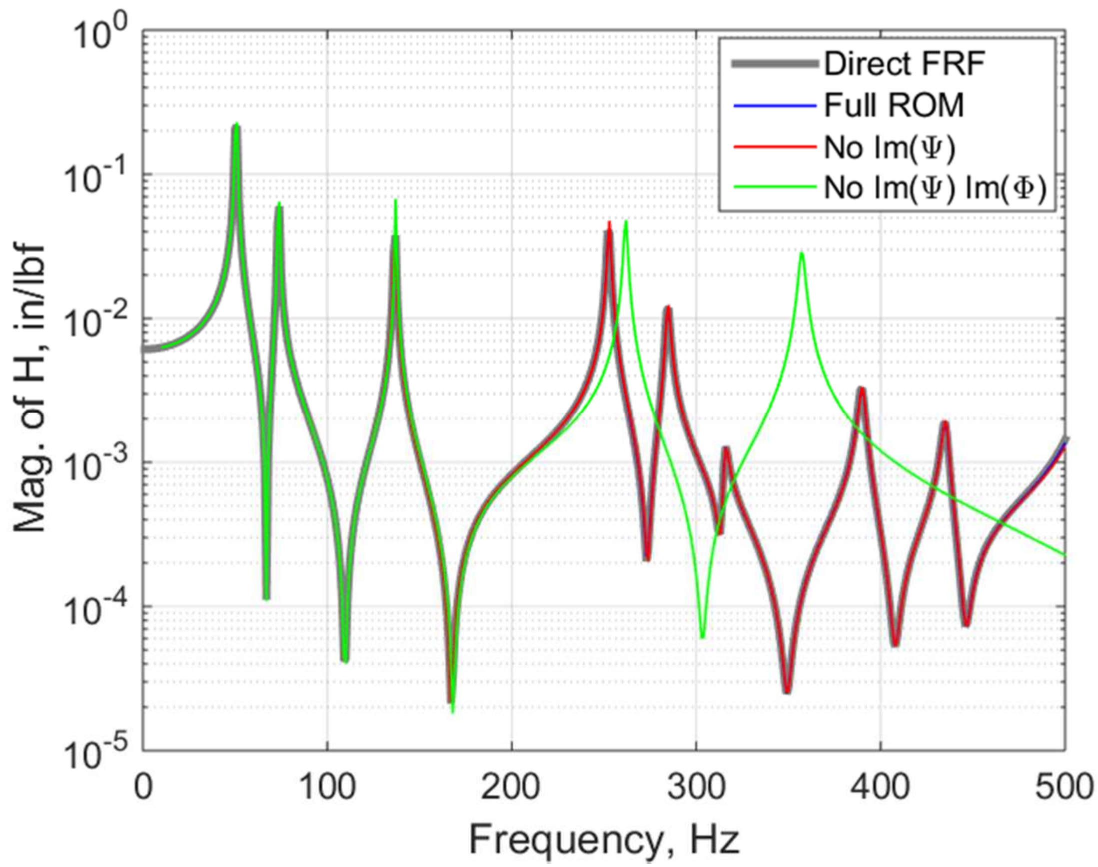


FRF Results Comparison



$$\begin{Bmatrix} \mathbf{q}^{(A)} \\ \mathbf{q}^{(B)} \end{Bmatrix} = \begin{Bmatrix} \mathbf{q}_k^{r,(A)} \\ \mathbf{q}_k^{i,(A)} \\ \cancel{\mathbf{x}_b^{i,(A)}} \\ \mathbf{x}_b^{r,(A)} \\ \mathbf{q}_k^{r,(B)} \\ \mathbf{q}_k^{i,(B)} \\ \cancel{\mathbf{x}_b^{i,(B)}} \\ \mathbf{x}_b^{r,(B)} \end{Bmatrix} = \mathbf{L} \begin{Bmatrix} \mathbf{q}_k^{r,(A)} \\ \mathbf{q}_k^{i,(A)} \\ \cancel{\mathbf{x}_b^{i,(A)}} \\ \mathbf{q}_k^{r,(B)} \\ \mathbf{q}_k^{i,(B)} \\ \cancel{\mathbf{x}_b^{i,(B)}} \\ \mathbf{x}_b^{r,(B)} \end{Bmatrix} = \mathbf{L}\ddot{\mathbf{q}}_u$$

