

# THE FLUID MECHANICS OF POLYURETHANE FOAM EXPANSION AND POLYMERIZATION

Rekha Rao, Lisa Mondy, Christine Roberts, Melissa Soehnel, Kevin Long, Victor Brunini, David Noble, Tyler Voskuilen, Mathew Celina

*Sandia National Laboratories*

*Albuquerque, NM*

James Tinsley

*Honeywell Kansas City Plant*

*Kansas City, MO*

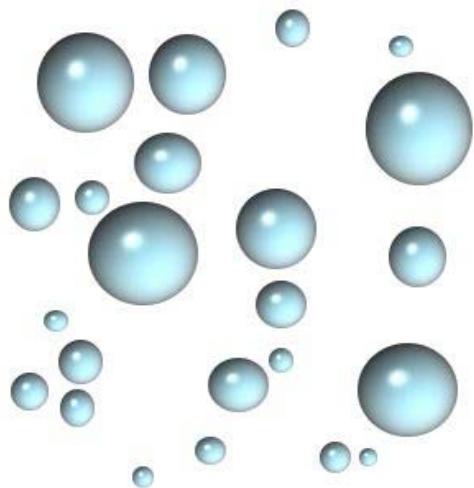
**XXIV International Congress on Theoretical and Applied Mechanics**  
Montreal, Canada  
August 21 - 26, 2016

SAND2016-????C

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# What is a Foam?

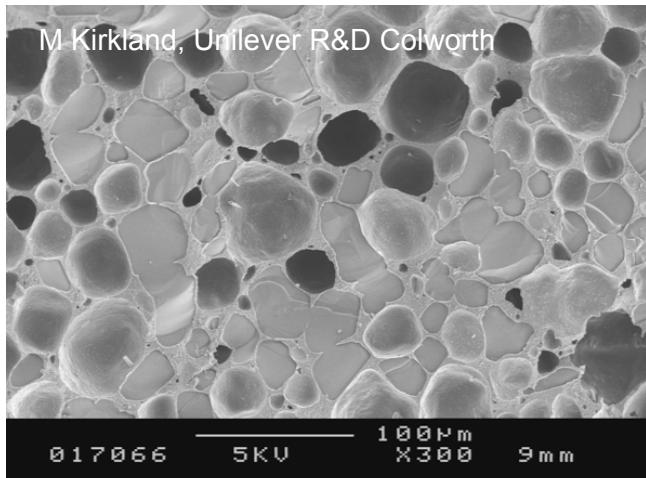


Bubbles

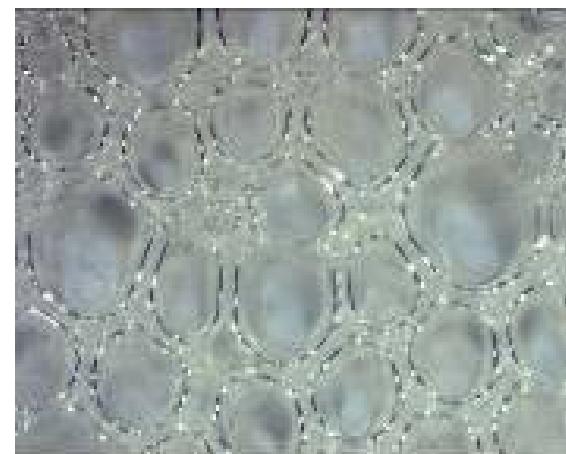


Whipped cream

- A multiphase material of gas bubbles in a liquid or solid matrix
- How do you make a foam?
  - Generate bubbles in a liquid
  - Stabilize them with particles, fat globules, or surfactant
  - Solidify liquid -freezing, polymerization, or phase change – if desired



Ice cream is a foam – that's why it is so much work to make



Epoxy foam is a collection of bubbles in polymer

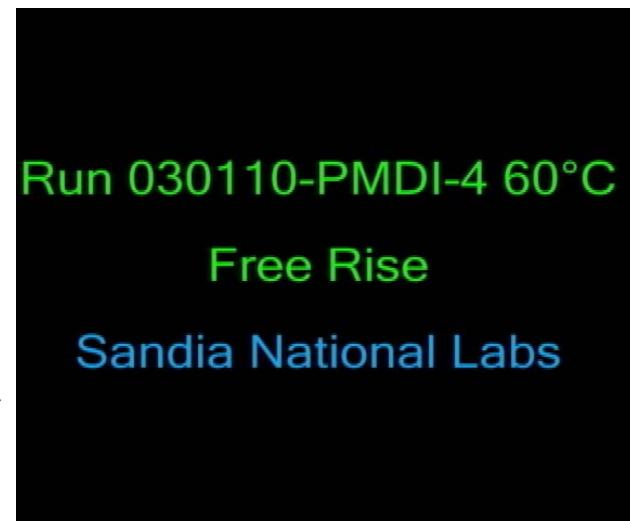
Foams need enough bubbles to jam, e.g. bubbles are touching or it is just a bubbly liquid

# Introduction

**Overarching Goal:** Cradle-to-grave model for foaming, vitrification, cure, aging  
Focus on moderate density PMDI foams



Injection,  
foaming and  
initial curing  
at lower T



Oven time  
at higher T  
to make  
sure it is  
fully cured



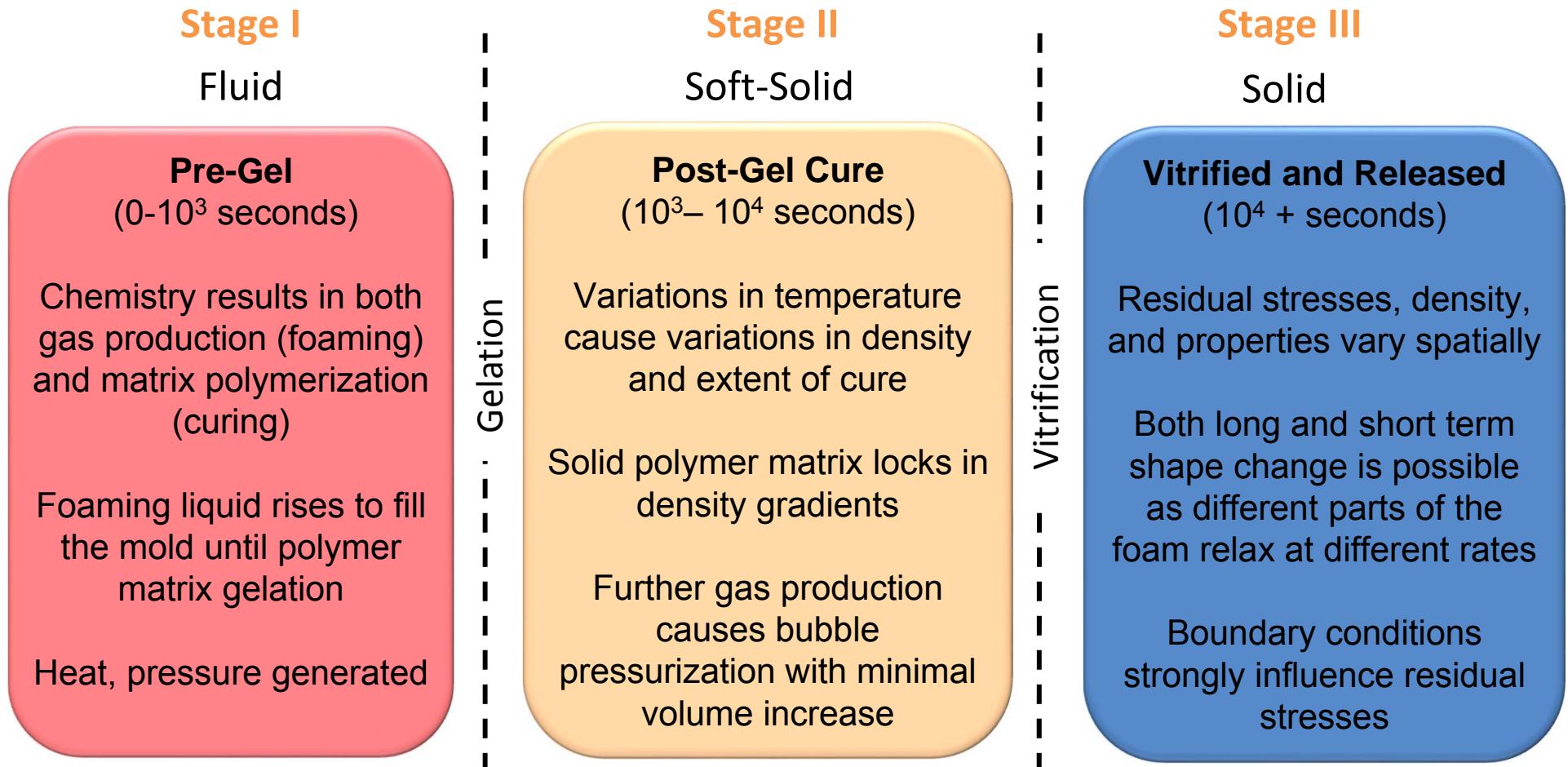
Remove  
from mold –  
predict cure  
and thermal  
stresses



Predict  
shape and  
size over  
years

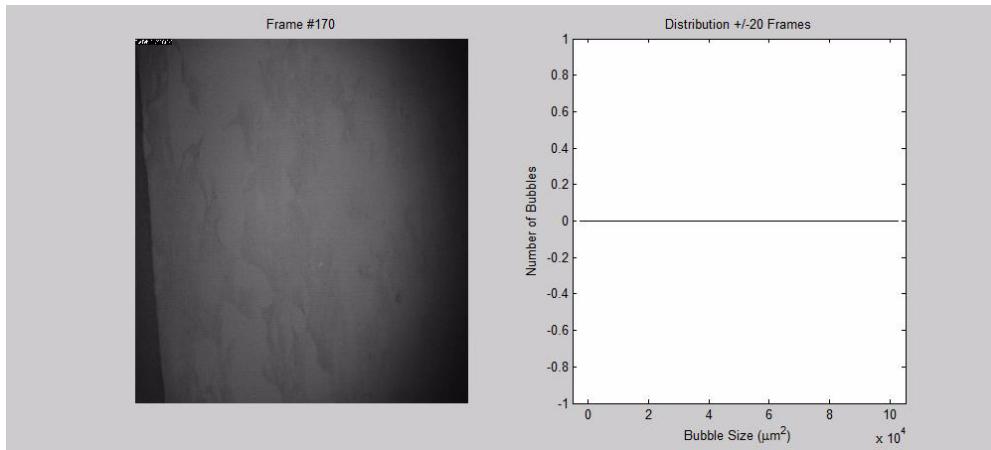


# Introduction

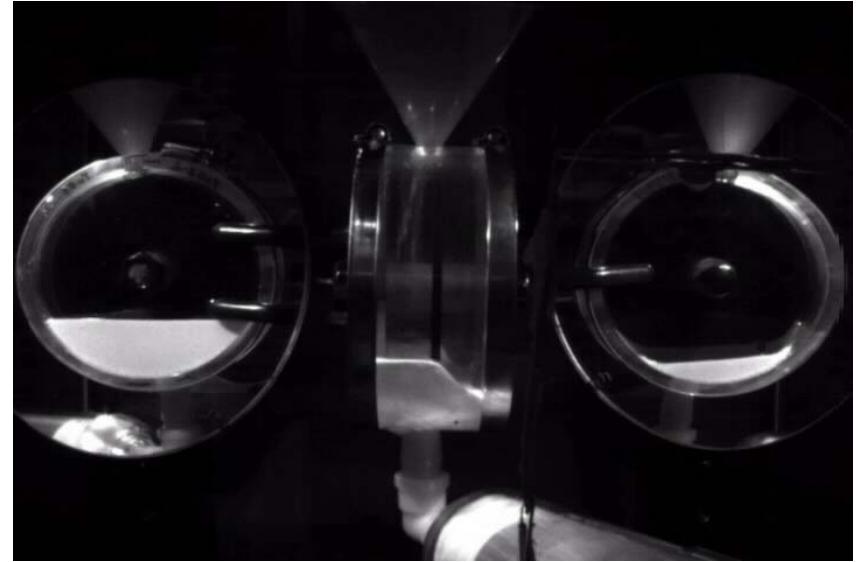


- Processing parameters at earlier stages will affect quality of part at later stages

# Foam Filling is Complex



Foam front moving past camera, with bubble sizes at transparent wall determined with image processing.



3 views of foam filling a mock AFS with several plates spaced unevenly. Vent location is critical to keep from trapping air.

- PMDI is used as an encapsulant for electronic components and lightweight structural parts, to mitigate against shock and vibration.
- **We would like to develop a computational model to help us understand foam expansion for manufacturing applications and how inhomogeneities effect the structural response of the final part, including long term shape stability.**
- Gas generation drives the foam expansion, changing the material from a viscous liquid to a multiphase material.
- Continuous phase is time- and temperature-dependent and eventually vitrifies to a solid.

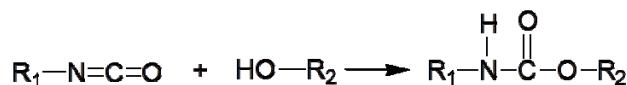
# Polyurethane (PMDI): Model Development

- We use a variety of physically and chemically blown foams. PMDI is used as an encapsulant for electronic components, to mitigate against shock and vibration, and to make light-weight structural parts.
- We would like to develop a computational model to help us understand foam expansion for manufacturing applications.
- Polyurethane is a chemically blown foam having two primary, competing simultaneous reactions:  $\text{CO}_2$  production and polymerization. Separating these reactions can be difficult.
- We use IR spectroscopy to track polymerization. IR does not provide a clear signal for the foaming reaction: Tracked with volume generation.

