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Container Surface Evaluation by Function Estimation

James Wendelberger
CCS-6

28 July 2017
Briefing

Abstract

Container images are analyzed for specific surface features, such as, pits, cracks, and corrosion. The detection of these features is confounded with complicating features. These complication features include: shape/curvature, welds, edges, scratches, foreign objects among others. A method is provided to discriminate between the various features. The method consists of estimating the image background, determining a residual image and post processing to determine the features present. The methodology is not finalized but demonstrates the feasibility of a method to determine the kind and size of the features present.

Outline

1. Background
2. Overview of the Methodology
 - i. Import Depth Image by Pixel
 - ii. Estimate the Image Background
 - iii. Estimate the Image with Background
 - iv. Difference the Estimates – Residual Surface
 - v. Evaluate Surface Features of the Residual Surface
 - vi. Report Image Statistics
3. 9 Examples on Actual Images
4. Lessons Learned
5. Future work

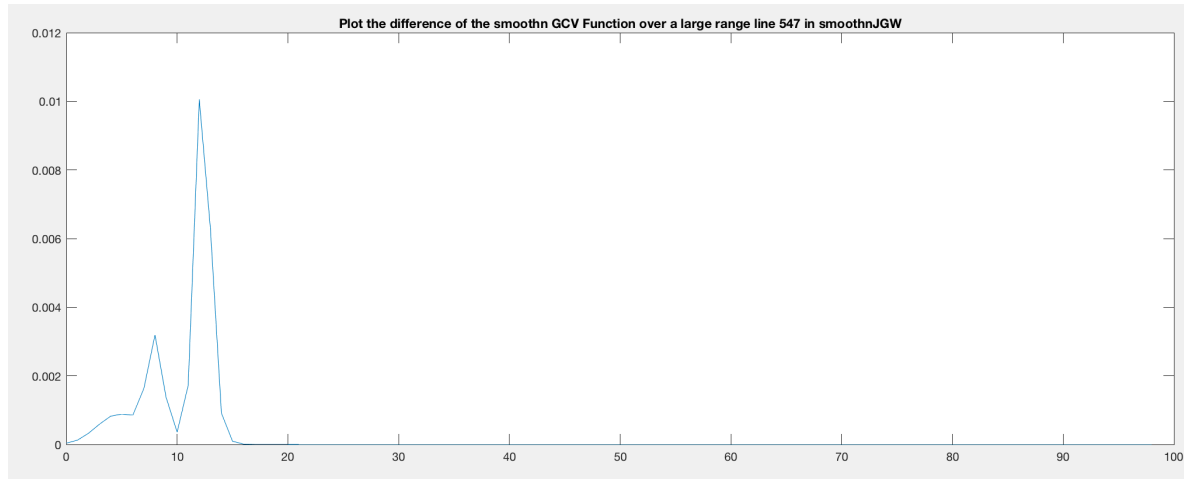
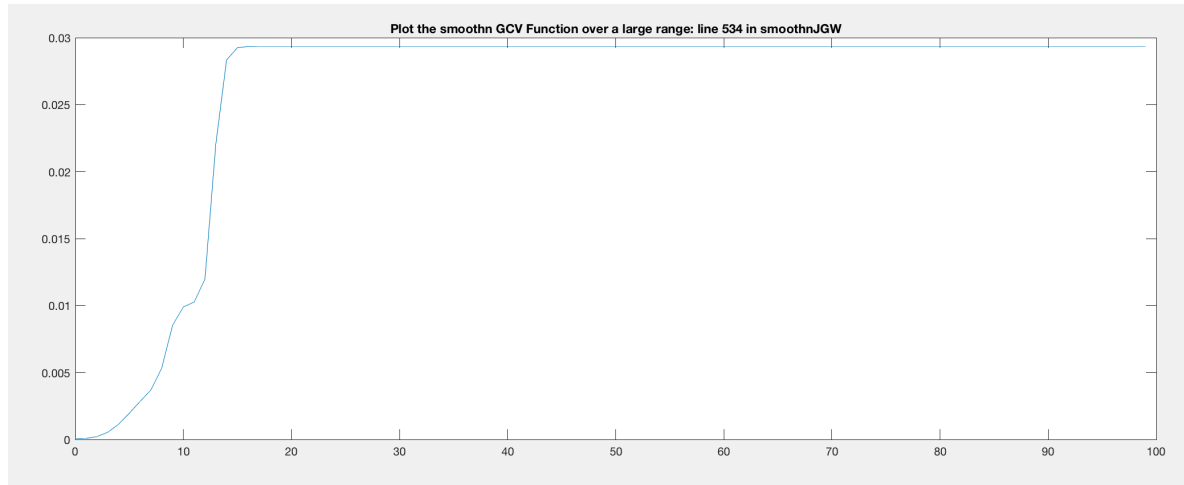
Background

1. Image Data Collected – Visible and Depth
 - i. Redundancies?
2. Detection of Relevant Features
 - i. Size – Depth, Area and Volume
3. Non detection/Exclusion of Irrelevant Features
 - i. Sample Edges, Sample Shape/Curvature
4. Automatic or Manual/Human Intervention
5. Stitching Required or Helpful?
6. Is a Solution to this Problem Feasible?

Methodological Overview

1. Import Depth Image by Pixel
2. Estimate the Image “Background” (Low Frequency)
3. Remove Background from Image – Residual Surface
4. Evaluate Surface Features of the Residual Surface
5. Report Image Statistics of the Residual Surface

Generalized Cross Validation



Nine Examples of Actual Images

Crack_HCL_Pol_no_O2_Sec1_noheader	12.6 MB
HCLO_SeC3_Stich_noheader	13.5 MB
HCLO_UnPol_no_O2...c4_Y1_X1_noheader	5.5 MB
MgCl2_Inside__Y1_X6_noheader	5.6 MB
MgCl2_Inside__Y15_X18_noheader	5.5 MB
MgCl2_Inside_noheader	16.4 MB
Pit_noheader	5.5 MB
S1_20X_6300X2500_noheader	5.5 MB
S19_5_3d_noheader	5.5 MB

Crack_HCL_Pol_no_O2_Sec1

1. 2571 nm/pixel
2. Z Calibration .1 nm per digit
3. Width 4096
4. Height 384
5. Stitched Images of a Crack in Section 1 of HCl Polished with No Oxygen

Crack_HCL_Pol_no_O2_Sec1

Original Data Contours with
Base Smoothing of: 0
No Smoothing, Interpolation

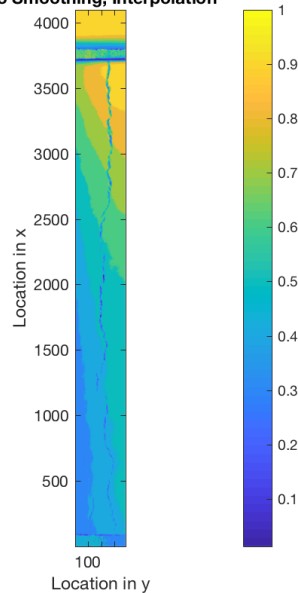


Image - Base Contours with
Base Smoothing of:100

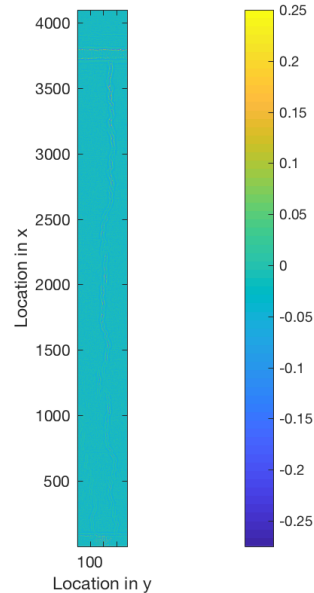


Image - Base Contours with
Base Smoothing of:1000000000

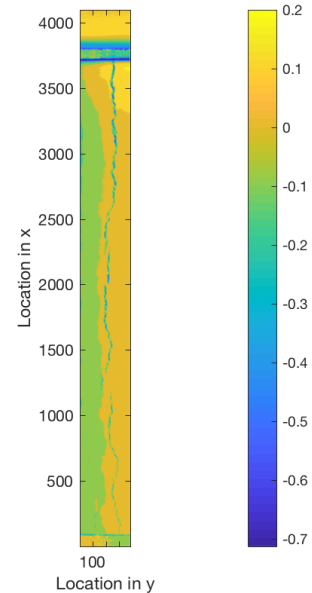
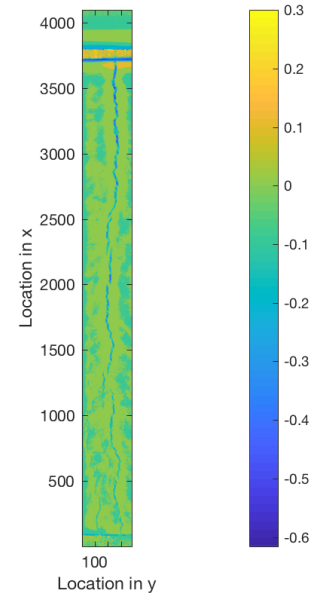
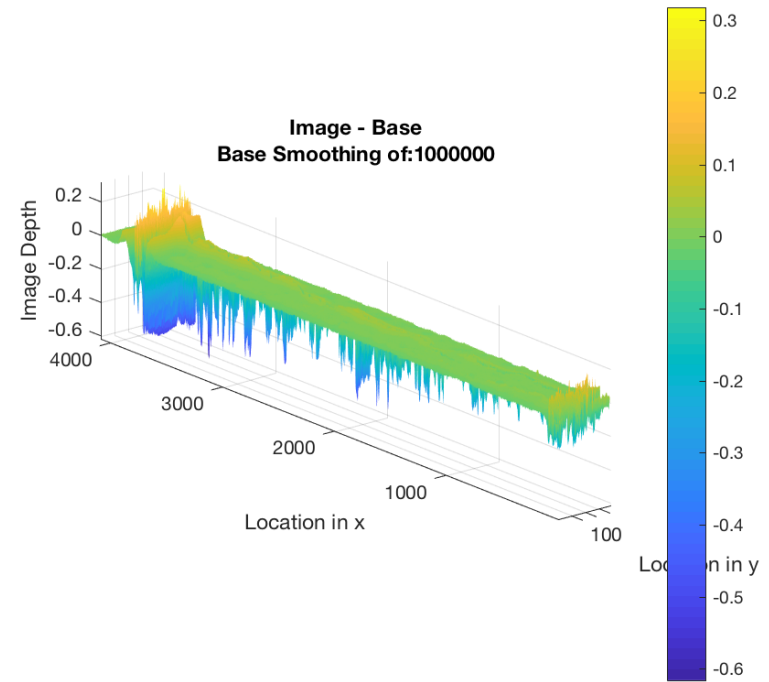
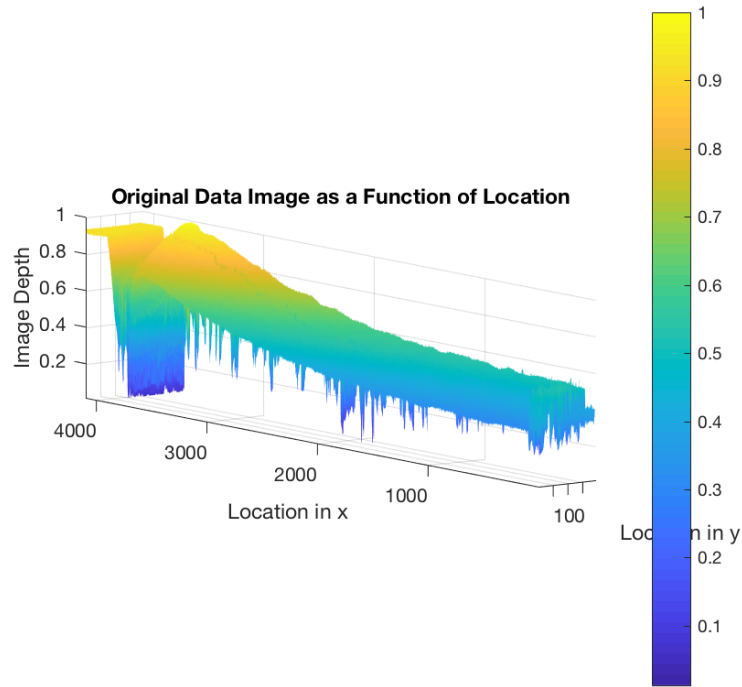


Image - Base Contours with
Base Smoothing of:1000000



Crack_HCL_Pol_no_O2_Sec1



Crack_HCL_Pol_no_O2_Sec1

Original Data Image as a Function of Location

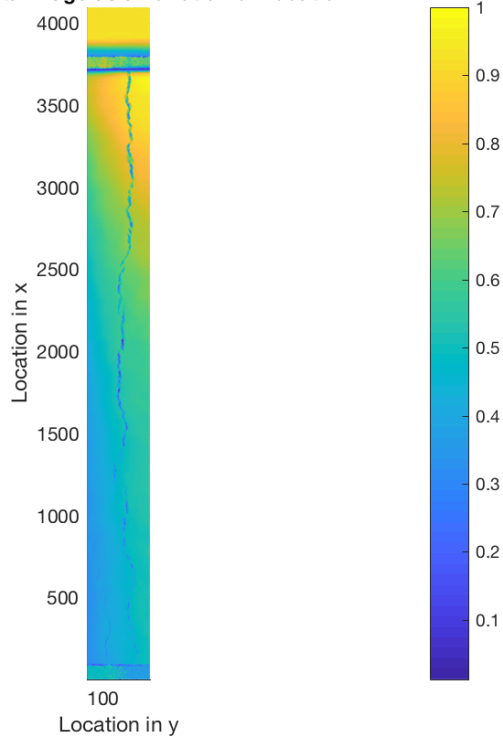
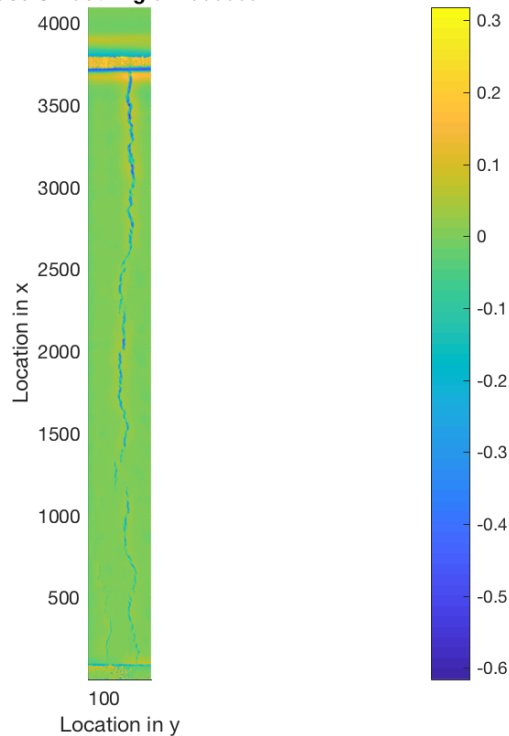
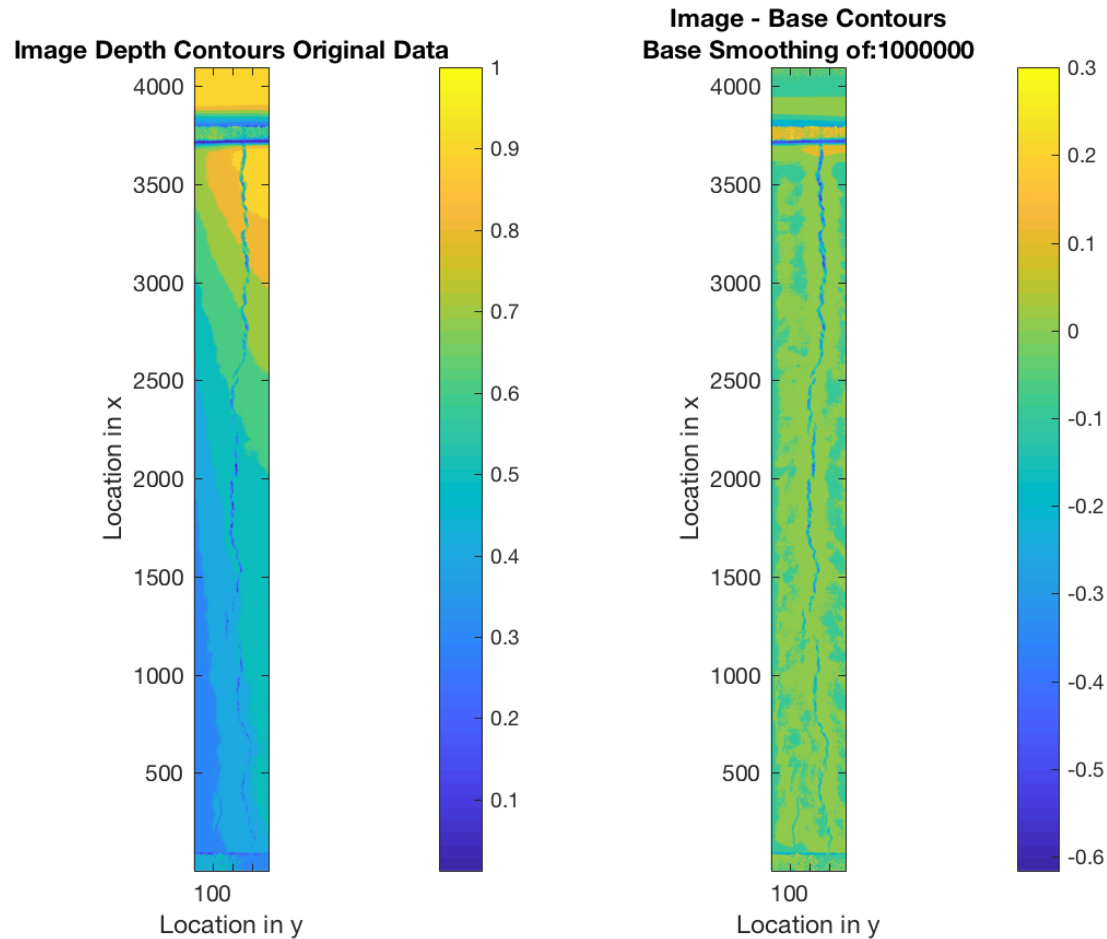


Image - Base
Base Smoothing of:1000000

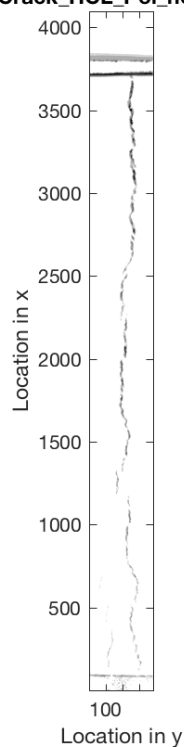


Crack_HCL_Pol_no_02_Sec1

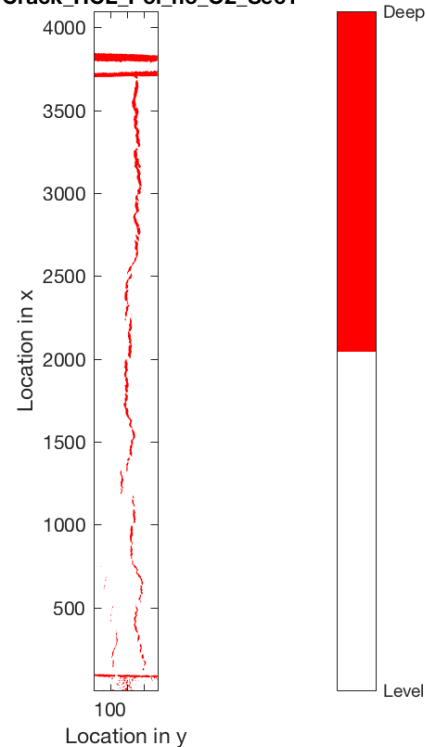


Crack_HCL_Pol_no_O2_Sec1

Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
Deep Area (in pixels): 78039
Deepest Pixel Depth: -0.61588
Volume Below Level: 13531.4836
File: Crack_HCL_Pol_no_O2_Sec1

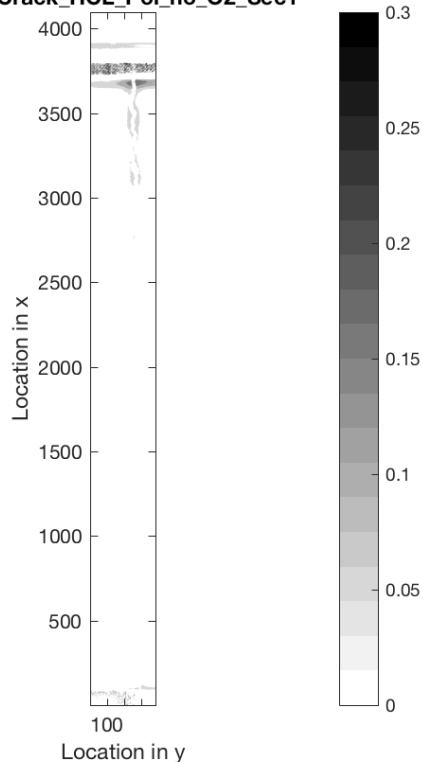


Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
Deep Area (in pixels): 78039
Deepest Pixel Depth: -0.61588
Volume Below Level: 13531.4836
File: Crack_HCL_Pol_no_O2_Sec1

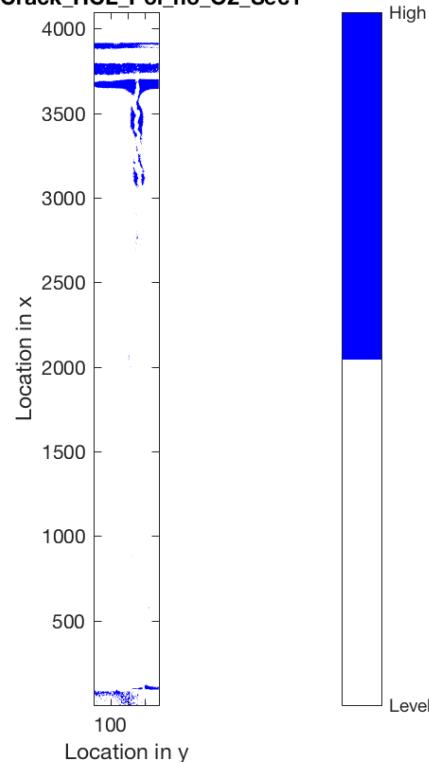


Crack_HCL_Pol_no_O2_Sec1

Height Above 0.05 of
Image - Base Contours
Base Smoothing of:1000000
High Area (in pixels): 72388
Highest Pixel Height: 0.31784
Volume Above Level: -6912.0912
File: Crack_HCL_Pol_no_O2_Sec1

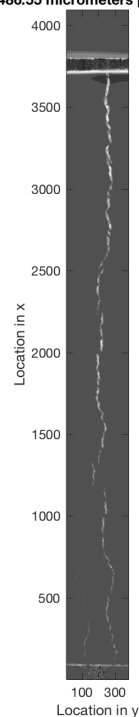


Height Above 0.05 of
Image - Base Contours
Base Smoothing of:1000000
High Area (in pixels): 72388
Highest Pixel Height: 0.31784
Volume Above Level: -6912.0912
File: Crack_HCL_Pol_no_O2_Sec1

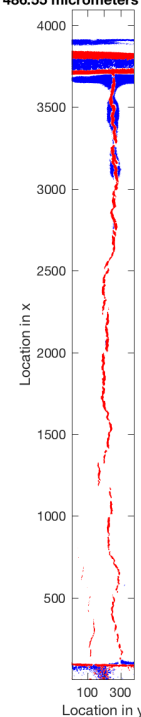


Crack_HCL_Pol_no_O2_Sec1

Height Above 0.05 and
Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
High Area (in pixels): 72388
Highest Pixel Height: 0.31784
Volume Above Level: -6912.0912
Deep Area (in pixels): 78039
Deepest Pixel Depth: -0.61588
Volume Below Level: 13531.4836
File: Crack_HCL_Pol_no_O2_Sec1
Unit Length in the z Direction:
486.55 micrometers per digit.

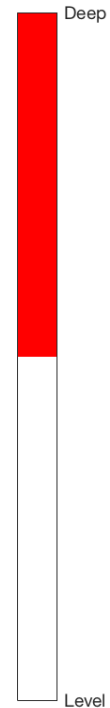
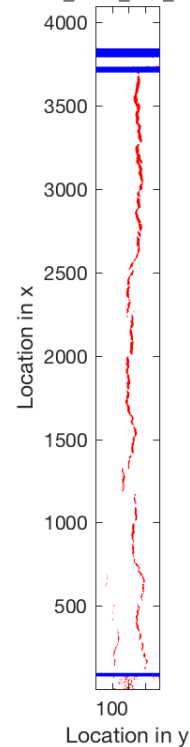


Height Above 0.05 and
Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
High Area (in pixels): 72388
Highest Pixel Height: 0.31784
Volume Above Level: -6912.0912
Deep Area (in pixels): 78039
Deepest Pixel Depth: -0.61588
Volume Below Level: 13531.4836
File: Crack_HCL_Pol_no_O2_Sec1
Unit Length in the z Direction:
486.55 micrometers per digit.



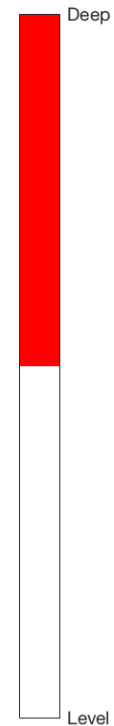
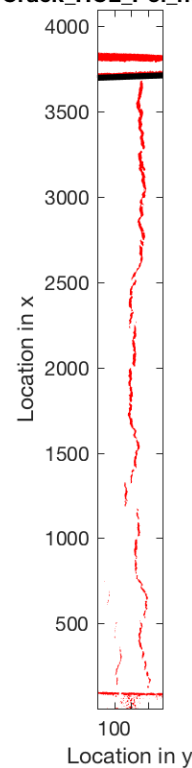
Crack_HCL_Pol_no_O2_Sec1

Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
Deep Area (in pixels): 78039
Deepest Pixel Depth: -0.61588
Volume Below Level: 13531.4836
File: Crack_HCL_Pol_no_O2_Sec1

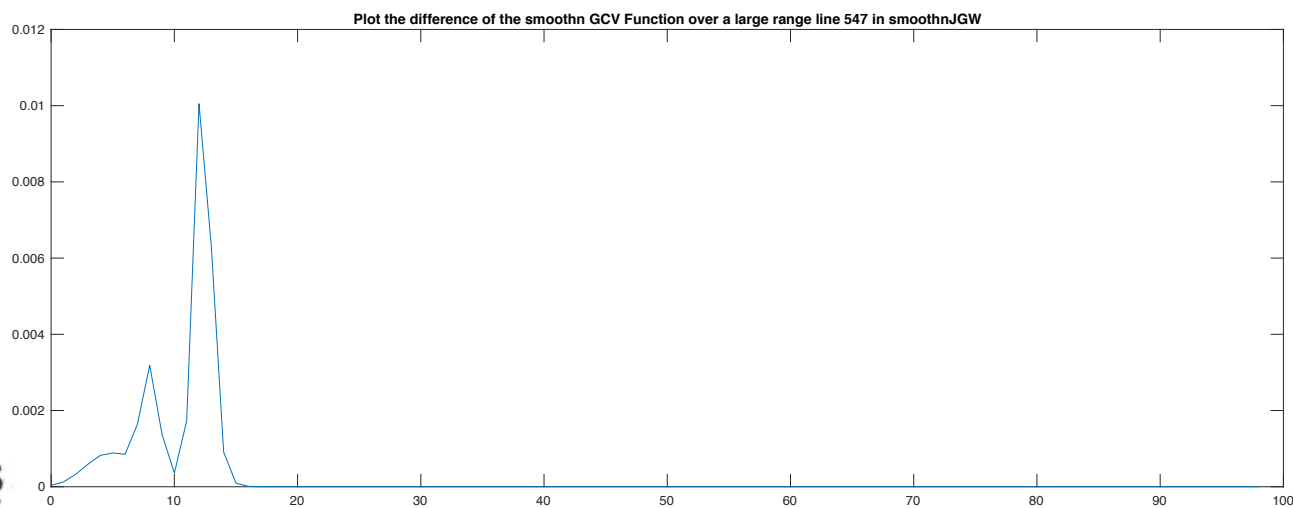
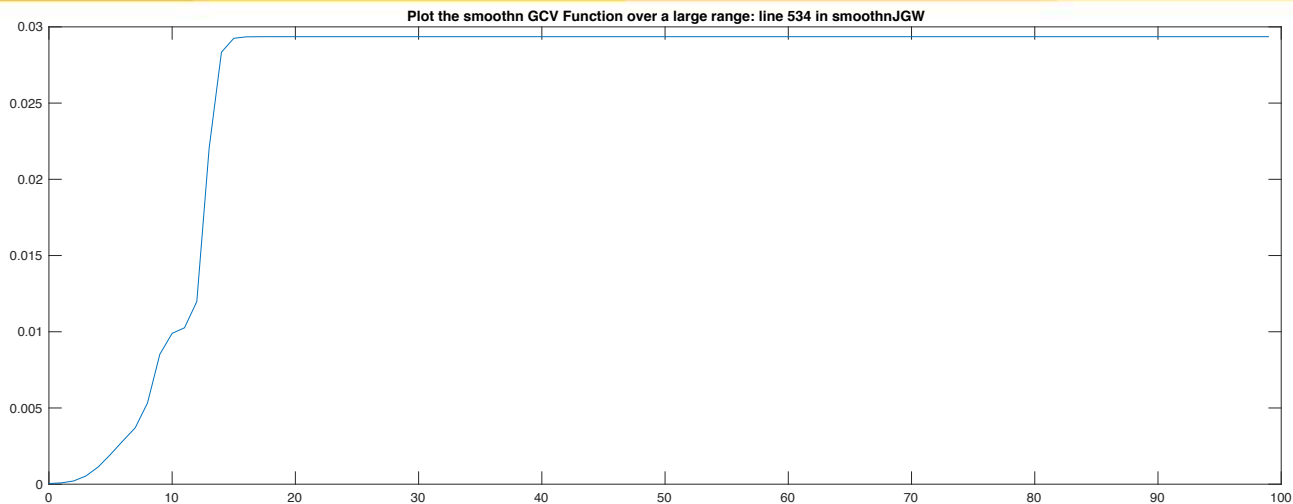