FRF Results Comparison



$$\begin{Bmatrix} \mathbf{q}^{(A)} \\ \mathbf{q}^{(B)} \end{Bmatrix} = \begin{Bmatrix} \mathbf{q}_k^{r,(A)} \\ \cancel{\mathbf{q}_k^{i,(A)}} \\ \cancel{\mathbf{x}_b^{i,(A)}} \\ \mathbf{x}_b^{r,(A)} \\ \mathbf{q}_k^{r,(B)} \\ \cancel{\mathbf{q}_k^{i,(B)}} \\ \cancel{\mathbf{x}_b^{i,(B)}} \\ \mathbf{x}_b^{r,(B)} \end{Bmatrix} = \mathbf{L} \begin{Bmatrix} \mathbf{q}_k^{r,(A)} \\ \cancel{\mathbf{q}_k^{i,(A)}} \\ \cancel{\mathbf{x}_b^{i,(A)}} \\ \mathbf{q}_k^{r,(B)} \\ \cancel{\mathbf{q}_k^{i,(B)}} \\ \cancel{\mathbf{x}_b^{i,(B)}} \\ \mathbf{x}_b^{r,(B)} \end{Bmatrix} = \mathbf{L}\ddot{\mathbf{q}}_u$$

FRF Results Comparison

FRFs of Plate Assembly		
Model	**Number of DOF**	**Solution Time**
Full FEA model	130,305	~27,300 s
Full ROM	3,495	1,834 s
No $\mathbf{Im(\Psi)}$	1,185	117.7 s
No $\mathbf{Im(\Psi)}$ **or** $\mathbf{Im(\Phi)}$	1,170	108.4 s

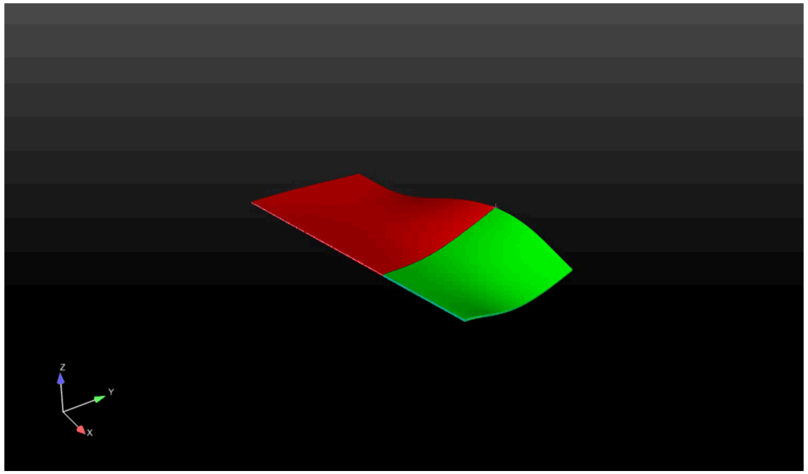
- ROM size determined by number of pseudo-static constraint modes

Conclusions

- Developed a **Craig-Bampton subsctructuring approach** to reduce the size of a finite element model with linear viscoelastic material behavior
  - Linearized complex fixed-interface modes
  - Pseudo-static constraint modes
- Mode selection based on **frequency of linearized complex mode** solution; FRF computed from ROM agrees well with full order model
- Obtained reduced assembly models with **non-viscous damping** from Prony series representation of the viscoelastic material

Any Questions?

- This research was supported by the Laboratory Directed Research and Development program at Sandia National Laboratories, a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.



- Contact Information
  - Rob Kuether: rjkueth@sandia.gov
  - Kevin Troyer: kltroye@sandia.gov

Extra Slides