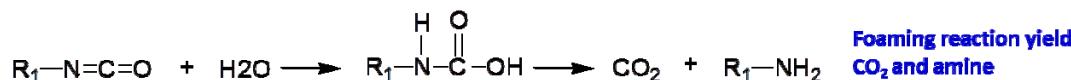


PU has a short pot-life: models can help reduce defects and improve filling process

## Two key reactions: Isocyanate reaction with polyols and water

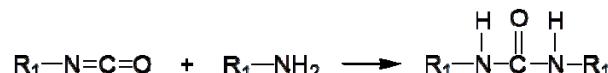


Urethane formation,  
crosslinking

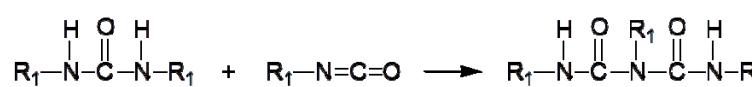


Foaming reaction yields  
 $\text{CO}_2$  and amine

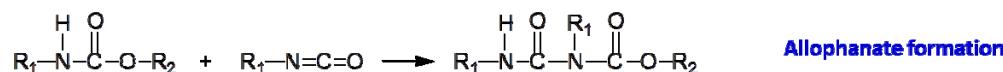
## Various follow up reactions: Isocyanate reaction with amine, urea and urethane



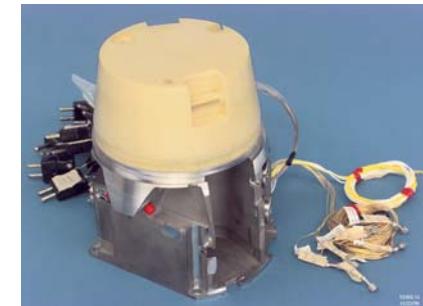
Urea formation



Bluret formation



Allophanate formation



Mock component encapsulated with PMDI from “KCP Encapsulation Design Guide” (Mike Gerdin, UUR)

# Kinetic Model Must Include CO<sub>2</sub> Generation and Polymerization Reaction

$$rate_1 = k_1 e^{-\Delta E_1 / RT} [isocyanate]^a [polyol]^b \quad \text{Polymerization}$$

$$rate_2 = k_2 e^{-\Delta E_2 / RT} [isocyanate]^c [H_2O]^d \quad \text{CO}_2 \text{ generation}$$

- Must track five species: water, polyol, polymer, carbon dioxide, and isocyanate, since we have competing primary reaction
- Use experiments to determine Arrhenius rate coefficients

$$\frac{D[CO_2]}{Dt} = +rate_2$$

$$\frac{D[H_2O]}{Dt} = -rate_2$$

$$\frac{D[isocyanate]}{Dt} = -rate_1 - rate_2$$

$$\frac{D[polyol]}{Dt} = -rate_1$$

$$\frac{D[polymer]}{Dt} = +rate_1$$

- Must provide initial conditions for all species
- Integrate rate equations as part of the simulation
- Density predicted from gas generation
- Our kinetics are unique because our formulation is different from literature polyurethanes

$$\rho_{gas} = \frac{PM_{CO_2}}{RT}$$

$$v = \frac{V_{gas}}{V_{liq}} = \frac{M_{CO_2} C_{CO_2}}{\rho_{gas}} \quad \phi_v = \frac{v}{1+v}$$

$$\rho_{foam} = \rho_{gas} \phi_v + \rho_{liq} (1 - \phi_v)$$

# Equations of Motion Include Evolving Material Models

Momentum equation and continuity have variable density, shear viscosity, and bulk viscosity

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\rho \mathbf{v} \bullet \nabla \mathbf{v} - \nabla p + \nabla \bullet (\mu_f (\nabla \mathbf{v} + \nabla \mathbf{v}^t)) - \nabla \bullet \lambda (\nabla \bullet \mathbf{v}) \mathbf{I} + \rho \mathbf{g}$$

$$\frac{D\rho_f}{Dt} + \rho_f \nabla \bullet \mathbf{v} = 0$$

Energy equation has variable heat capacity and thermal conductivity including a source term for heat of reaction for foaming and curing reactions

$$\rho C_{pf} \frac{\partial T}{\partial t} + \rho C_{pf} \mathbf{v} \bullet \nabla T = \nabla \bullet (k \nabla T) + \rho \varphi_e \Delta H_{rxn} \frac{\partial \xi}{\partial t}$$

Extent of reaction equation for polymerization: condensation chemistry

$$\frac{\partial \xi}{\partial t} = \left( \frac{1}{(1+wa)^\beta} \right) \left( k_0 \exp\left(-\frac{E}{RT}\right) \right) (b + \xi^m) (1 - \xi)^n$$

Molar concentration equations for water and carbon dioxide

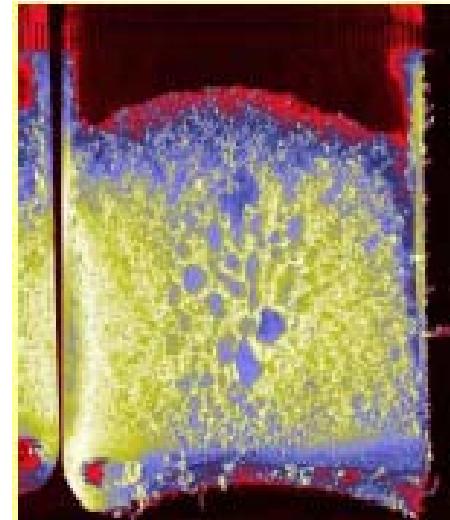
$$\frac{dC_{H_2O}}{dt} = -k_{H_2O} C_{H_2O}^n$$

$$C_{H_2O} = \frac{\rho_{foam} x_{H_2O}}{M_{H_2O}}$$

$$\frac{dC_{CO_2}}{dt} = +k_{H_2O} C_{H_2O}^n$$

$$C_{CO_2} = \frac{\rho_{foam} x_{CO_2}}{M_{CO_2}}$$

$$k_{H_2O} = A_{H_2O} \exp(-E_{H_2O} / RT)$$



NMR imaging shows coarse microstructure (Altobelli, 2006)

# Complex Material Models Vary with Cure, Temperature, and Gas Fraction

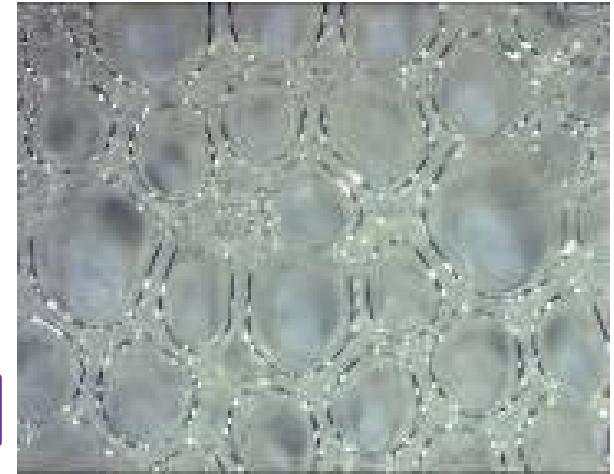
Foaming reaction predicts moles of gas from which we can calculate density

$$\rho_{gas} = \frac{PM_{CO_2}}{RT}$$

$$v = \frac{V_{gas}}{V_{liq}} = \frac{M_{CO_2} C_{CO_2}}{\rho_{gas}} \quad \phi_v = \frac{v}{1+v}$$

$$\rho_{foam} = \rho_{gas} \phi_v + \rho_{liq} (1 - \phi_v)$$

Compressibility built into this model via the ideal gas law for gas density



Thermal properties depend on gas volume fraction and polymer properties

$$k = \frac{2}{3} \left( \frac{\rho}{\rho_e} \right) k_e + \left( 1 - \frac{\rho}{\rho_e} \right) k_v$$

$$C_{pf} = C_{pl} \phi_l + C_{pv} \phi_v + C_{pe} \phi_e$$

Shear and bulk viscosity depends on gas volume fraction, temperature and degree of cure

$$\mu = \mu_0 \exp\left(\frac{\phi_v}{1-\phi_v}\right) \quad \mu_0 = \mu_0^0 \exp\left(\frac{E_\mu}{RT}\right) \left(\frac{\xi_c^p - \xi^p}{\xi_c^p}\right)^{-q}$$
$$\lambda = \frac{4}{3} \mu_0 \frac{(\phi_v - 1)}{\phi_v}$$

M. Mooney, *J. Colloid Sci.*, **6**, 162-170 (1951).

Foam is a collection of bubbles in curing polymer

- Experiments to determine foaming and curing kinetics as well as parameters for model
- Equations solved with the finite element method using a level set to determine the location of the free surface (Rao et al., IJNMF, 2012)

Gibson, L. J.; M. F. Ashby. Cambridge University Press, Cambridge, UK, 1990

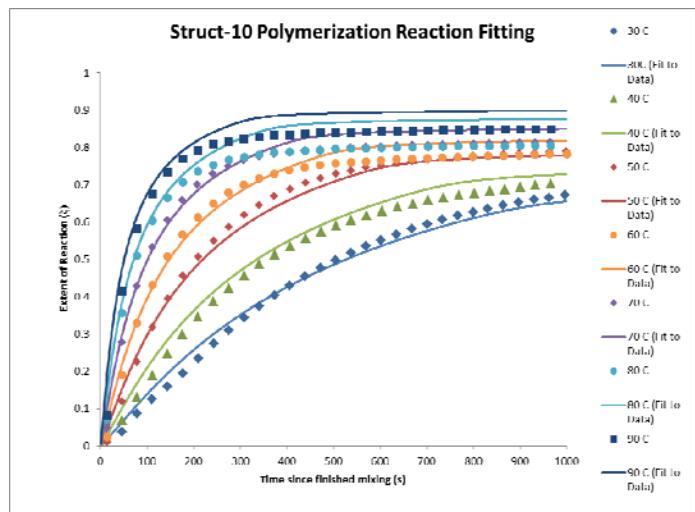
# Extent of Reaction for Polymerization

- Fit the rate and the extent of reaction to IR data to a standard equation form
- Fit  $T_g$  to both rheology and DSC data:  $T_g$  changes as cure progresses making this complex

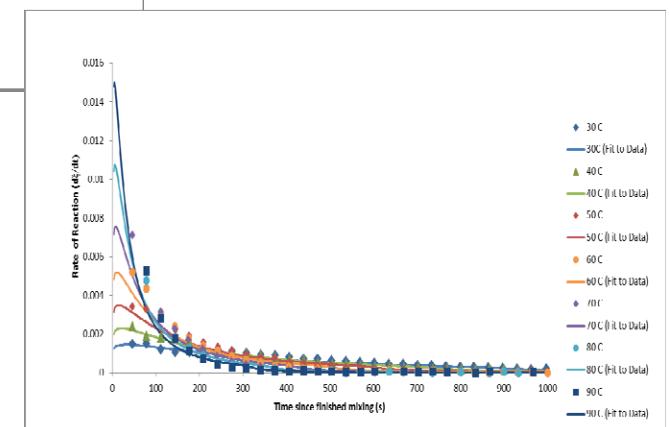
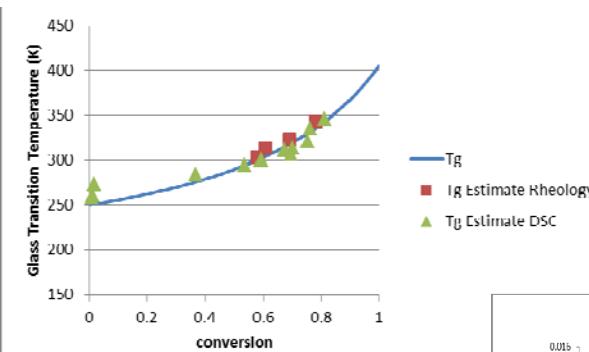
$$\frac{\partial \xi}{\partial t} = \left( \frac{1}{(1+wa)^\beta} \right) \left( k_0 \exp\left(-\frac{E}{RT}\right) \right) (b + \xi^m) (1 - \xi)^n$$

