

# Fractional progress toward understanding the fractional diffusion limit: The electromagnetic response of spatially correlated geomaterials

Chester J Weiss<sup>\*</sup>, G Didem Beskardes<sup>2</sup> and Mark E Everett<sup>3</sup>

<sup>\*</sup>Geophysics Dept, Sandia National Laboratories, cjweiss@sandia.gov  
<sup>2</sup>Dept of Earth and Planetary Sciences, University of New Mexico, gjweiss@unm.edu  
<sup>3</sup>Geosciences Dept, Virginia Tech <sup>3</sup> Dept of Geology and Geophysics, Texas A&M University

## 1. Abstract

Whereas geophysical measurements of rock properties are the only viable approach for large-scale reconnaissance and monitoring of the subsurface and its changing state, these properties (mass density, seismic wavespeed, electrical conductivity, etc) are often a proxy for other properties of greater interest (e.g. permeability, fracture density and orientation, mineralogy). The challenge in geophysics is thus twofold: proper accounting of these properties of interest in the petrophysics of predictive "forward" simulations; and, accurate extraction of these properties from gross, bulk-averaged maps ("inversion" solutions) of the geophysical proxies. In particular, the bulk electrical properties of rocks – much like their hydrologic properties – are driven strongly by small-scale features that can both "shunt" and "short" the underlying transport physics. In other words, for these problems what matters are the details and how the details connect across orders of magnitude in length scale.

In this presentation we review the observational evidence for anomalous electromagnetic diffusion in near-surface geophysical exploration and how such evidence is consistent with a detailed, spatially-correlated geologic medium. To date, the inference of multi-scale geologic correlation is drawn from two independent methods of data analysis. The first of which is analogous to seismic move-out, where the arrival time of an electromagnetic pulse is plotted as a function of transmitter/receiver separation. The "anomalous" diffusion is evident by the fractional-order power law behavior of these arrival times, with an exponent value between unity (pure diffusion) and 2 (lossless wave propagation). The second line of evidence comes from spectral analysis of small-scale fluctuations in electromagnetic profile data which cannot be explained in terms of instrument, user or random error. Rather, the power-law behavior of the spectral content of these signals (i.e. power versus wavenumber) and their increments reveals them to lie in a class of signals with correlations over multiple length scales, a class of signals known formally as fractional Brownian motion. Numerical results over simulated geology with correlated electrical texture – representative of, for example, fractures, sedimentary bedding or metamorphic lineation – are consistent with the (albeit limited, but growing) observational data, suggesting a possible mechanism and modeling approach for a more realistic geology. Furthermore, we show how similar simulated results can arise from a modeling approach where geologic texture is economically captured by a modified diffusion equation containing exotic, but manageable, fractional derivatives. These derivatives arise physically from the generalized, convolutional form for the electromagnetic constitutive laws and thus have merit beyond mere mathematical convenience. In short, we are zeroing in on the anomalous, fractional diffusion limit from two converging directions: a zooming down of the macroscopic (fractional derivative) view; and, a heuristic homogenization of the atomistic (brute force discretization) view.