Crack_HCL_Pol_no_O2_Sec1

Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
Deep Area (in pixels): 78039
Deepest Pixel Depth: -0.61588
Volume Below Level: 13531.4836
File: Crack_HCL_Pol_no_O2_Sec1

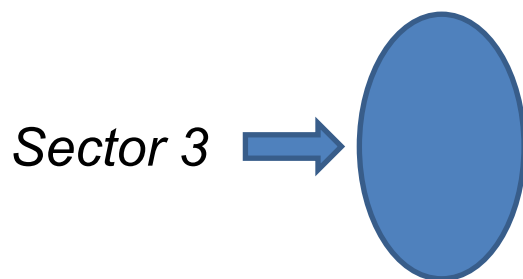


Crack_HCL_Pol_no_O2_Sec1



HClO_SeC3_Stich

1. 2759 nm/pixel
2. Z Calibration .1 nm per digit
3. Width 3392
4. Height 512
5. Stitched Images of Sector 3 of HClO Teardrop



HClO_SeC3_Stich

Original Data Contours with
Base Smoothing of: 0
No Smoothing, Interpolation

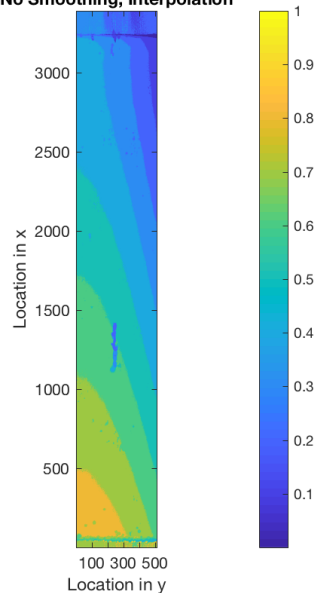


Image - Base Contours with
Base Smoothing of:100

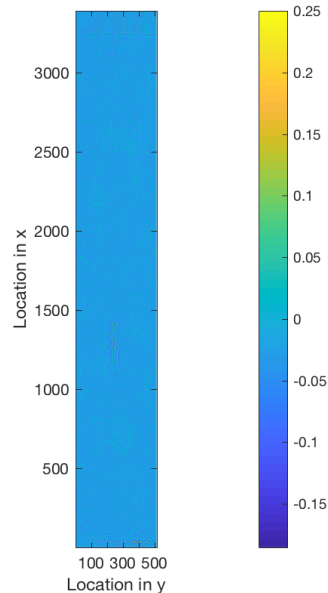


Image - Base Contours with
Base Smoothing of:1000000000

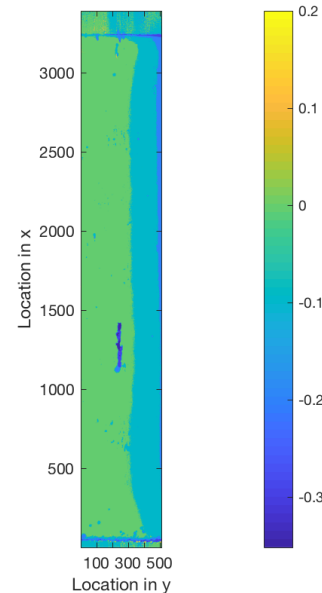
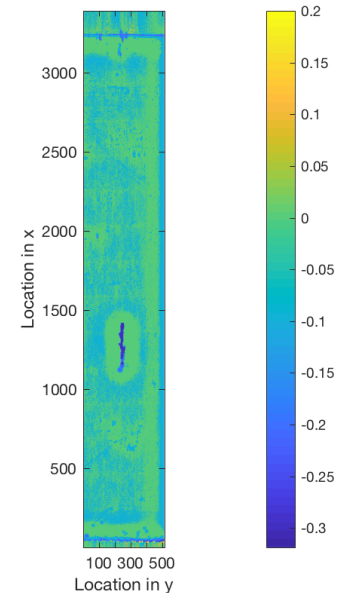
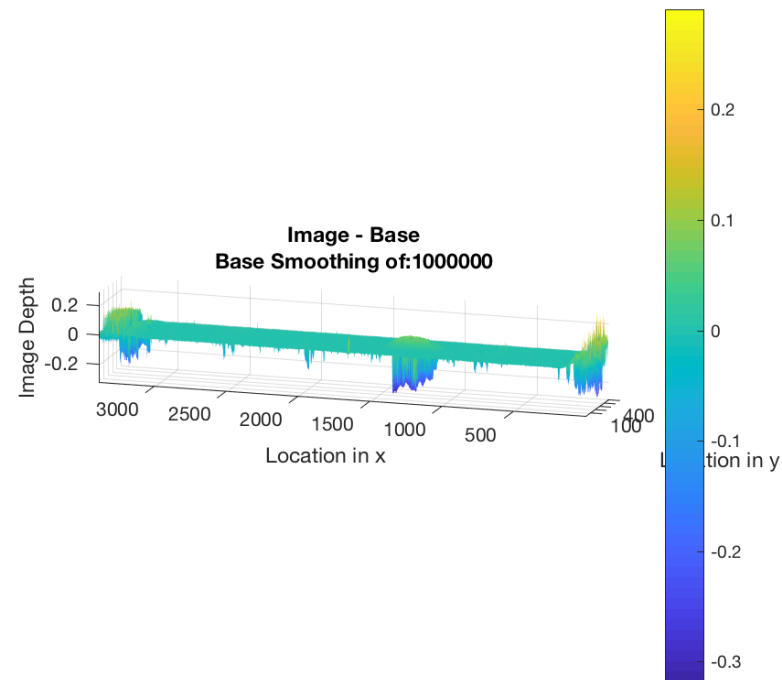
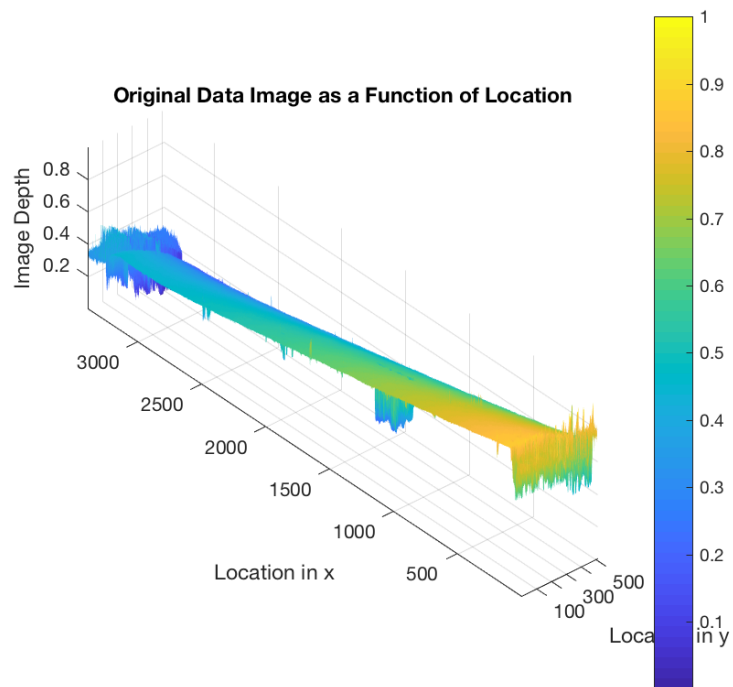


Image - Base Contours with
Base Smoothing of:1000000



HClO_SeC3_Stich



HClO_SeC3_Stich

Original Data Image as a Function of Location

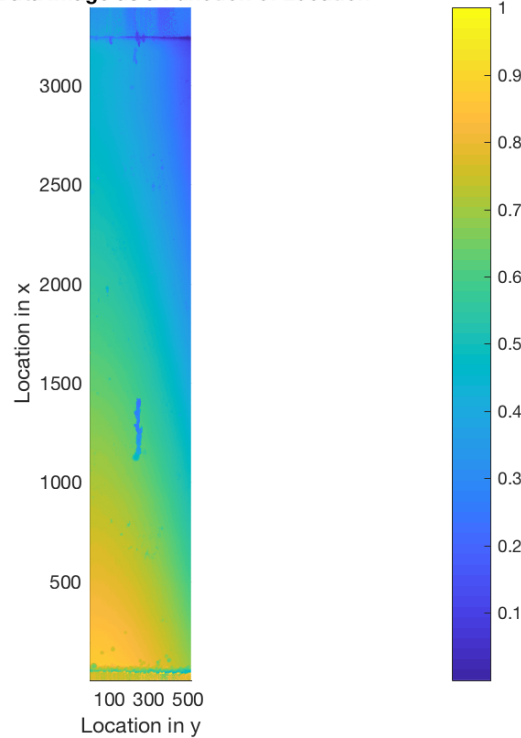
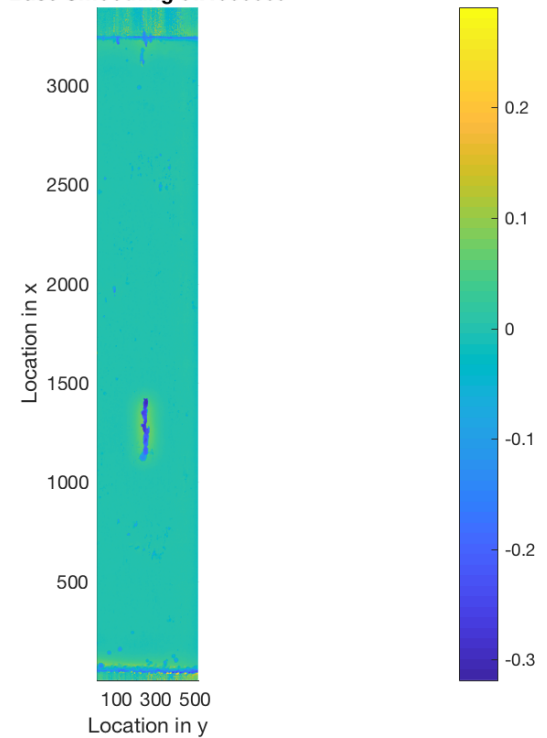


Image - Base
Base Smoothing of:1000000



HClO_SeC3_Stich

Image Depth Contours Original Data

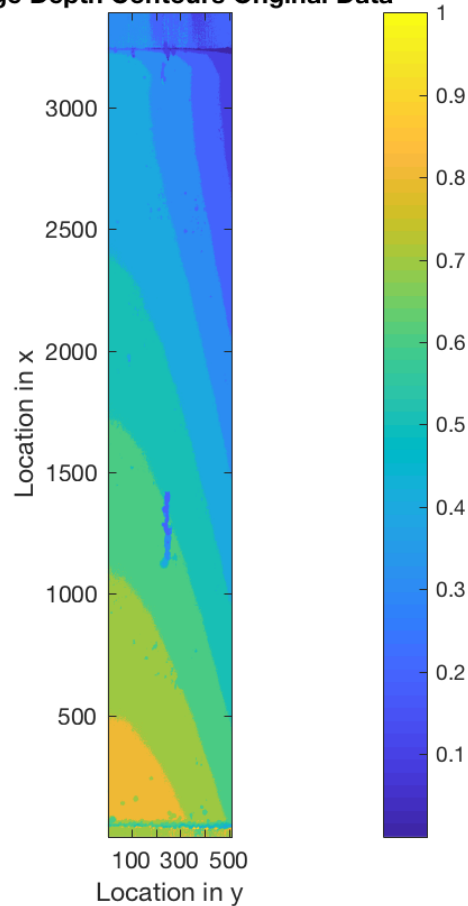
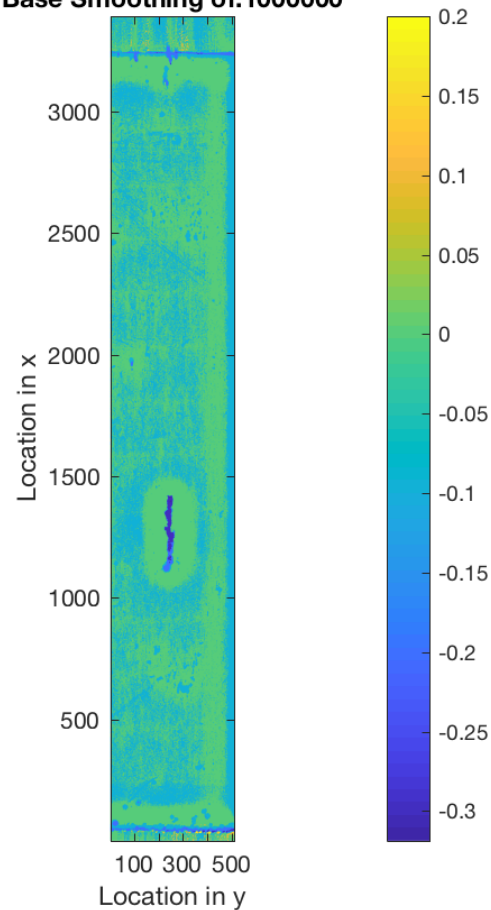
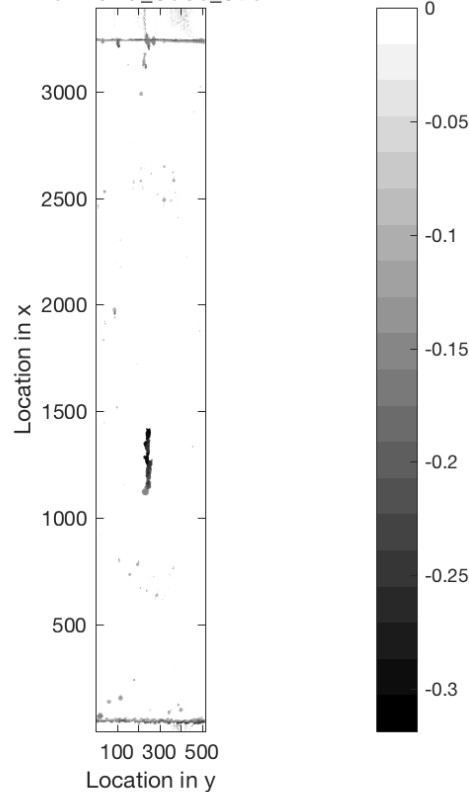


Image - Base Contours
Base Smoothing of:1000000

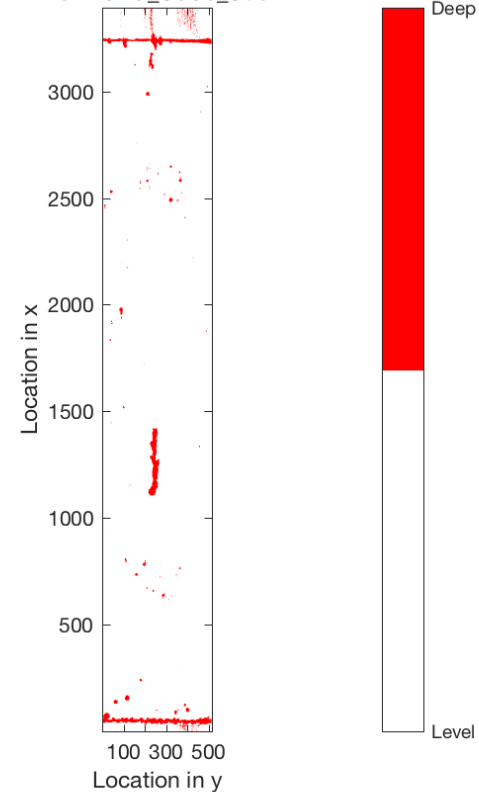


HCLO_SeC3_Stich

Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
Deep Area (in pixels): 36590
Deepest Pixel Depth: -0.31865
Volume Below Level: 4530.0397
File: HCLO_SeC3_Stich

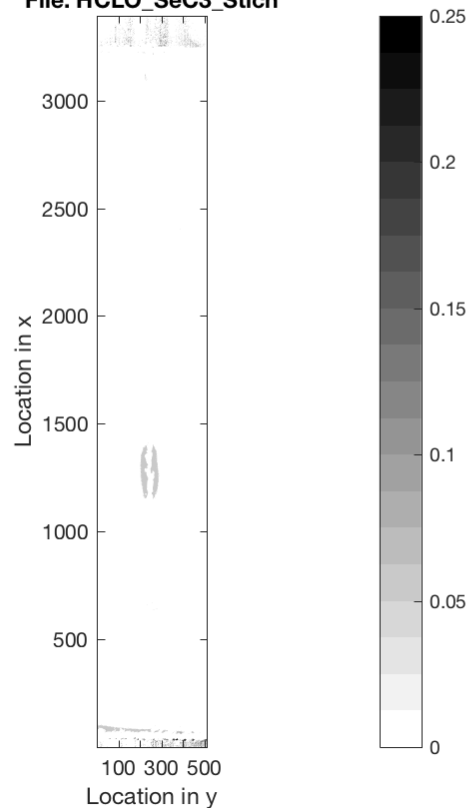


Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
Deep Area (in pixels): 36590
Deepest Pixel Depth: -0.31865
Volume Below Level: 4530.0397
File: HCLO_SeC3_Stich

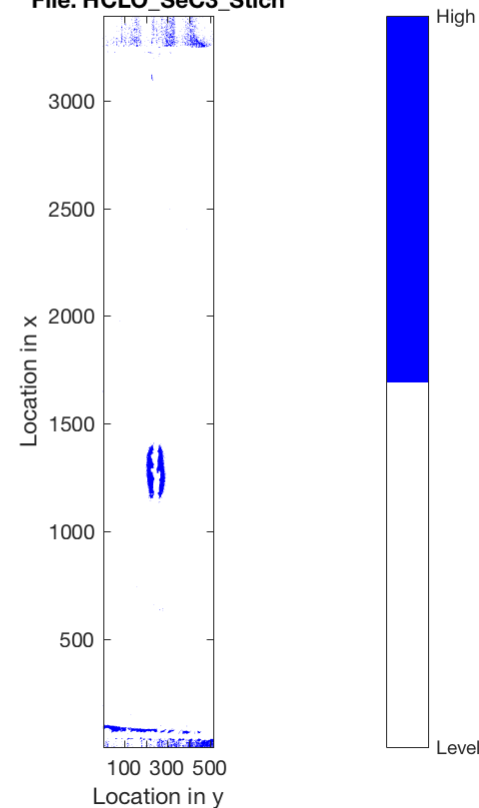


HCLO_SeC3_Stich

Height Above 0.05 of
Image - Base Contours
Base Smoothing of:1000000
High Area (in pixels): 31529
Highest Pixel Height: 0.29064
Volume Above Level: -2392.6612
File: HCLO_SeC3_Stich

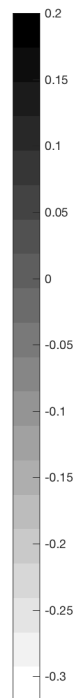
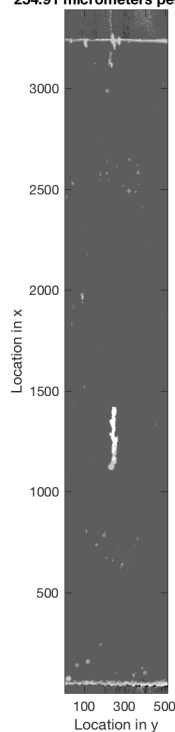


Height Above 0.05 of
Image - Base Contours
Base Smoothing of:1000000
High Area (in pixels): 31529
Highest Pixel Height: 0.29064
Volume Above Level: -2392.6612
File: HCLO_SeC3_Stich

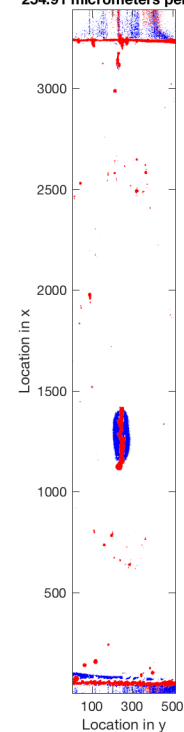


HClO_SeC3_Stich

Height Above 0.05 and
Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
High Area (in pixels): 31529
Highest Pixel Height: 0.29064
Volume Above Level: -2392.6612
Deep Area (in pixels): 36590
Deepest Pixel Depth: -0.31865
Volume Below Level: 4530.0397
File: HClO_SeC3_Stich
Unit Length in the z Direction:
254.91 micrometers per digit.

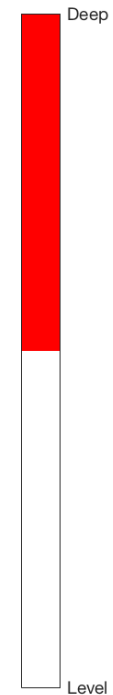
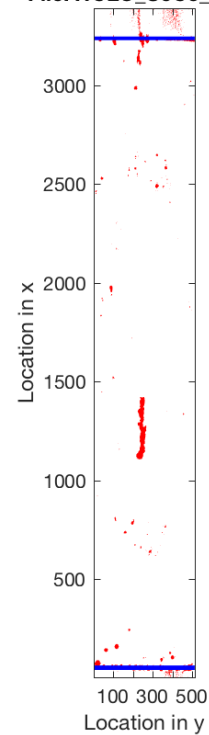


Height Above 0.05 and
Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
High Area (in pixels): 31529
Highest Pixel Height: 0.29064
Volume Above Level: -2392.6612
Deep Area (in pixels): 36590
Deepest Pixel Depth: -0.31865
Volume Below Level: 4530.0397
File: HClO_SeC3_Stich
Unit Length in the z Direction:
254.91 micrometers per digit.



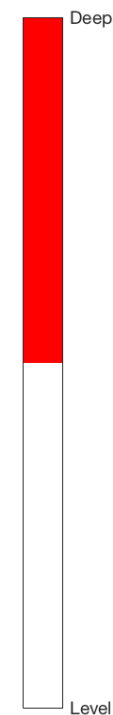
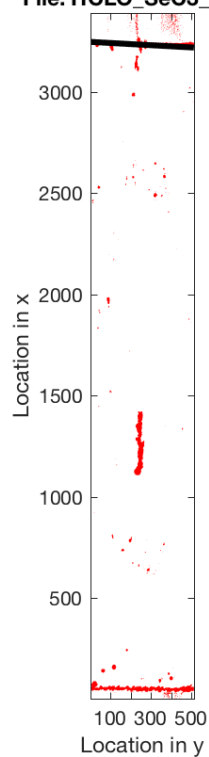
HCLO_SeC3_Stich

Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
Deep Area (in pixels): 36590
Deepest Pixel Depth: -0.31865
Volume Below Level: 4530.0397
File: HCLO_SeC3_Stich

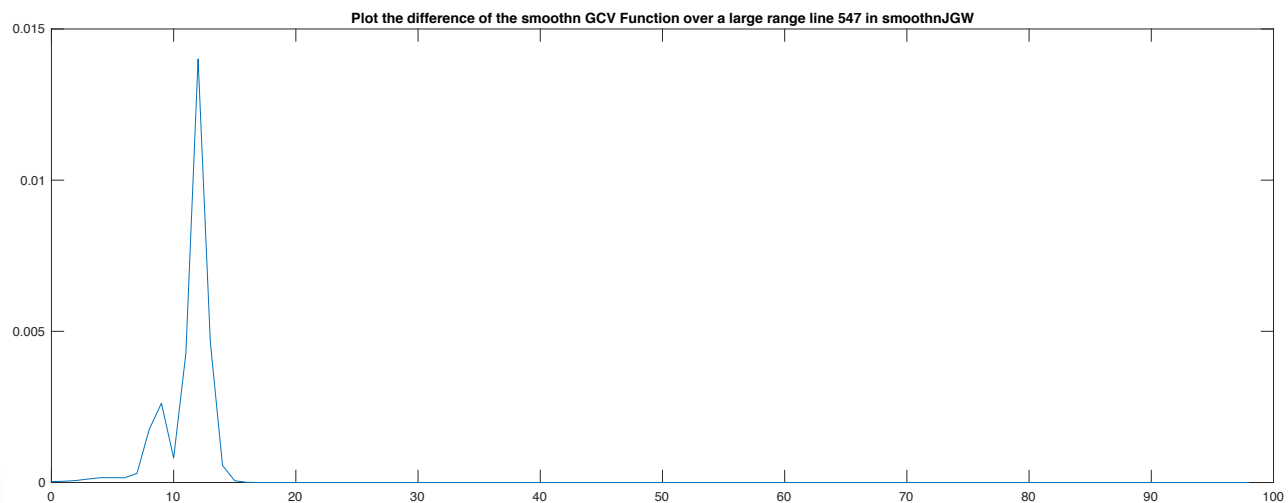
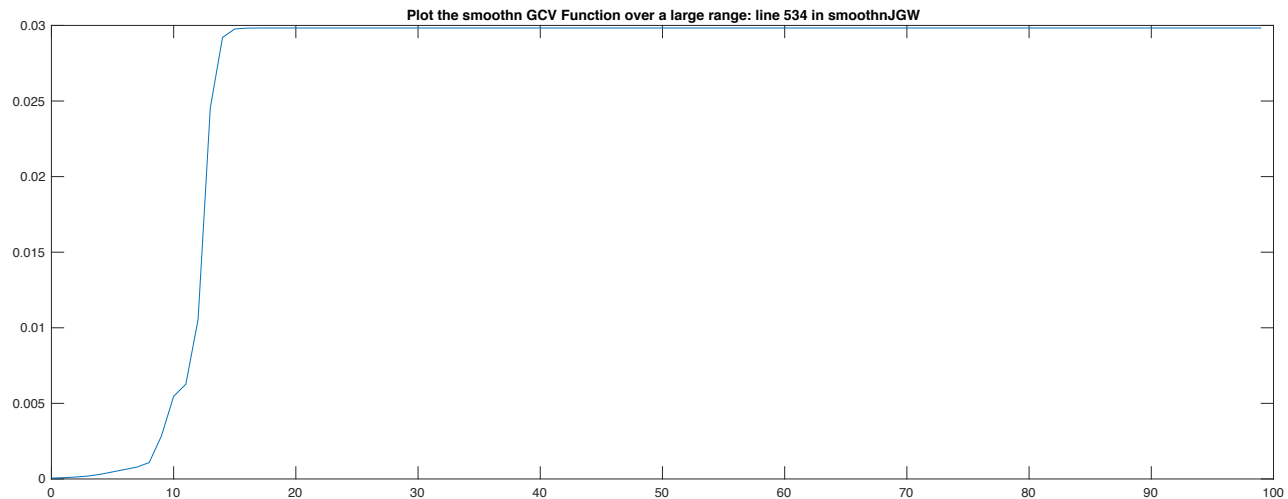


HClO_SeC3_Stich

Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
Deep Area (in pixels): 36590
Deepest Pixel Depth: -0.31865
Volume Below Level: 4530.0397
File: HClO_SeC3_Stich

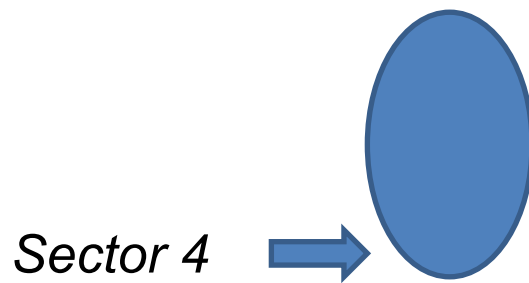


HClO_SeC3_Stich

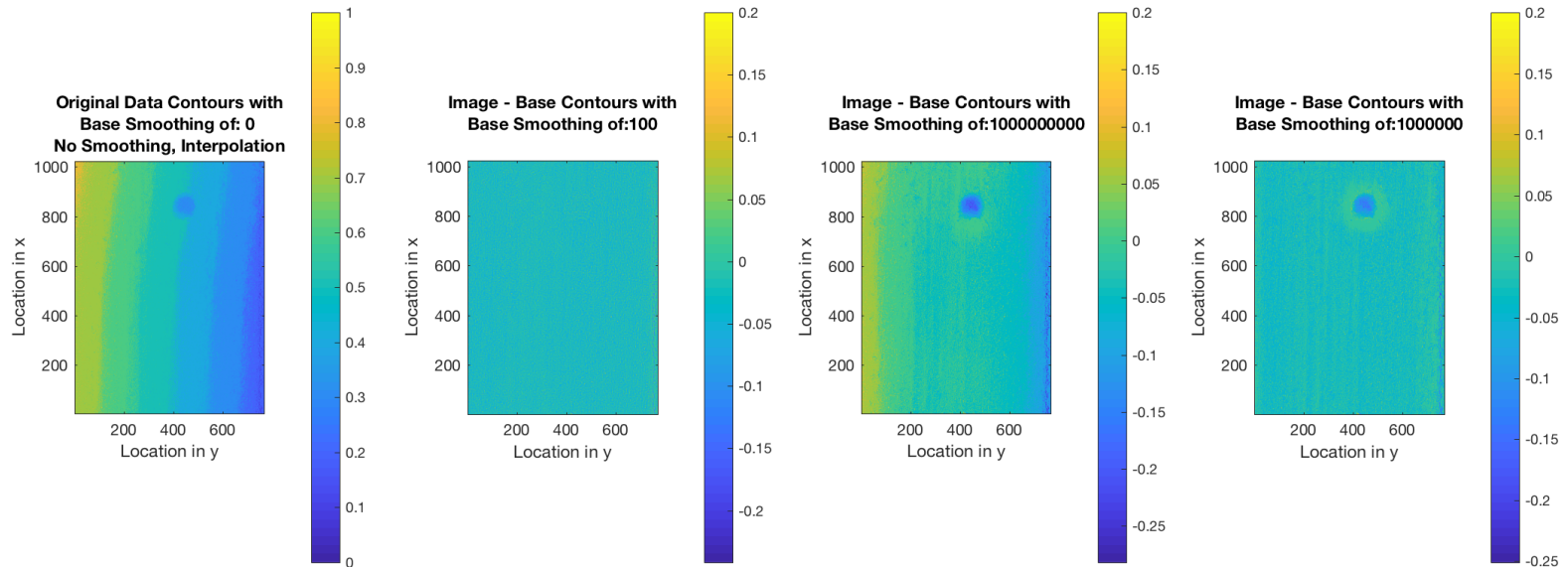


HCLO_UnPol_no_O2_Sec4_Y1_X1

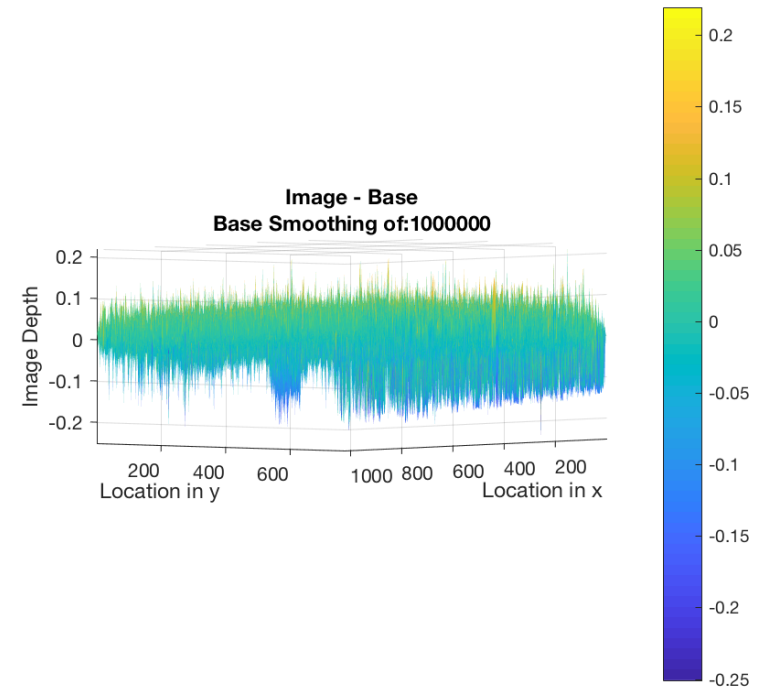
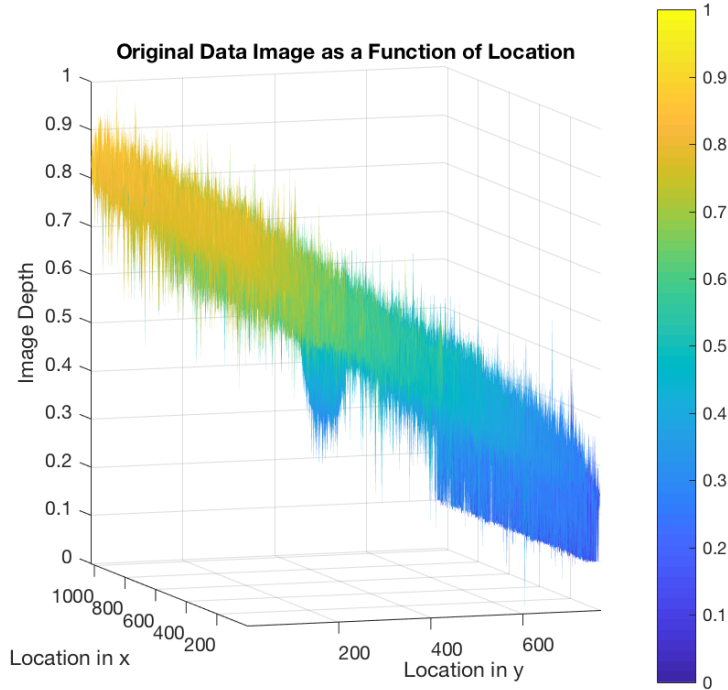
1. 690 nm/pixel
2. Z Calibration .1 nm per digit
3. Width 1024
4. Height 768
5. Stitched Images of Sector 4 of HCLO Unpolished No Oxygen In Location Y1, X1 of a Teardrop



