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Title: The effect of capturing the correct turbulence dissipation rate in BHR

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The Effect of Capturing the Correct Turbulence Dissipation Rate in BHR

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Abstract
In this manuscript, we discuss the shortcoming of a quasi-equilibrium assumption made in the BHR closure model. Turbulence closure models generally assume fully developed turbulence, which is not applicable to 1) non-equilibrium turbulence (e.g. change in mean pressure gradient) or 2) laminar-turbulence transition flows. Based on DNS data, we show that the current BHR dissipation equation [modeled based on the fully developed turbulence phenomenology] does not capture important features of non-equilibrium flows. To demonstrate our thesis, we use the BHR equations to predict a non-equilibrium flow both with the BHR dissipation and the dissipation from DNS. We find that the prediction can be substantially improved, both qualitatively and quantitatively, with the correct dissipation rate. We conclude that a new set of non-equilibrium phenomenological assumptions must be used to develop a new model equation for the dissipation to accurately predict the turbulence time scale used by other models.

Background
Homogeneous variable density turbulence (HVDT) is described as a series of blobs of different density fluids (depicted in Figure 1) that when set into an acceleration field undergo motion and mixing.

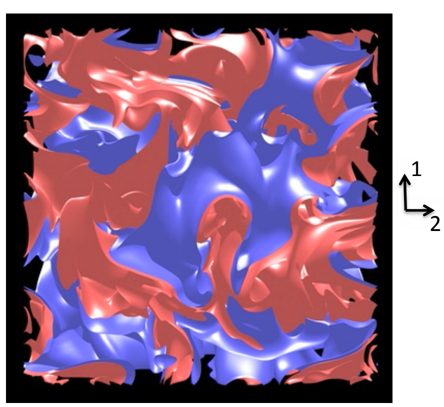

Figure 1: Initial configuration for HVDT [1-3]. The heavy (blue) and light (red) fluids are initially segregated and turbulence is produced as the fluids start moving in opposite direction due to buoyancy.

- In 2011, Stalsberg-Zarling and Gore [4] showed that BHR2 (a turbulence mix model with an evolution equation for the mix parameter b) was needed to match HVDT, however they did not discuss the coefficients used to match the direct numerical simulations (DNS). The set of coefficients for BHR2 that was eventually decided upon shows that this data set is not well matched (see Figure 2).
- In 2016, Schwarzkopf et al. [5] showed that BHR3 reasonably matched to DNS statistics for HVDT using BHR3, also given in Figure 2. This match required a two-turbulent length scale (where the turbulent length scale can be thought of as a large vortex and is defined as $S = K^{3/2}/\varepsilon$) to match a large set of data. However, even the two-length scale model (as assumed) has limitations because it is slaved to the turbulence energy model.

The purpose of this manuscript is to describe the effect of turbulence dissipation (related to the turbulent length scale) on various statistics.

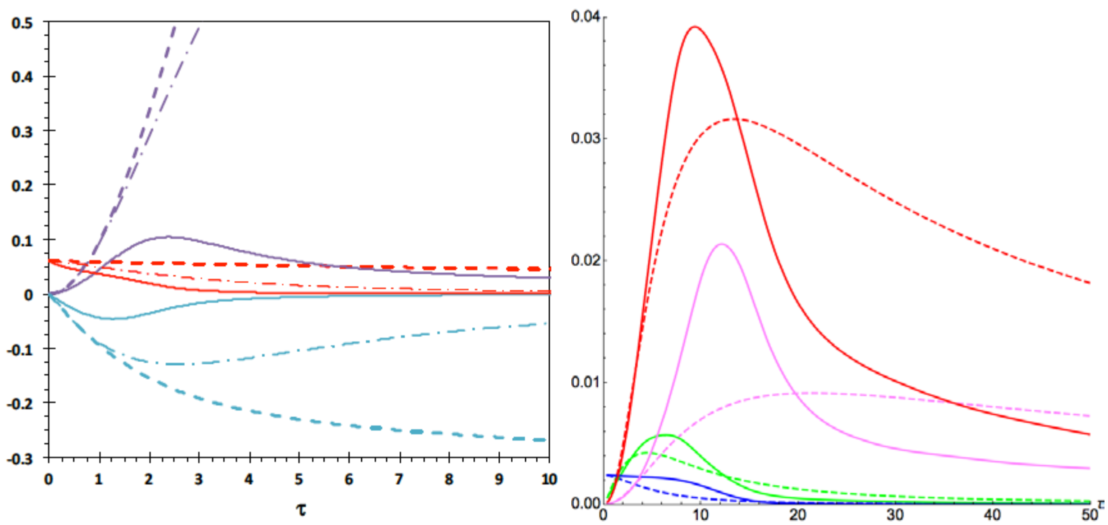

Figure 2: (LEFT) Comparison of K (purple), a (blue), and b (red) for At. = 0.25 HVDT (solid lines – DNS, dashed lines – BHR2, dash-dot lined – BHR3); (RIGHT) Comparison of R11 (red), R22 (magenta), a (green), and b (blue) for At. = 0.05 HVDT (solid lines – DNS, dashed lines – BHR3.1).