## 3. Anomalous Diffusion Theory at the Macroscopic Level

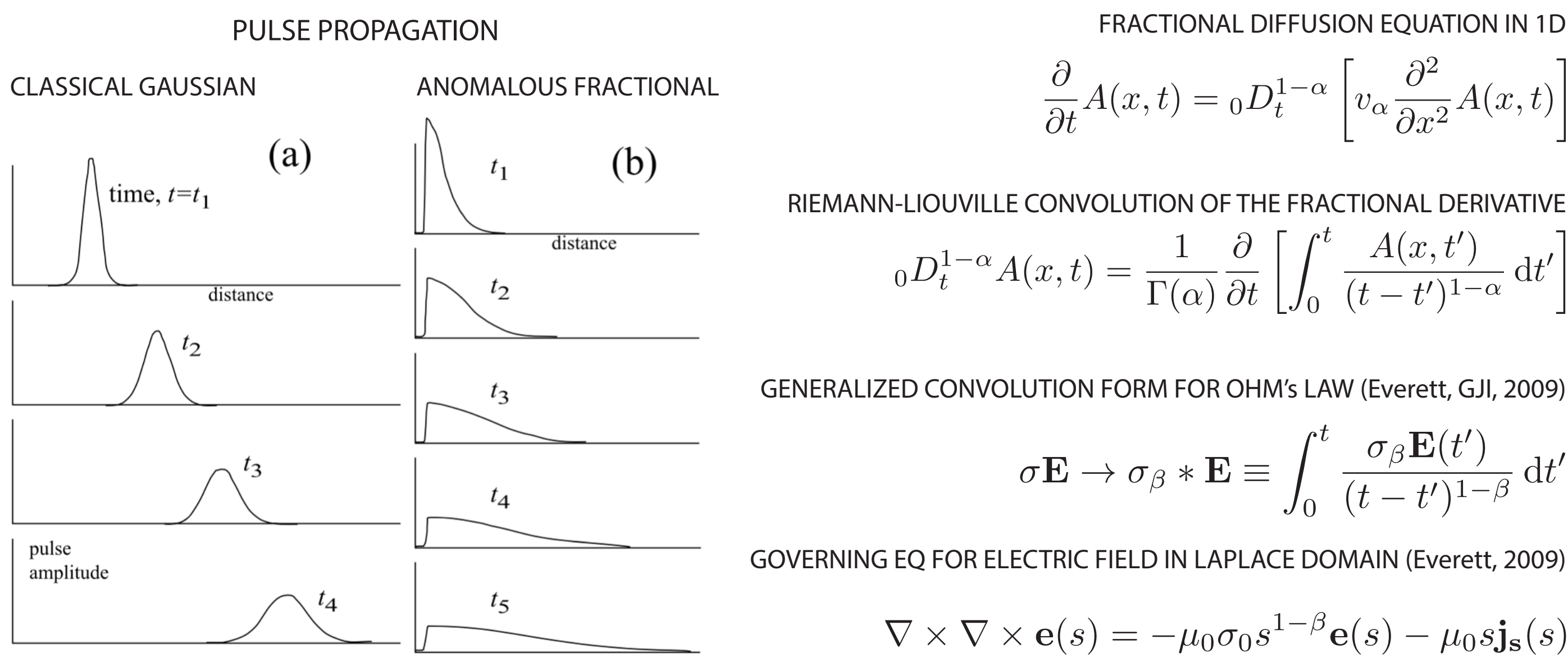