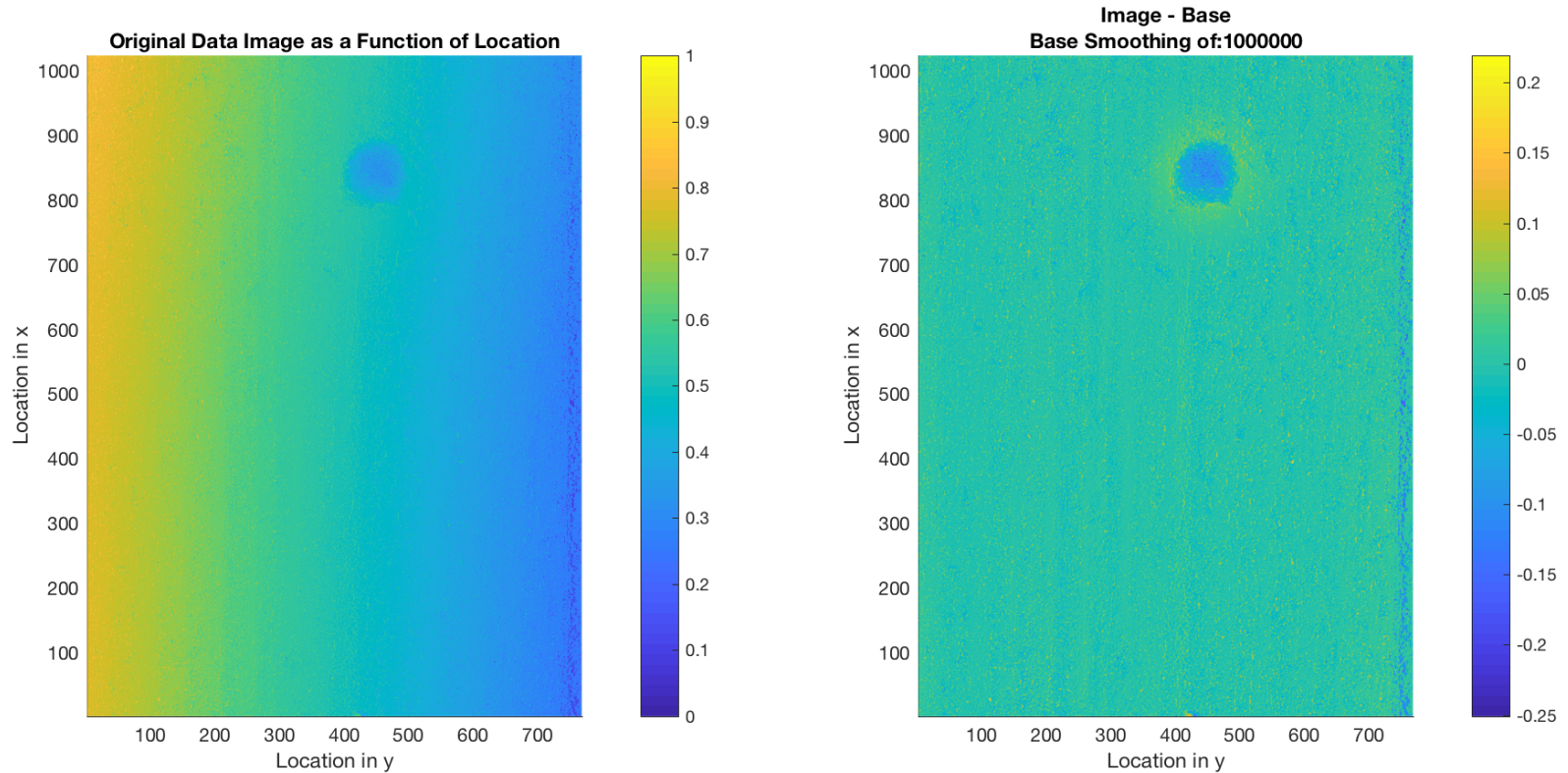
HCLO_UnPol_no_O2_Sec4_Y1_X1



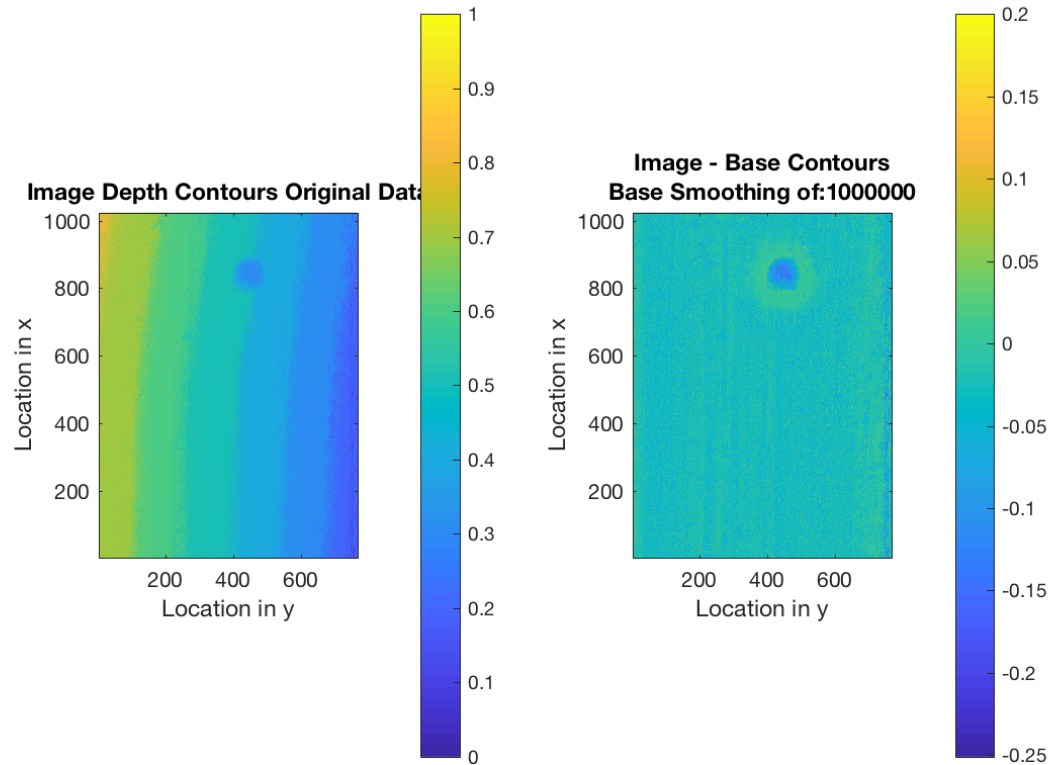
HCLO_UnPol_no_O2_Sec4_Y1_X1



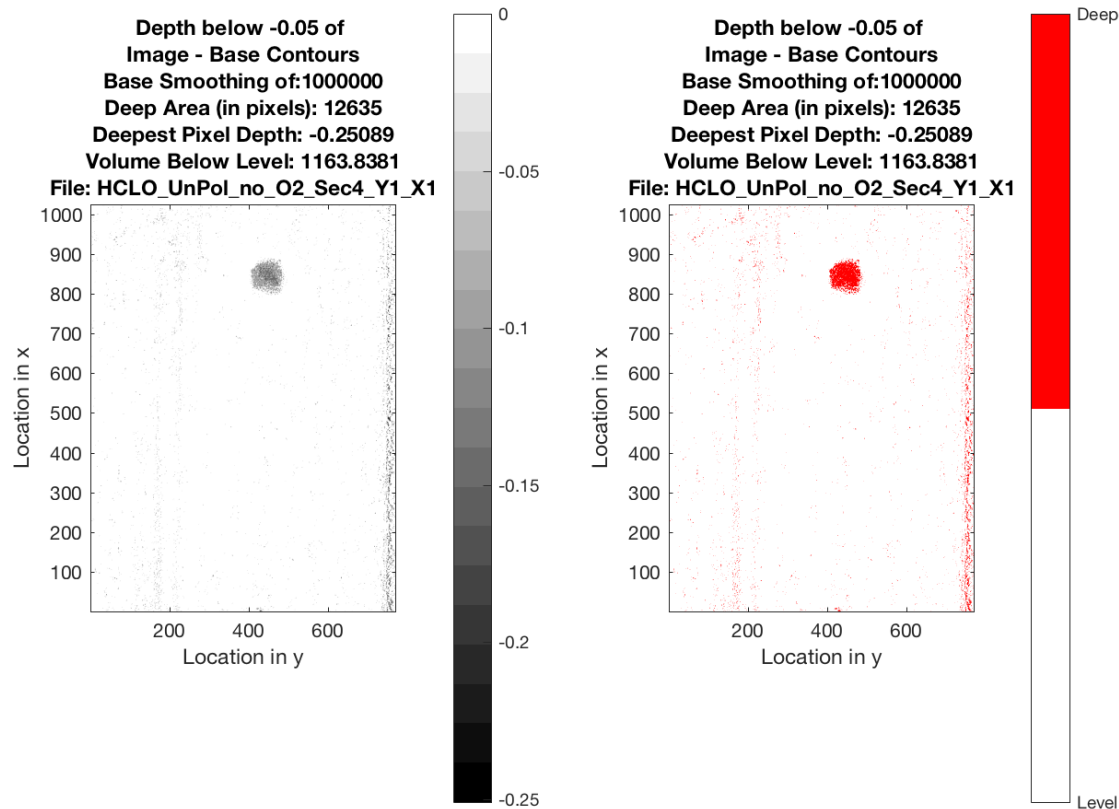
HClO_UnPol_no_O2_Sec4_Y1_X1



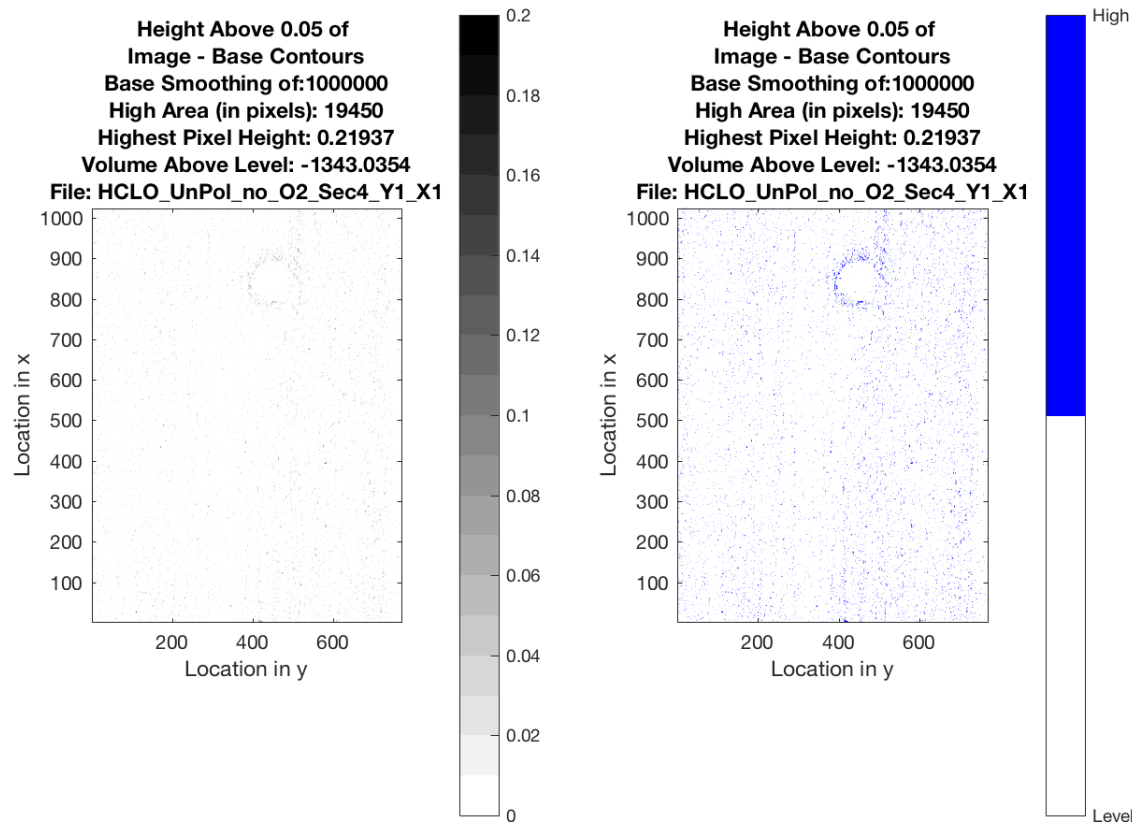
HCLO_UnPol_no_O2_Sec4_Y1_X1



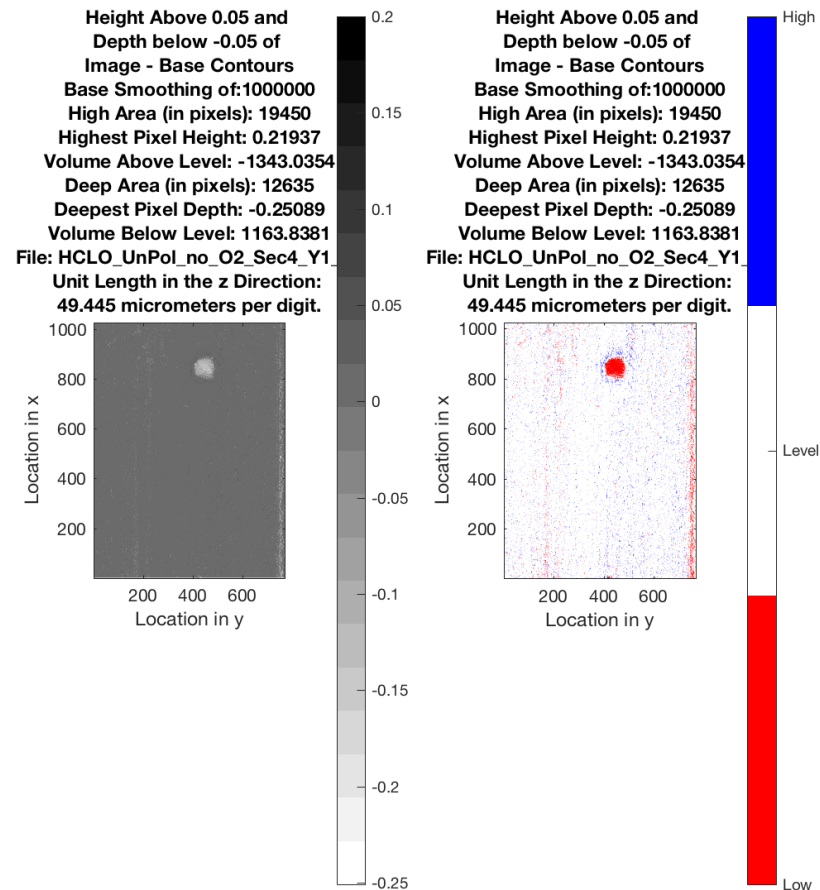
HCLO_UnPol_no_O2_Sec4_Y1_X1



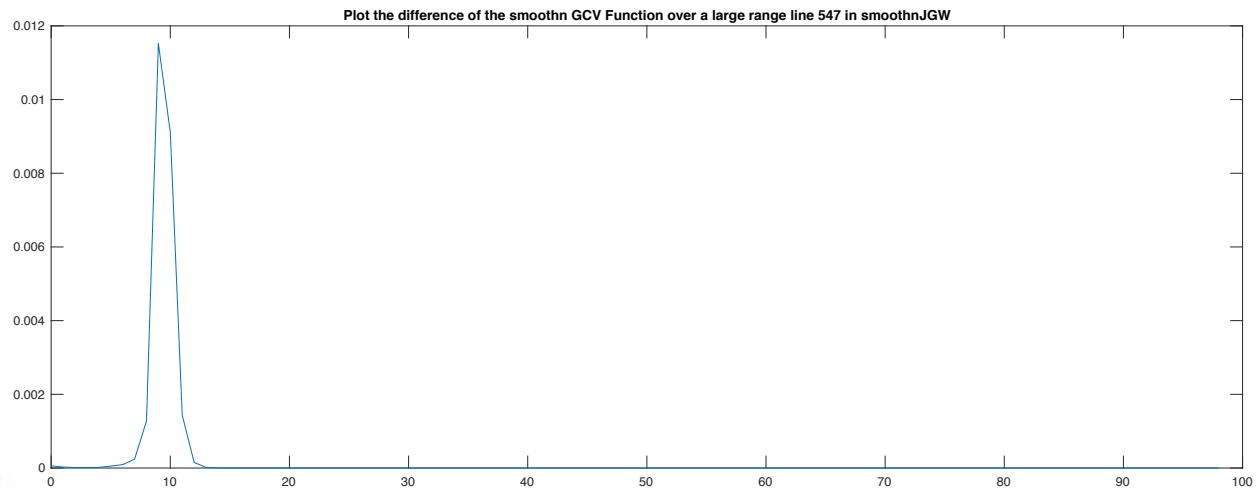
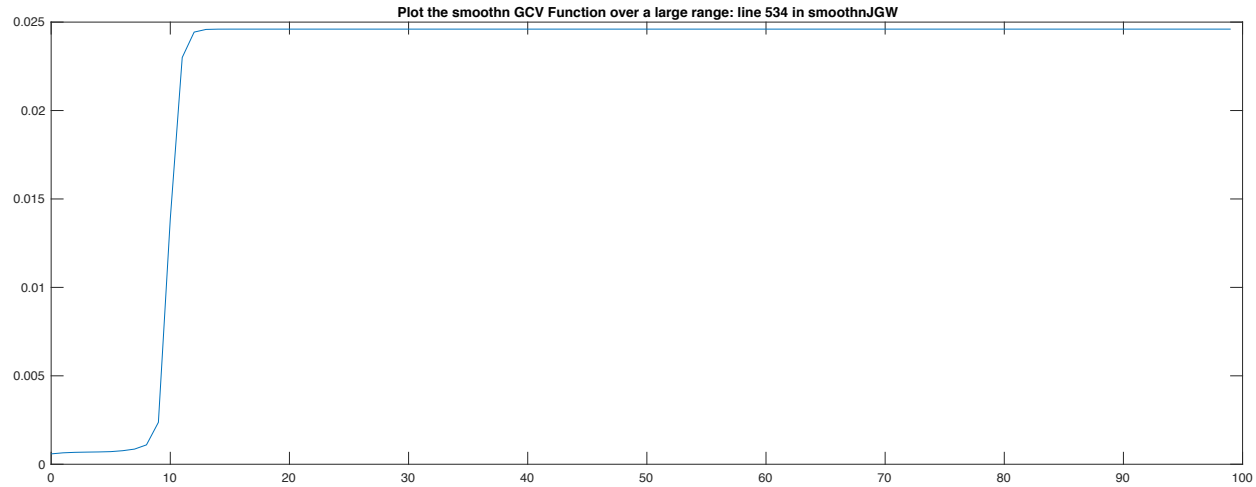
HCLO_UnPol_no_O2_Sec4_Y1_X1



HCLO_UnPol_no_O2_Sec4_Y1_X1



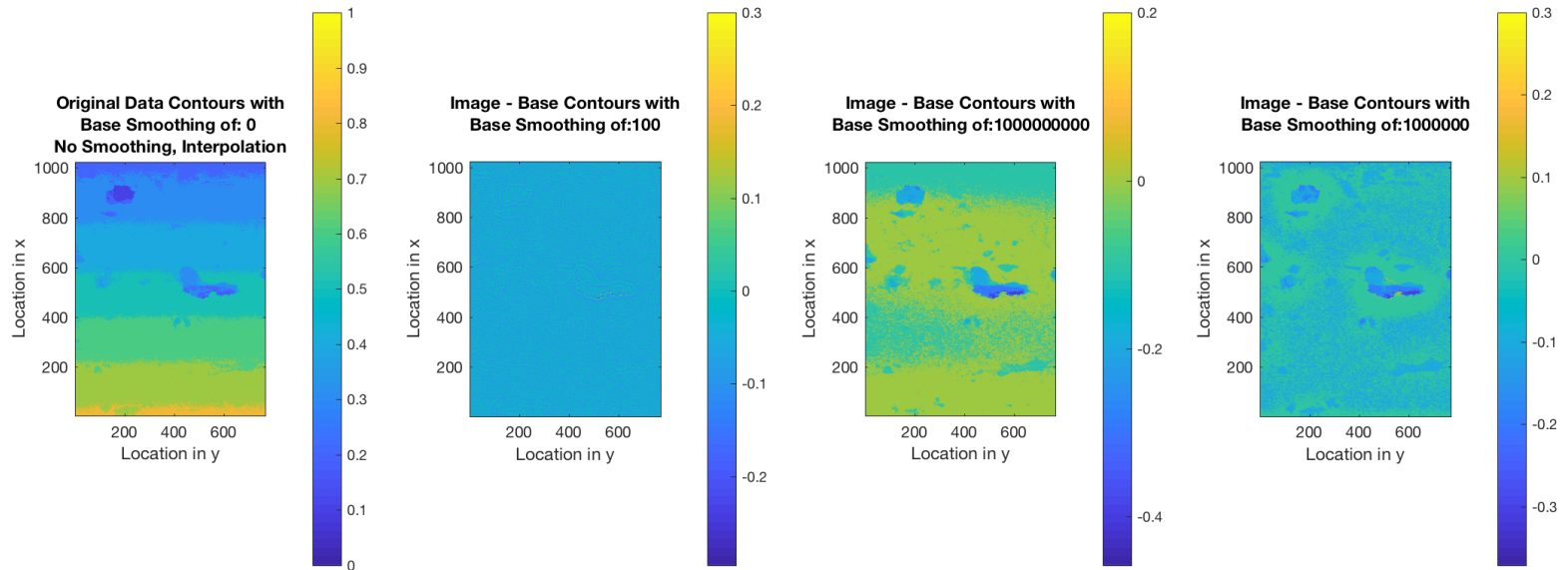
HCLO_UnPol_no_O2_Sec4_Y1_X1



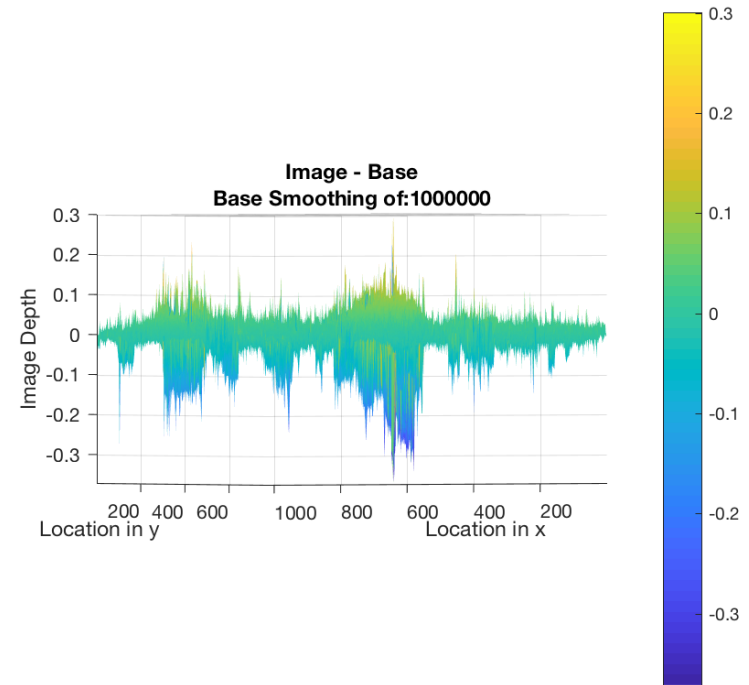
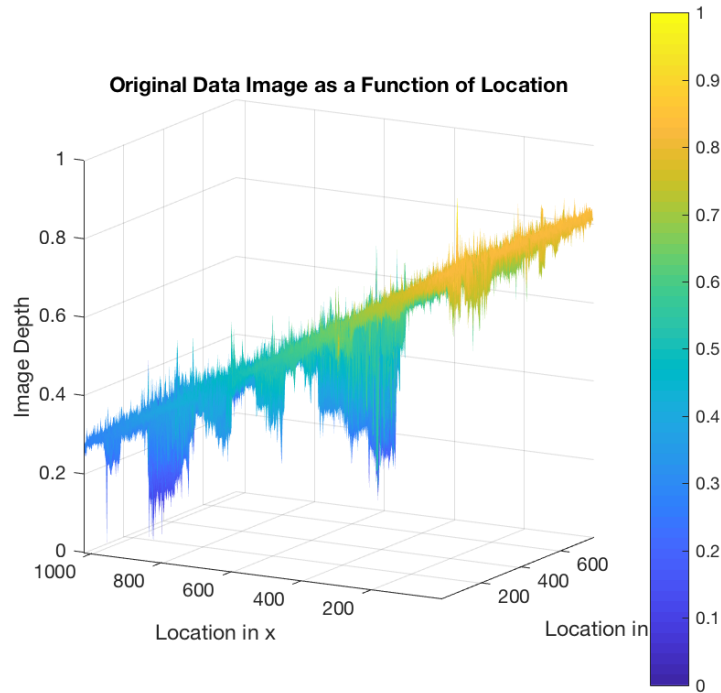
MgCl₂_Inside__Y1_X6

1. 690 nm/pixel
2. Z Calibration .1 nm per digit
3. Width 1024
4. Height 768
5. Magnesium Chloride In Location Y1, X6 Inside Can at 20x Magnification

