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Yield Surface Effects on Stability and Failure

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Yield Surface Descriptions

For analysis with continuum plasticity models, isotropic models are generally the first, and often the only, model used. For some applications, like sheet metal forming, models with anisotropy are chosen. Thin materials usually have some anisotropy due to forming, and this anisotropy is often included in analyses. However, given that there are many anisotropic plasticity models, why would we choose one over another? To decide this more analysis of anisotropic yield surfaces is necessary.

We restrict ourselves to rate-independent plasticity models that assume associated flow. We consider four yield surface descriptions: von Mises, Hosford, Hill, and Barlat (Yld2004-18p). The first two are isotropic and the last two are orthotropic.

von Mises :

$$\phi(\boldsymbol{\sigma}) = \sqrt{\frac{1}{2}\left[(\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 + (\sigma_1-\sigma_2)^2\right]}$$

Hosford :

$$\phi(\boldsymbol{\sigma}) = \left\{\frac{1}{2}\left[|\sigma_2-\sigma_3|^a + |\sigma_3-\sigma_1|^a + |\sigma_1-\sigma_2|^a\right]\right\}^{1/a}$$

Hill :

$$\phi(\boldsymbol{\sigma}) = \sqrt{F(\hat\sigma_{22}-\hat\sigma_{33})^2 + G(\hat\sigma_{33}-\hat\sigma_{11})^2 + H(\hat\sigma_{11}-\hat\sigma_{22})^2 + 2L\hat\sigma_{23}^2 + 2M\hat\sigma_{31}^2 + 2N\hat\sigma_{12}^2}$$

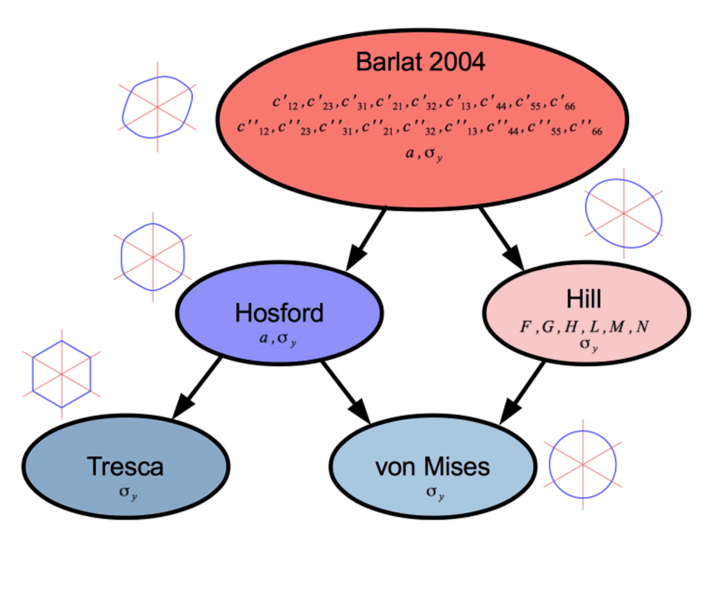
Barlat :

$$\begin{aligned}\phi(\boldsymbol{\sigma}) = \Big\{\frac{1}{4}\Big[&|s'_1-s''_1|^a + |s'_1-s''_2|^a + |s'_1-s''_3|^a \\ &+ |s'_2-s''_1|^a + |s'_2-s''_2|^a + |s'_2-s''_3|^a \\ &+ |s'_3-s''_1|^a + |s'_3-s''_2|^a + |s'_3-s''_3|^a\Big]\Big\}^{1/a}\end{aligned}$$