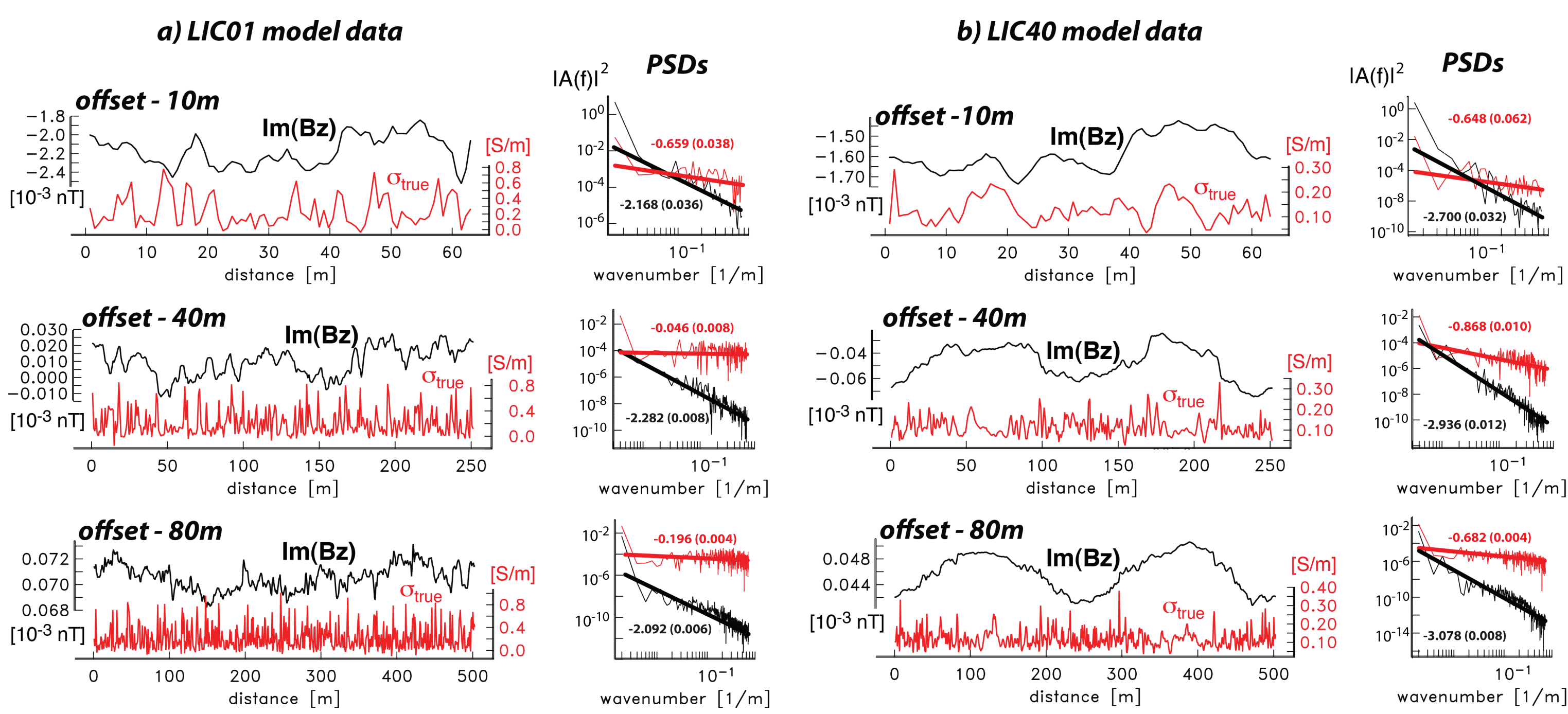
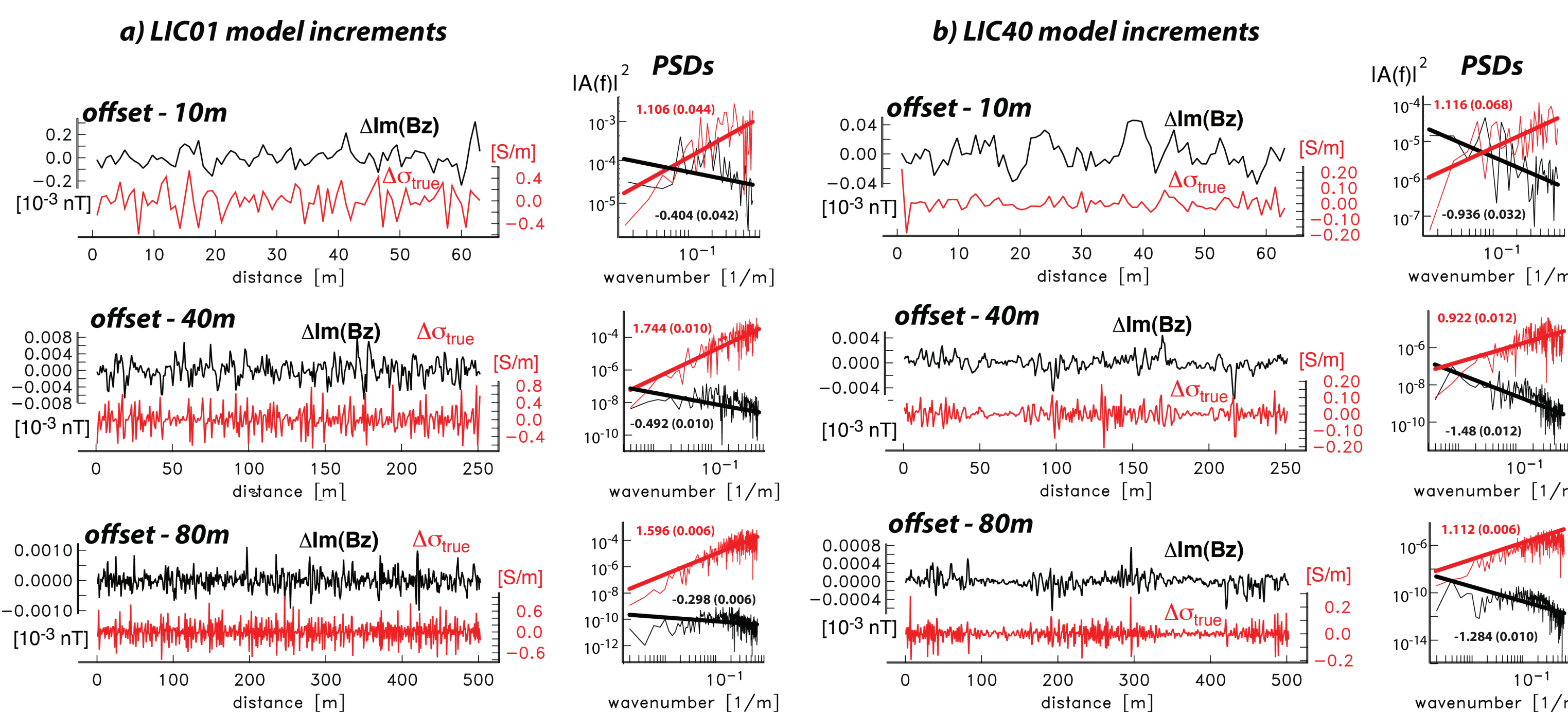