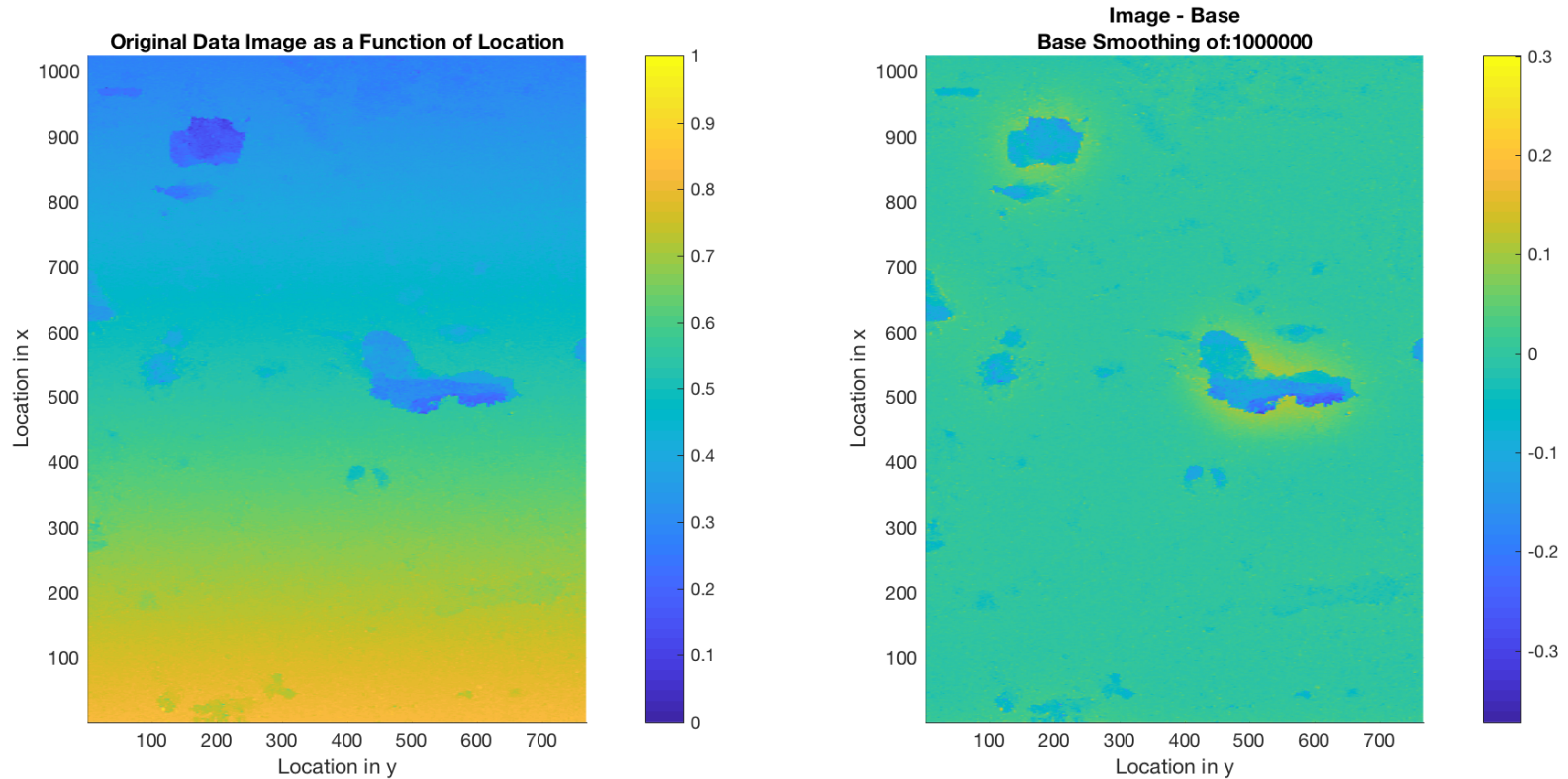
MgCl2_Inside__Y1_X6



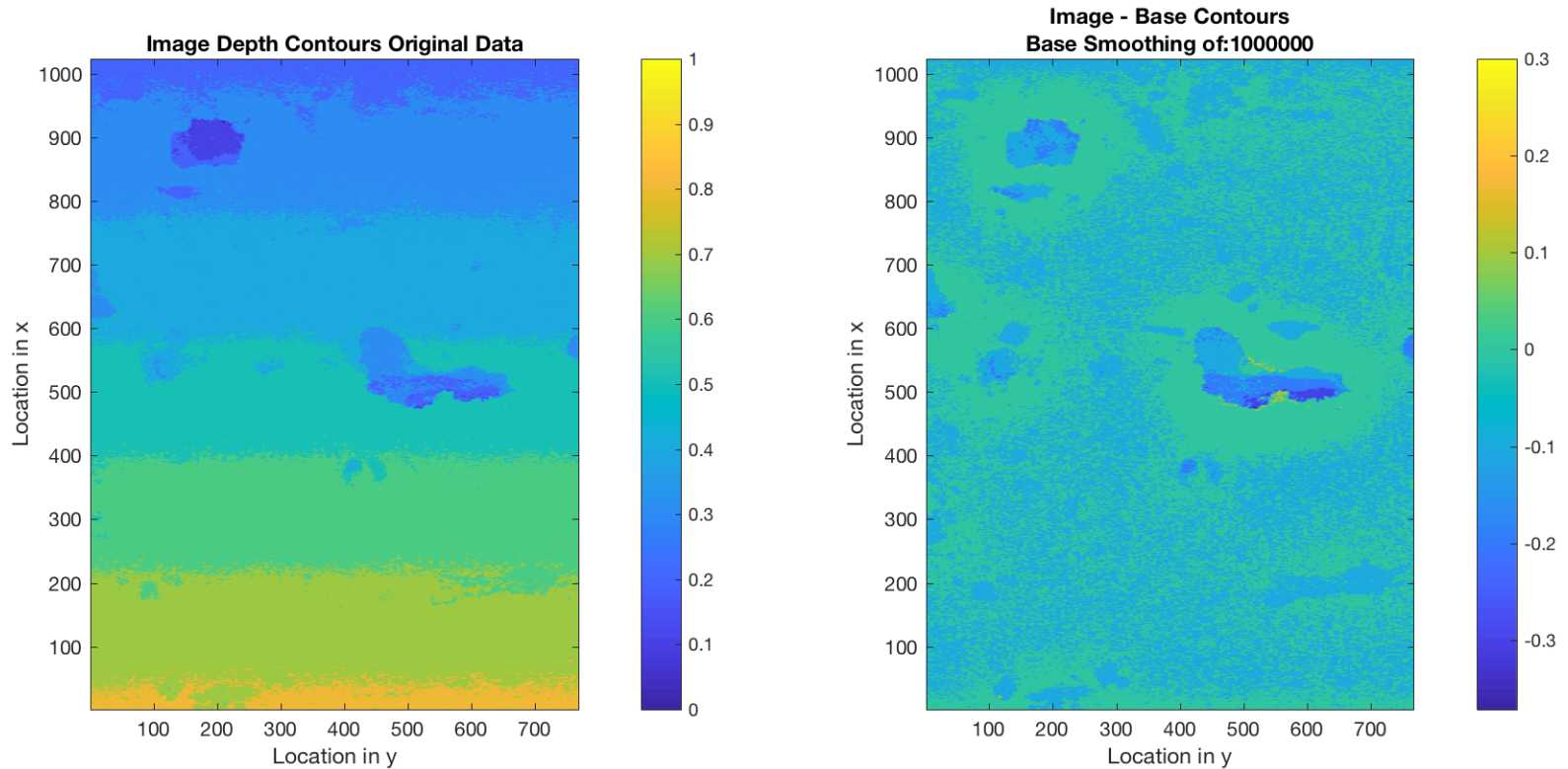
MgCl2_Inside__Y1_X6



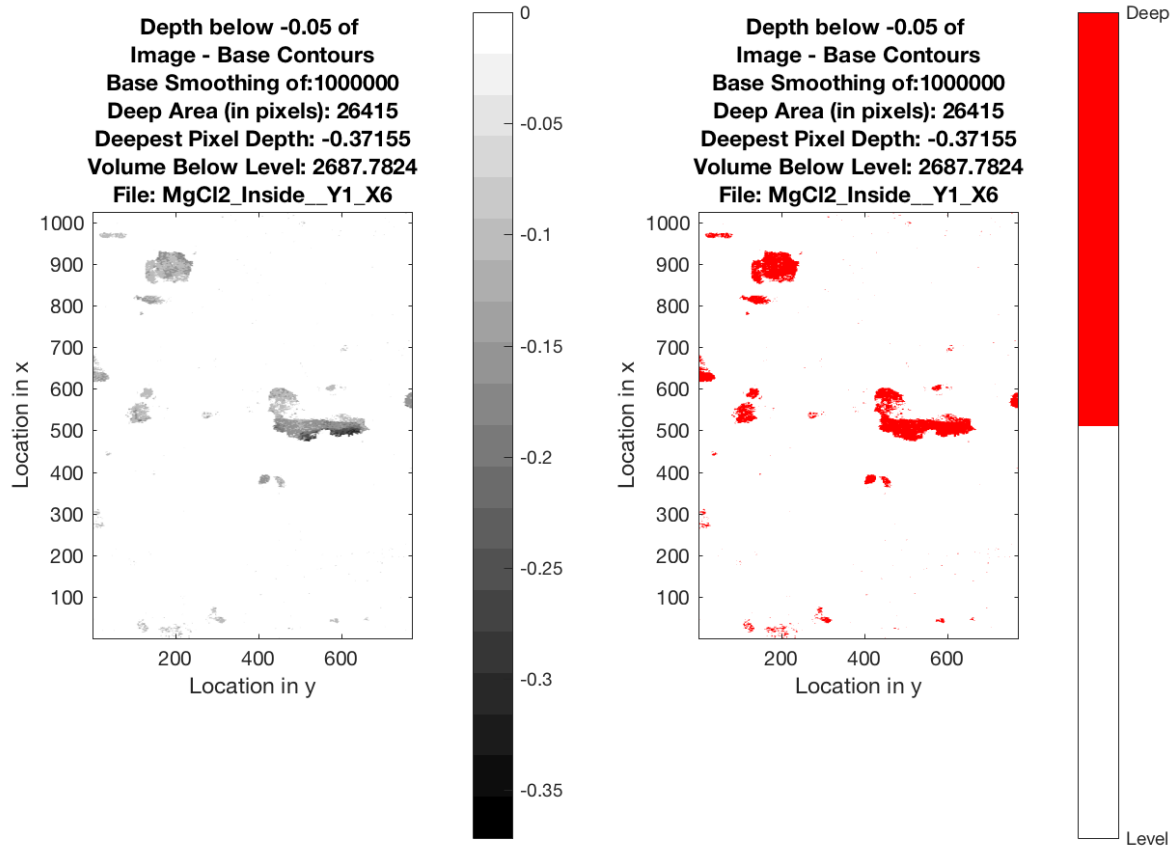
MgCl2_Inside__Y1_X6



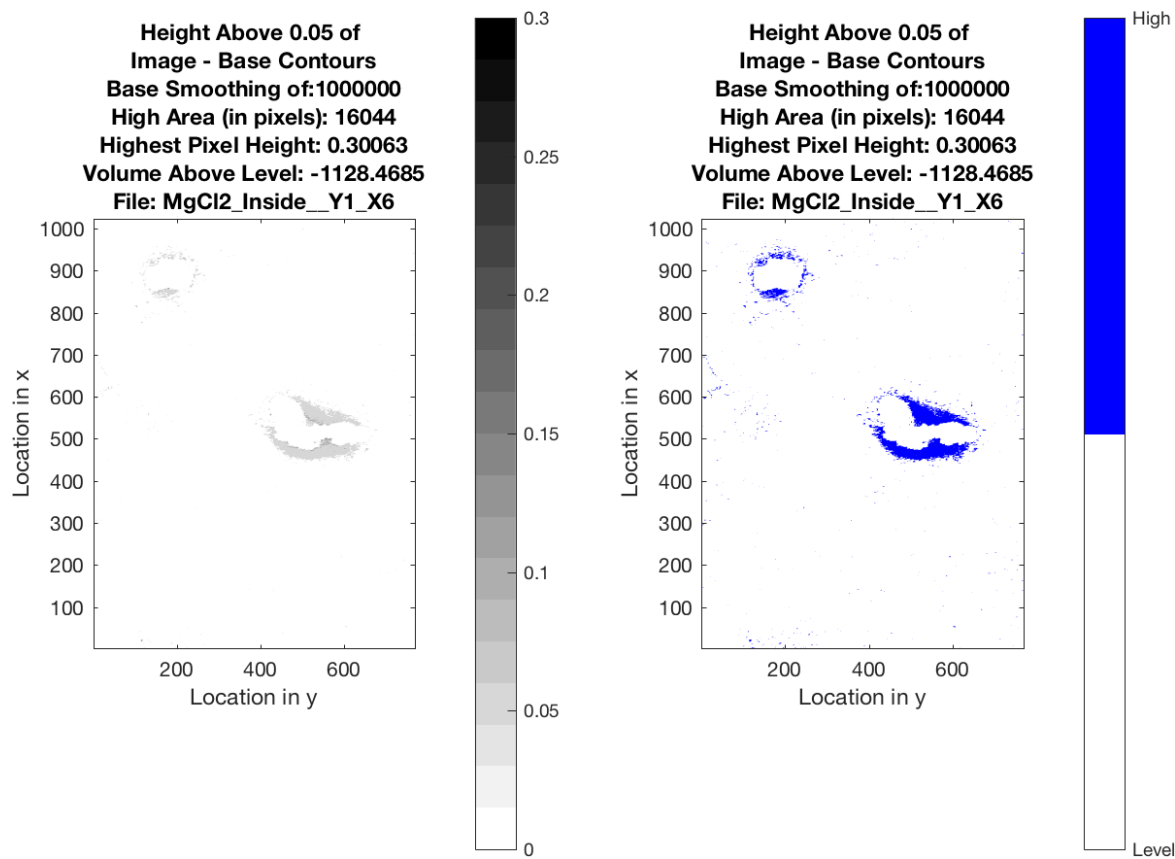
MgCl2_Inside__Y1_X6



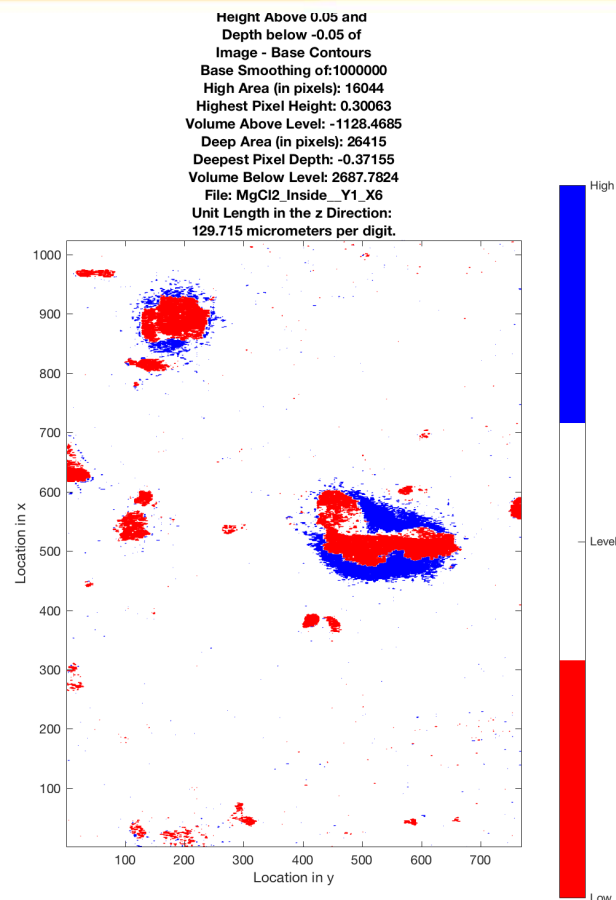
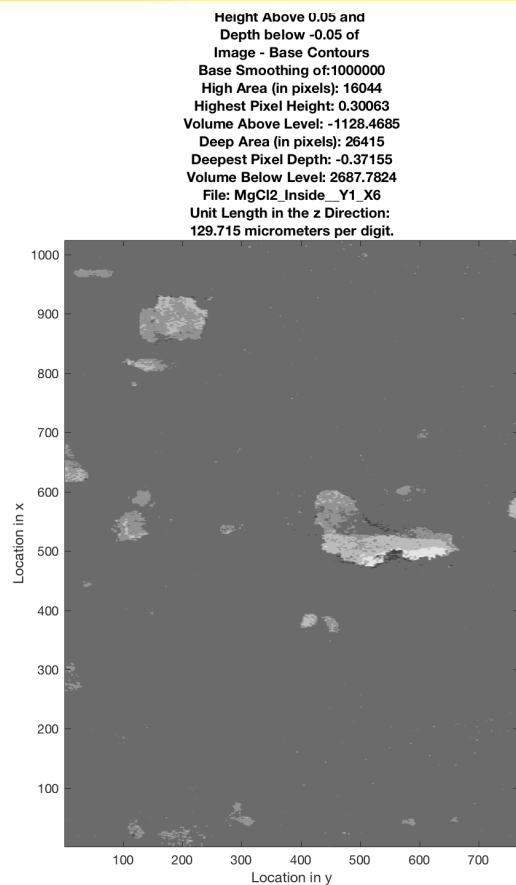
MgCl2_Inside__Y1_X6



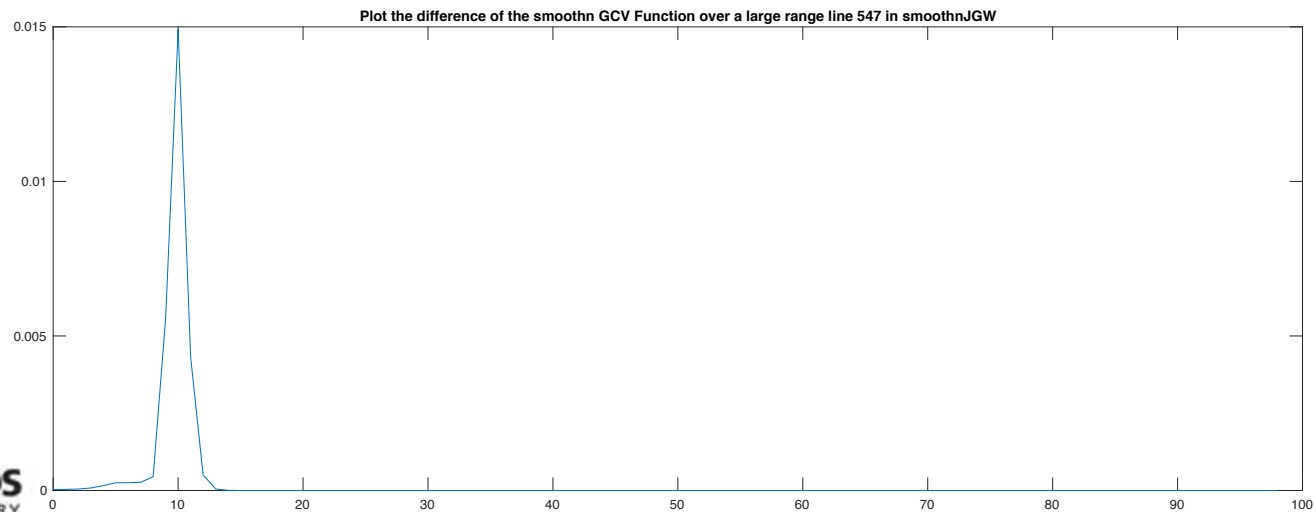
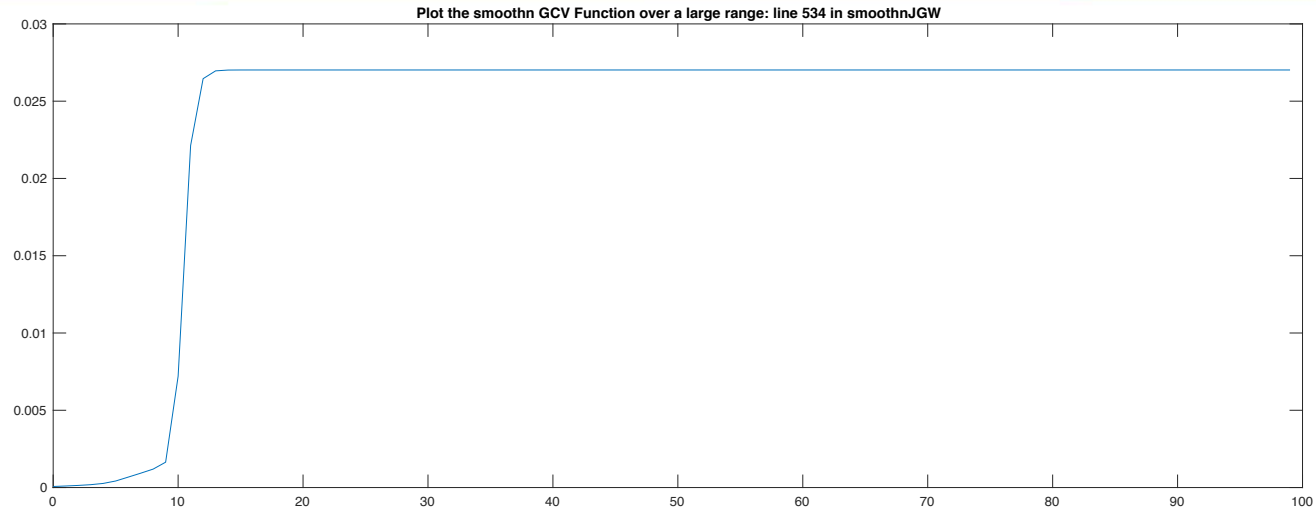
MgCl2_Inside__Y1_X6



MgCl2_Inside__Y1_X6



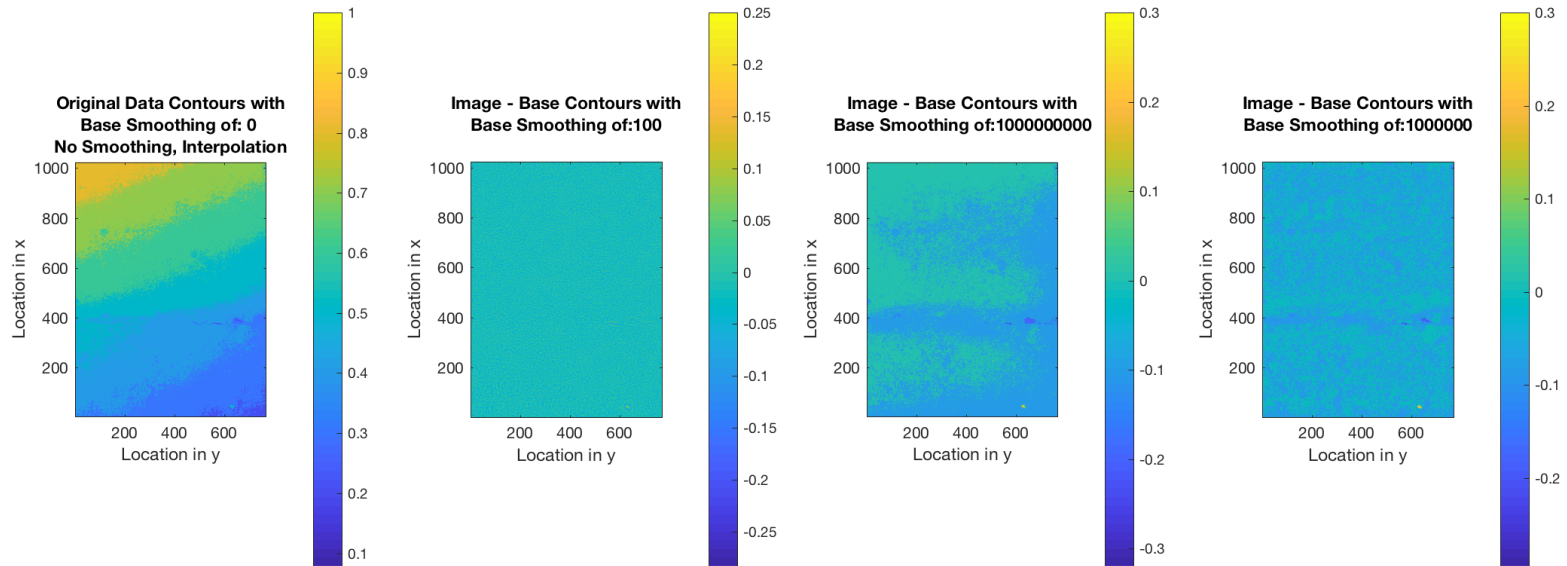
MgCl2_Inside__Y1_X6



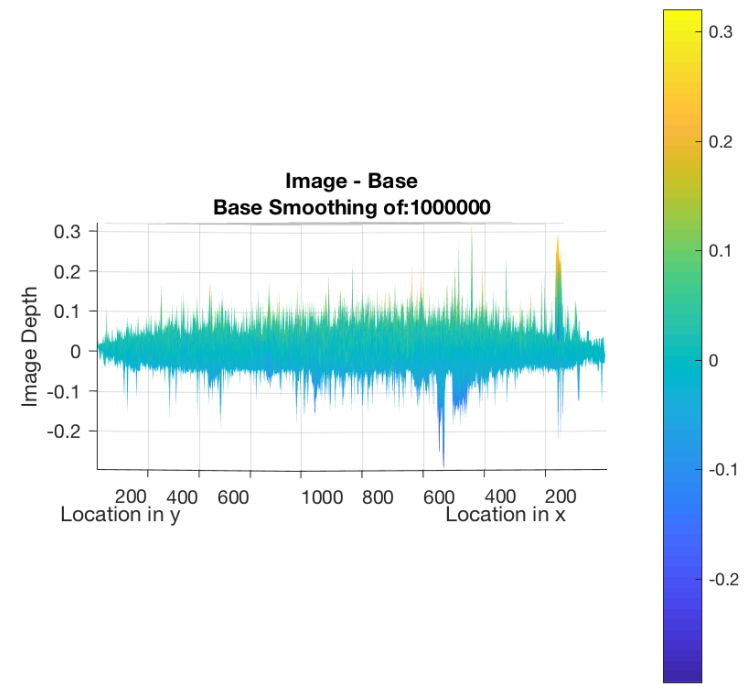
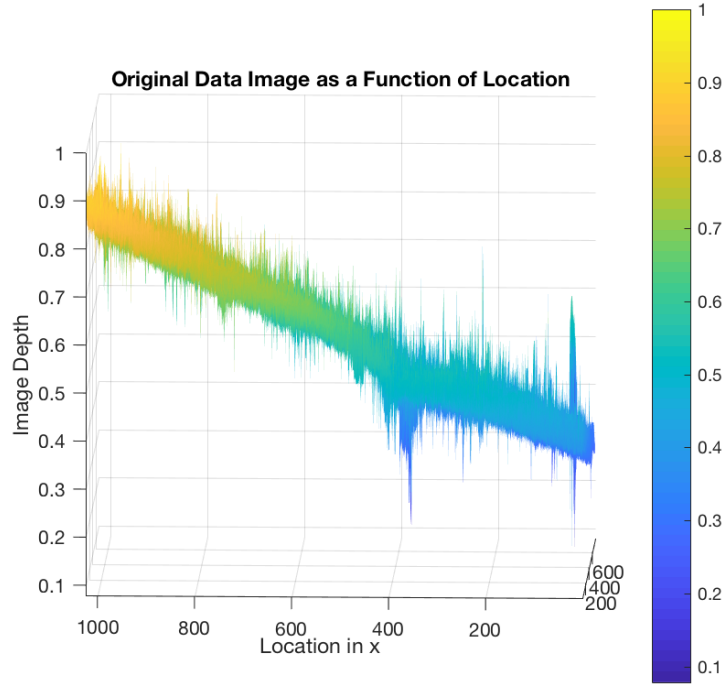
MgCl₂_Inside__Y15_X18

1. 690 nm/pixel
2. Z Calibration .1 nm per digit
3. Width 1024
4. Height 768
5. Magnesium Chloride In Location Y15, X18 Inside Can at 20x Magnification