$$\mathbf{s}' = \mathbf{L}' : \boldsymbol{\sigma}$$

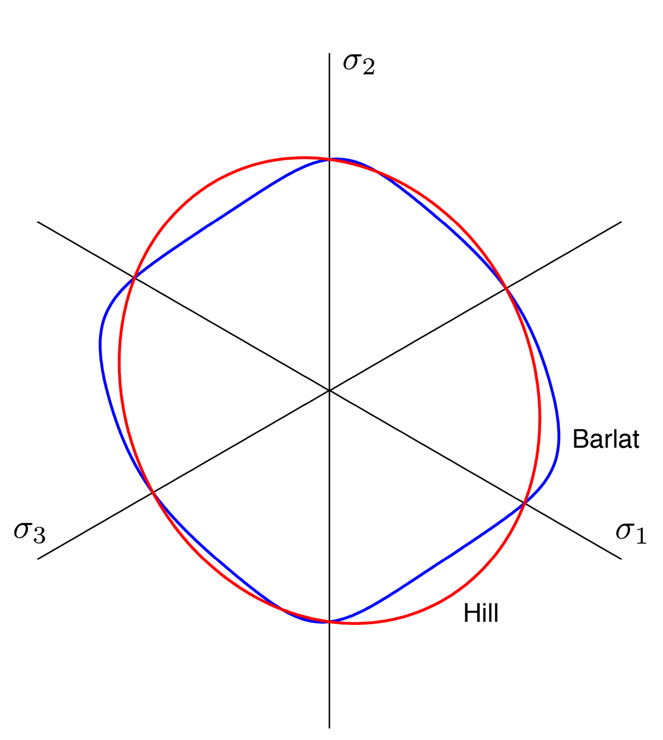
$$\mathbf{s}'' = \mathbf{L}'' : \boldsymbol{\sigma}$$

Yield Surface Hierarchy



The yield surfaces used in this work provide a hierarchy of yield surfaces. The orthotropic Barlat model is the most flexible, being able to fit a number of yield stresses and flow directions. The model can also be reduced to the orthotropic Hill model, or the isotropic Hosford model. From these models the more common Tresca and von Mises models can be used.

For this work we assume the Barlat model is the highest fidelity model. Using the parameterization of 2090-T3 Aluminum from Barlat, et. al., Yld2004-18p (*Int. J. Plast.*, **21**, 2005, 1009-1039) we "fit" the Hill, Hosford, von Mises, and Tresca models.



The Hill fit to the Barlat model gives the same yield stress for uniaxial loading in the principal material directions and pure shear relative to the principal material directions.

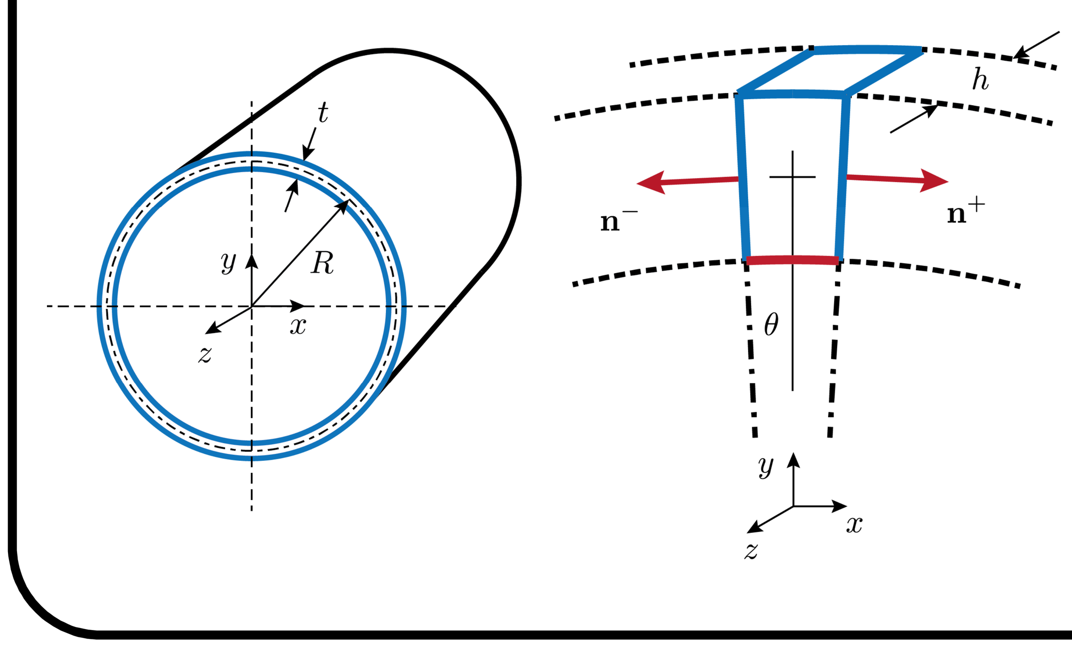
The flow directions for uniaxial stress are not the same for the two models. For other stress paths the yield and flow directions are, in general, different.