The effect of the dissipation rate

The equation for the dissipation rate of turbulence is complicated. A Reynolds averaged version for uniform density, single phase/fluid is given below [Bernard and Wallace, 2002]),

$$\frac{D\varepsilon}{Dt} = -2\nu\overline{\frac{\partial u_j'}{\partial x_i}\frac{\partial u_j'}{\partial x_k}}\frac{\partial \bar{u}_j}{\partial x_k} - 2\nu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}\frac{\partial \bar{u}_i}{\partial x_j} - 2\nu\overline{u_k'\frac{\partial u_i'}{\partial x_j}}\frac{\partial^2 \bar{u}_i}{\partial x_k \partial x_j} - 2\nu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_j}\frac{\partial u_k'}{\partial x_j}}$$
$$-\frac{2\nu}{\rho}\frac{\partial}{\partial x_i}\left(\overline{\frac{\partial P}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}\right) - \nu\frac{\partial}{\partial x_k}\left(\overline{u_k'\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}\right) + \nu\nabla^2\varepsilon - 2\nu^2\left(\overline{\frac{\partial^2 u_i'}{\partial x_j \partial x_k}\frac{\partial^2 u_i'}{\partial x_j \partial x_k}}\right) \quad (1)$$

where u_i' is the fluctuating velocity, $\bar{u}_i$ is the mean velocity, ν is the fluid viscosity, and P is the fluctuating pressure. Because of the complexity, a simple model (derived within the aerospace engineering society) has been adopted, which is based on the assumption that the turbulence dissipation rate scales with the turbulence kinetic energy, such as

$$\frac{D\varepsilon}{Dt} \propto \frac{\varepsilon}{k}\frac{Dk}{Dt} \text{ and } \frac{Dk}{Dt} = P - \varepsilon \quad (2)$$

where P here is the production rate and ε is the dissipation rate of turbulence energy. The assumed model, given in Eqn. (2), basically implies that the turbulence dissipation rate time scale is similar to the time scale for turbulent kinetic energy. In other words, as turbulent kinetic energy is produced, the rate of dissipation instantly responds (i.e. equilibrium turbulence); this is not the case for Eqn. (1). In non-equilibrium turbulence, the dissipation rate of turbulence may be suppressed or enhanced relative to production, such as in the transition regime.

In fully developed equilibrium turbulence the turbulence energy cascades from large scale vortices to small (viscous) scales at which it is transferred to heat. In such flows the dissipation at the small scales is determined by the rate at which energy is fed to the large scales and cascaded down to small scales by nonlinear effects, the so-called cascade. This idea is the underlying assumption in many turbulence models having the form of Eqn. (2).

Flows of interest at LANL differ from this fully developed turbulence notion in a number of respects. Within flows that are transitioning to turbulence, such as the onset of RT, there is no range of scales of motion cascading energy from large to small scales and the dissipation cannot be set equal to a large scale "cascade" rate as there is no fully developed spectrum of nonlinearly interacting modes reflecting a balance. In fully developed turbulence, undergoing a sudden acceleration or a shock, the energy input at the large-scale end of the nonlinear cascade undergoes a rapid change. Due to the viscosity, the energy input takes some time for the nonlinear cascade mechanisms to deliver that change to the small dissipative scales of the motion.

In either a) the turbulence transition or b) the sudden change of the large-scales of the motion the dissipation lags the changes in cascade rate at the large-scales, and dissipation models built on the notions of a fully developed turbulence, as in equation (2), are inadequate. The HVDT case provides evidence that the energy production and the energy dissipation do not track each other as assumed in the phenomenological model in Eqn. (2). In Figure 3 we see that $P >> \varepsilon$, indicating that the nonlinear effects have not yet communicated the large-scale energy to the small-scale dissipation. The effect of the lagging dissipation rate relative to production of turbulence energy is also evident in Figure 4, where the DNS turbulent kinetic energy and dissipation rate are compared with BHR turbulent kinetic energy and dissipation rate. One can see that BHR cranks up the dissipation rate much earlier in time than DNS due to the model assumptions based on the presence of a cascade, whereas DNS has a delayed dissipation rate during the development of the inertial range.