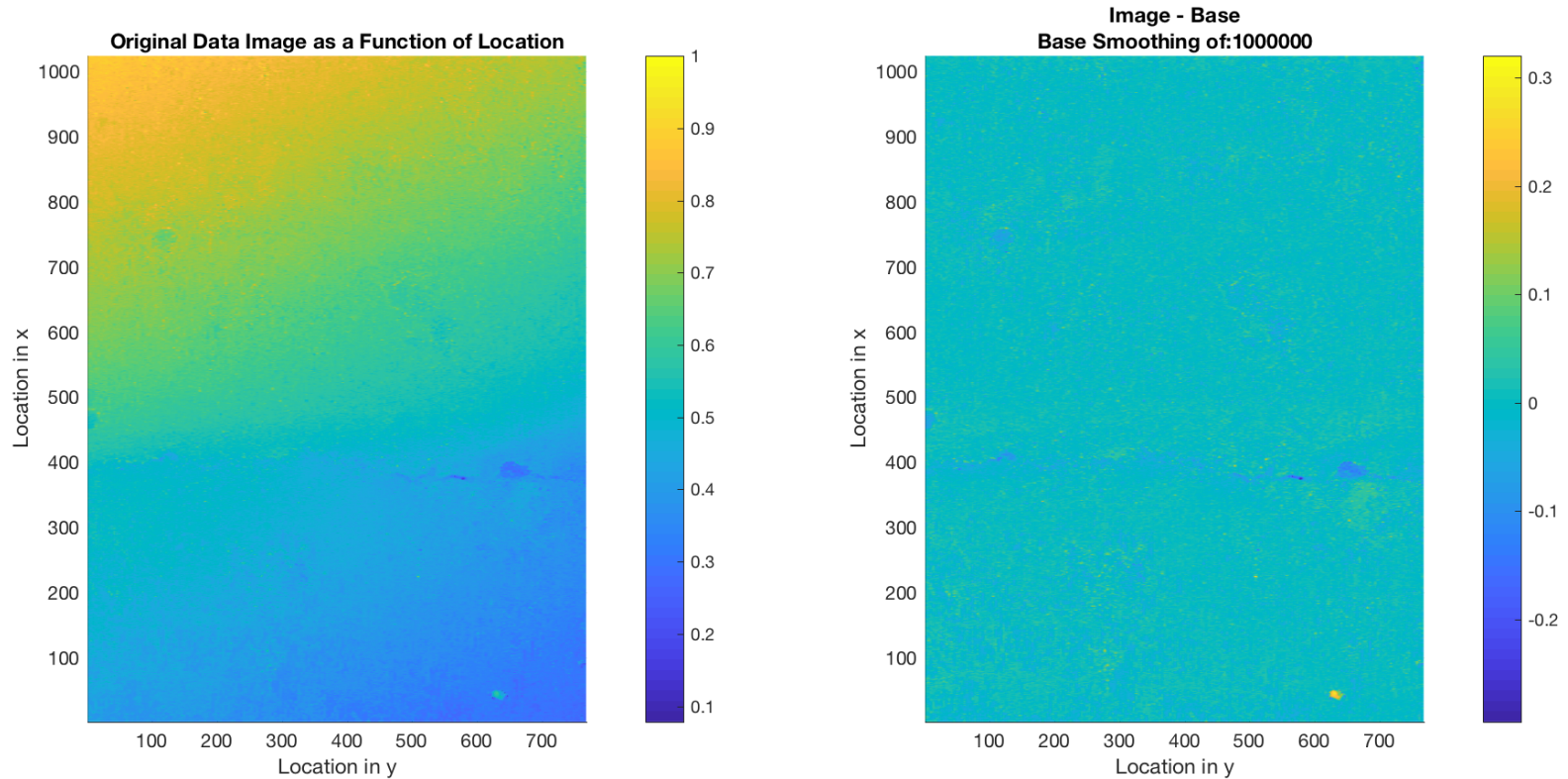
MgCl2_Inside__Y15_X18



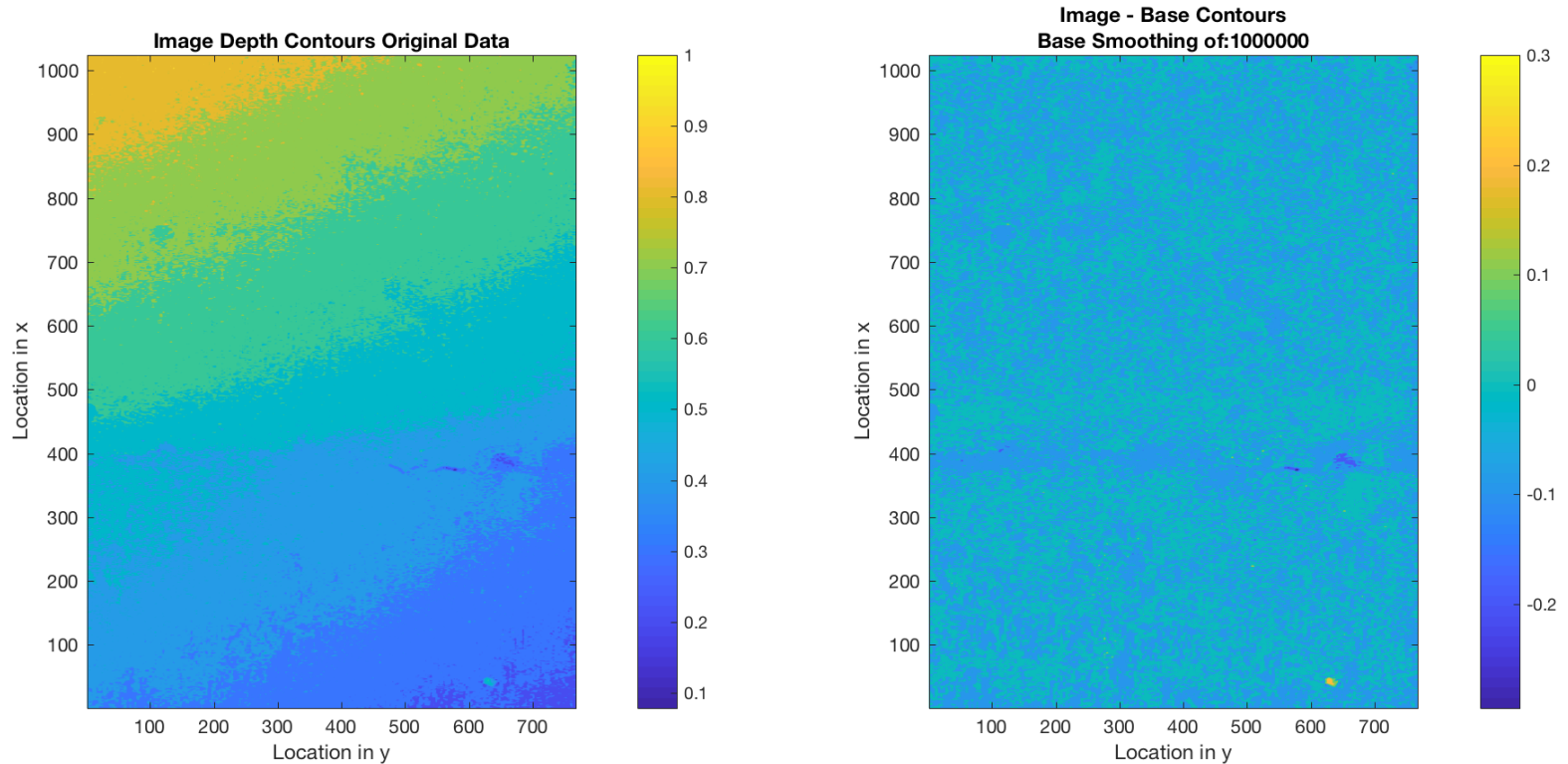
MgCl2_Inside__Y15_X18



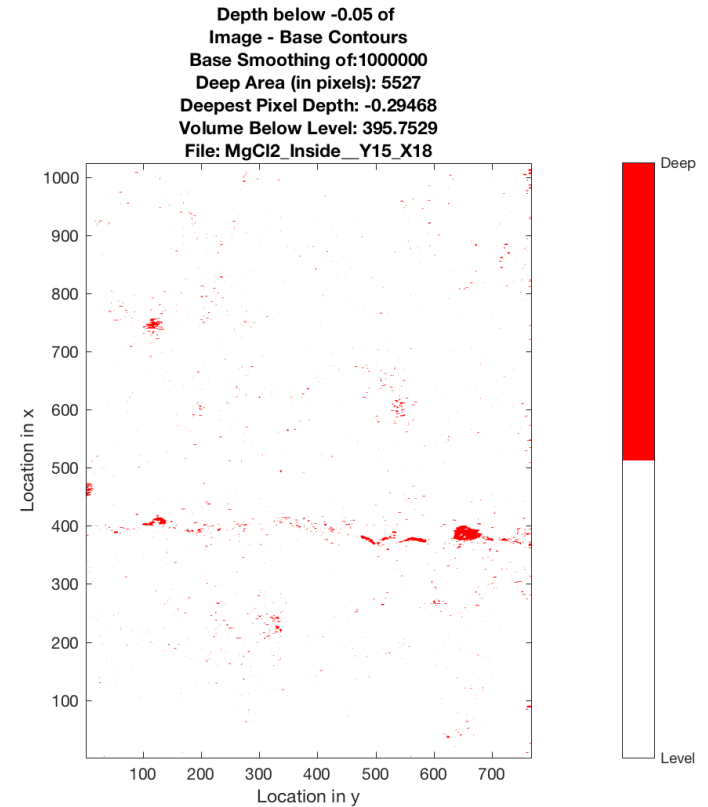
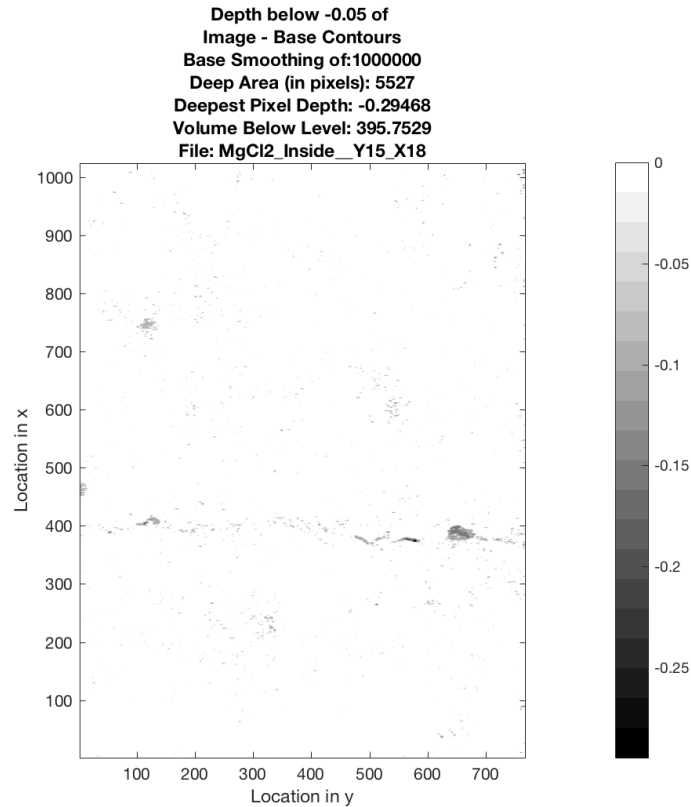
MgCl2_Inside__Y15_X18



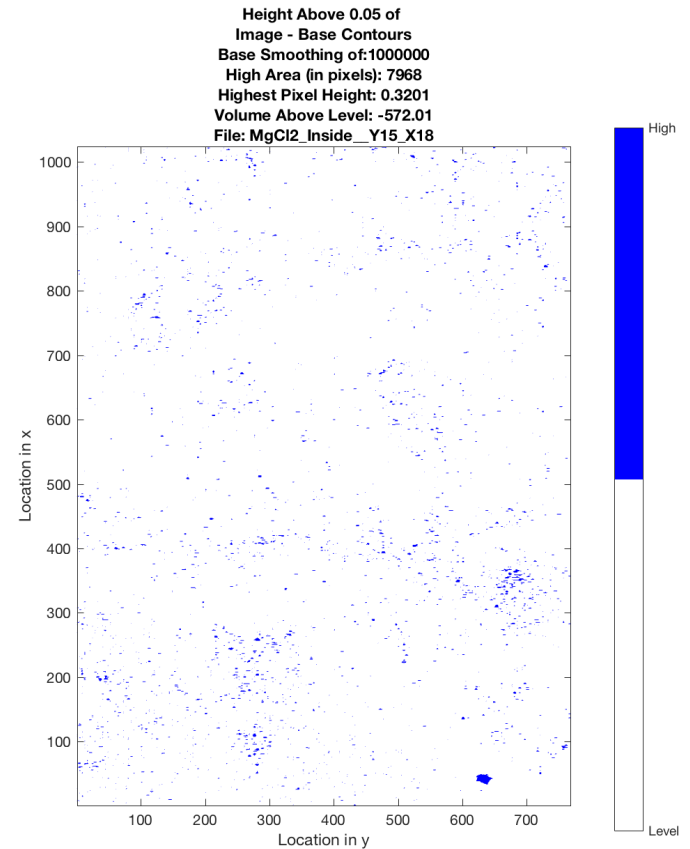
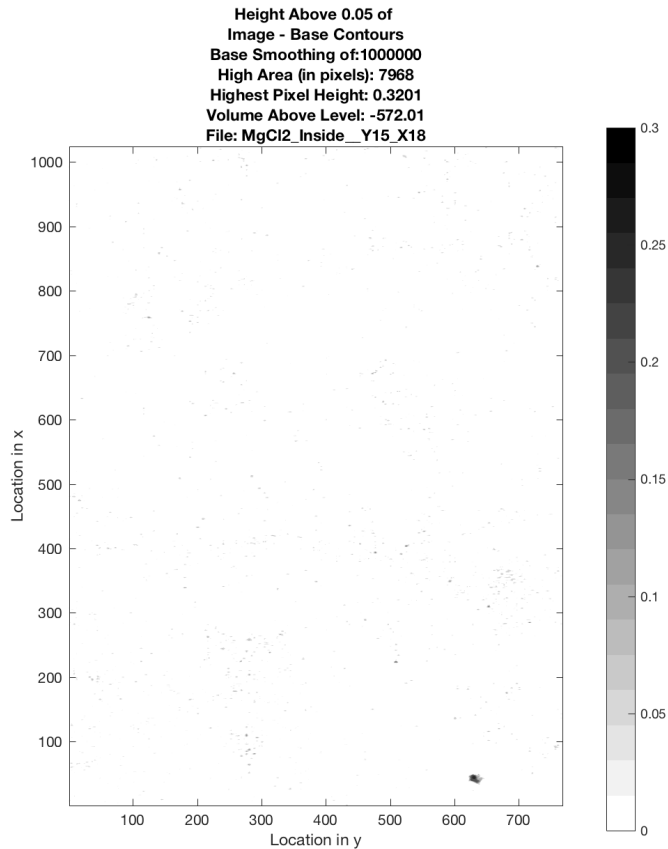
MgCl2_Inside__Y15_X18



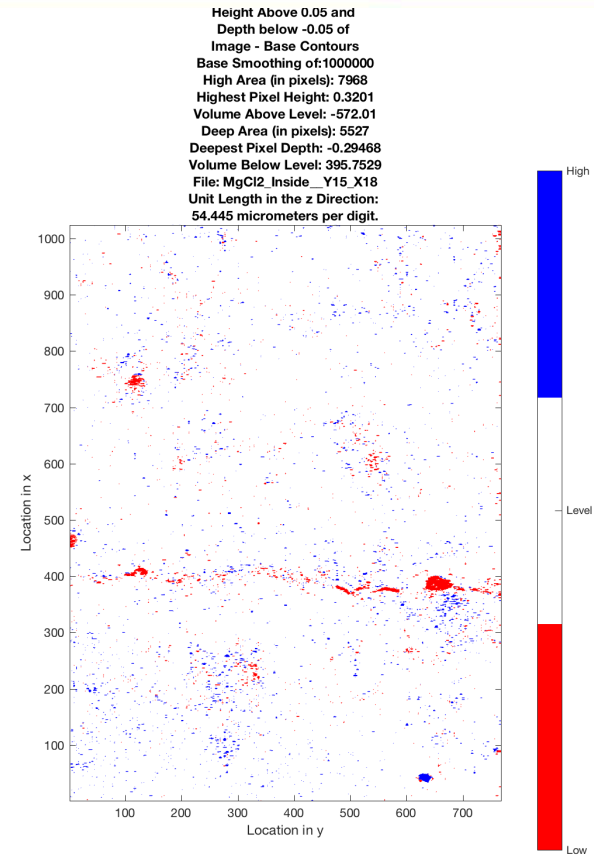
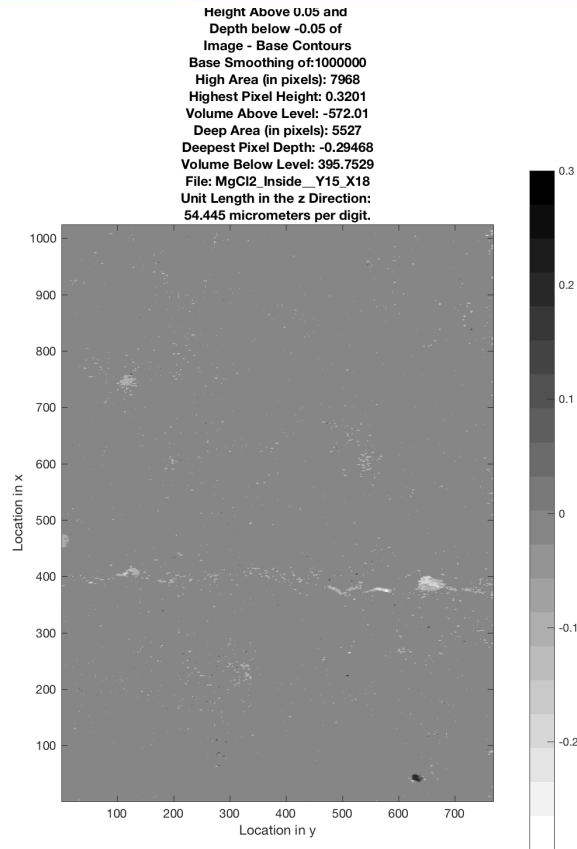
MgCl2_Inside__Y15_X18



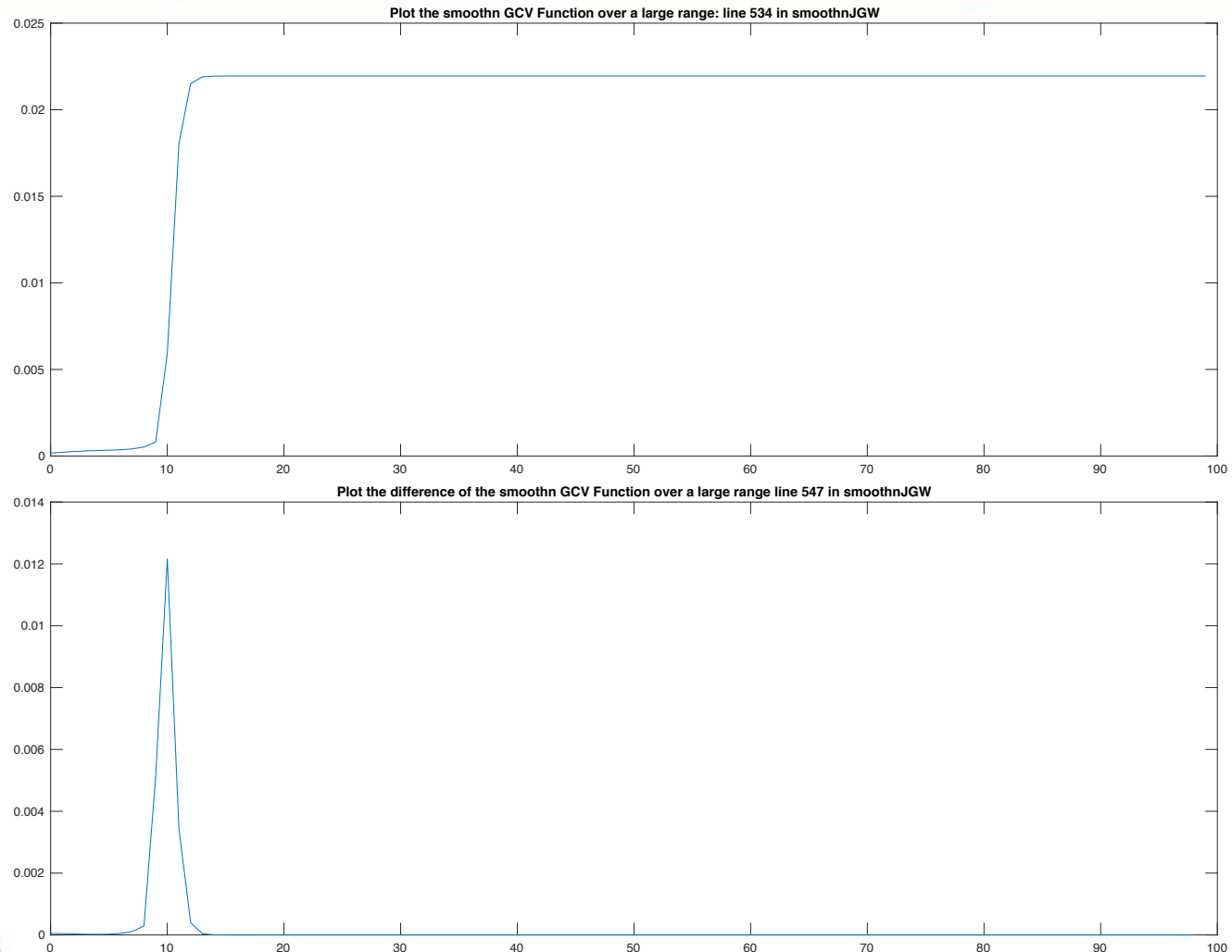
MgCl2_Inside__Y15_X18



MgCl2_Inside__Y15_X18



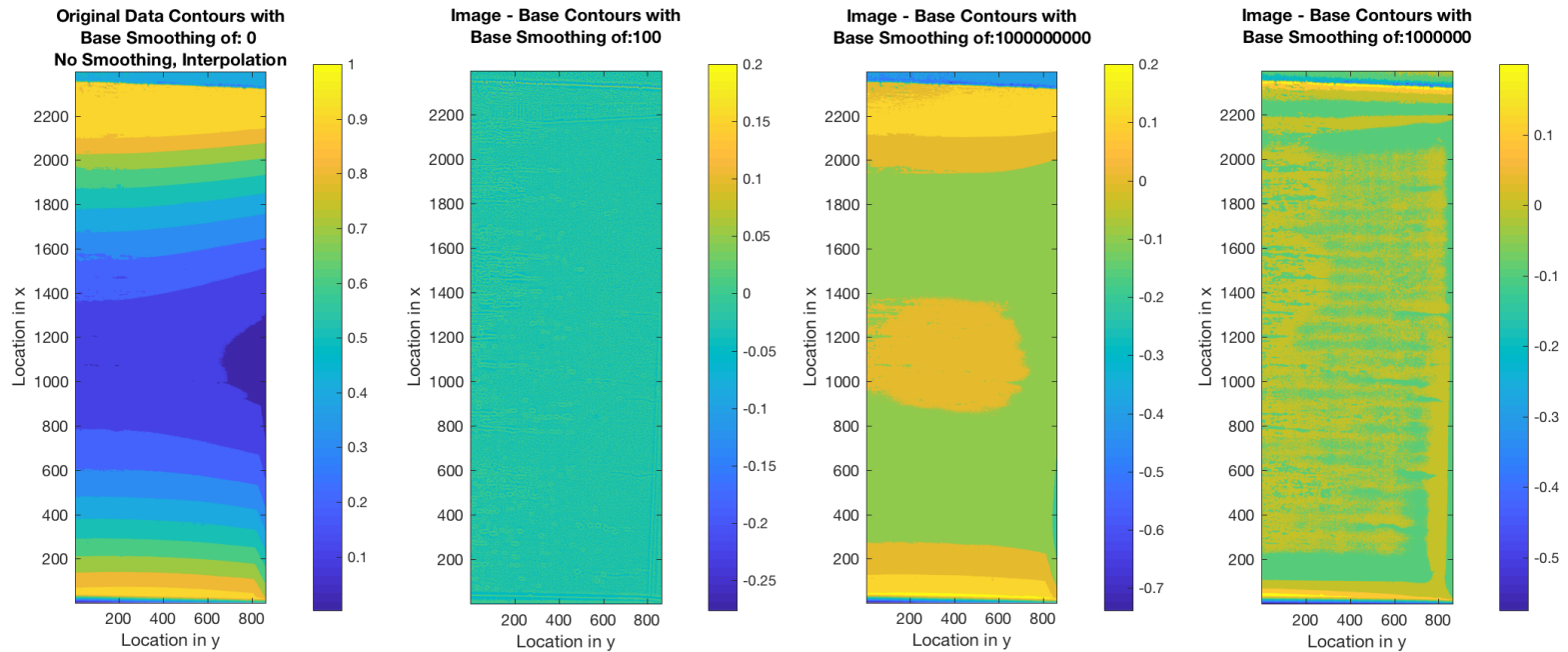
MgCl2_Inside__Y15_X18



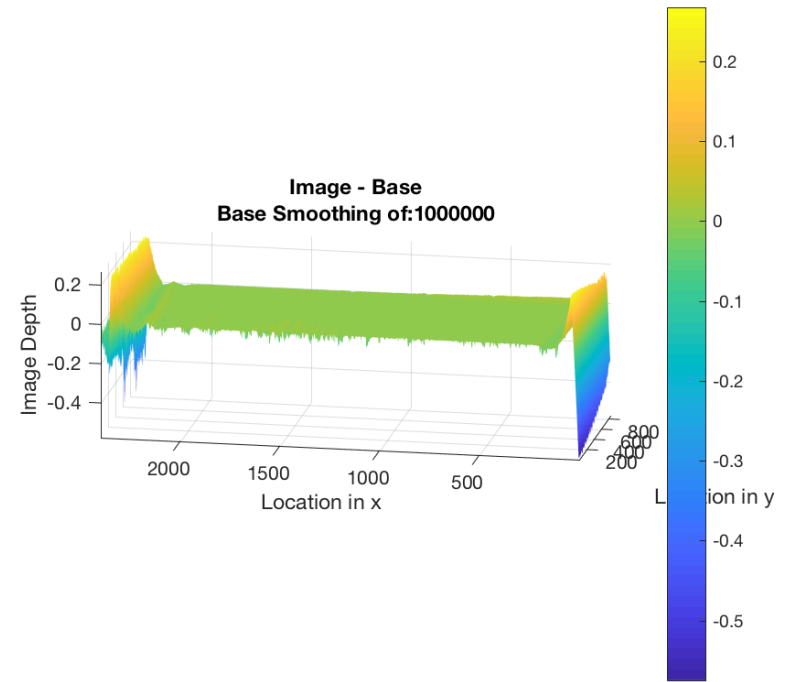
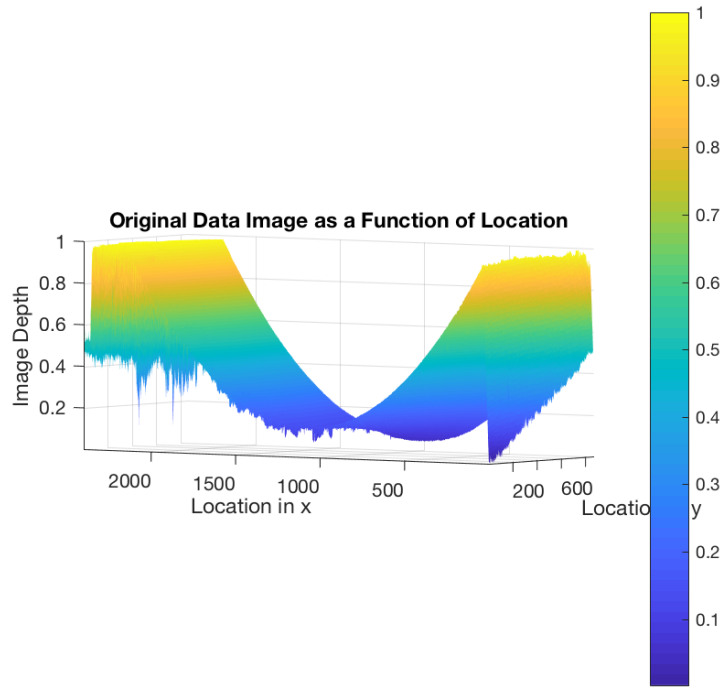
MgCl₂_Inside_

1. 8276 nm/pixel
2. Z Calibration .1 nm per digit
3. Width 2399
4. Height 863
5. Magnesium Chloride Stitched Image Inside Can

MgCl2_Inside_



MgCl2_Inside_



MgCl₂_Inside_

Original Data Image as a Function of Location

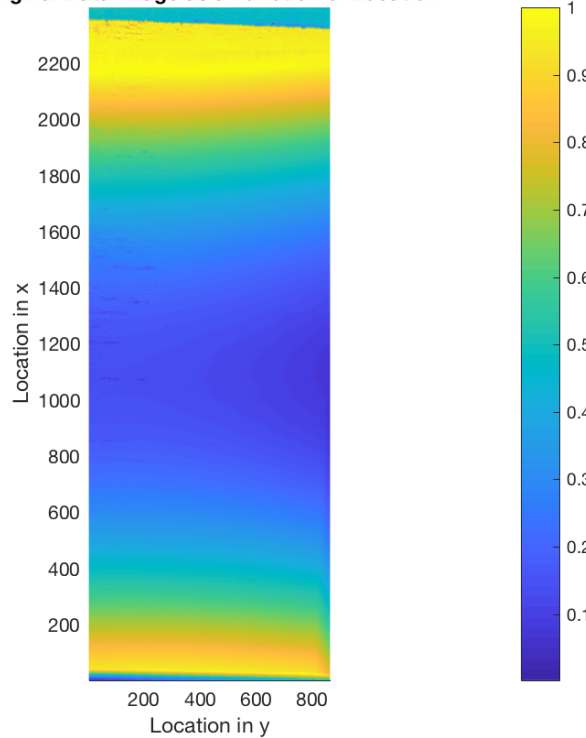
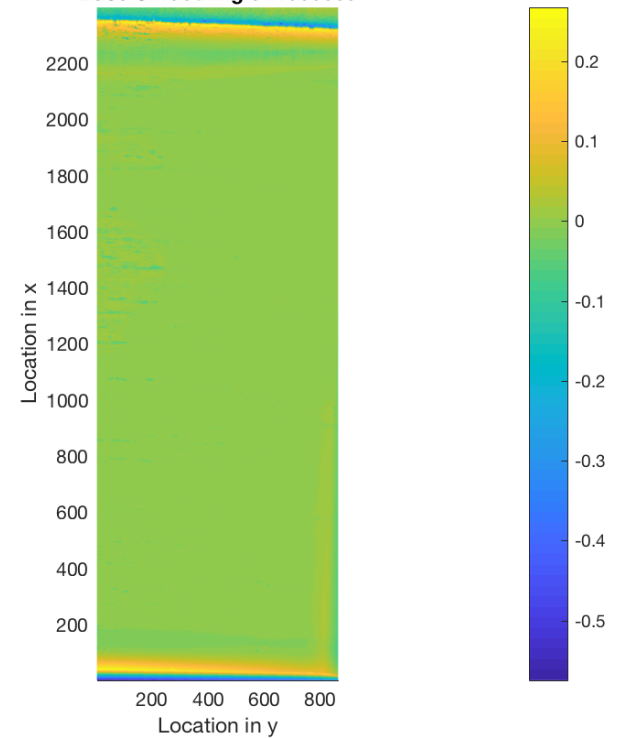


Image - Base
Base Smoothing of:1000000



MgCl₂_Inside_

Image Depth Contours Original Data

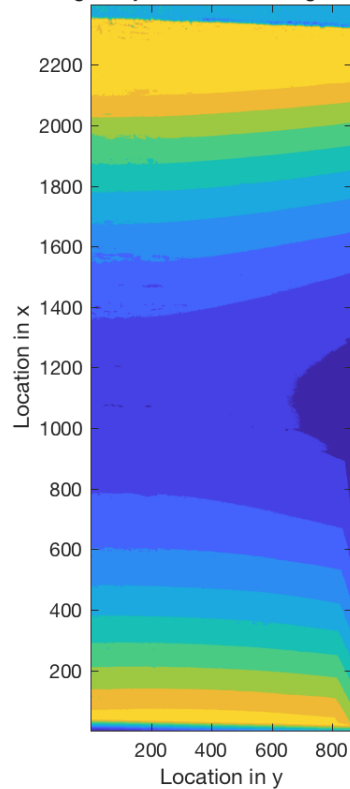
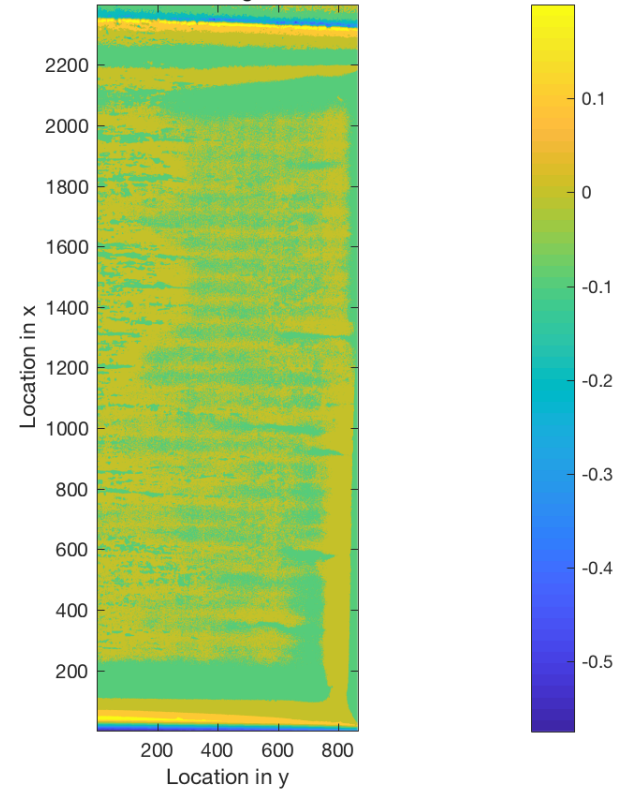
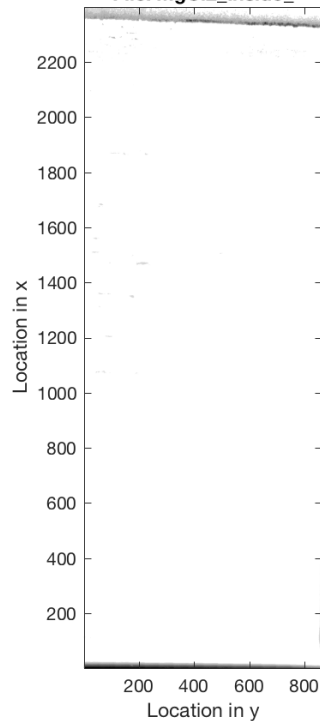