Figure 1. (a) Propagation of a Gaussian pulse  $G(x,t)$  undergoing classical diffusion. (b) Propagation of a CTRW pulse  $A(x,t)$  undergoing anomalous diffusion (after the work of Scher and Montroll [1975]).

## 5. Spectral Analysis of Simulated Azimuthal Variations



(ABOVE) Ground conductivity (true) and the CSEM responses (1600 Hz,  $\text{Im}(B_z)$ ) profiles along a constant-offset circular survey paths and their corresponding power spectral densities (PSDs) for two different conductivity models. Superimposed on the PSDs are least-squares-fit, annotated with values of the slope and jackknife error estimates. (a): Model conductivity and the vertical B profiles (LIC01), and their PSDs for a randomly populated model (convolution length,  $N=1$ ) for 10 m, 40 m, 80 m TX-RX intercoil spacing. (b): The true and the CSEM profiles (LIC40 model), and their PSDs for the x-lined geologic texture model (convolution length,  $N=40$ ) for 10 m, 40 m, 80 m TX-RX intercoil spacing. PSD slopes of the underlying conductivity model (red) have magnitude less than unity, representative of stationary signals with no long-range correlations. A pure white noise signal would have slope equal to zero. In contrast, PSD slopes of the vertical B field (black) do exhibit power-law behavior with a slope greater than unity and thus, long-range correlations.



(ABOVE) Increments of conductivity and CSEM profiles (far above) and their corresponding PSDs. A signal resulting from fractional Brownian motion (fbm) exhibits long range correlations (PSD slopes greater than unity), but with stationary increments. Observe that the CSEM response (black curves) of the LIC01 model is strongly fbm whereas the response of the LIC40 model is suggestive of an fbm signal.