$$\log_{10} a = \frac{-C_1(T - T_g)}{C_2 + T - T_g}$$

$$T_g = \frac{T_{g0}(1 - \xi) + A\xi T_{g\infty}}{(1 - \xi + A\xi)}$$



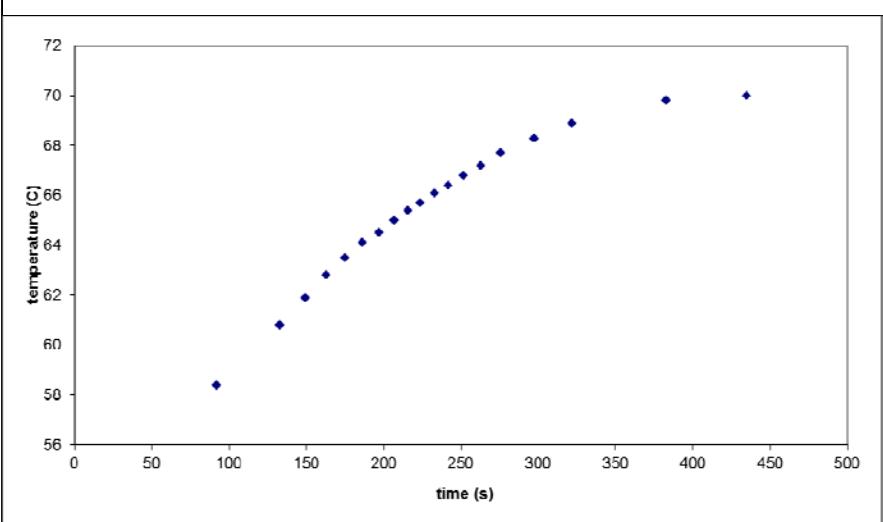
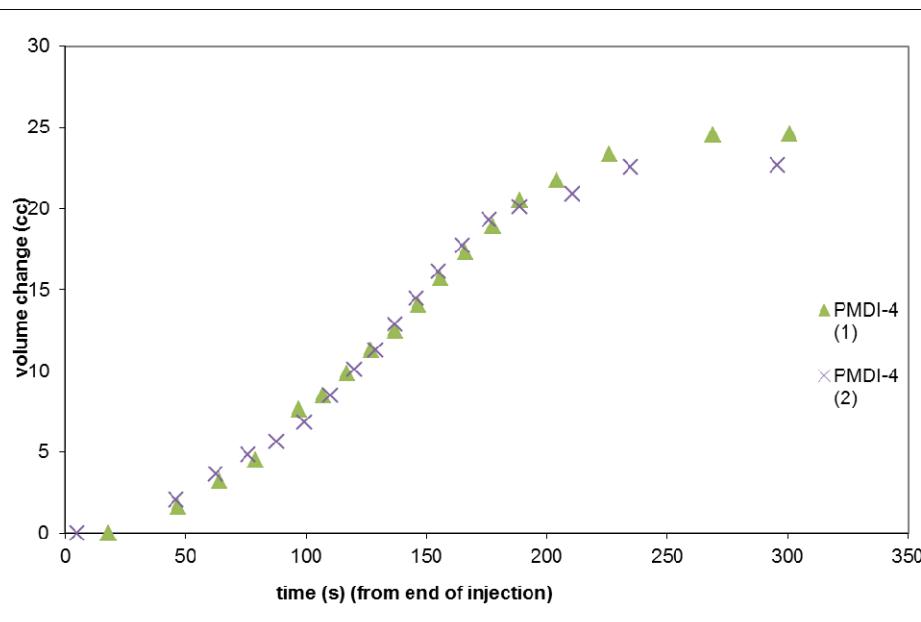
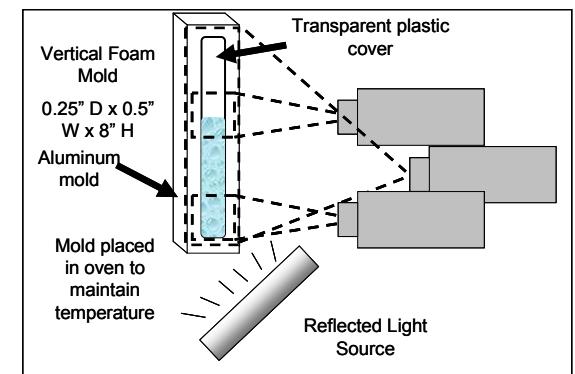
- New form captures arrest of reaction below the glass transition temperature ( $T_g$  evolves with extent of reaction)



Rate and extent of reaction fit to data, where parameters of the model, including  $T_g$  are optimized for lower temperatures expected in the process. The apparent time-to-gel from rheology is correlated with extent to give a  $T_g$  with conversion. Similar analysis can be done with DSC and results are consistent.

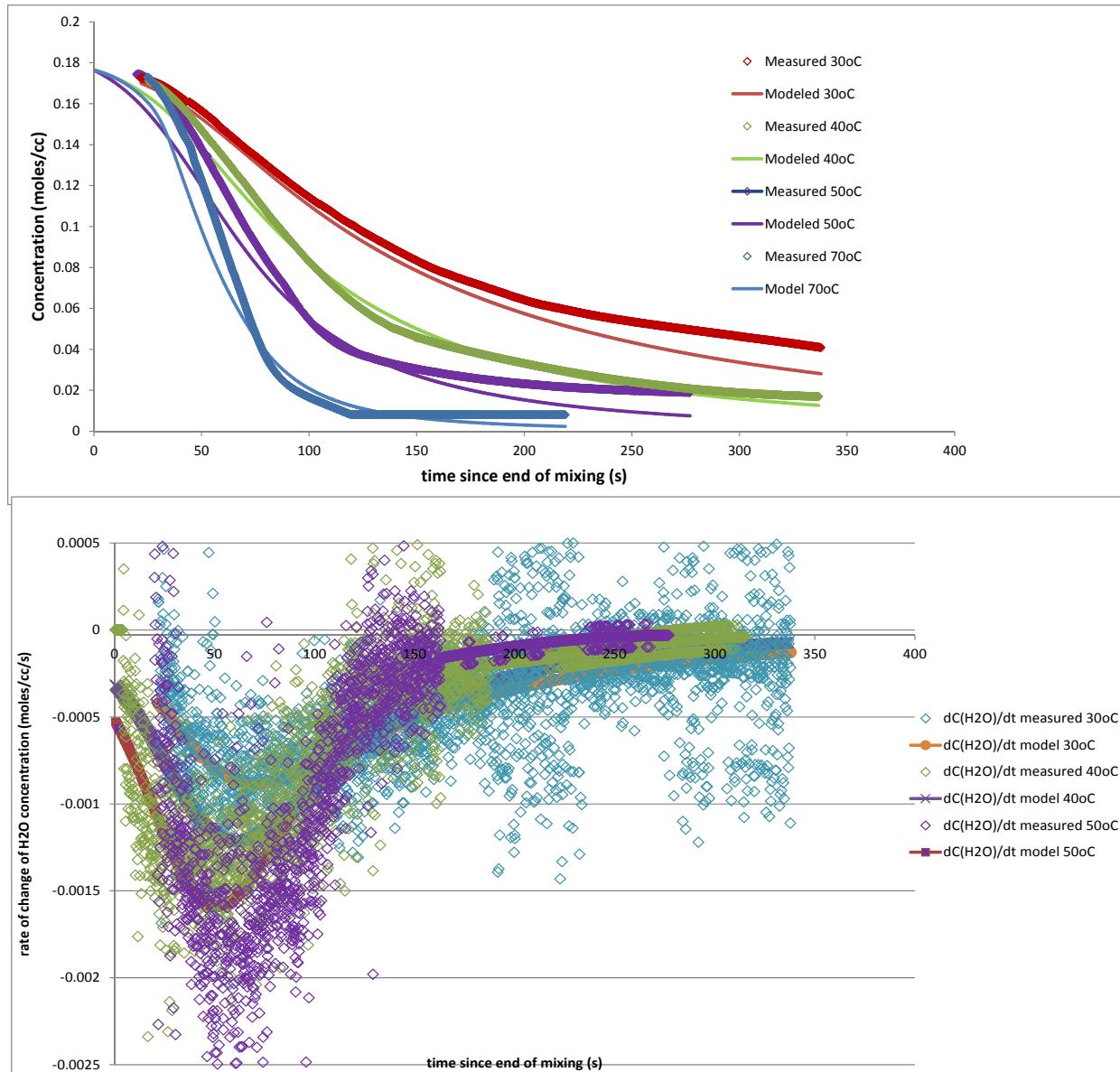
# Measure Height Change in Simple Geometry to Quantify Foaming Reaction

- Data have most uncertainty at early times because reaction is occurring during mixing and injections, but bubbles are being destroyed in these processes, too.
- We can only measure height change after these processes.
- CO<sub>2</sub> loss from bubble breakage at top surface? BUT bottom line: engineering model to predict volume change
- The foam cannot be preheated, so during the foam rise the temperature is not steady.



# Kinetics of CO<sub>2</sub> Generation

- Fit the concentration of water and its rate of disappearance simultaneously



$$\frac{dC_{H_2O}}{dt} = -Nk_{H_2O} C_{H_2O}^n$$

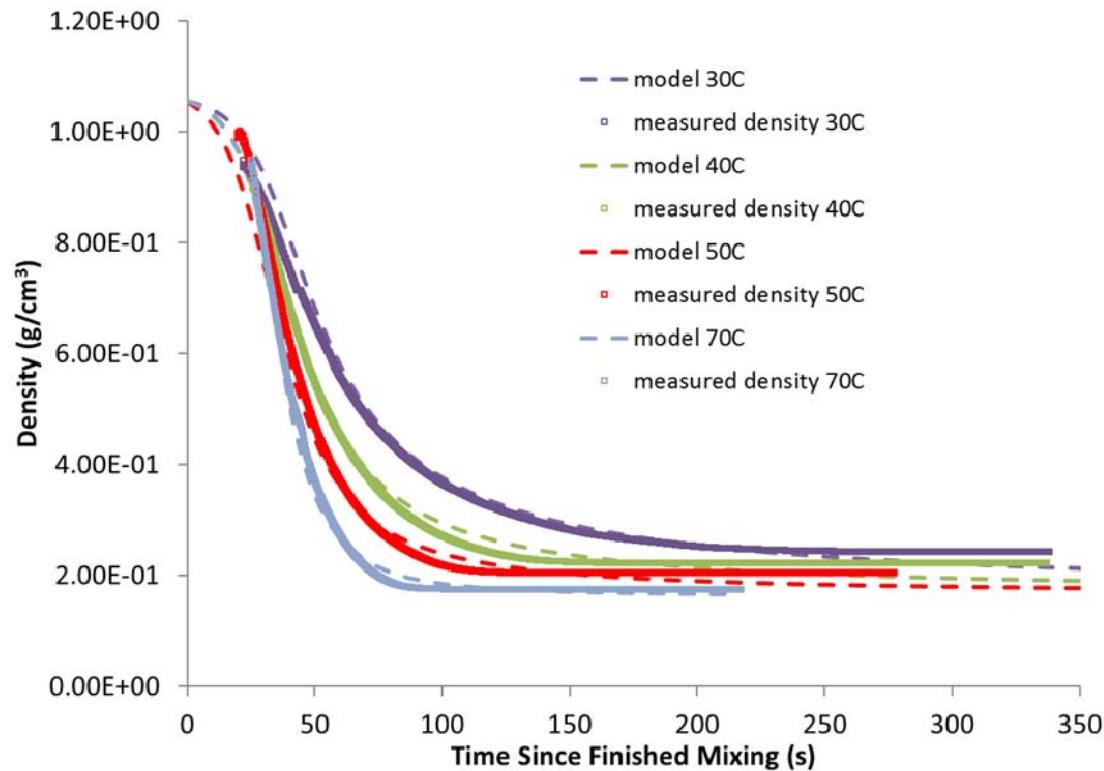
$$\frac{dC_{CO_2}}{dt} = +Nk_{H_2O} C_{H_2O}^n$$

$$N = 0.5 \left\{ 1 + \tanh \left( \frac{t - t_{nucleation}}{t_{scale}} \right) \right\}$$

- Apparent water concentration shows a change in slope
- Model must capture this
- Physically it relates to the solubility of the carbon dioxide in the polymer
- Must super saturate before nucleation and growth

# Kinetics of CO<sub>2</sub> Generation

Predictions of density using a nucleation time of 40s and a time scale of 20s compared to measured density with time in the channel for various temperatures.



$$\frac{dC_{H_2O}}{dt} = -Nk_{H_2O} C_{H_2O}^n$$
$$\frac{dC_{CO_2}}{dt} = +Nk_{H_2O} C_{H_2O}^n$$

- Experiments give us average density
- Hard to determine evolving density gradients
- Measure density gradients from post-test experiments

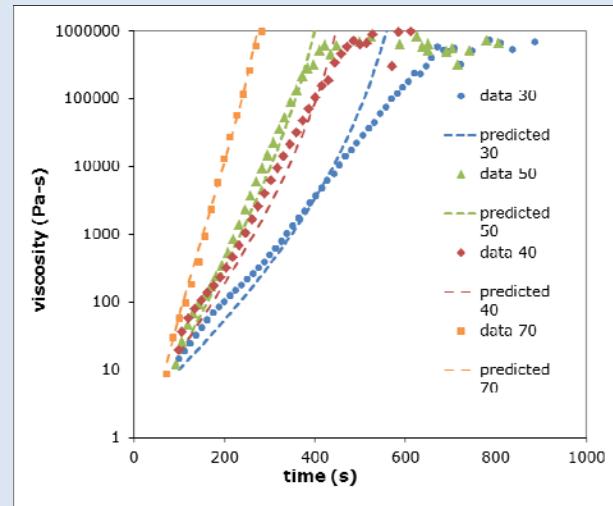
# Model Foam Viscosity as $f(\xi, \phi)$

## Start with continuous phase viscosity only

- IR kinetics + dry formulation rheology (two sets of experiments) give an approximation of the curing continuous phase rheology
- Relate time of gel point to  $\xi$  to find  $\xi_c$ .

$$\mu_{polymer} = \mu_0^0 \left( \frac{\xi_c - \xi}{\xi_c} \right)^{-6} \quad \xi_c = 0.86$$

$$\mu_0^0 = 600 e^{-1549/RT} \text{ Pa-s}$$

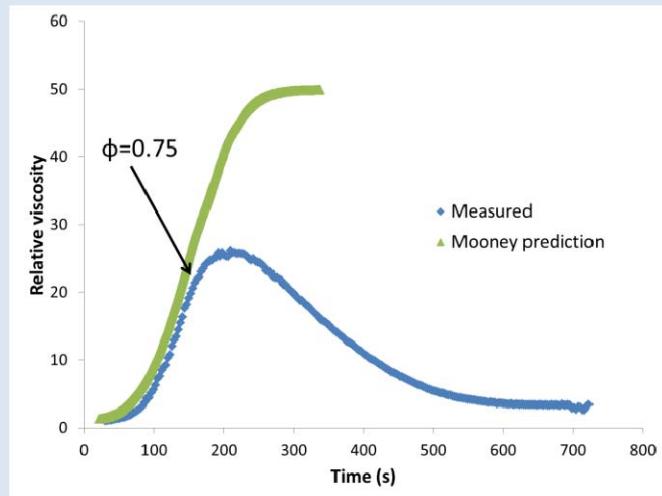


## Relate foam viscosity to continuous phase viscosity

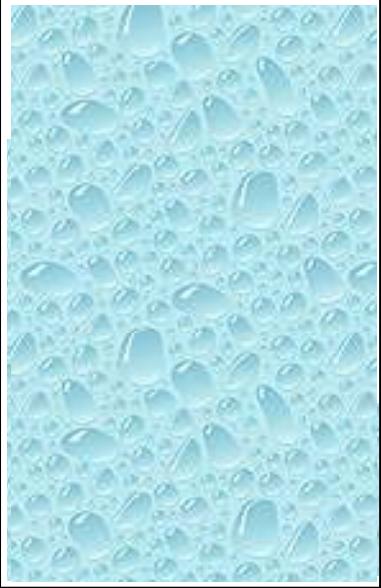
- Foam rise + wet formulation rheology (two sets of experiments) give an approximation of the rheology as a function of gas fraction
- Mooney prediction (for  $\phi_{gas} < 0.5$ )

$$\mu_\phi = \mu_{polymer} \exp \left( \frac{\phi_g}{1 - \phi_g} \right)$$

- For  $\phi_{gas} > 0.75$  estimate  $\mu_{foam} = \mu_{cure} * f(\xi)$



# Coupled Finite Element Method/Level Set to Solve Foam Dynamics



- Given fluid velocity field,  $u(x,y,z)$ , evolution on a fixed mesh is according to:

$$\frac{\partial \phi}{\partial t} + u \cdot \nabla \phi = 0$$

- Purely hyperbolic equation ... fluid particles on  $\phi(x,y,z) = 0$  should stay on this contour indefinitely
  - Does not preserve  $\phi(x,y,z)$  as a distance function
  - Introduces renormalization step.