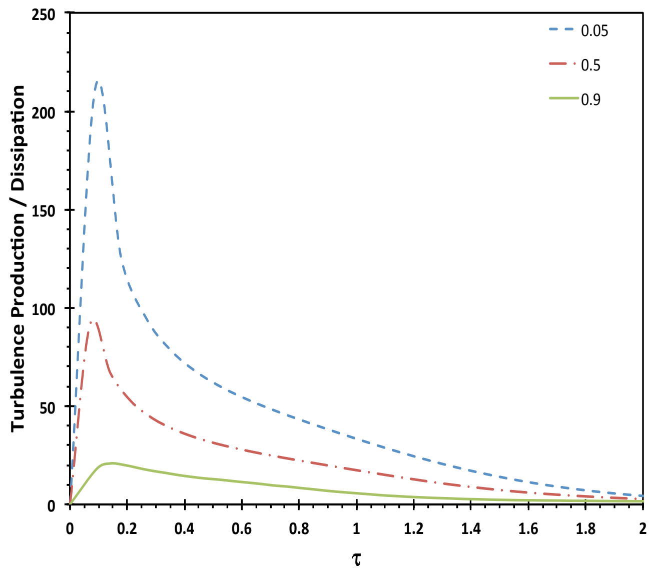

Figure 3: The ratio of production of turbulence energy over dissipation rate showing the rate of change, showing that the dissipation rate is low at the onset and cranks up later in time.

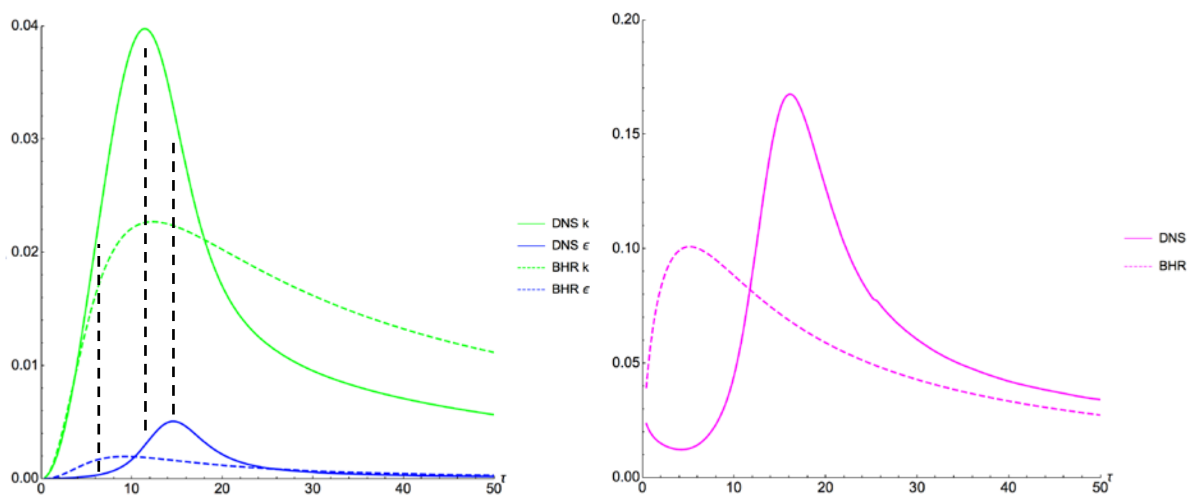

Figure 4: Comparison of turbulence energy, dissipation rate, and the inverse turbulence time scale showing the shift in timing where the peak dissipation rate occurs.

The impact of getting the turbulent dissipation correct

To better understand the effect of getting the dissipation rate correct in BHR, we substituted the DNS turbulent length scale into the modeled form of the BHR equations, and the result is shown in Figure 5. Overall the match in terms of trend is much better than that shown in Figure 2. The peaks were better matched by modifying the coefficient for the decay model in the a-equation and the rapid distortion coefficient in the Reynolds stress equation. This match suggests that models for the decay of b, or the decay of a, or the pressure strain correlation, are reasonable and can be properly tuned if the turbulence time scale is correct. Overall the match of other turbulence statistics is much better than the original model, given in Figure 2.