## 2. Observational Evidence for Spatial Hierarchy

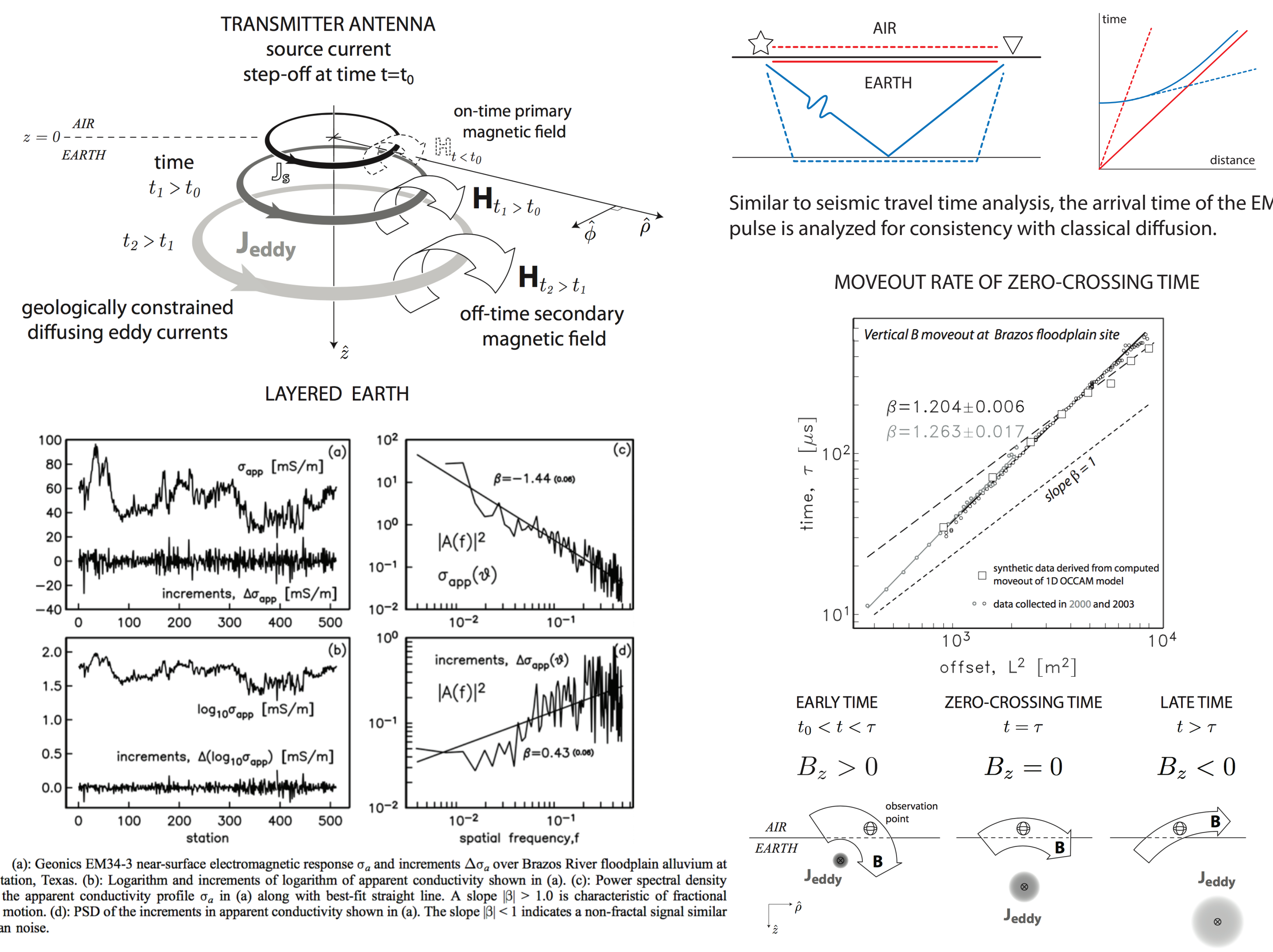
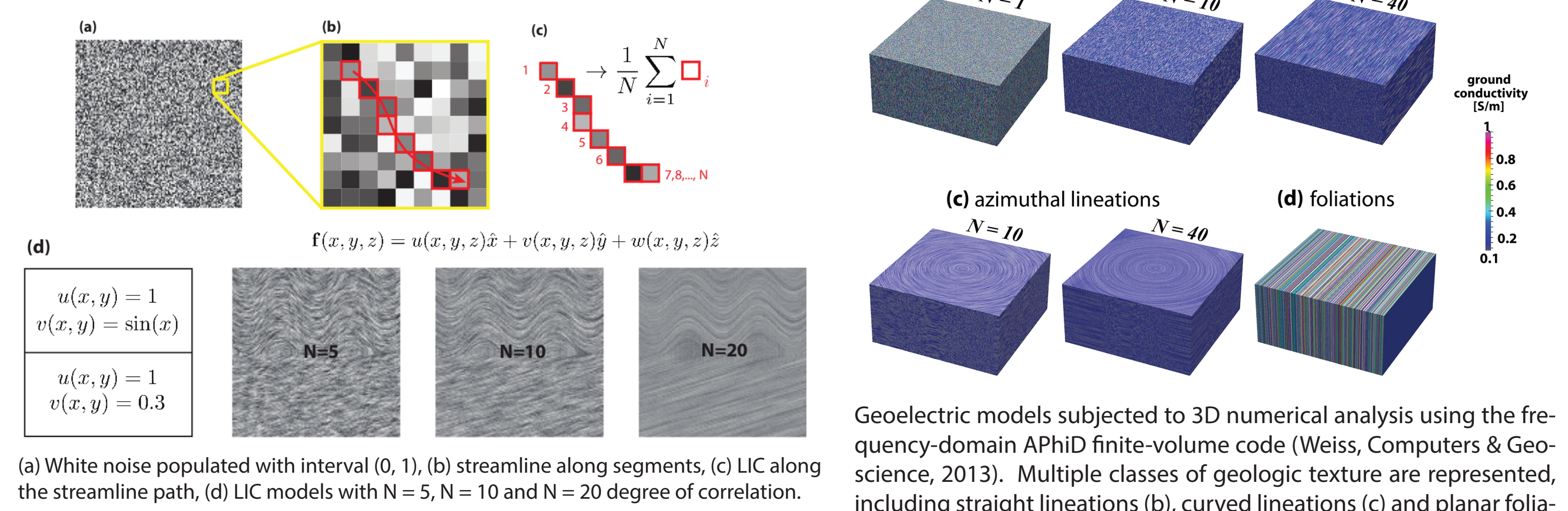