The hardening curve assumed for every model uses a Voce model.

$$\sigma(\bar\varepsilon^p) = \sigma_y + A\left(1 - e^{-B\bar\varepsilon^p}\right)$$

$$\sigma_y = 200\ \mathrm{MPa}$$
$$A = 200\ \mathrm{MPa}$$
$$B = 20$$

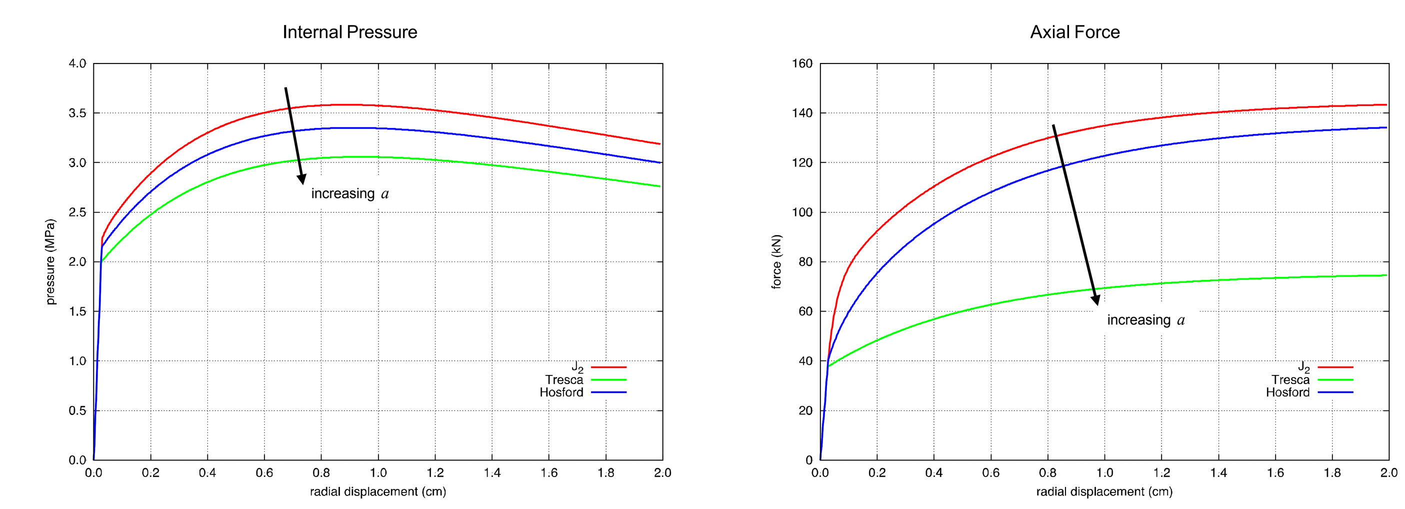
Boundary Value Problem
Internally Pressurized Cylinder



Under internal pressurization, a cylinder will expand until a maximum pressure is reached causing an instability. This problem can be modeled using a plane strain, axisymmetric analysis.