Image - Base Contours
Base Smoothing of:1000000

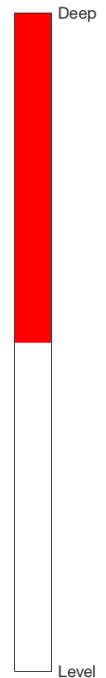
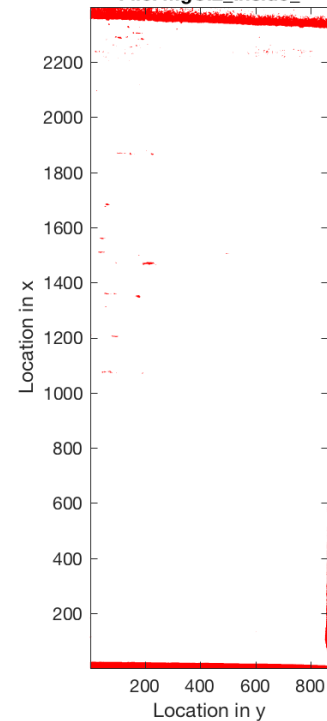


MgCl2_Inside_

Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
Deep Area (in pixels): 53074
Deepest Pixel Depth: -0.57468
Volume Below Level: 9098.7155
File: MgCl2_Inside_

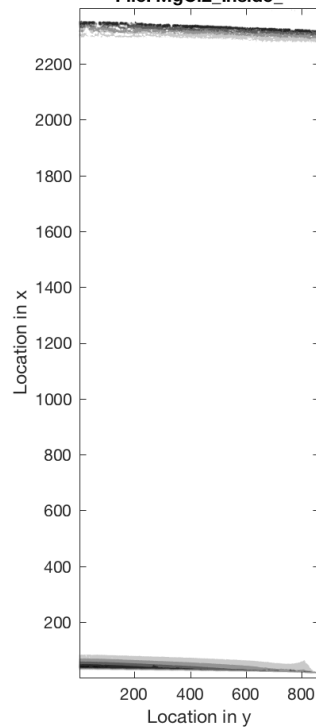


Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
Deep Area (in pixels): 53074
Deepest Pixel Depth: -0.57468
Volume Below Level: 9098.7155
File: MgCl2_Inside_

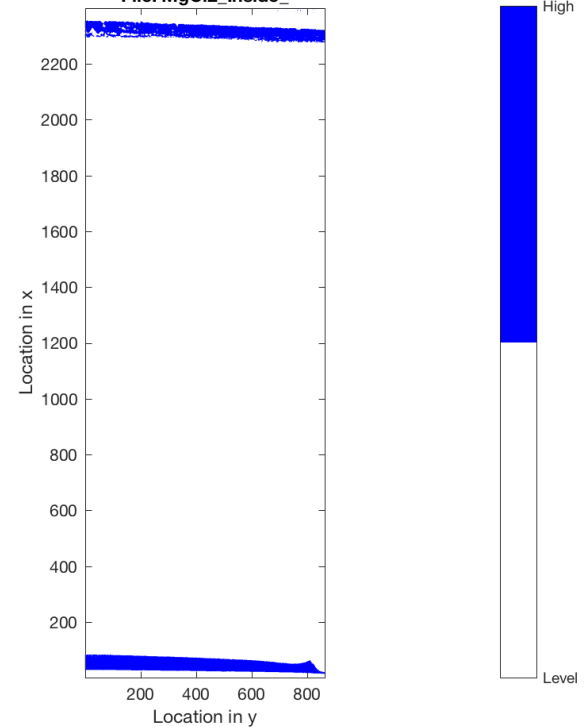


MgCl2_Inside_

Height Above 0.05 of
Image - Base Contours
Base Smoothing of:1000000
High Area (in pixels): 78005
Highest Pixel Height: 0.26744
Volume Above Level: -9724.2967
File: MgCl2_Inside_

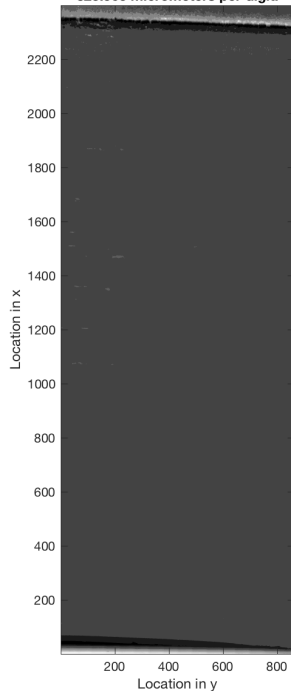


Height Above 0.05 of
Image - Base Contours
Base Smoothing of:1000000
High Area (in pixels): 78005
Highest Pixel Height: 0.26744
Volume Above Level: -9724.2967
File: MgCl2_Inside_

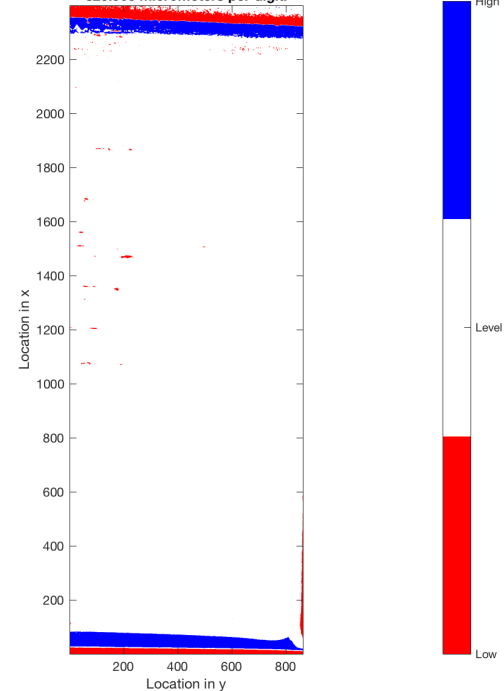


MgCl2_Inside_

Height Above 0.05 and
Depth below -0.05 of
Image - Base Contours
Base Smoothing of: 1000000
High Area (in pixels): 78005
Highest Pixel Height: 0.26744
Volume Above Level: -9724.2967
Deep Area (in pixels): 53074
Deepest Pixel Depth: -0.57468
Volume Below Level: 9098.7155
File: MgCl2_Inside_
Unit Length in the z Direction:
823.565 micrometers per digit.

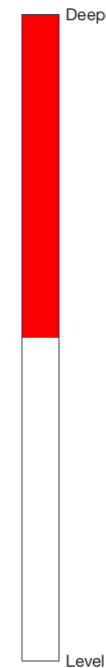
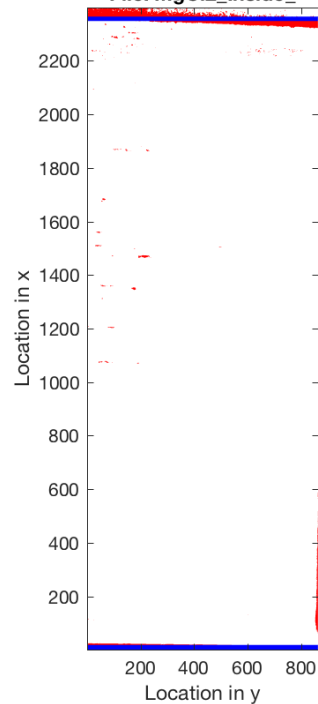


Height Above 0.05 and
Depth below -0.05 of
Image - Base Contours
Base Smoothing of: 1000000
High Area (in pixels): 78005
Highest Pixel Height: 0.26744
Volume Above Level: -9724.2967
Deep Area (in pixels): 53074
Deepest Pixel Depth: -0.57468
Volume Below Level: 9098.7155
File: MgCl2_Inside_
Unit Length in the z Direction:
823.565 micrometers per digit.

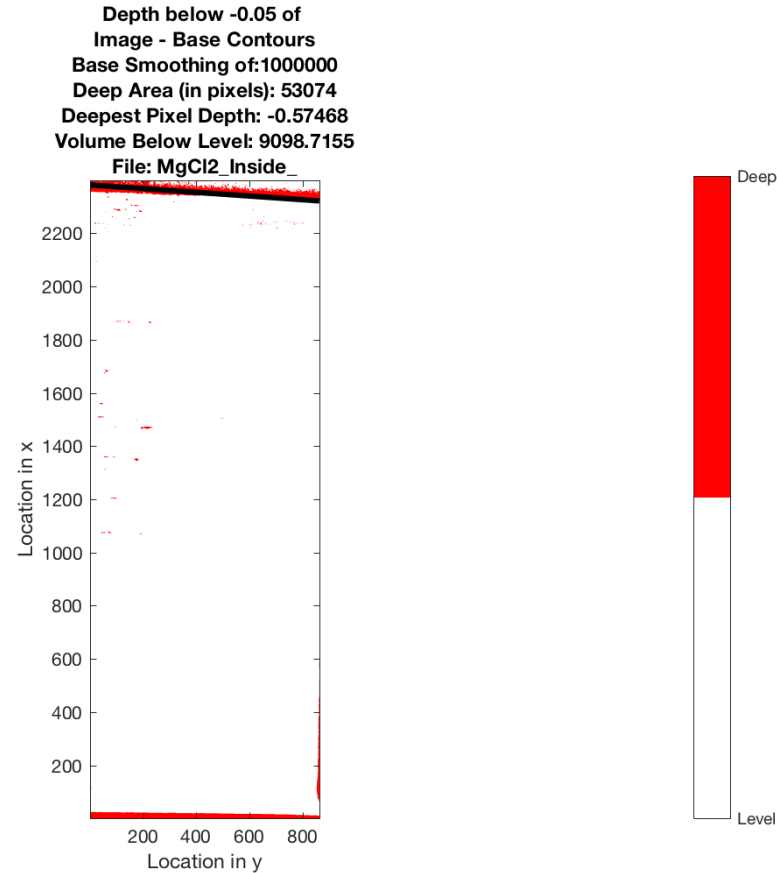


MgCl2_Inside_

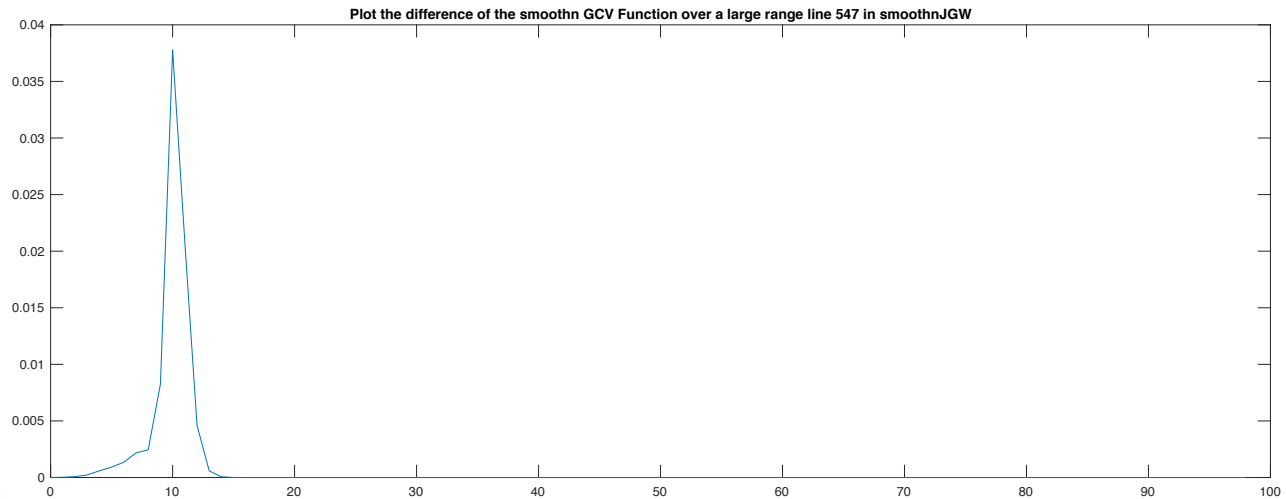
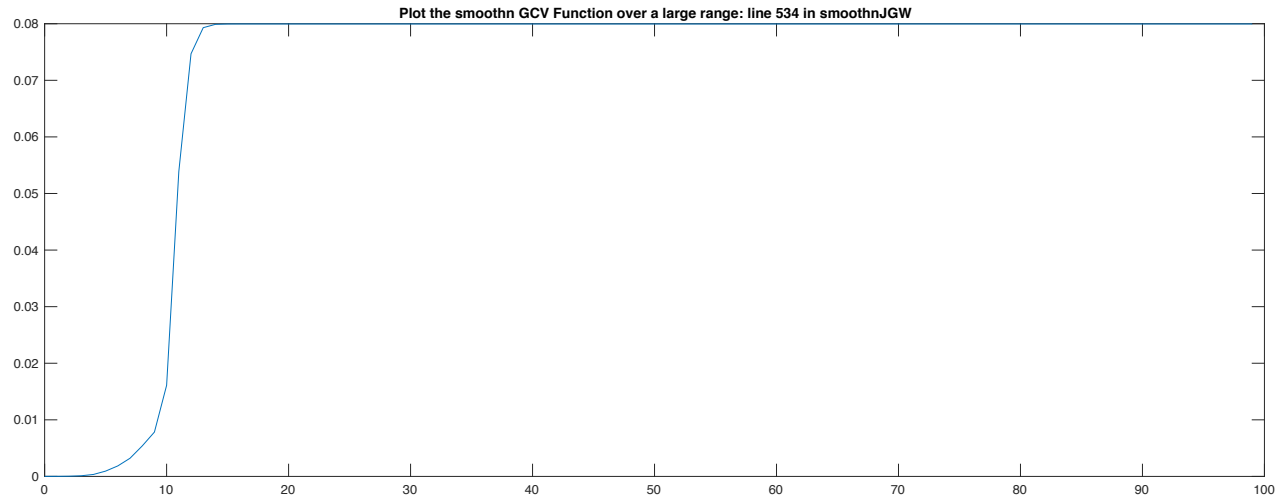
Depth below -0.05 of
Image - Base Contours
Base Smoothing of: 1000000
Deep Area (in pixels): 53074
Deepest Pixel Depth: -0.57468
Volume Below Level: 9098.7155
File: MgCl2_Inside_



MgCl2_Inside_



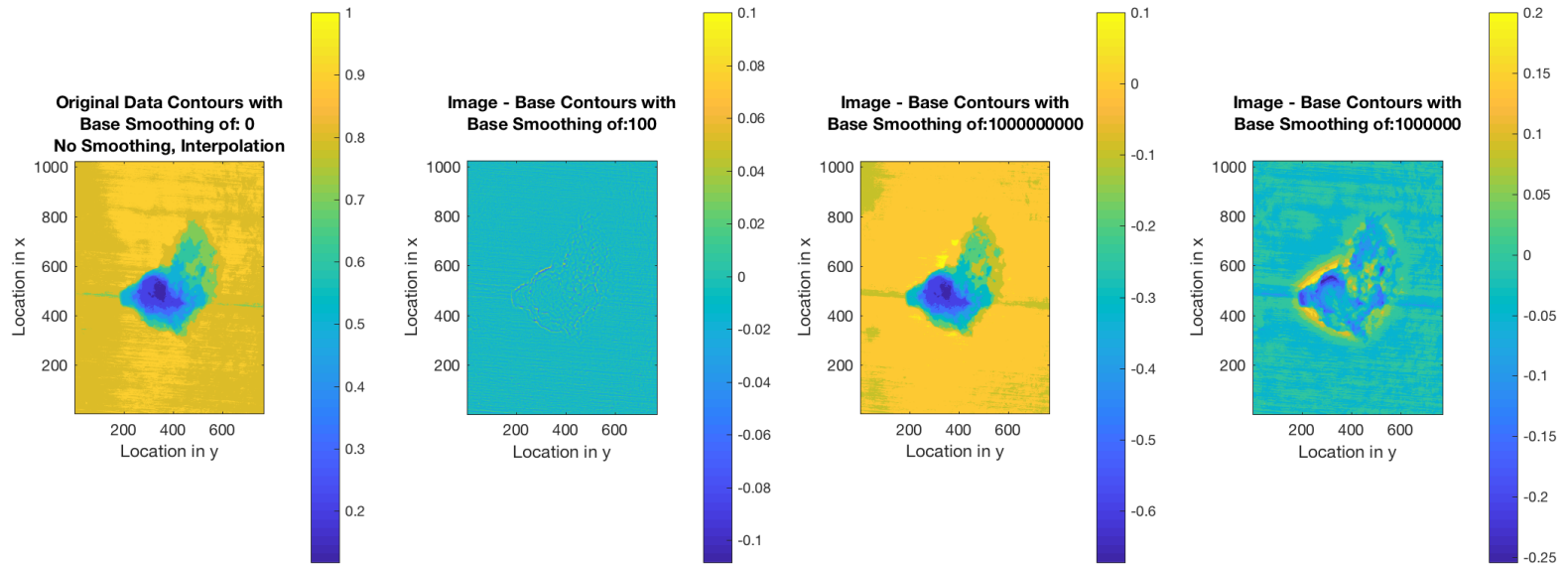
MgCl2_Inside_



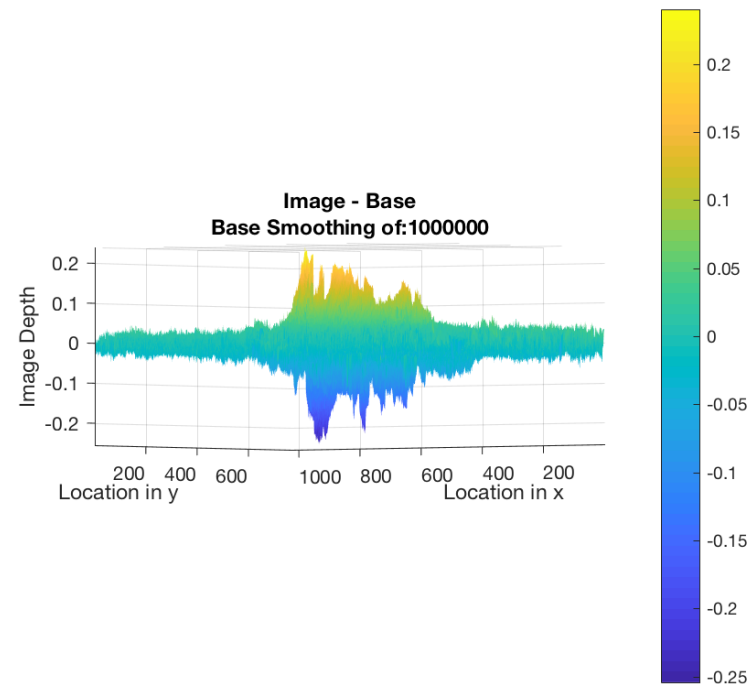
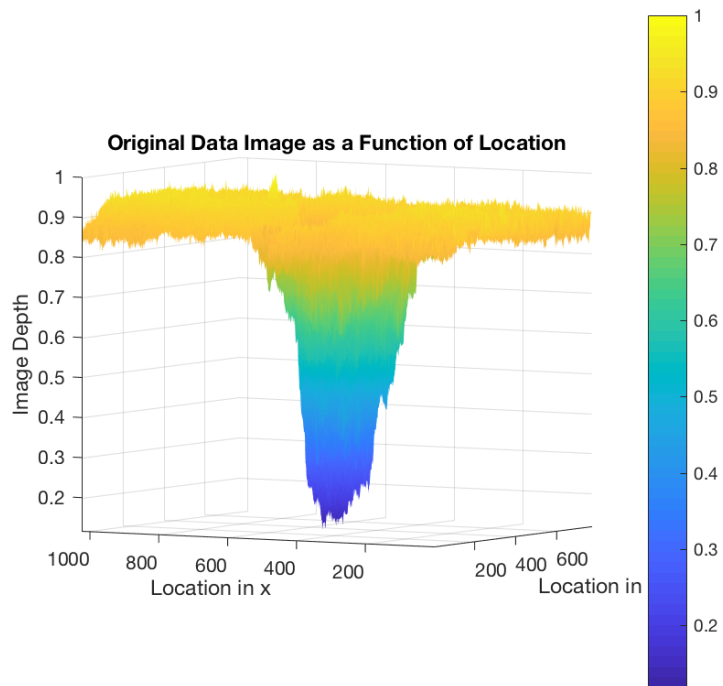
Pit

1. 689 nm/pixel
2. Z Calibration .1 nm per digit
3. Width 1024
4. Height 768
5. Large Pit from the Baseline Surface with 20x magnification

Pit



Pit



Pit

Original Data Image as a Function of Location

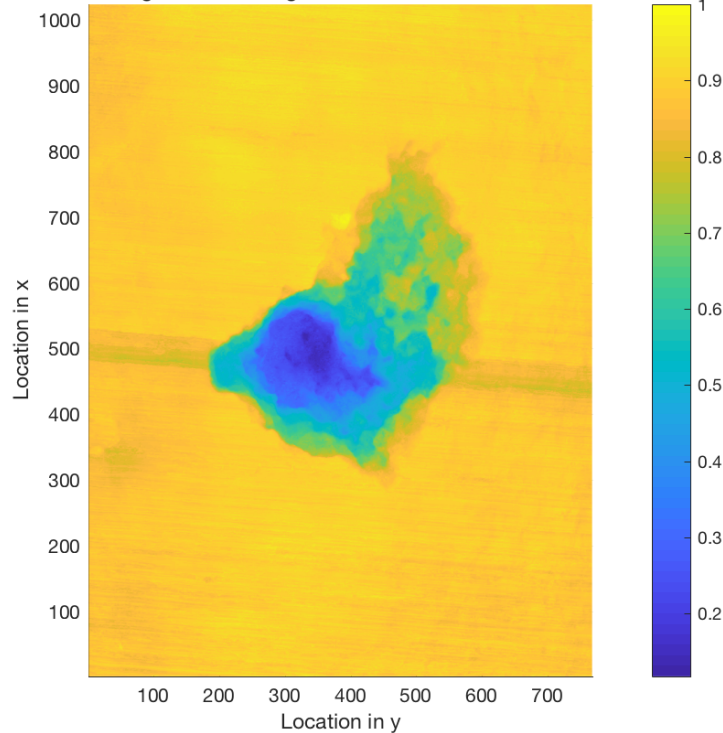
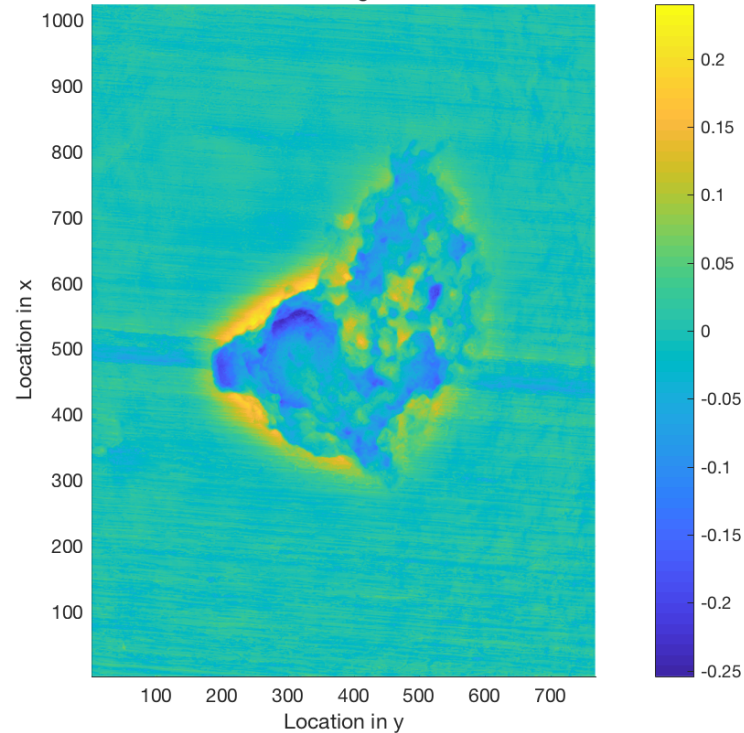
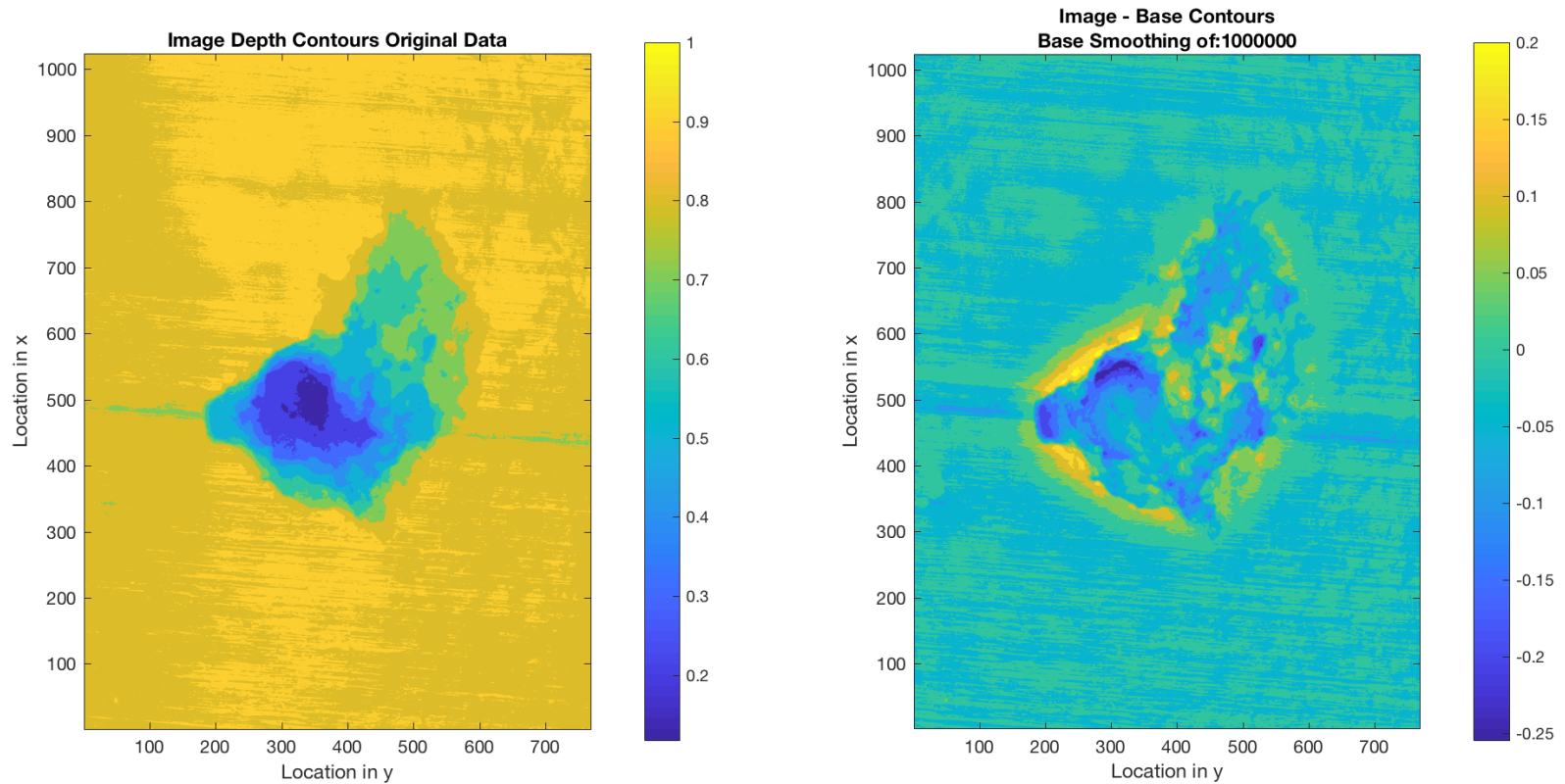


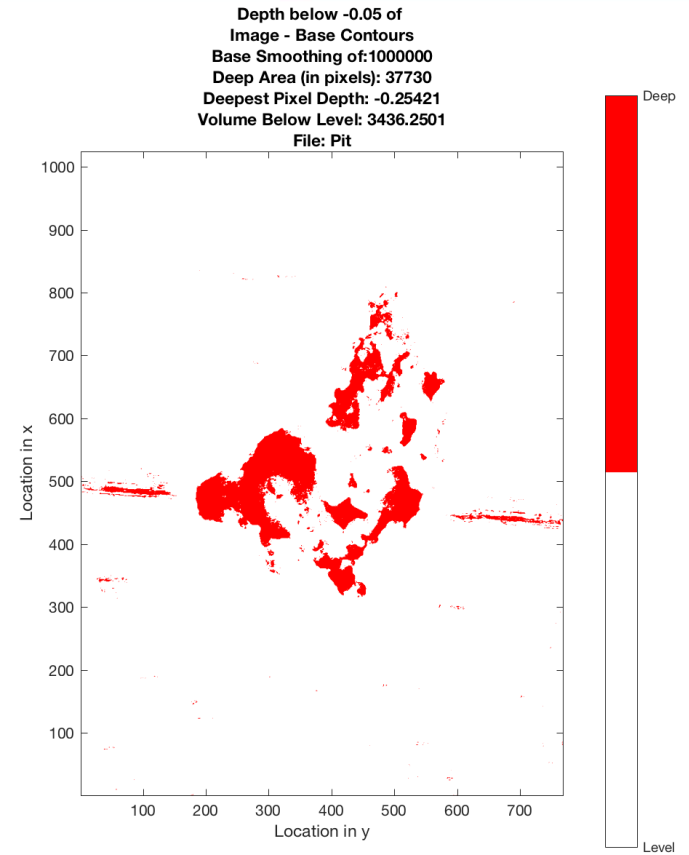
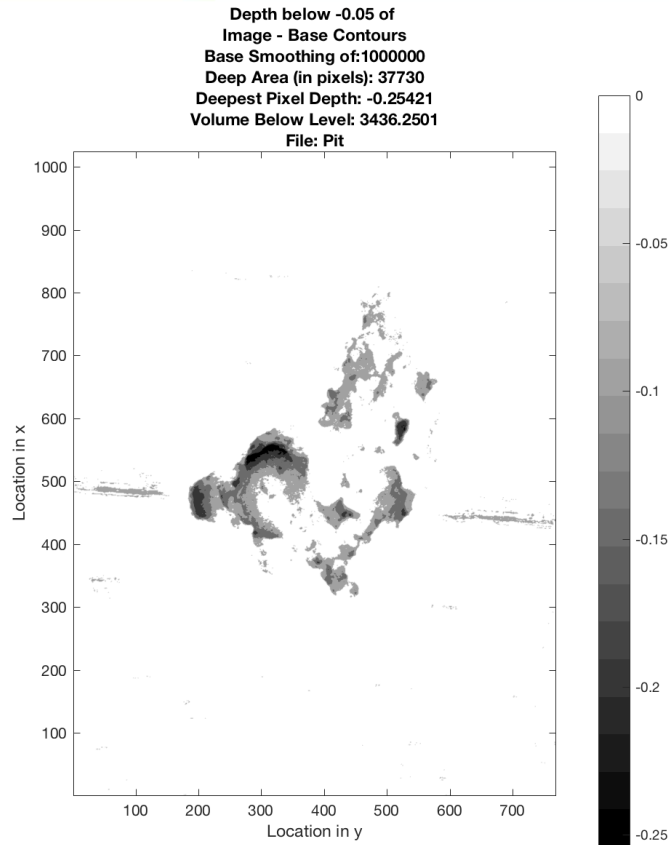
Image - Base
Base Smoothing of:1000000