Figure 3. (a): Geonics EM34-3 near-surface electromagnetic response  $\sigma_{app}$  and increments  $\Delta\sigma_{app}$  over Brazos River floodplain alluvium at College Station, Texas. (b): Logarithm and increments of logarithm of apparent conductivity shown in (a). (c): Power spectral density (PSD) of the apparent conductivity profile  $\sigma_{app}$  in (a) along with best-fit straight line. A slope  $|\beta| > 1.0$  is characteristic of fractional Brownian motion. (d): PSD of the increments in apparent conductivity shown in (a). The slope  $|\beta| < 1$  indicates a non-fractional signal similar to Gaussian noise.

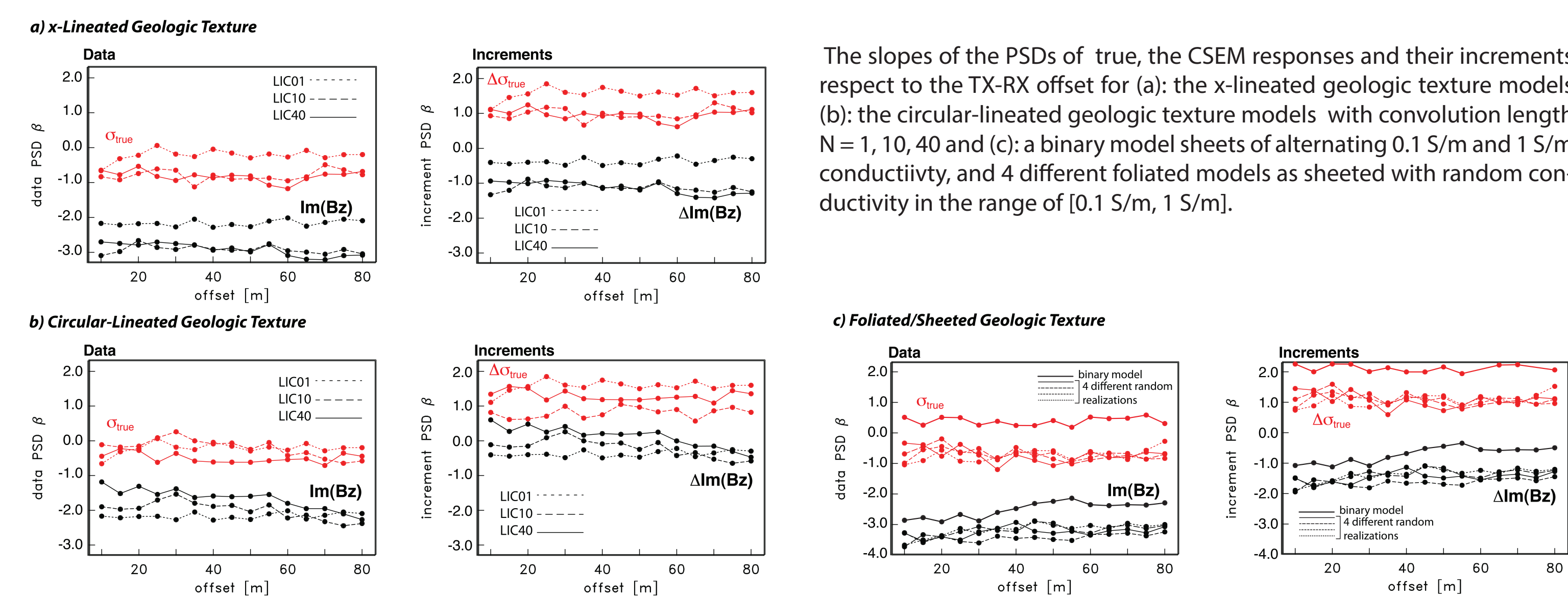
## 4. Building a Multi-scale Macroscopic Model - LIC