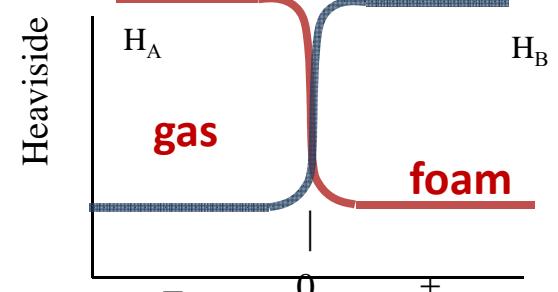
- Equations of motion, kinetics and energy balance averaged based on level set,  $\phi$

$$H_A \rho_A \frac{D u}{D t} + H_B \rho_B \frac{D u}{D t} = -\nabla P + H_A \nabla \cdot (\mu_A \dot{\gamma}) + H_B \nabla \cdot (\mu_B \dot{\gamma}) + (H_A \rho_A + H_B \rho_B) g + I.T.,$$

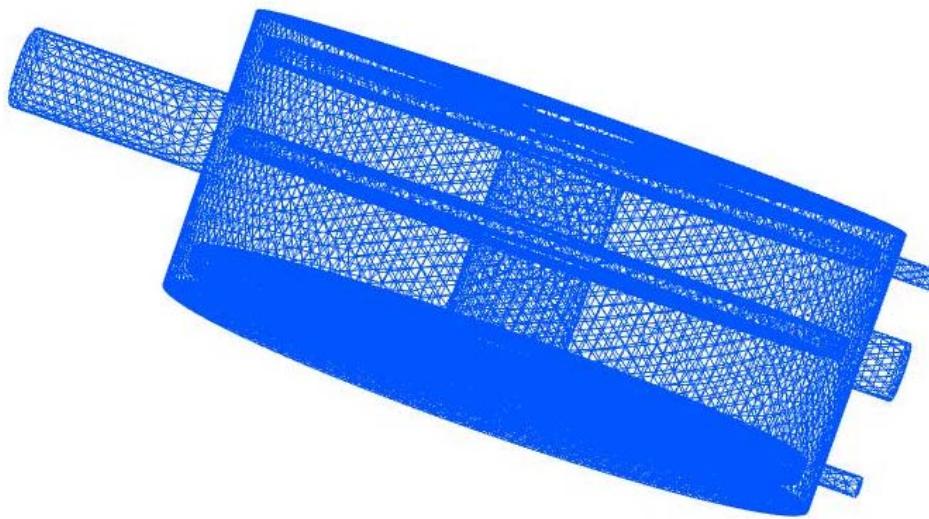
$$H_A \frac{D \rho_A}{D t} + H_B \frac{D \rho_B}{D t} + (H_A \rho_A + H_B \rho_B) \nabla \cdot u = 0$$

$$H_A + H_B = 1$$

Rao et al, IJNMF, 2012



# Idealized Foam Encapsulation Part: Board Would Contain Electronics in Real Part

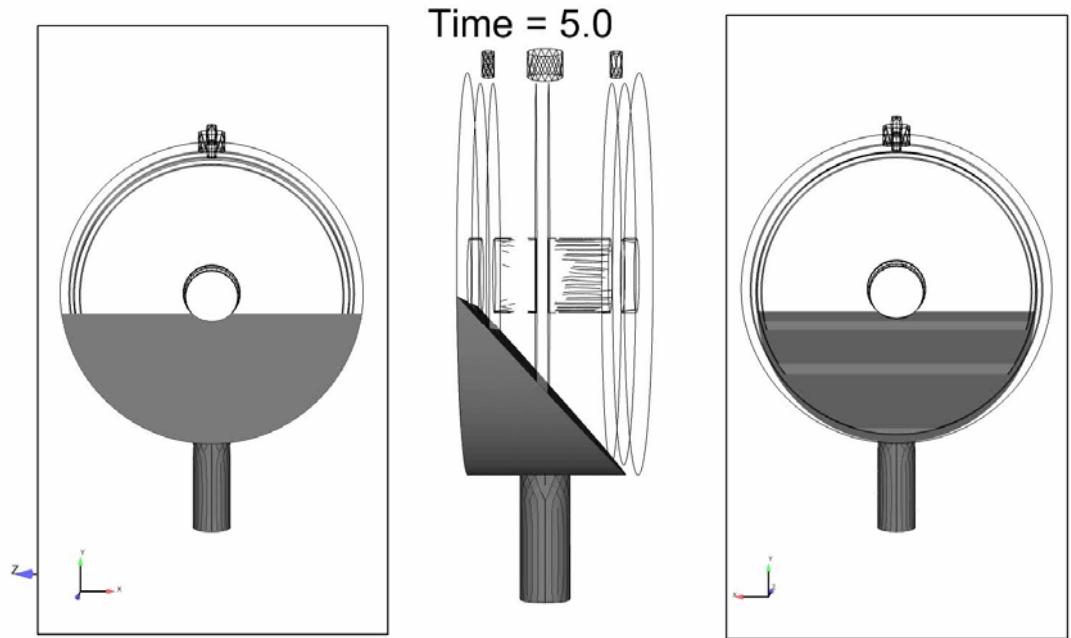
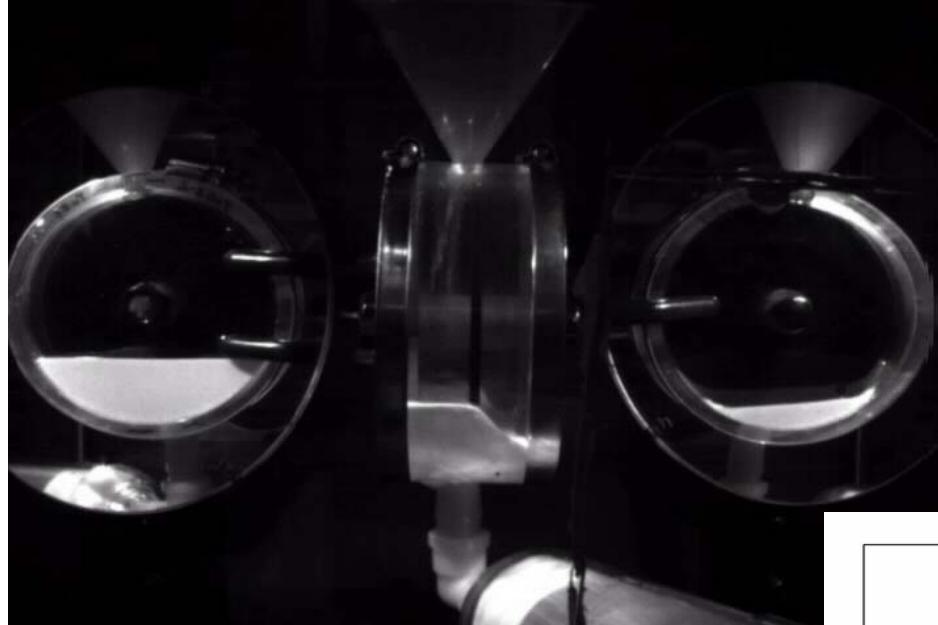


- Mold is preheated to ten degrees hotter than the foam
- Inflow is asymmetric and fills thinner area first
- Boards have different thicknesses of foam
- Three vents are used to improve filling
- Foam slips at the wall using a Navier slip condition with Beta = .001
- Gas slips ten times more than the foam

$$n \cdot \underline{\tau} \cdot n = \frac{1}{\beta} (v - v_s) \cdot n$$

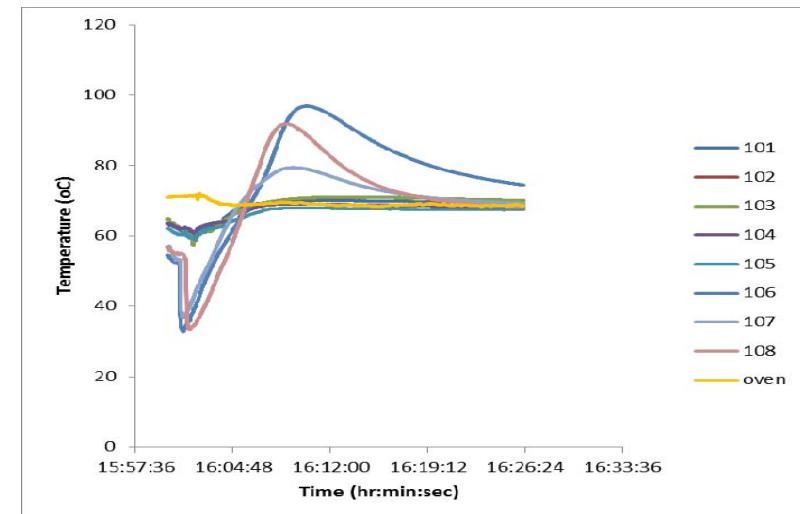
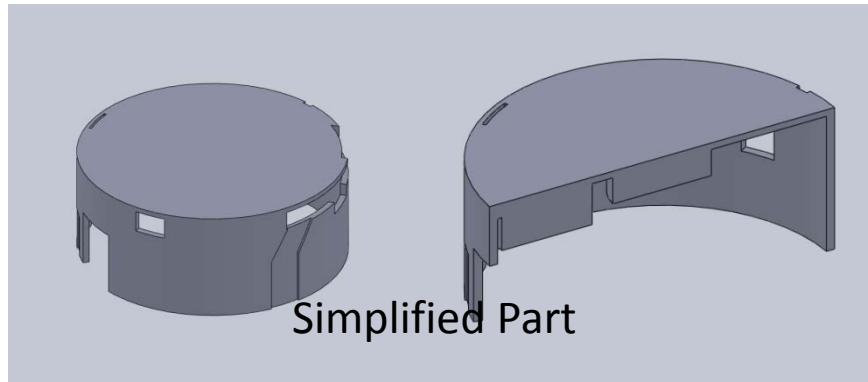
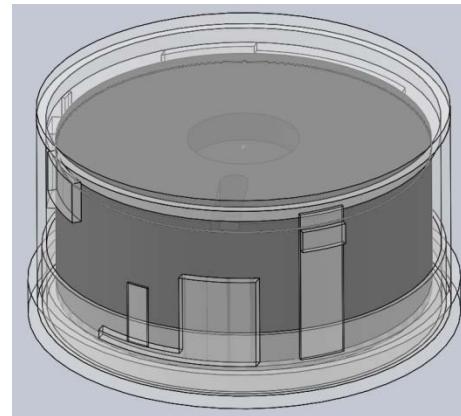
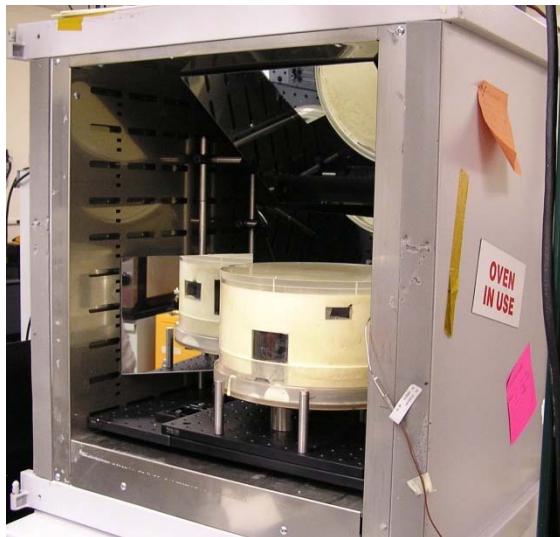
$$\beta = (\beta_{gas} - \beta_{foam}) H(\phi) + \beta_{foam}$$