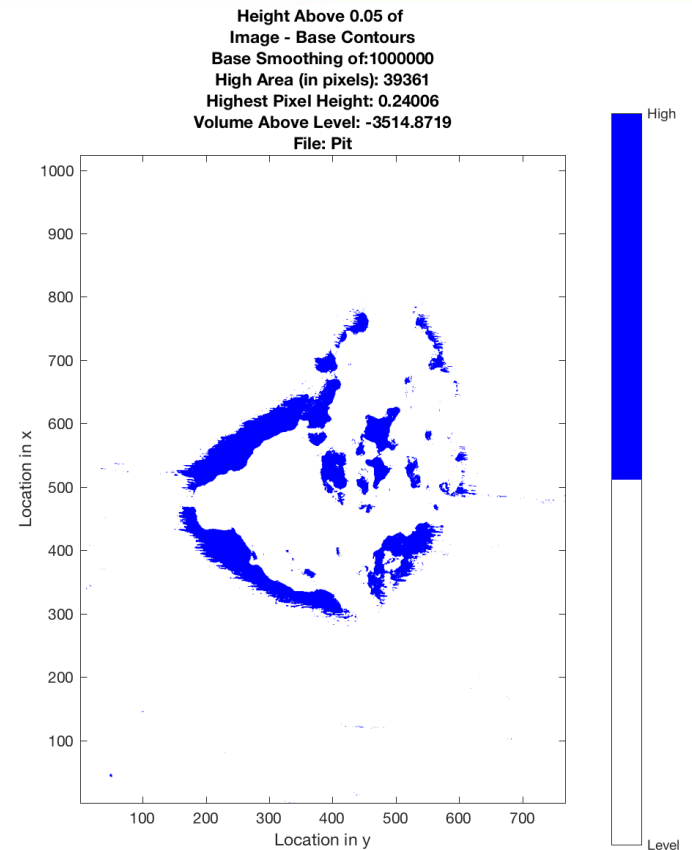
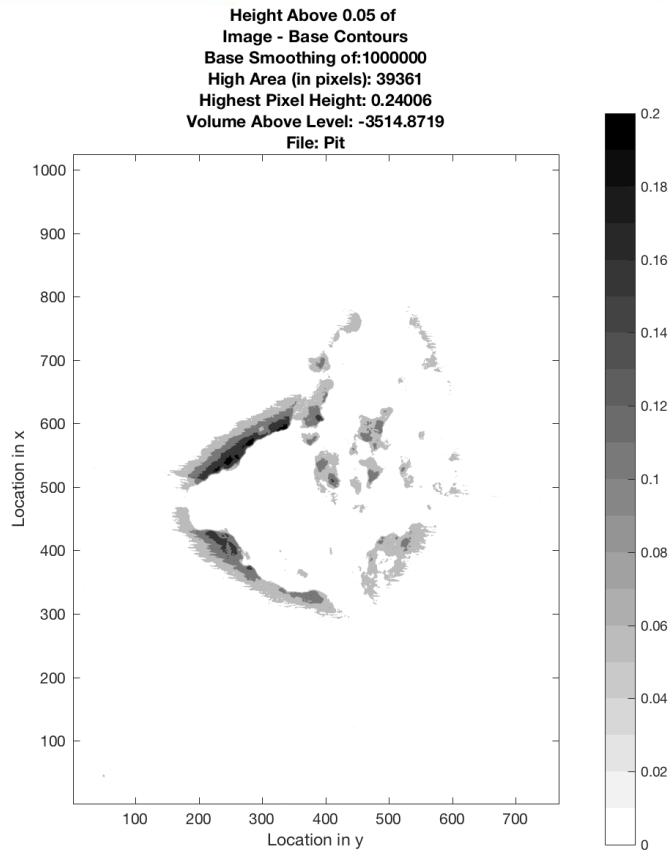
Pit



Pit

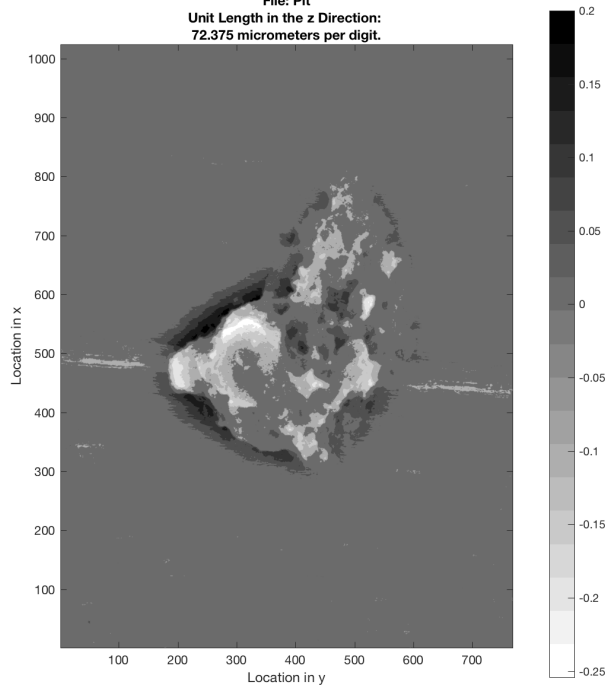


Pit

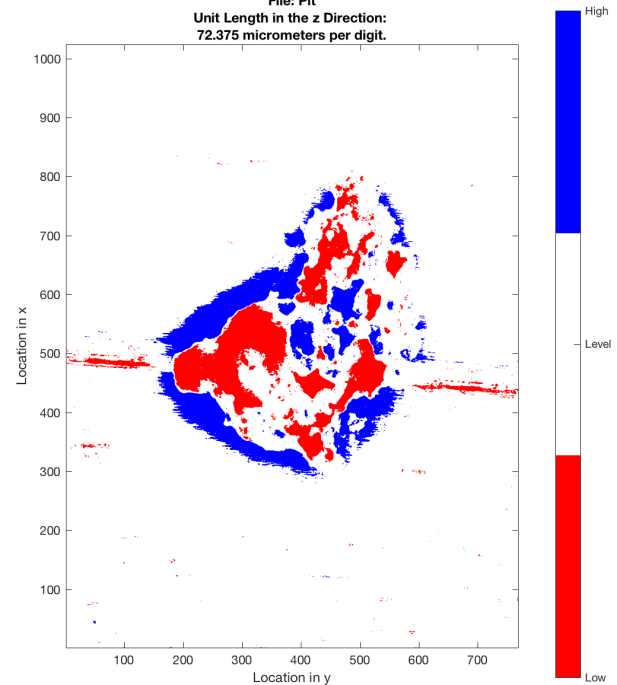


Pit

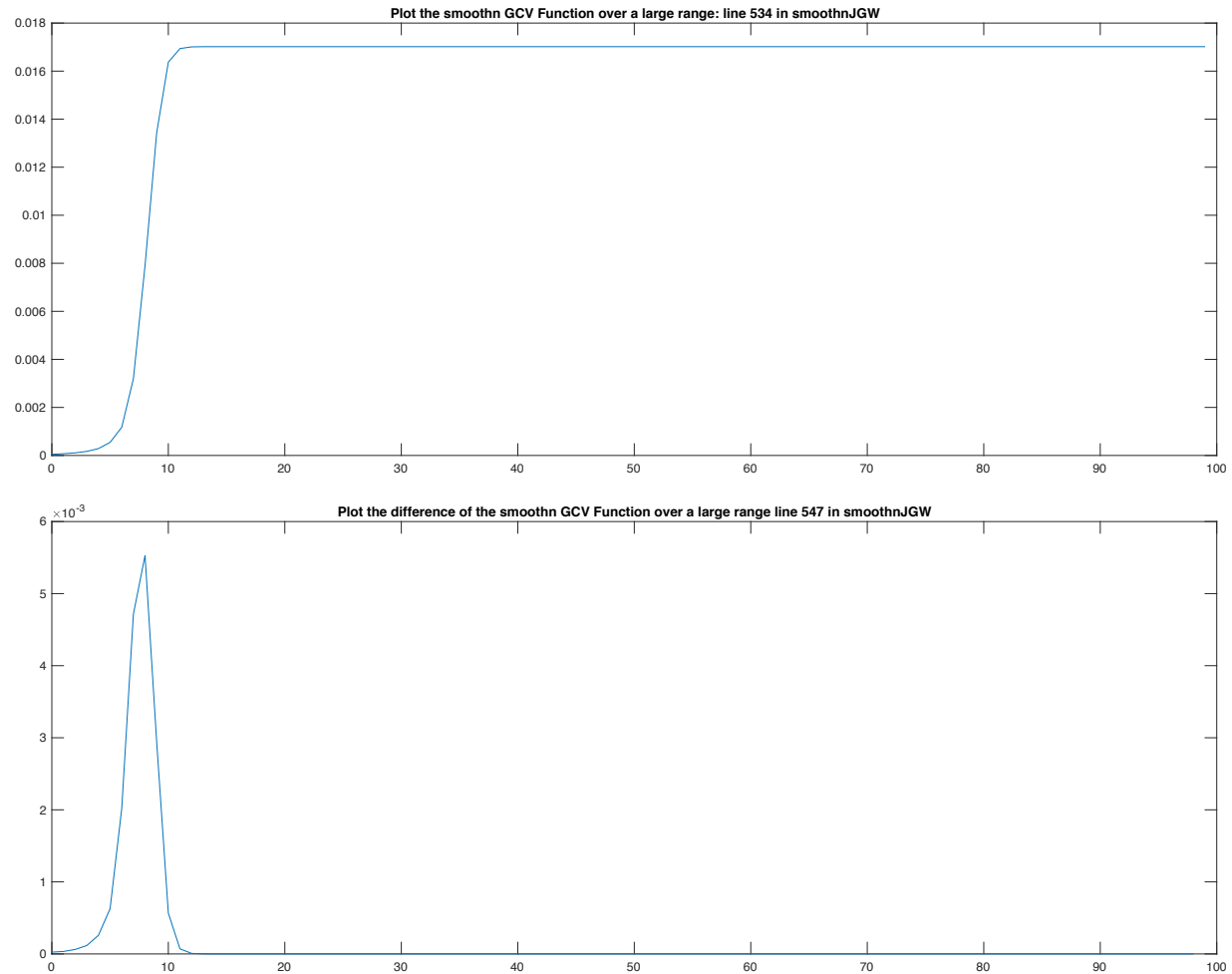
Height Above 0.05 and
Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
High Area (in pixels): 39361
Highest Pixel Height: 0.24006
Volume Above Level: -3514.8719
Deep Area (in pixels): 37730
Deepest Pixel Depth: -0.25421
Volume Below Level: 3436.2501
File: Pit
Unit Length in the z Direction:
72.375 micrometers per digit.



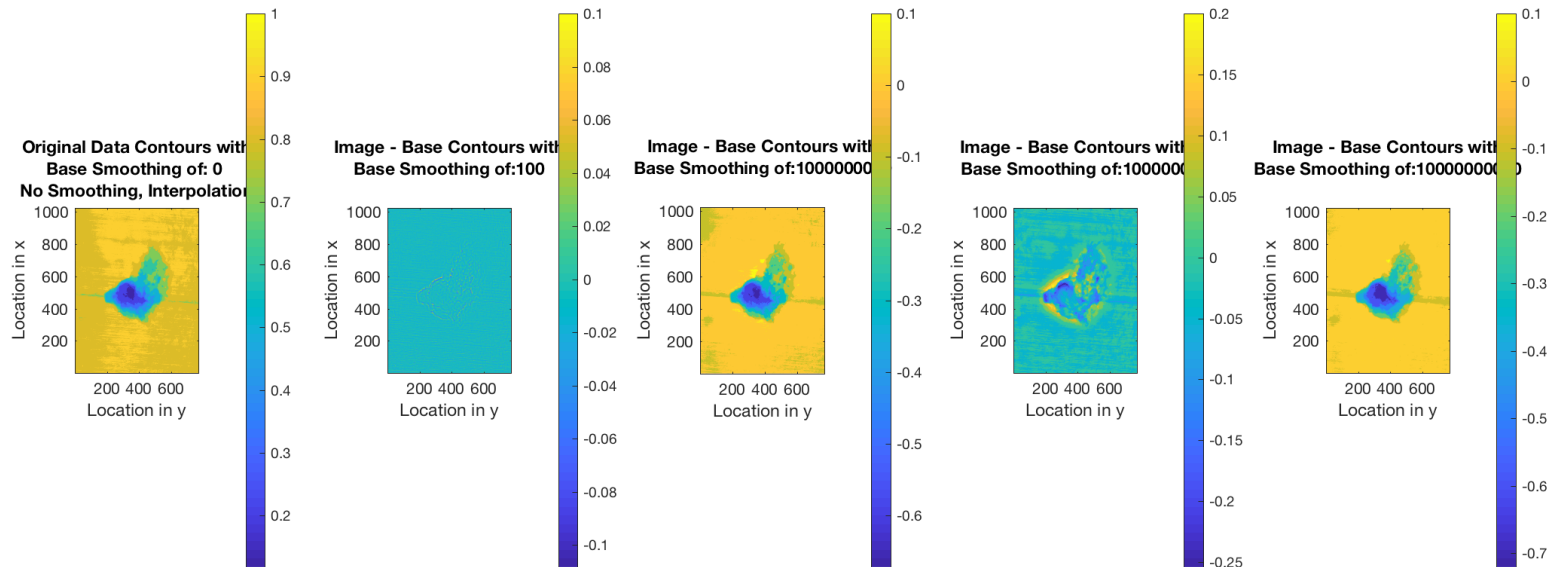
Height Above 0.05 and
Depth below -0.05 of
Image - Base Contours
Base Smoothing of:1000000
High Area (in pixels): 39361
Highest Pixel Height: 0.24006
Volume Above Level: -3514.8719
Deep Area (in pixels): 37730
Deepest Pixel Depth: -0.25421
Volume Below Level: 3436.2501
File: Pit
Unit Length in the z Direction:
72.375 micrometers per digit.



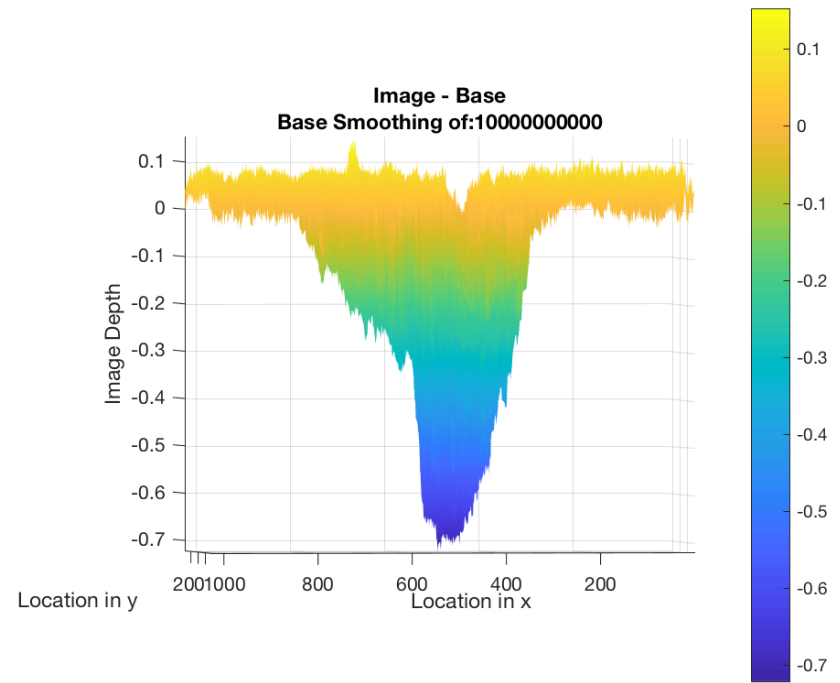
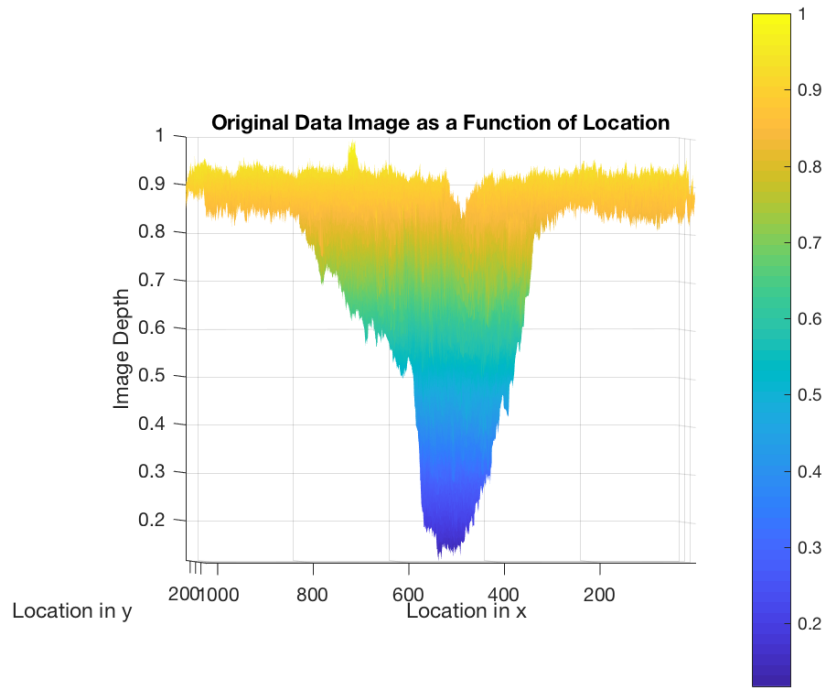
Pit



Pit



Pit



Pit

Original Data Image as a Function of Location

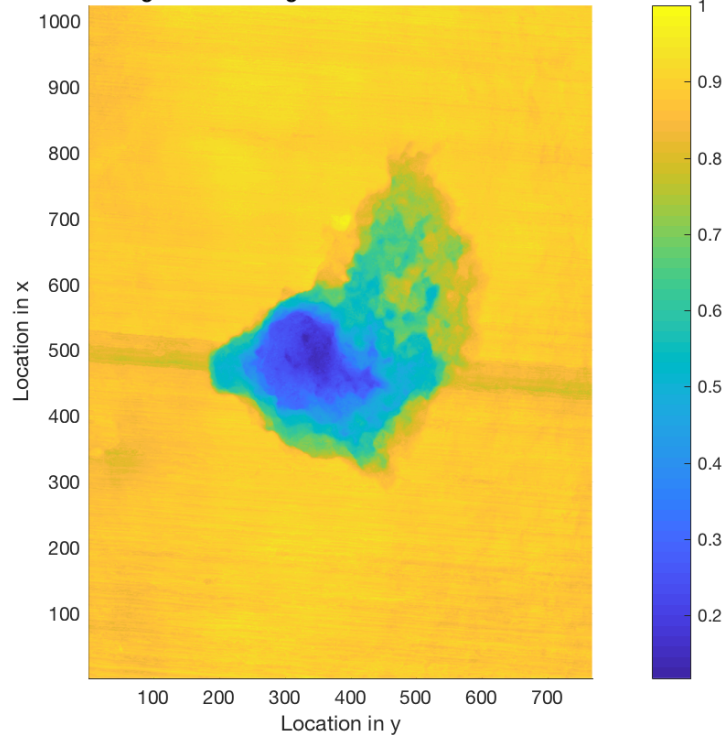
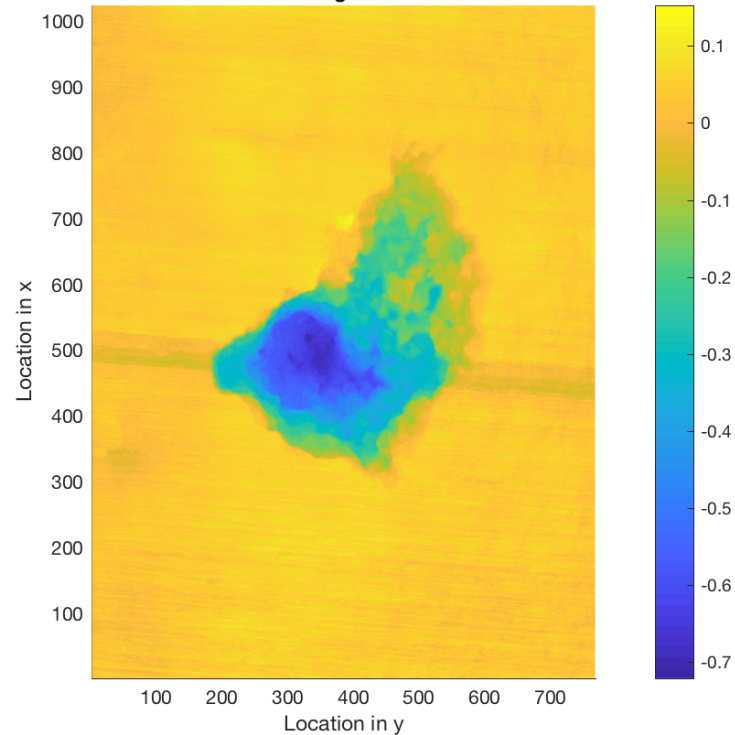
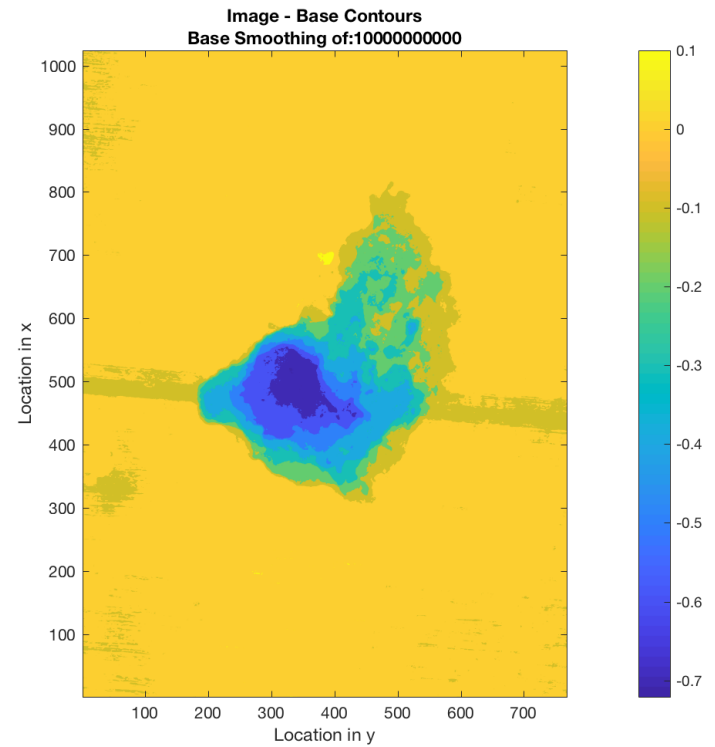
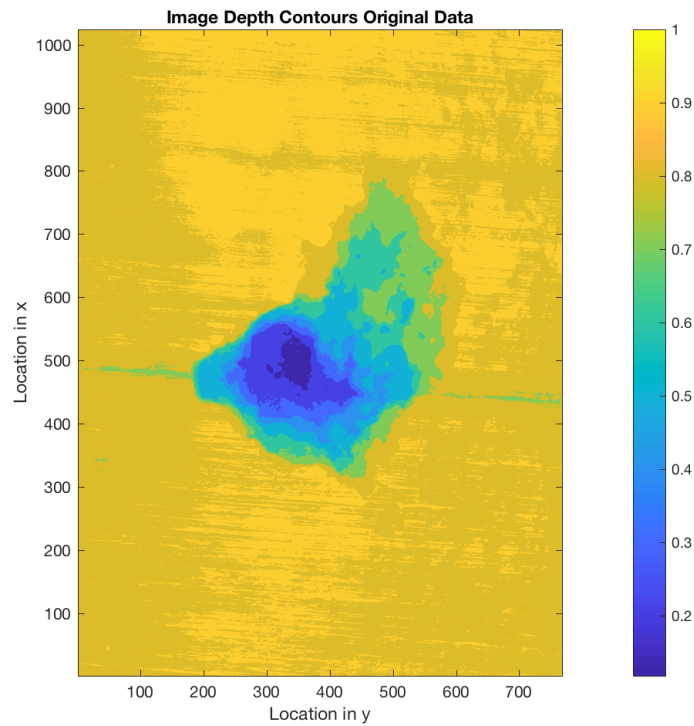


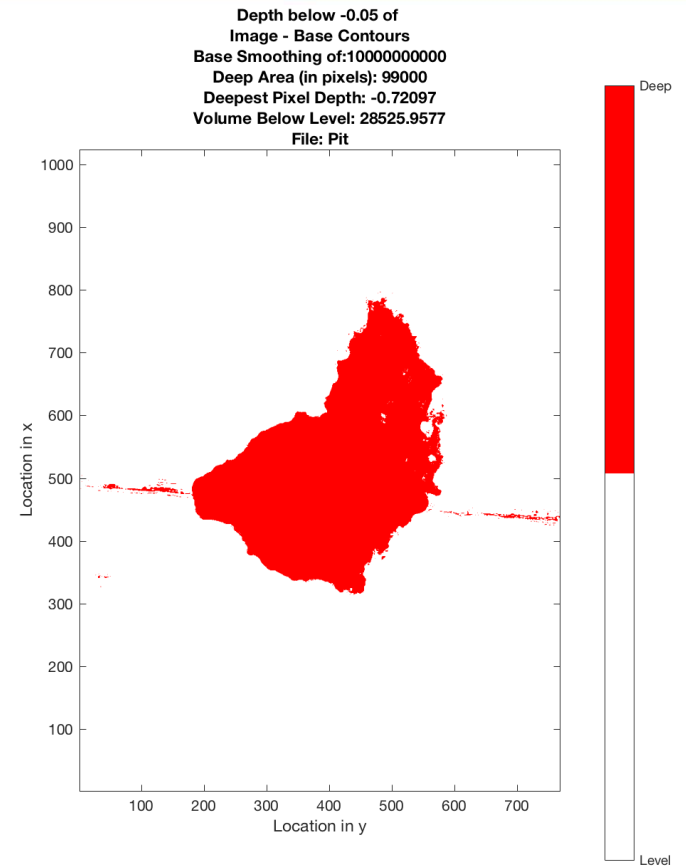
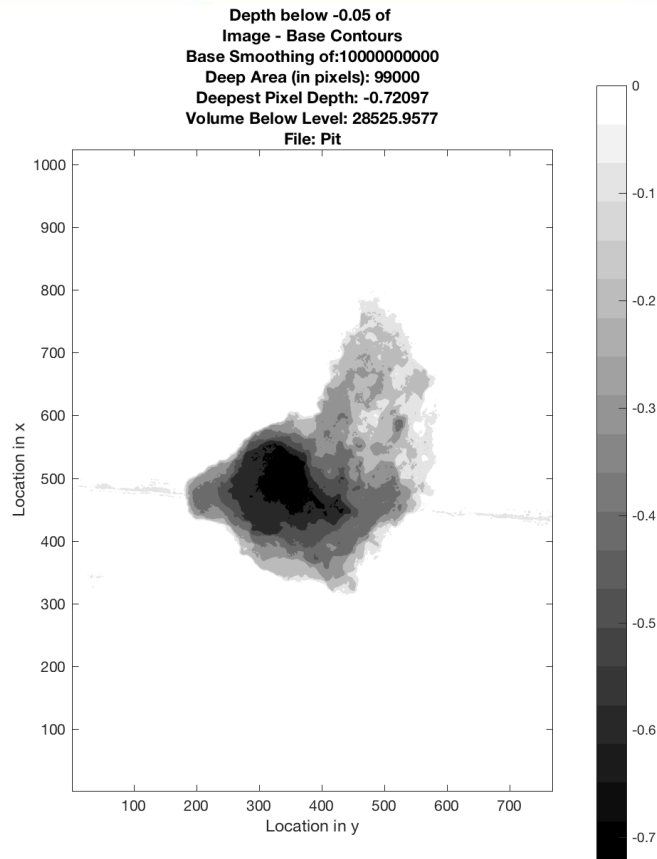
Image - Base
Base Smoothing of:10000000000



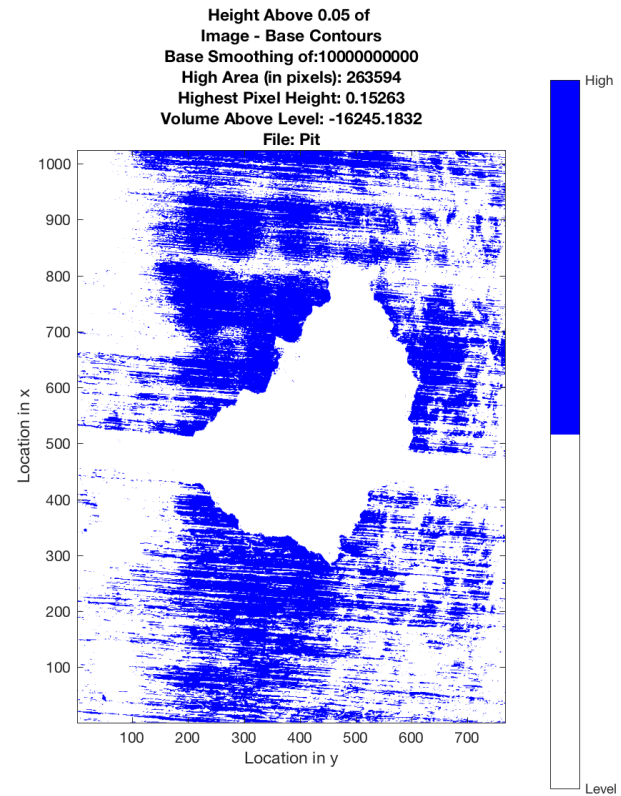
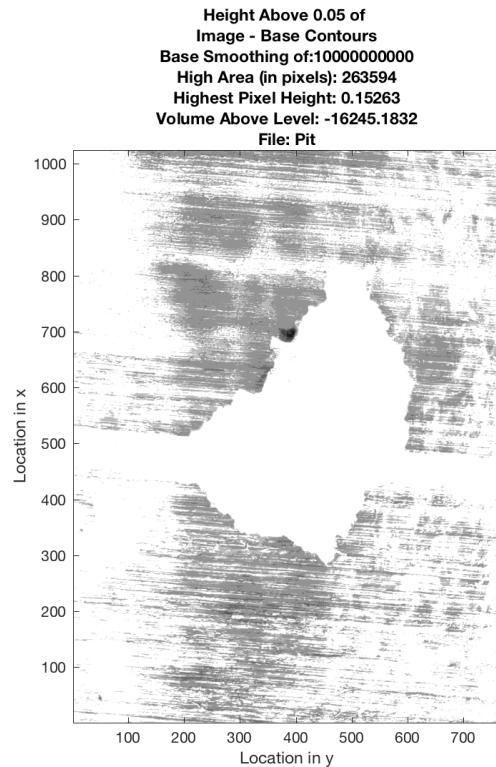
Pit