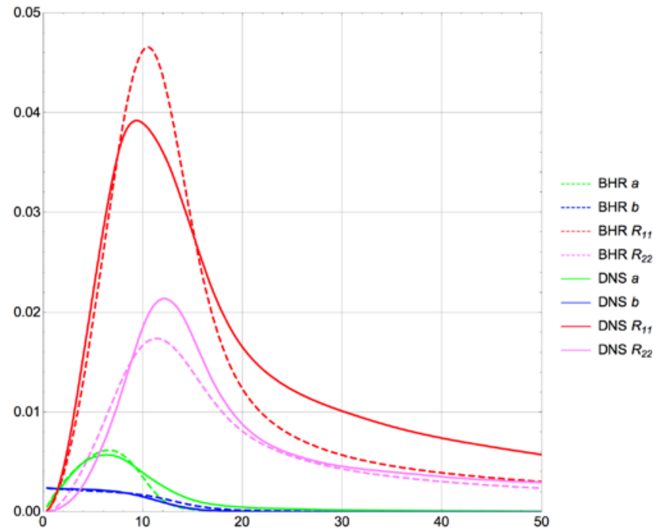

Figure 5: Comparison of turbulence statistics for At. = 0.05 HVDT with the DNS turbulent length scale substituted for the BHR turbulent length scale (solid lines – DNS, dashed lines – BHR3.1).

Comparison of turbulent time scales

With the correct dissipation rate (i.e. feeding in the DNS turbulent length scale into the BHR model), we now see that the BHR inverse turbulent time scale is nearly identical to the DNS inverse turbulent time scale, see Figure 6. This is a substantial improvement in relative comparison and suggests that once the turbulence time scale is captured, the Reynolds stress model is reasonable for this case.

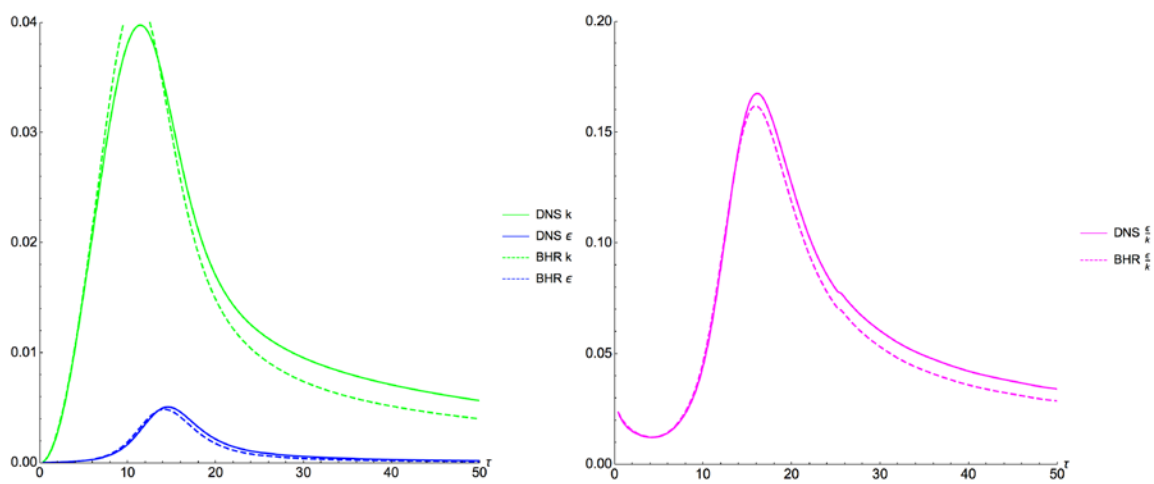

Figure 6: Comparison of BHR and DNS turbulence energy, dissipation rate, and the inverse turbulence time scale if the correct dissipation rate were modeled in BHR.

The reason the BHR model is independent of fluid viscosity is because of the assumption of the presence of the inertial range and the cascade of energy from large to viscous scales. When the assumption is made that turbulence dissipation rate scales with turbulent kinetic energy, the presence of a fully developed cascade is also assumed or implied (i.e. the flow is in a self-similar state). The actual dissipation rate, i.e. Eqn. (1), is dependent on viscosity. In order to capture transition-type flows, an evolution equation for compressible turbulence dissipation rate that is not slaved to the turbulence kinetic energy equation will be required. Modeling the right physics in the right regime is crucial to predictability.

Conclusions

- We show that the primary reason for the inaccuracy during the transition in HVDT is the inability to properly capture the evolution of the turbulent length scale in BHR.
- The BHR turbulent length scale was assumed to scale with the turbulence energy, which is valid in a fully-developed, self-similar turbulence regime, but does not account for the lag in dissipation in transitional flows.
- We have demonstrated that if BHR can capture the correct turbulent length scale, it will be more predictive in transient flows of programmatic interest.
- An evolution equation for the Favre averaged turbulence dissipation rate is needed to correctly capture the turbulent length and time scales in non-equilibrium flows of interest.

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