# Foam Filling Simulation of Complex Part with Plates

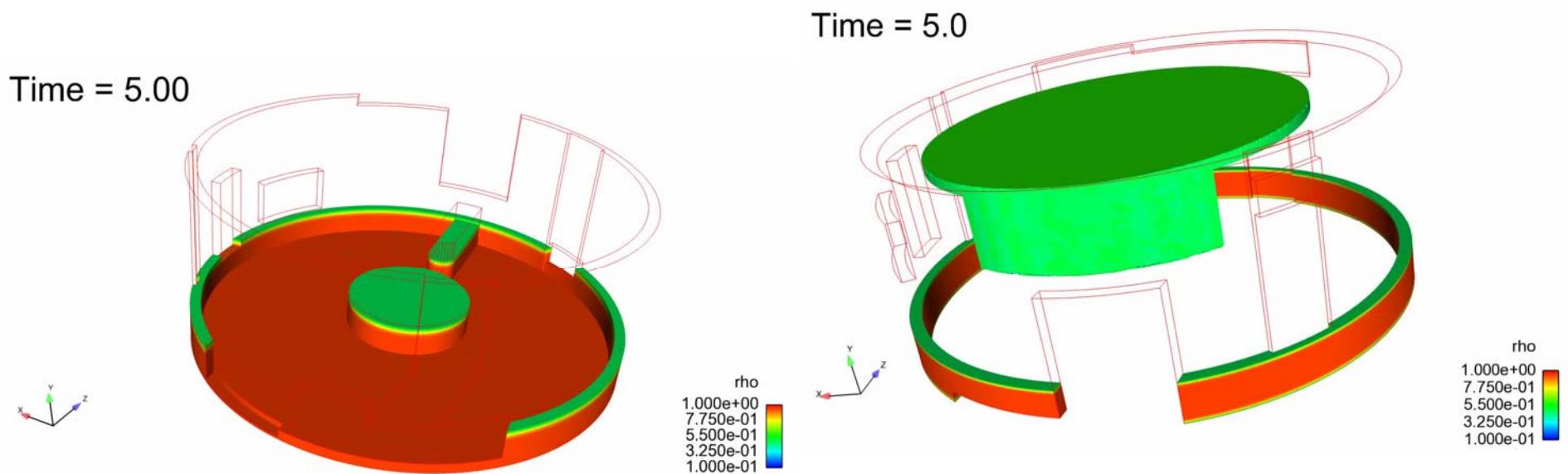


# Simplified Structural Support Mold Tests

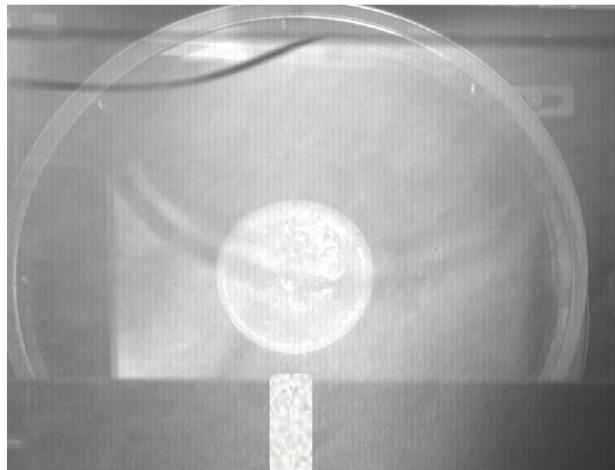
- Legacy mold that had trouble filling
- KC gave us a solid model of the part; we inverted it to design a transparent mold
- Temperature instrumented with four camera views



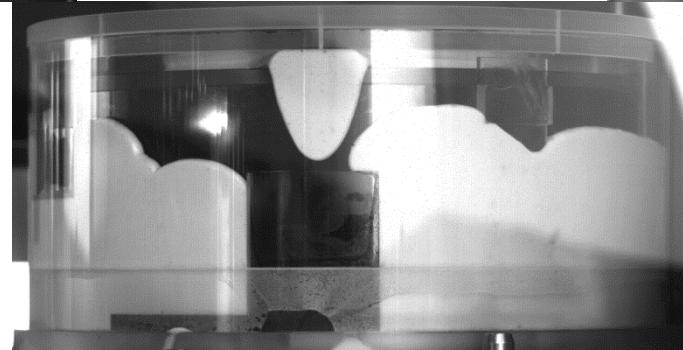
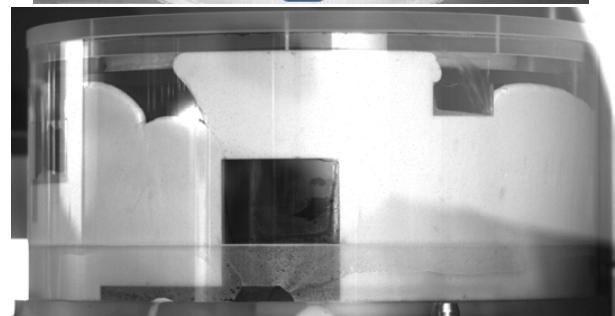
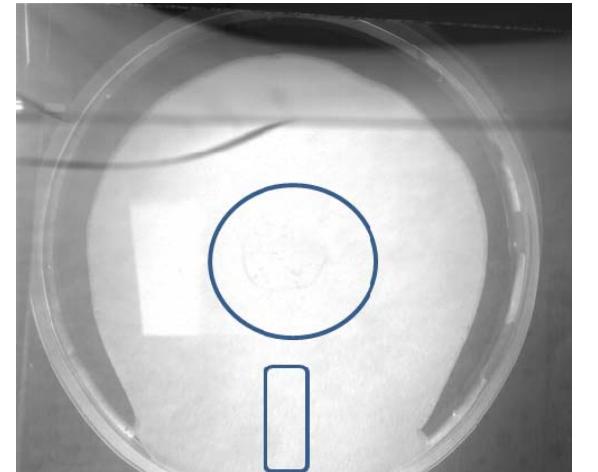
# Computational Modeling of Foam Expansion Can Help Design a Mold Filling Process



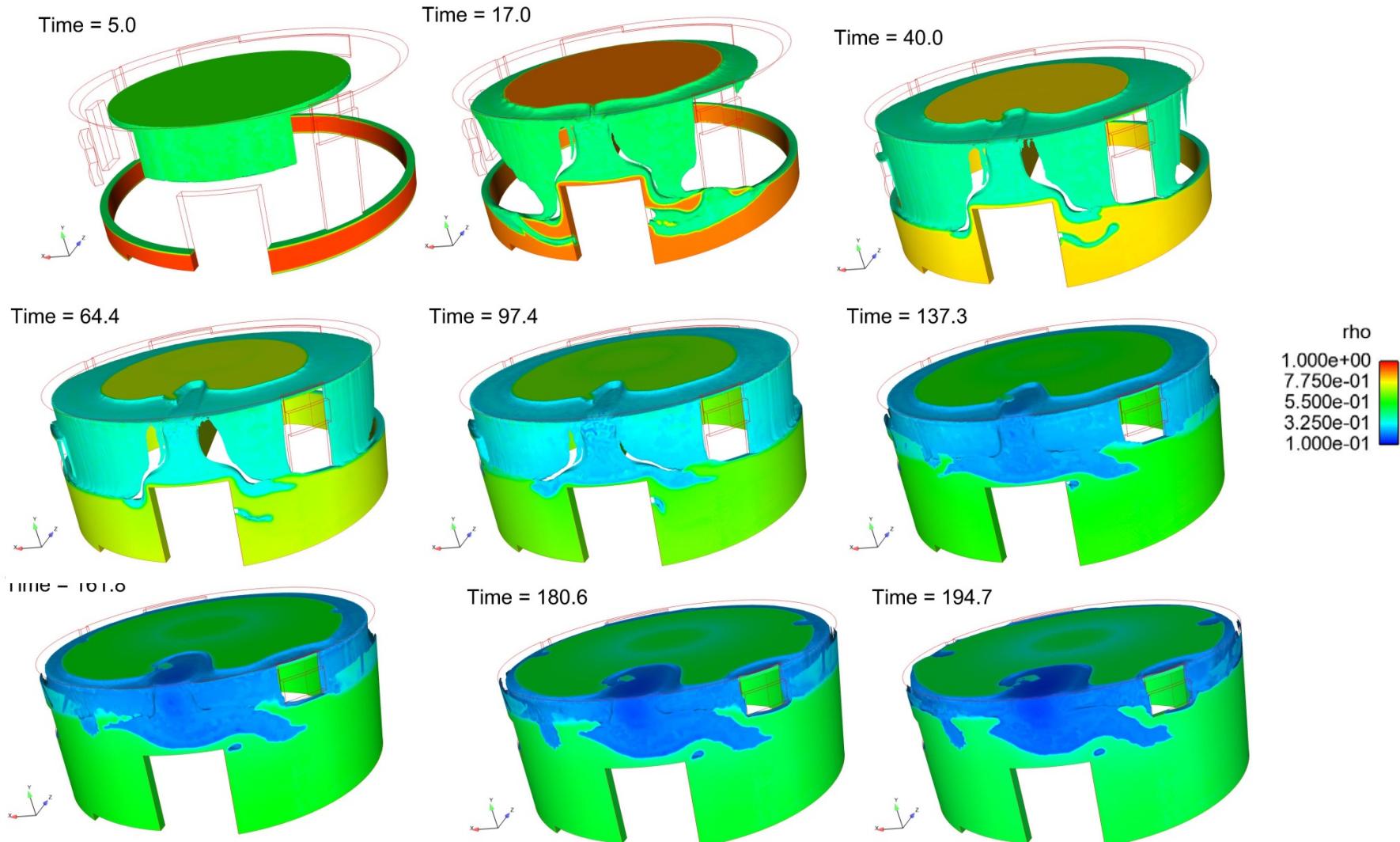
# Filling Method Creates Knit Lines



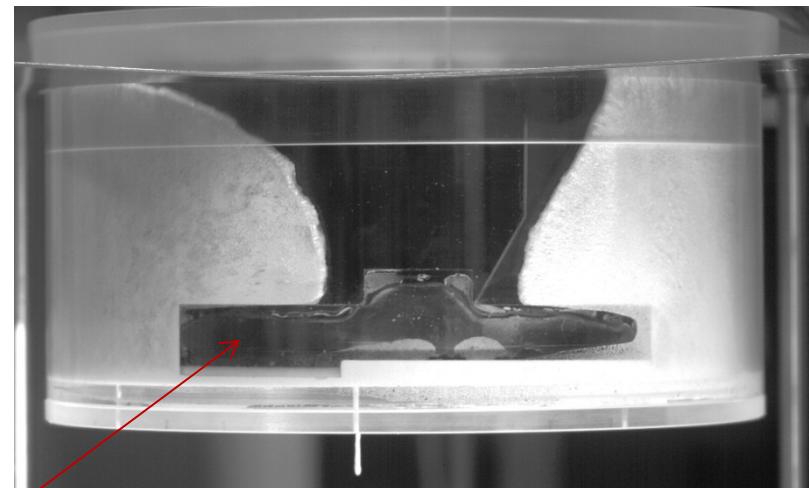
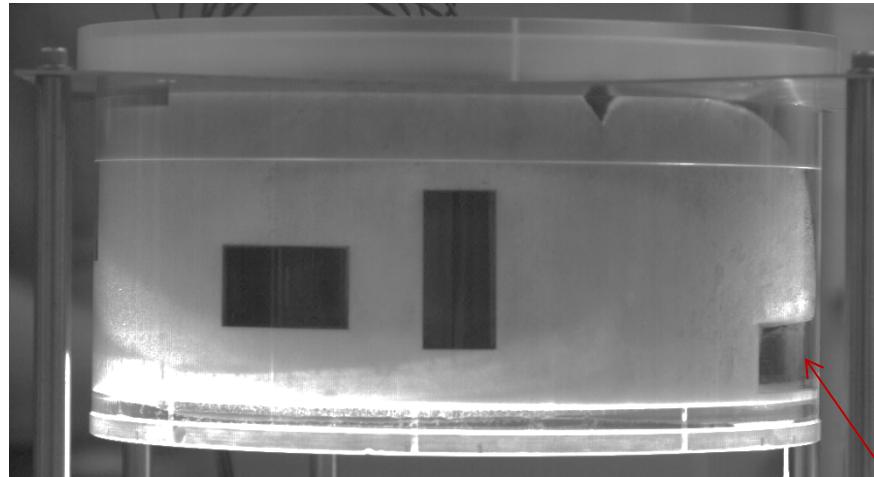
Foaming material is originally placed in top rectangular and cylindrical reservoirs and in bottom rim reservoir, to simulate legacy KC filling method