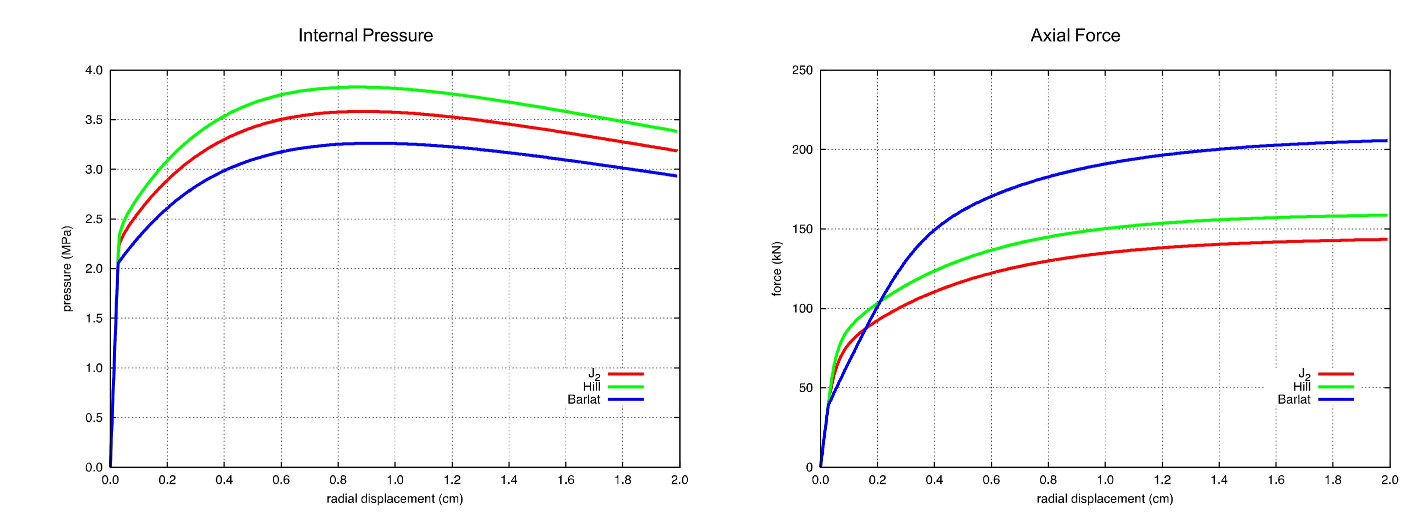
The details of the maximum load depend on the material response. The details of the plasticity model, including yield and flow directions, has a large effect on the results.

Isotropic Models



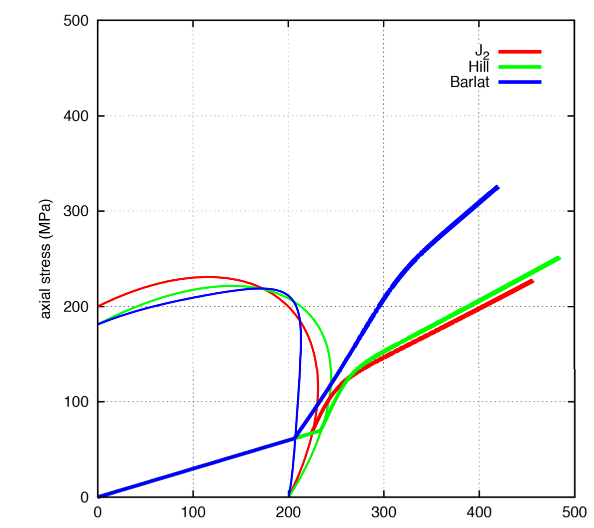
The Hosford yield surface (a = 8) between von Mises (a = 4) and Tresca ($a \longrightarrow \infty$) gives a solution between von Mises and Tresca. The boundary value problem with an isotropic model is relatively easy to interpret. As the exponent, a, increases the internal pressure and axial force decrease.

Orthotropic Models



Two orthotropic yield descriptions give two very different behaviors. For plasticity models the yield **and** the flow directions are important. The stress paths in plane stress space for this boundary value problem are important. They are determined, in part, by the flow direction from the plasticity model.

If the Barlat model is assumed to give the highest fidelity representation of the material response, then the other models, viewed as approximations to the actual response, have varying degrees of success. Of note is the fact that the Hill model – the other orthotropic model – gives the worst prediction of the maximum internal pressure.



Conclusion

For metals exhibiting anisotropic yield, the form of the yield surface can have large effects on the results of an analysis. Using a hierarchy of yield surfaces we can systematically examine and understand the effects of anisotropic yield and plastic flow. For this particular material, and the plasticity models used to model the material, the computed maximum pressures can be off by as much as 15%. These differences are a result of differences in yield in biaxial stress, and the plastic flow direction.

One area of study this opens up is uncertainty quantification and model form error. Can the Barlat model provide measures for the appropriate use of other models for a given material and/or a boundary value problem?

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