LIC - LINE INTEGRAL CONVOLUTION: construct a spatially correlated geoelectric model by convolving a random distribution along streamlines of a given vector field. Correlation length,  $N$ , controls the roughness of the resulting model.



Geoelectric models subjected to 3D numerical analysis using the frequency-domain APhID finite-volume code (Weiss, Computers & Geoscience, 2013). Multiple classes of geologic texture are represented, including straight lineations (b), curved lineations (c) and planar foliations (d).

## 6. Spectral Dependence on Transmitter/Receiver Offset



The slopes of the PSDs of true, the CSEM responses and their increments respect to the TX-RX offset for (a): the x-lined geologic texture models (b): the circular-lined geologic texture models with convolution length  $N=1, 10, 40$  and (c): a binary model sheets of alternating 0.1 S/m and 1 S/m conductivity, and 4 different foliated models as sheeted with random conductivity in the range of [0.1 S/m, 1 S/m].

## 7. Comments, Observations and Conclusions

### Notes on mesh design

In our simulations, the EM-34 transmitter is approximated by a square current loop,  $0.75 \times 0.75$  m lying directly on the air-earth interface at the origin of the computational grid. The loop is excited with an alternating current of amplitude 1.0 A at frequency 1600 Hz. The Earth region of the conductivity model is discretized using a  $400 \times 400 \times 50$  uniform mesh with cell-size  $0.75 \times 0.75 \times 3$  m, within the limits of [150 m, 150 m], [150 m, 150 m] and [0 m, 150 m] in the x, y and z directions, respectively. The EM-34 survey profile is designed using a circular RX path with fixed TX at the center. This is done to avoid any effect of the direction of the survey profile relative to the lineation direction. For each model, both the ground conductivity (true) and the resulting CSEM responses (herein reported in the form of apparent ground conductivity) are sampled at the mesh resolution (0.75 m) along the circular paths for 10, 40 and 80 m intercoil spacings.

### Understanding the origins of the fbm signal - a qualitative assessment

- The LIC convolution procedure introduces a short-range dependence/correlation of conductivity
- The forward-modeling step from the conductivity to the EM response is a kind of spatial averaging that provides the medium-range dependence (over the "footprint" of instrument).
- The deterministic trend (i.e. "texture" represented the vector-field LIC convolution kernel) provides the long-range dependence.
- Thus, the final result: signals with dependencies over short, medium and long scales, i.e. an fbm signal.

### What to do next?

- Challenge the fbm/LIC theory by growing the database of EM observations in textured geologic settings.
- Develop strategies for evaluating the EM response from multiple layers textural overprinting - e.g. sedimentary texture overlain by oblique fracture set.
- Explore and map out the transition length scale where the physics from brute force discretization, such as done here, is more efficiently represented by the continuum model of fractional operators.
- Devise strategies for inverting EM data, where appropriate, in terms of its statistical distribution/correlation parameters.