# Computational Modeling of Foam Expansion Can Help Design a Mold Filling Process



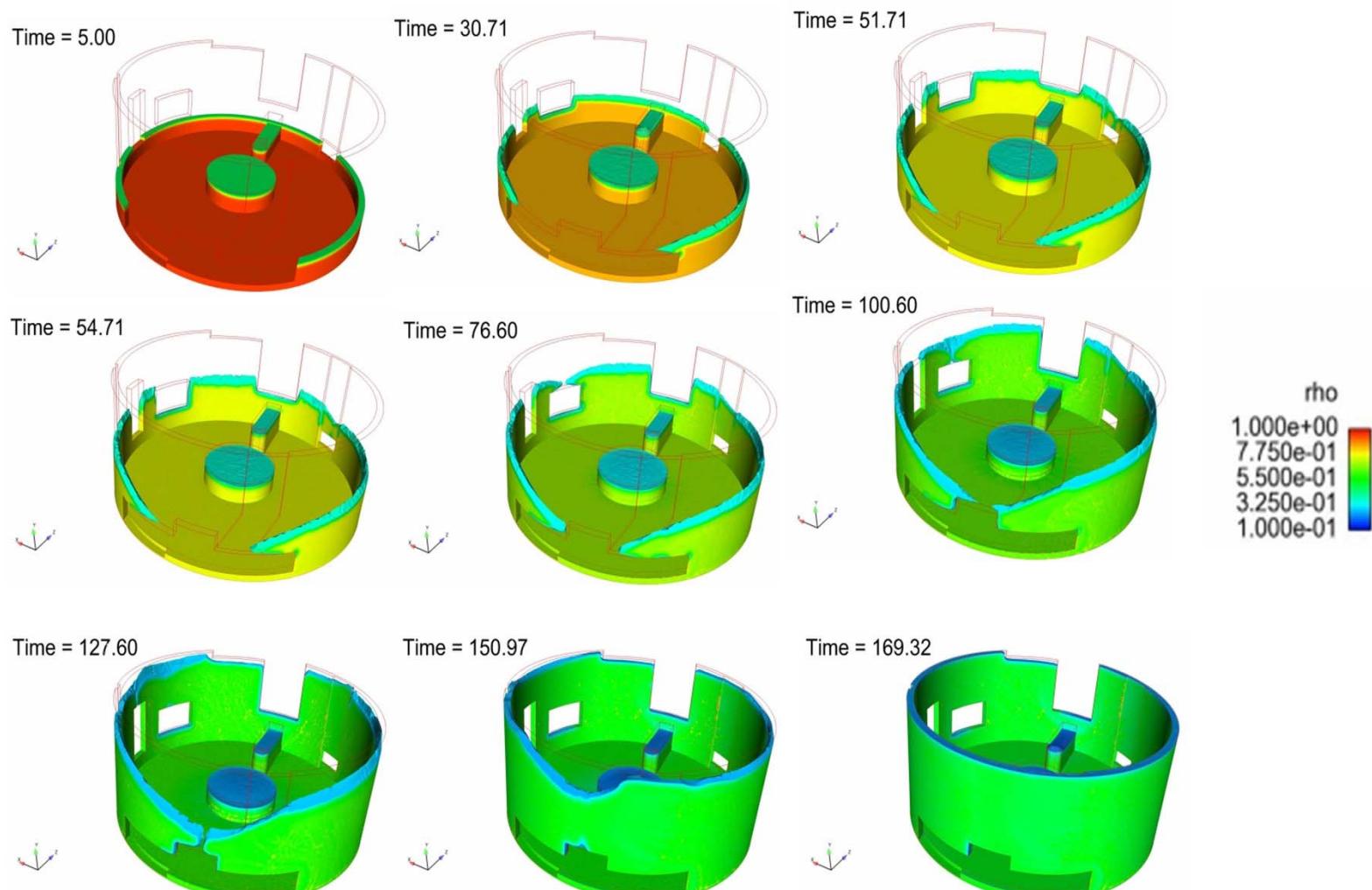
## Last Place to Fill Now on Other Side of Largest Feature



Largest feature

Short shot: less foam than encapsulation test 1, to see where last places to fill would occur. Reaction proceeded faster gelling foam before could finish rising.

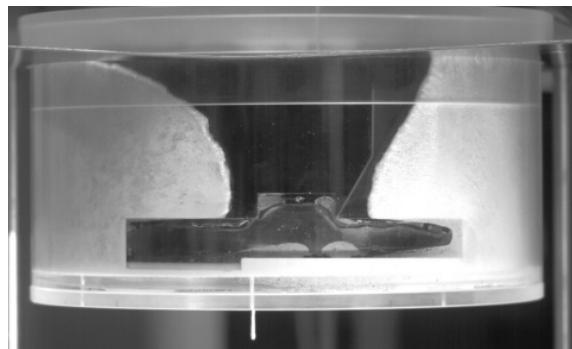
# Computational Modeling of Foam Expansion Can Help Design a Mold Filling Process



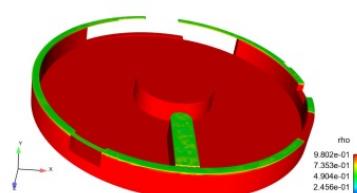
# Model Give More Physics than Just the Filling Locations

Models developed for foam filling and curing  
=> density/cure

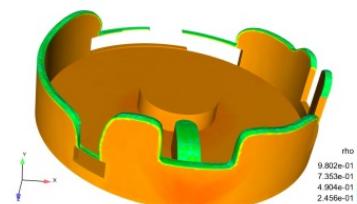
- The model allows us to look inside the mold
- New kinetics show water depletion and CO<sub>2</sub> variations
- Density variations are seen in the mold
- Foam exotherms significantly even and early times



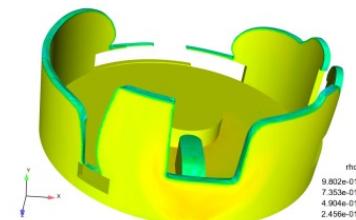
Time = 24.531



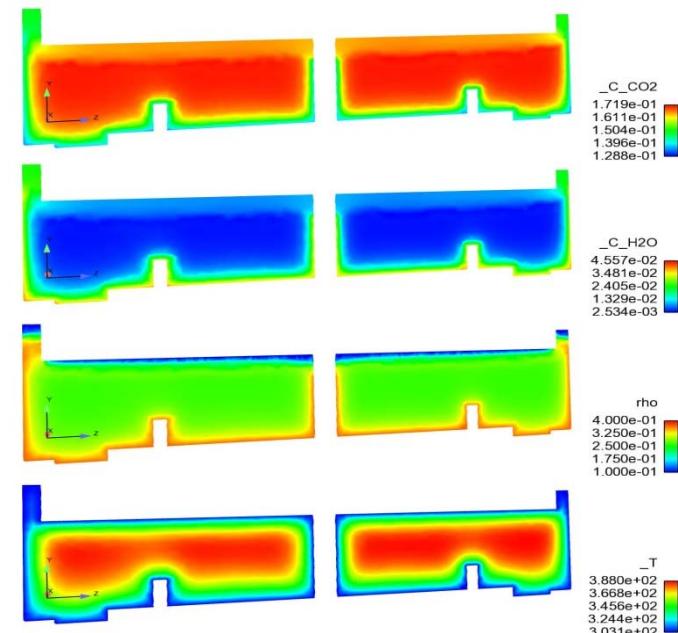
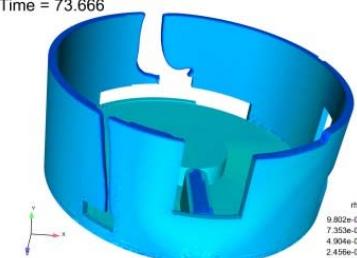
Time = 29.315



Time = 32.136

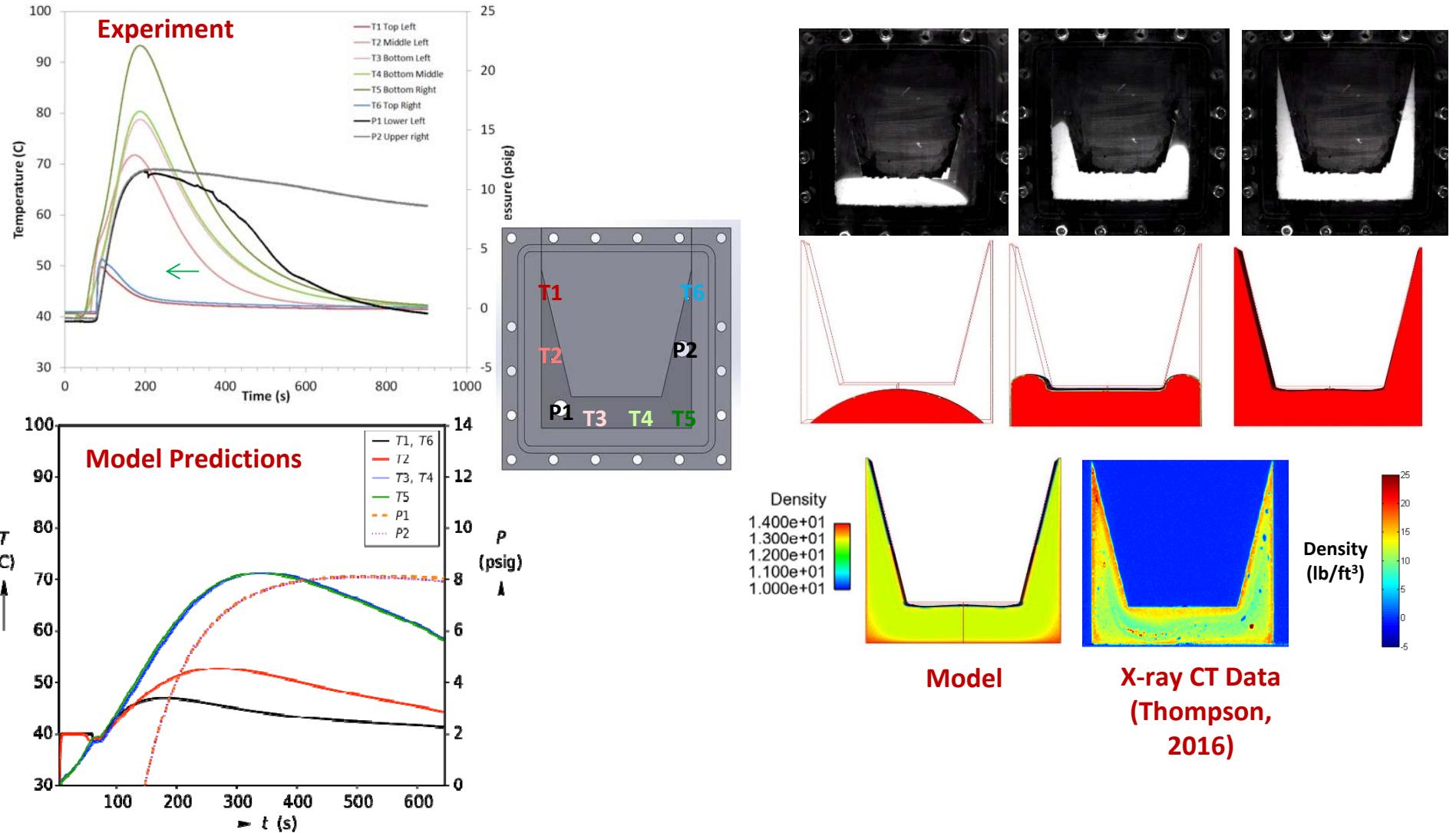


Time = 73.666

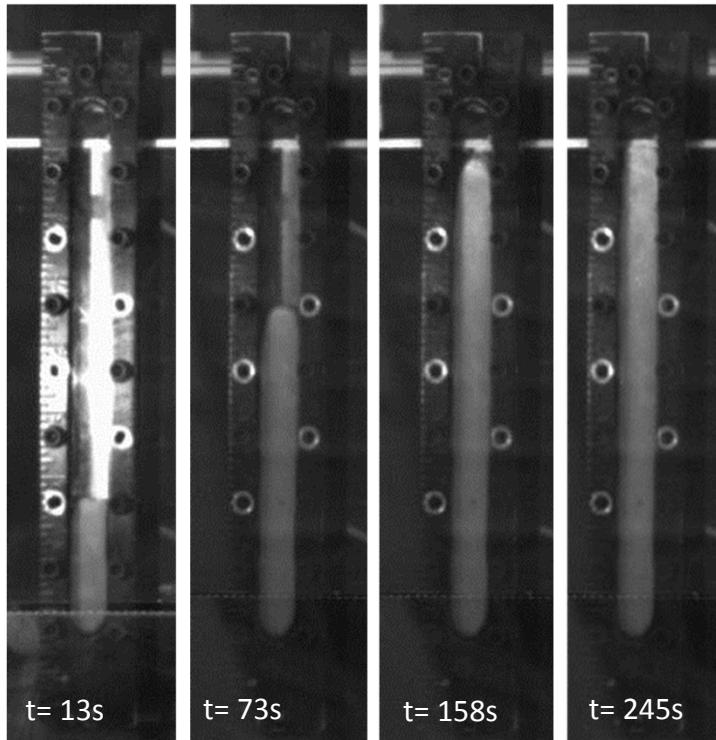


# Foaming U-shaped Staple Mold

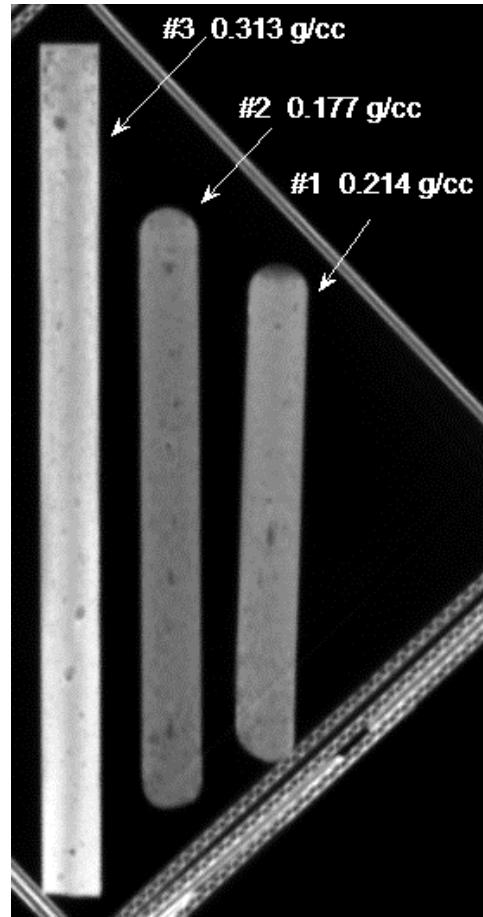
- Over many repeats, temperature, pressure, and flow profile are remarkably repeatable
- Imperfectly symmetric fill common
- Pressure rises as foam expands, relaxes at lower corner and stays positive at P2.



# Density Study for Structural Foam PMDI-10



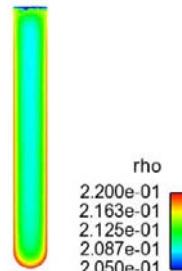
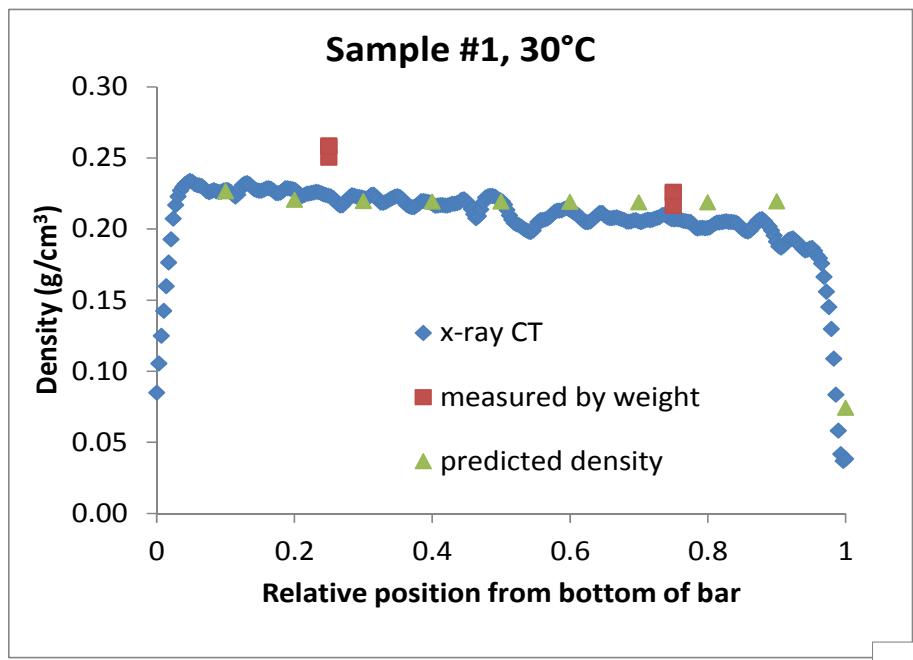
Foam expanding in a mold at 30°C. Time shown on frames is after the end of mixing the resin and the curative together for 45 seconds.



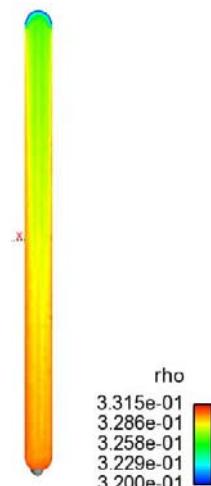
X-ray image of PMDI-10 foam bars: 1) free rise at 30°C, 2) free rise at 50°C, 3) over packed (1.5) at 30°C

- Can the model predict the effects of over packing seen experimentally?
- Over-packed sample shows higher density and greater density variation
- 17% for free rise and 31% for over-packed foam bars

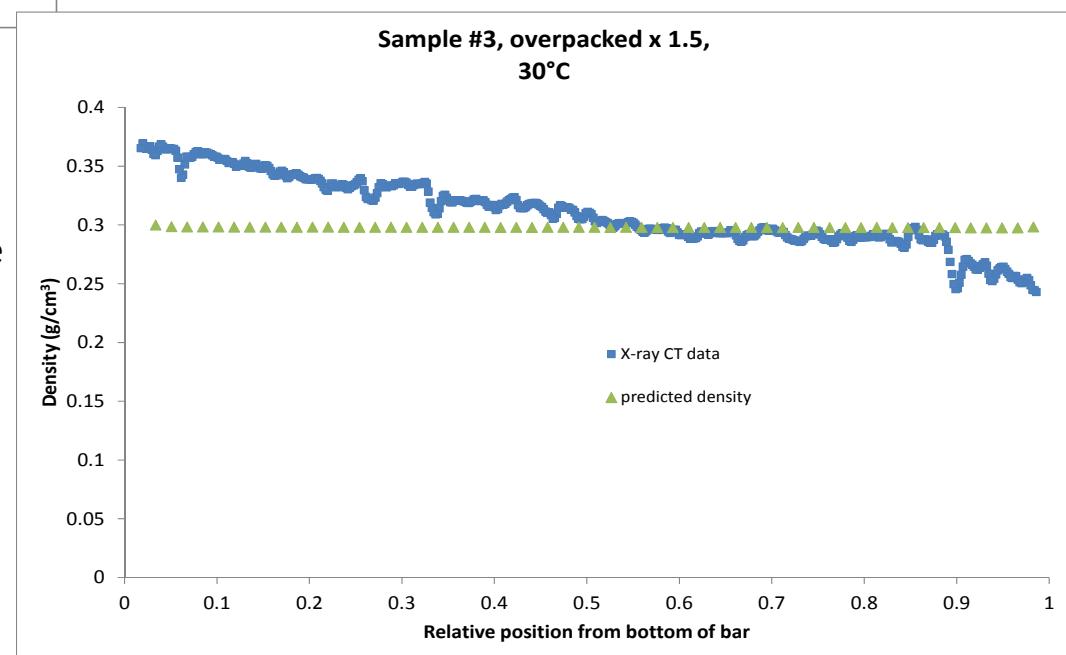
# Density Study for Structural Foam PMDI-10



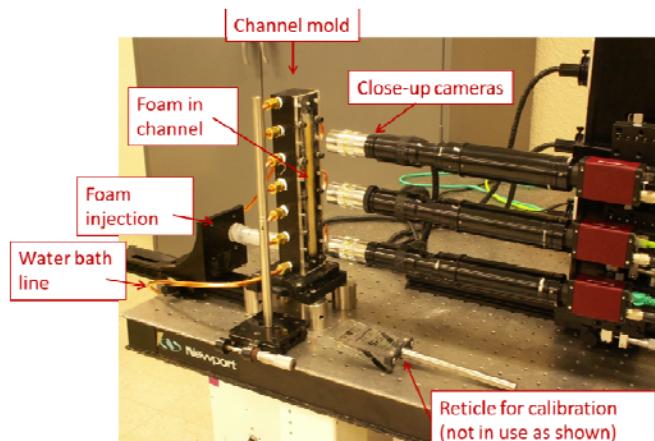
- Free rise foam density gradients. Plots are shown at the centerline of the foam cylinder
- Cylinder is under filled to give the free rise density



- Over packed (1.5) foam density gradients. Plots are shown at the centerline of the foam cylinder
- Self-closing vent lets air out, but keeps foam in for pressurization

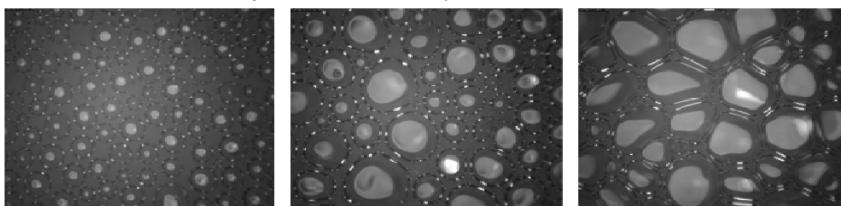


# Study of the Evolution of Bubble Size

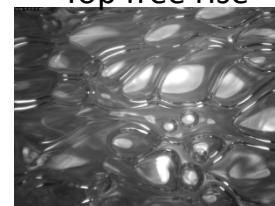


- Three cameras record bubbles at transparent wall (top, middle, and bottom of a column) as foam fills the column
- Light area in pictures below are where the wall is wetted by the bubble – edges are dark lines dashed with bright spots (makes difficult to automatically analyze)
- Image processing developed to analyze – checks by hand shows software good until late times when the bubbles distort severely
- Bubbles nominally about 200-300 microns in diameter
- Size and shape evolve in time, depend on temperature, foam density
- Over packing the foam helps keep the bubbles small and round
- Under packed foam often ends up with highly distorted bubbles near leading front

PMDI-4 free rise (bottom camera)



Top free rise



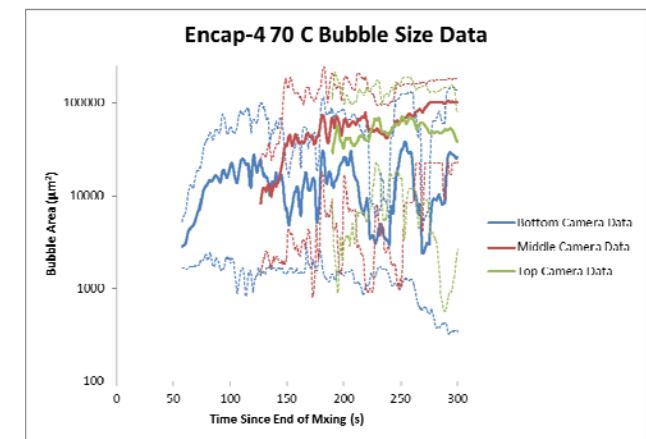
PMDI-4 packed to 8pcf (bottom camera)



Time=79.5 s

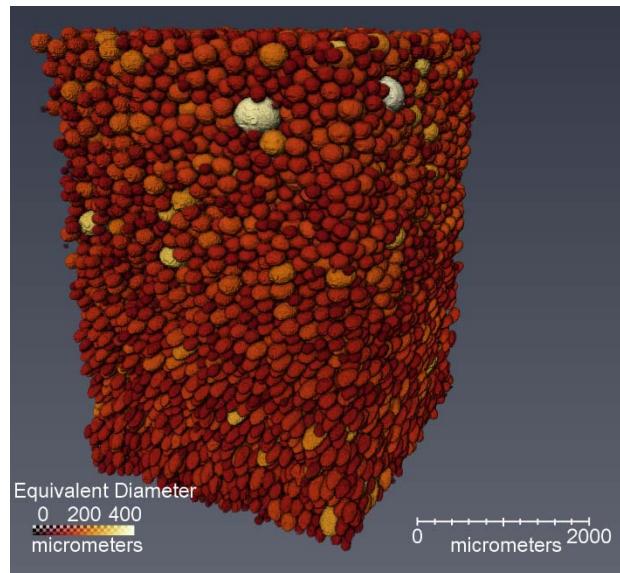
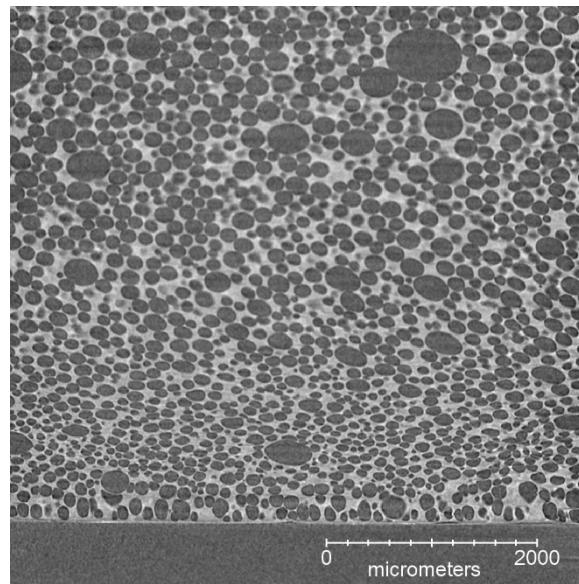
Time=152 s

Time=266 s since end of mixing

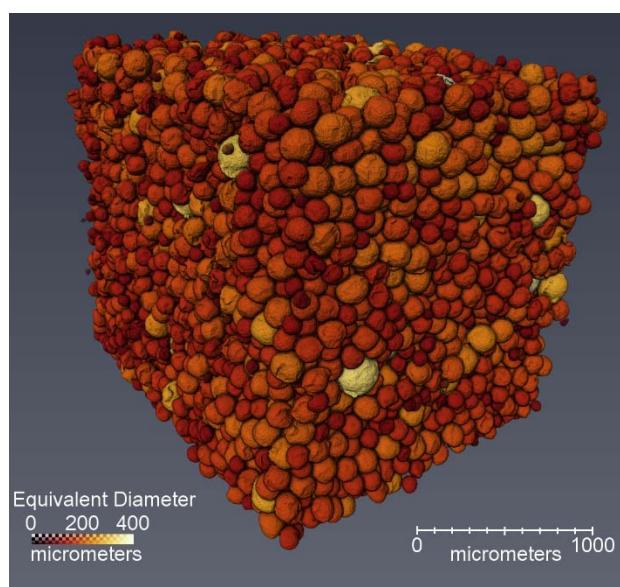
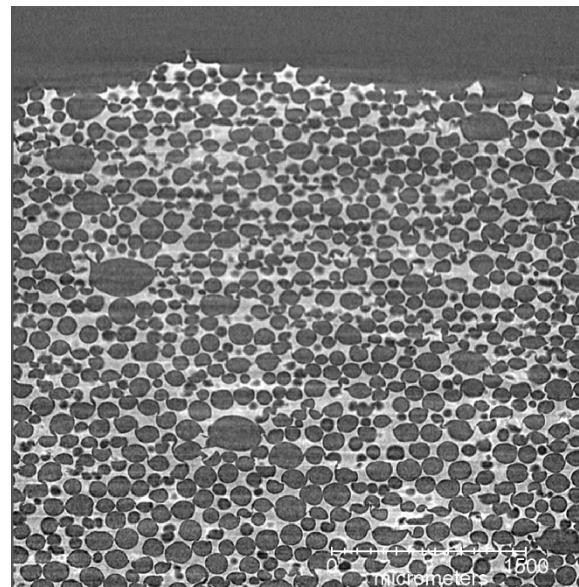


Results of image processing. Solid lines are mean value. Dotted lines indicate top and bottom 10% of values to indicate spread.

# CT Microstructure of Bubbles from Large Complex Mold



Sample 1 top

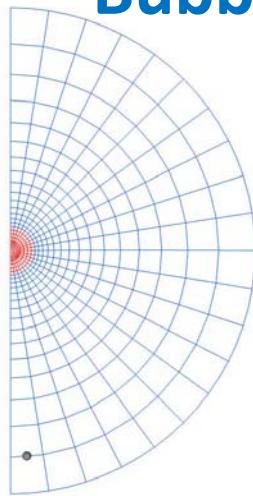


Sample 1 bottom

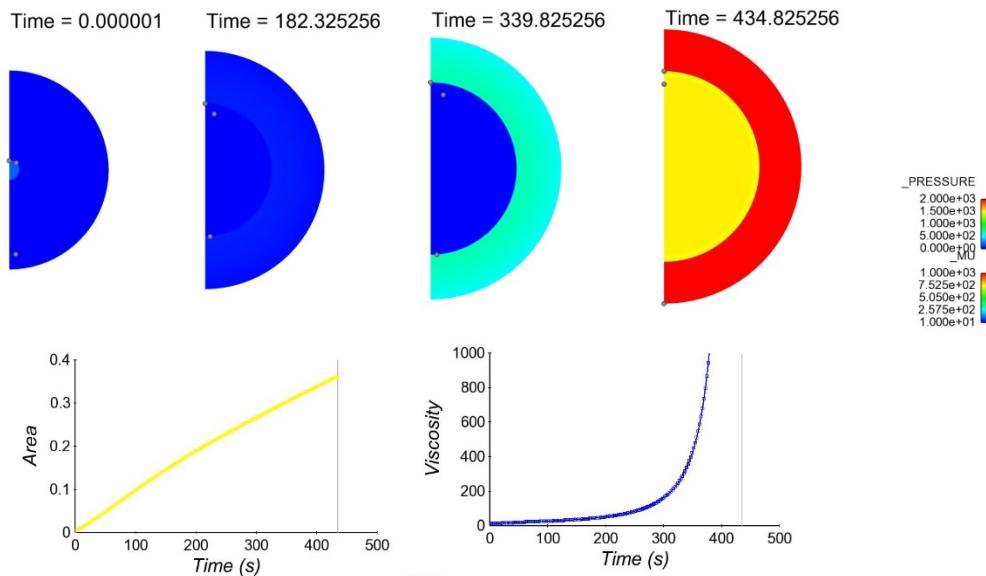
## Foam microstructure

- Polydisperse bubble sizes
- Shear near boundaries cause elongated ellipsoidal bubbles

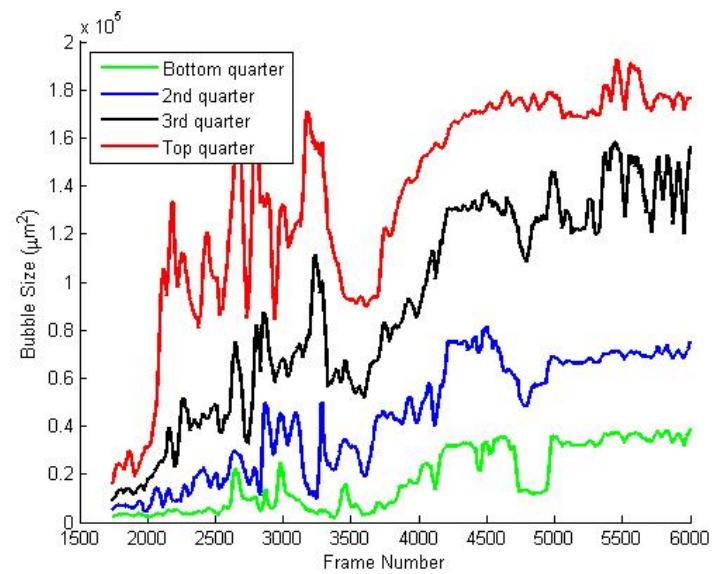
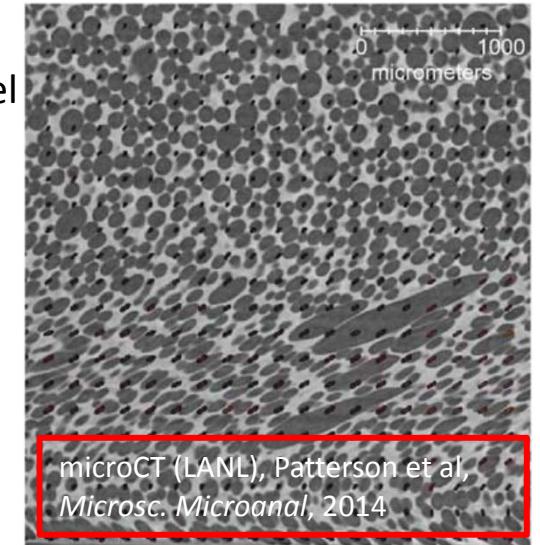
# Bubble Expansion in a Polymerizing Fluid



- Bubble grows as  $\text{CO}_2$  enters the bubble (VLE model)
- Growth is halted abruptly once the polymer reaches the gel point and the viscosity diverges
- Post-gelation, bubble pressurization is observed
- ALE mesh is robust over shape change
- Data shows the correct trends when compared to experiment



$$\rho \left( \frac{3}{2} \dot{R}^2 + \ddot{R} R \right) = p_{\text{gas}} - p_{\text{liq}} - 2 \frac{\sigma}{R} - 4 \eta_{\text{polymer}} \frac{\dot{R}}{R}$$



# Including Bubble-Scale Effects

$$\frac{\partial C_{H_2O}}{\partial t} + \nabla \cdot \vec{v} C_{H_2O} = D_{H_2O} \nabla^2 C_{H_2O} - (1 - \varphi) k_{H_2O} C_{H_2O}^n \quad \text{Existing equation with minor mods}$$

$$\frac{\partial C_{CO_2}}{\partial t} + \nabla \cdot \vec{v} C_{CO_2} = D_{CO_2} \nabla^2 C_{CO_2} + (1 - \varphi) k_{H_2O} C_{H_2O}^n - S_{pg} \quad \text{Existing equation with mods including source}$$

$$\frac{\partial C_{CO_2}^g}{\partial t} + \nabla \cdot \vec{v} C_{CO_2}^g = S_{pg} \quad \text{New equation similar to liquid}$$

$$\frac{\partial \rho_{gas}}{\partial t} + \vec{v} \cdot \nabla \rho_{gas} = -\rho_{gas} S_v + M_{CO_2} S_{pg} \quad \text{New equation for bubble gas density}$$

$$\frac{\partial n}{\partial t} = \nabla \cdot (\vec{v} n) \quad \text{New equation for bubble number density}$$

$$\varphi(t) = \frac{\rho_{foam} Y_g}{\rho_{gas}} = \frac{M_{CO_2} C_{CO_2}^g}{\rho_{gas}} \quad \rho_f = (\rho_{gas} - \rho_{liq}) \varphi(t) + \rho_{liq}$$

$$R_{av} = \left[ \frac{3}{4\pi} \frac{\varphi}{n} \right]^{1/3}, \quad S_{av} = \left[ \frac{3}{4\pi} \frac{1}{n} \right]^{1/3}$$

$$\Delta r = \frac{(S_{av}^4 - R_{av}^4) - R_{av} (S_{av}^3 - R_{av}^3)}{S_{av}^3 - R_{av}^3}$$

$$\eta_{polymer} = \eta_0^0 \exp\left(\frac{E_\mu}{RT}\right) \left(\frac{\xi_c^p - \xi^p}{\xi_c^p}\right)^{-q}$$

Source terms from bubble scale:

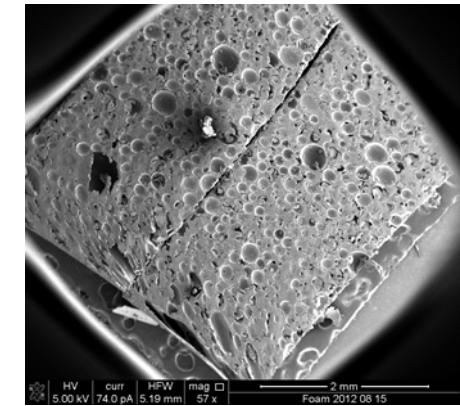
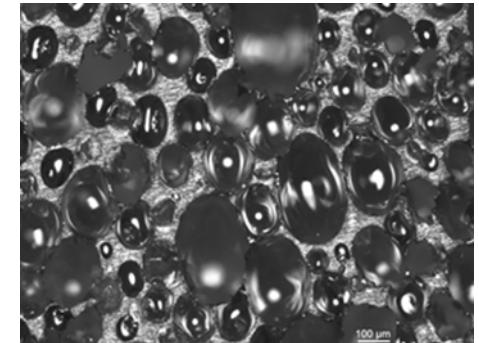
$$S_v = \frac{1}{4\eta_{polymer}} \left( (\rho_{gas} \mathfrak{R} T / M_{CO_2} - p) - \frac{2\sigma}{R_{av}} \right)$$

$$S_{pg} = 3 \frac{\varphi}{R_{av}} D_{CO_2} \frac{C_{CO_2} - K_H \rho_{gas} \mathfrak{R} T / M_{CO_2}}{\Delta r}$$

# Conclusions and Future Work

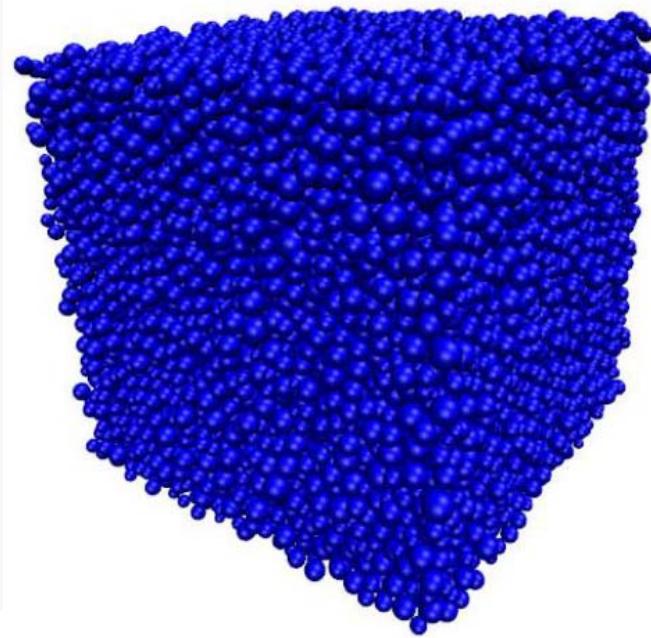
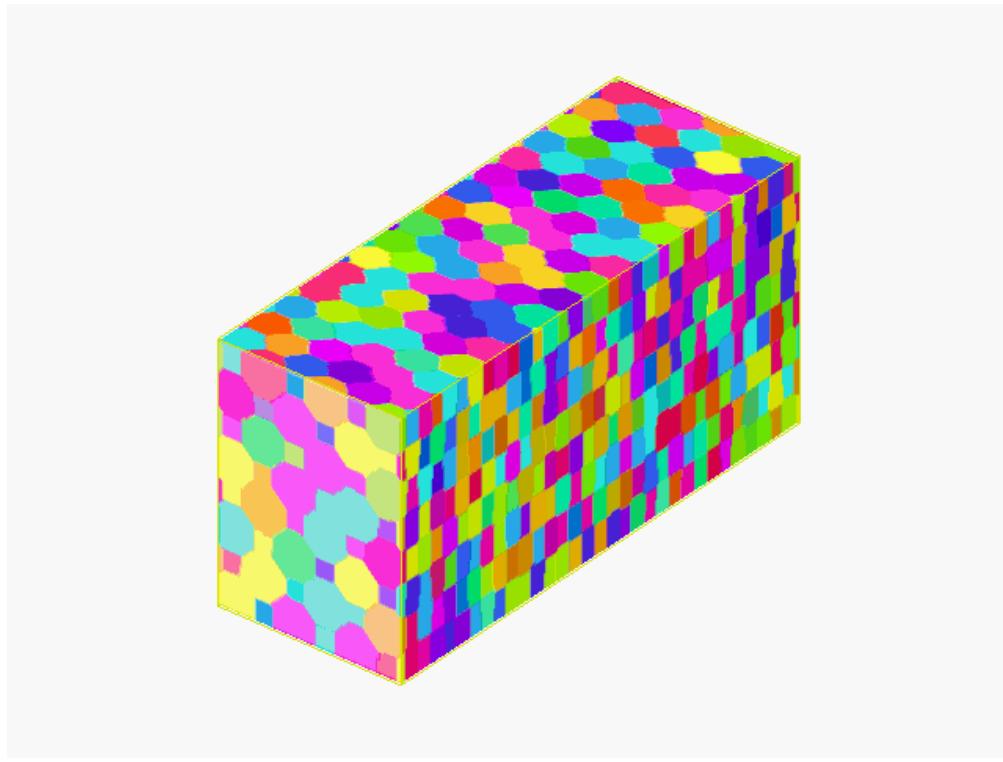
- **Current model is adequate for production calculation**
  - Determining metering, initial placement, voids, gate, and vent location
  - Investigate encapsulation of new geometries of interest
  - Current model is “first order.” We are working to make the model more predictive
- **Next generation model need to include**
  - Equation of state for density approach for gas phase
  - Two-phase CO<sub>2</sub> generation model: solubilized CO<sub>2</sub> in the polymer and CO<sub>2</sub> gas in the bubbles
- **Include local bubble size and bubble-scale interactions**
  - Predict bubble size with Rayleigh-Plesset equation
  - From the bubble size and number density, predict foam density
  - Bubble-scale modeling to include gelation and gas pressure in density model to make it more predictive
  - Drainage/creaming term could help make density model more representative of experiments

SEM of foam showing polydispersity



Bubble at walls are elongated and show coarsening

# Questions?



Pott's model of foam  
bubbles in shear flow  
(Veena Tikare, SNL)

Polydisperse bubble  
microstructure generated with  
LAMMPS and Aria/CDFEM  
(Dan Bolintineanu , SNL)