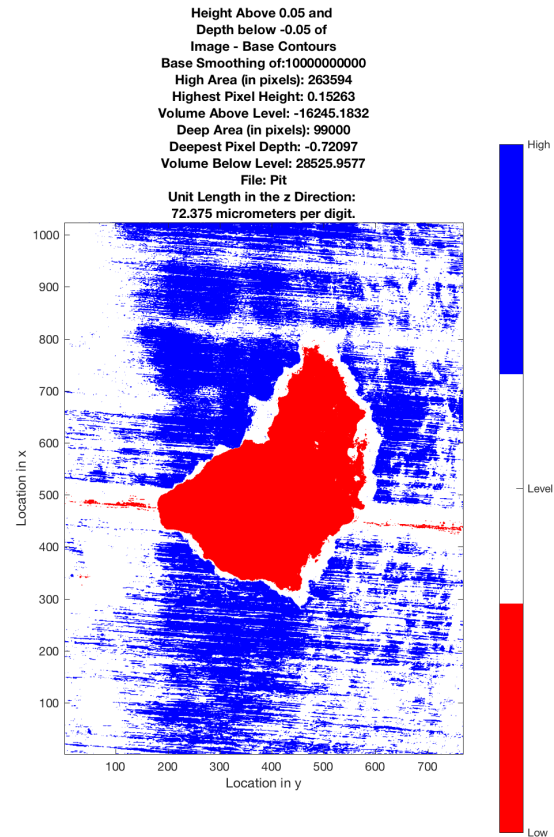
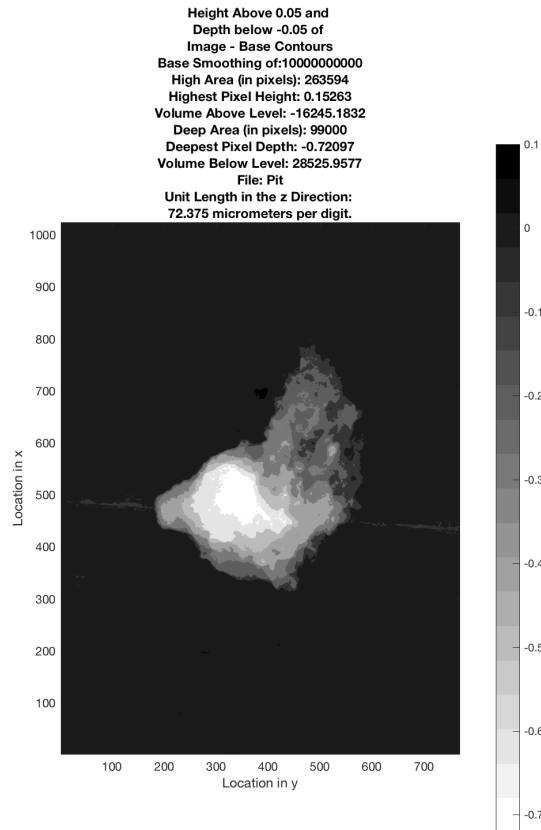
Pit



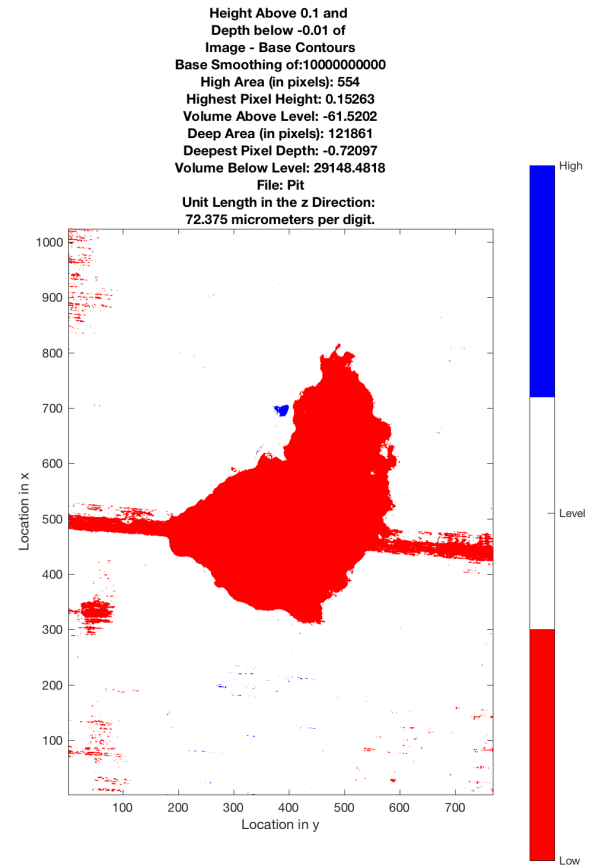
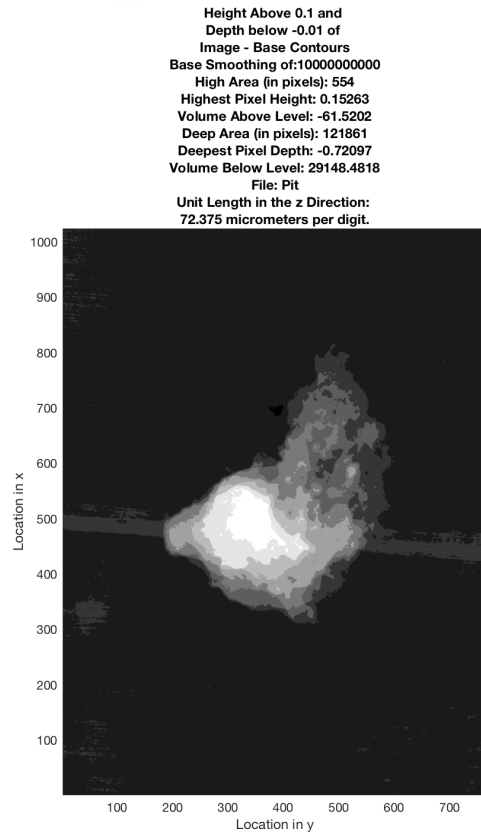
Pit



Pit



Pit



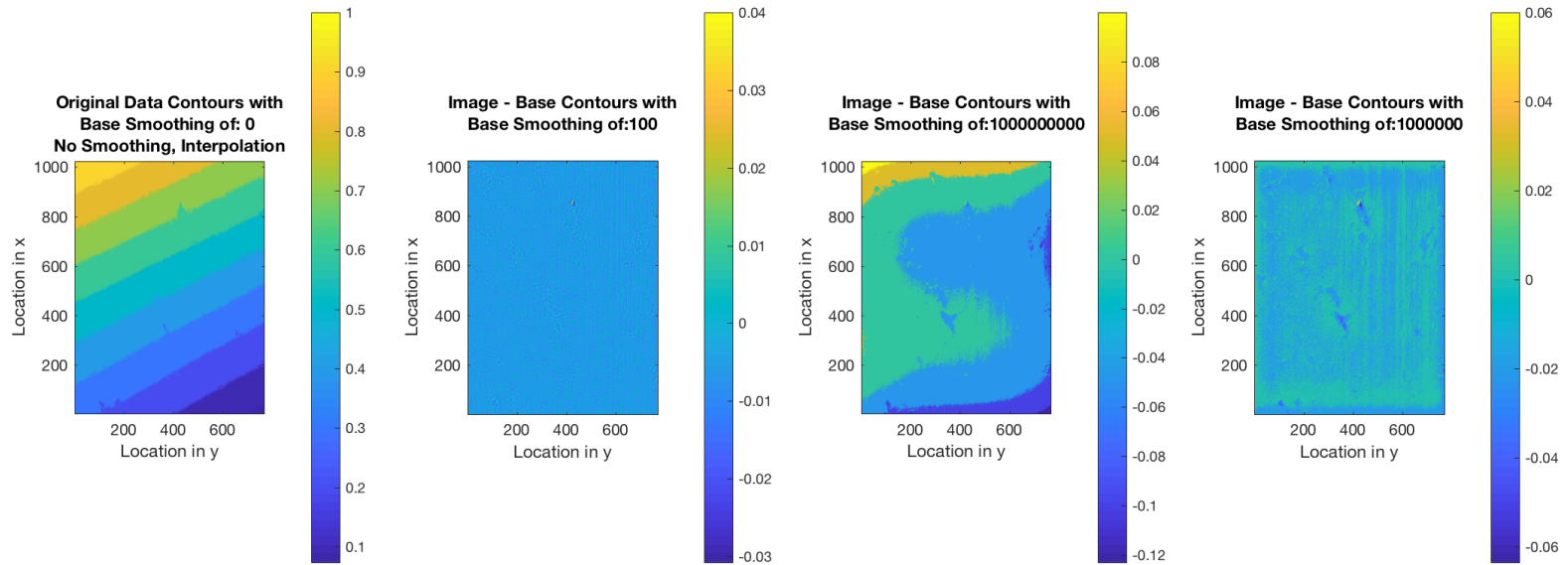
Pit

Pit

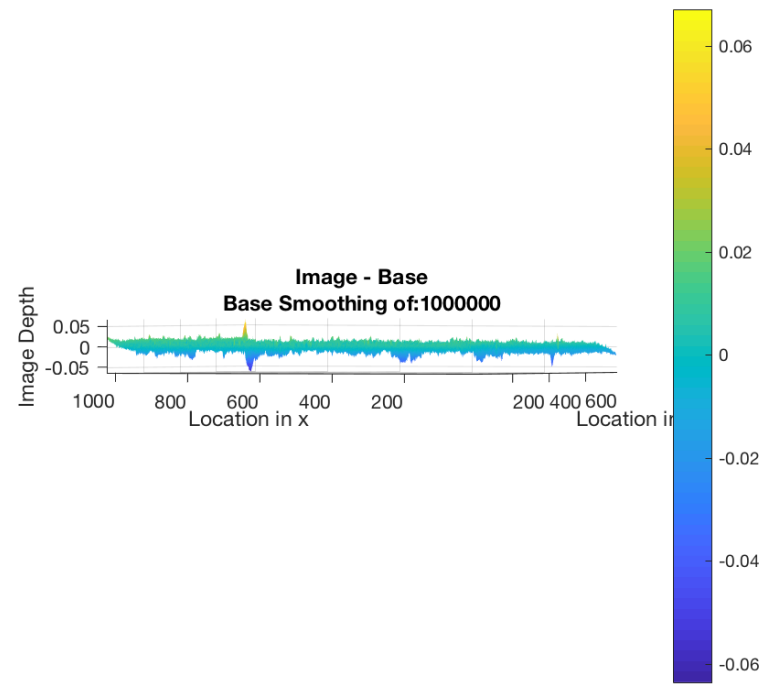
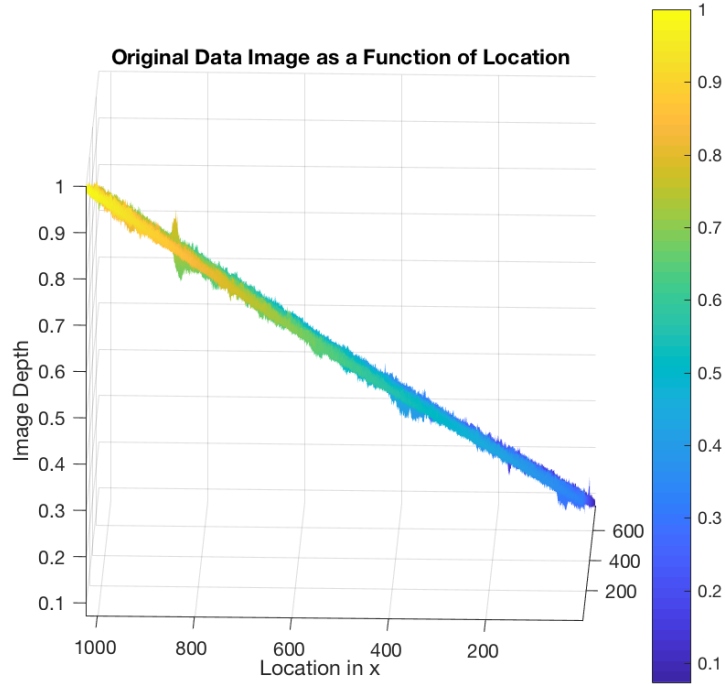
S1_20X_6300X2500

1. 690 nm/pixel
2. Z Calibration .1 nm per digit
3. Width 1024
4. Height 768
5. Sample 1 of Baseline Data with 20x magnification at Location 6300 by 2500

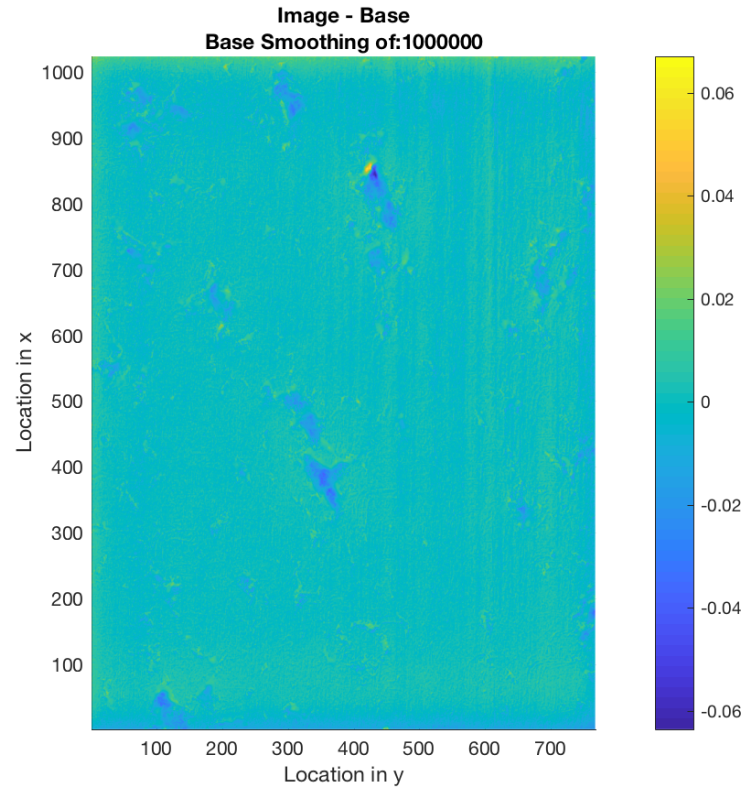
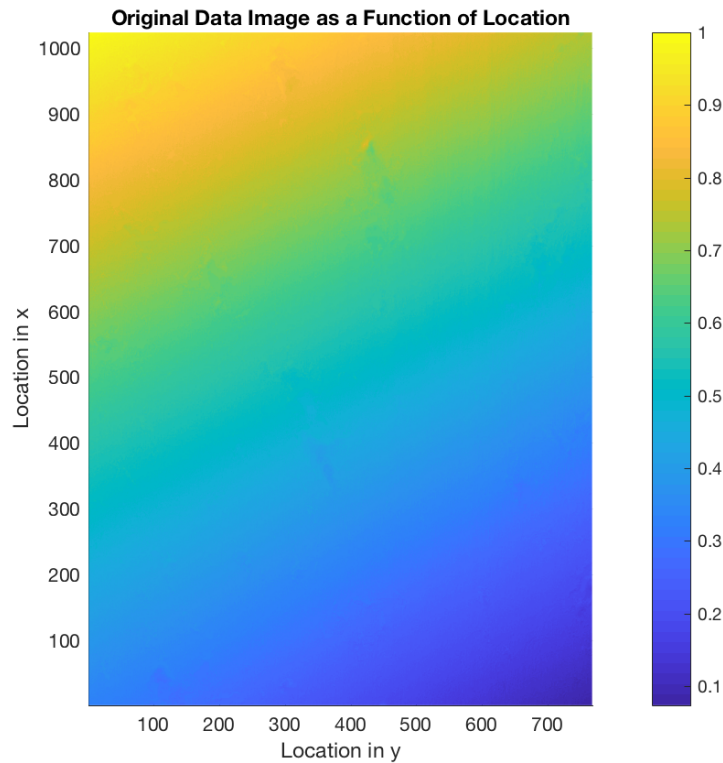
S1_20X_6300X2500



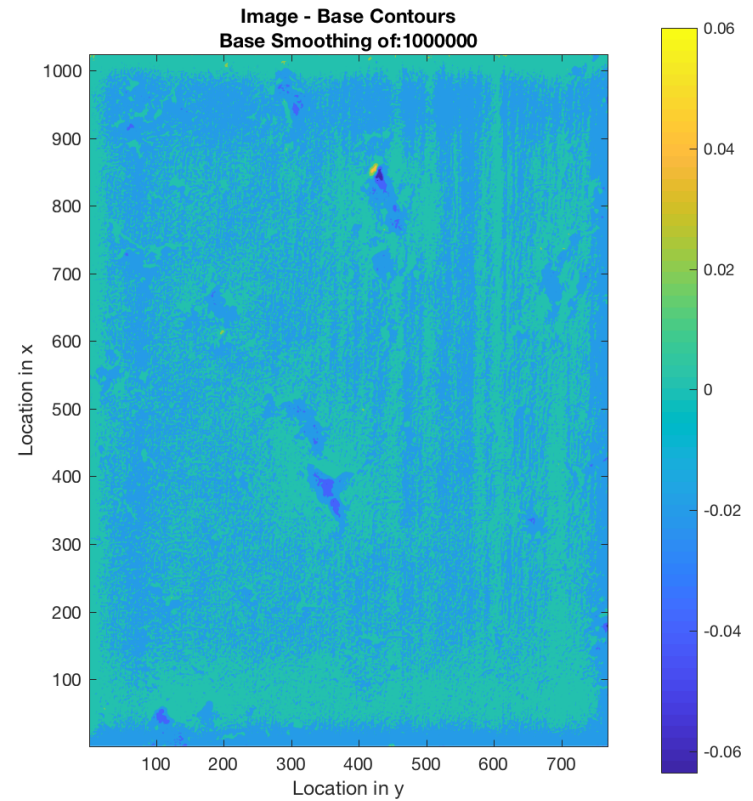
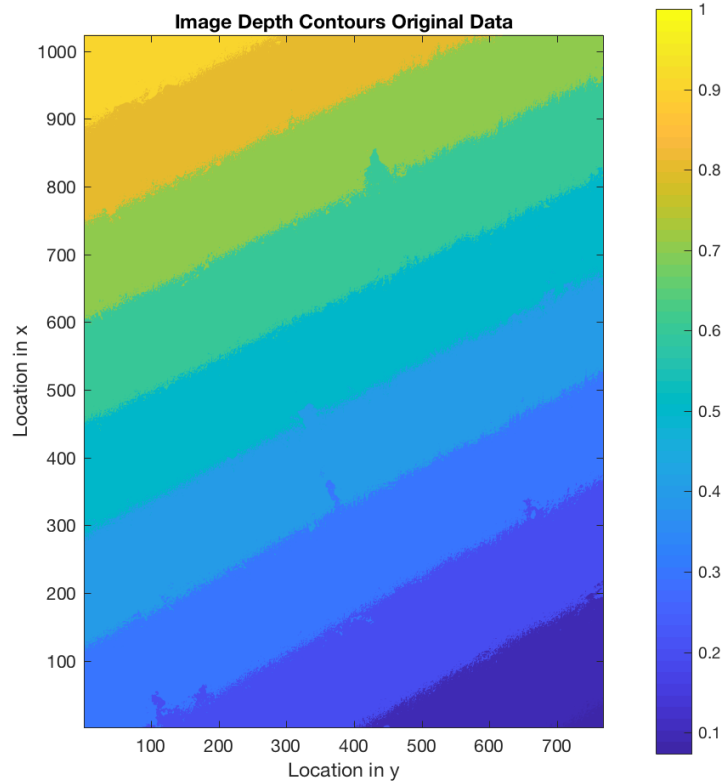
S1_20X_6300X2500



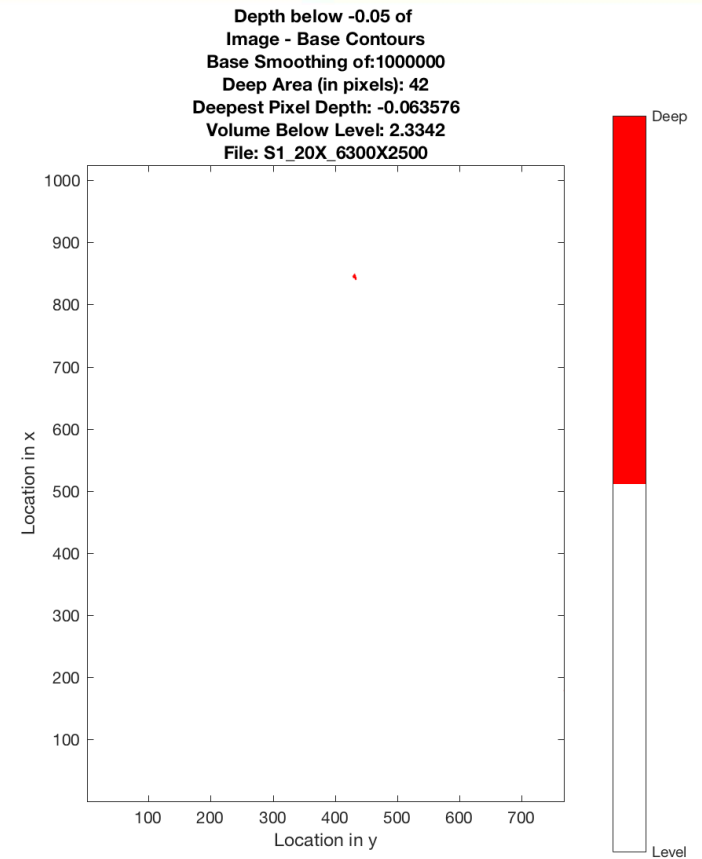
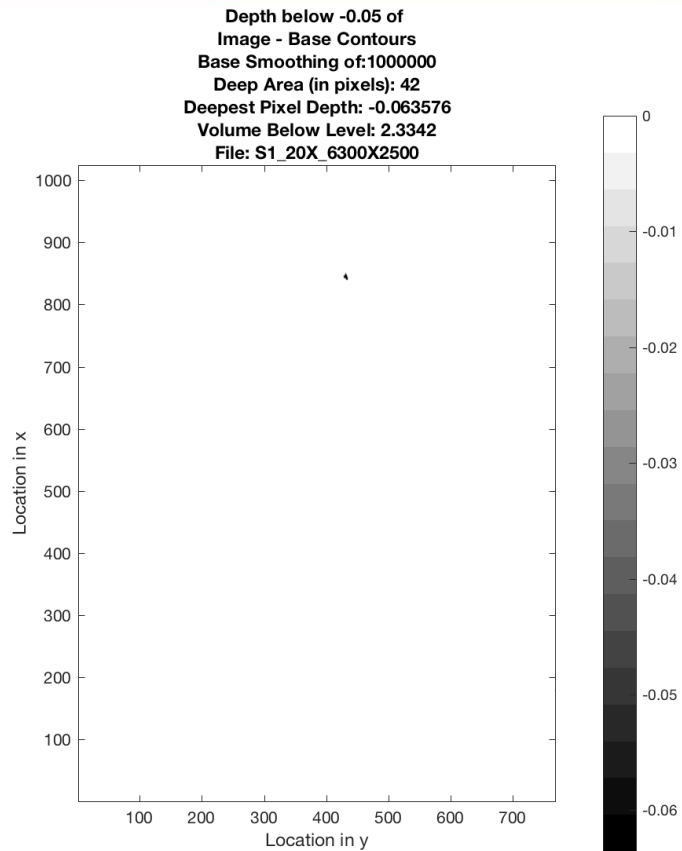
S1_20X_6300X2500



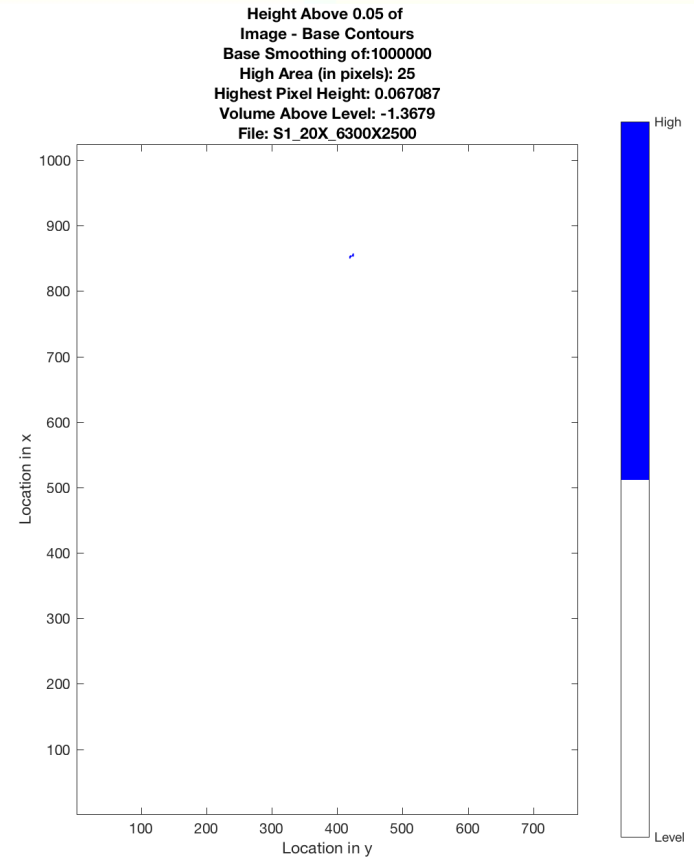
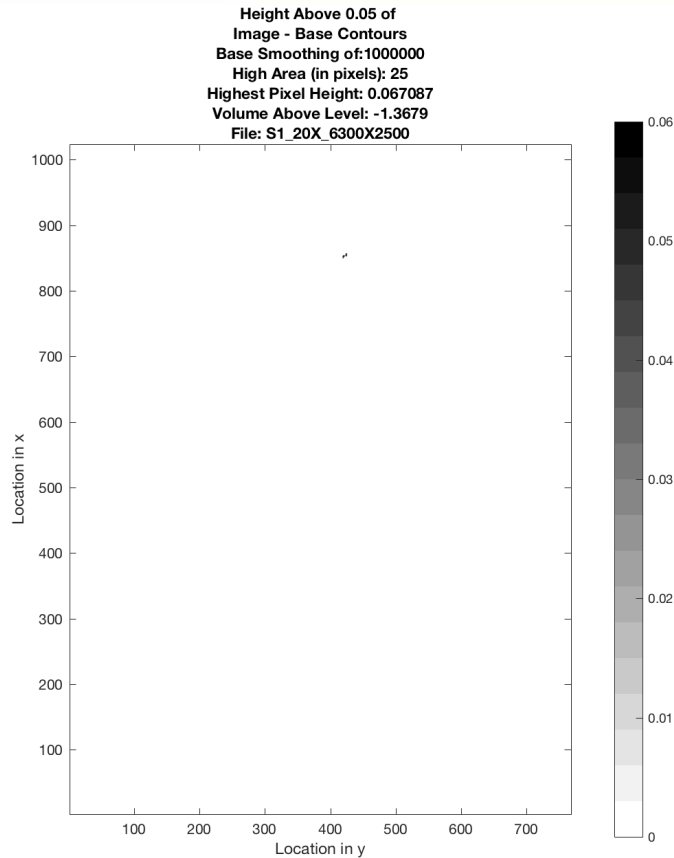
S1_20X_6300X2500



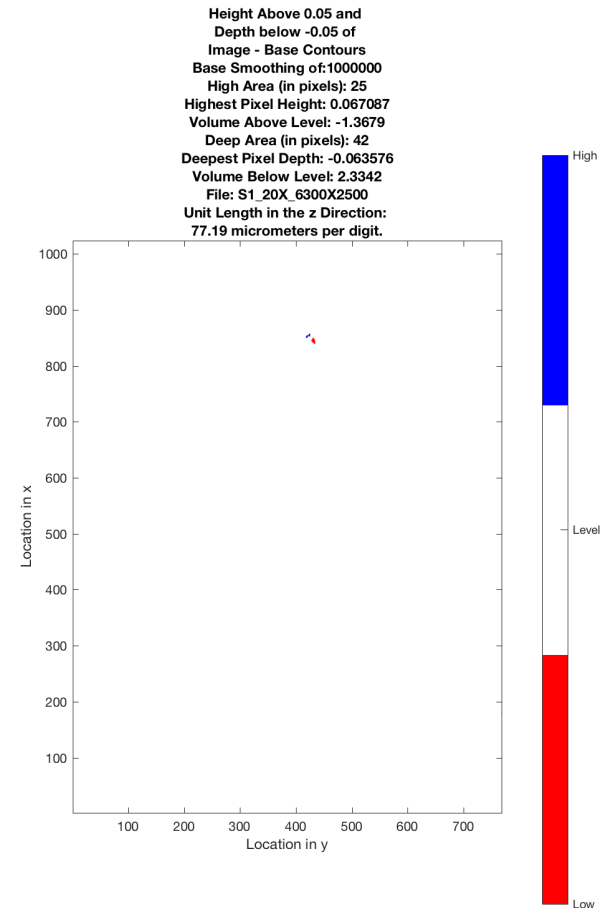
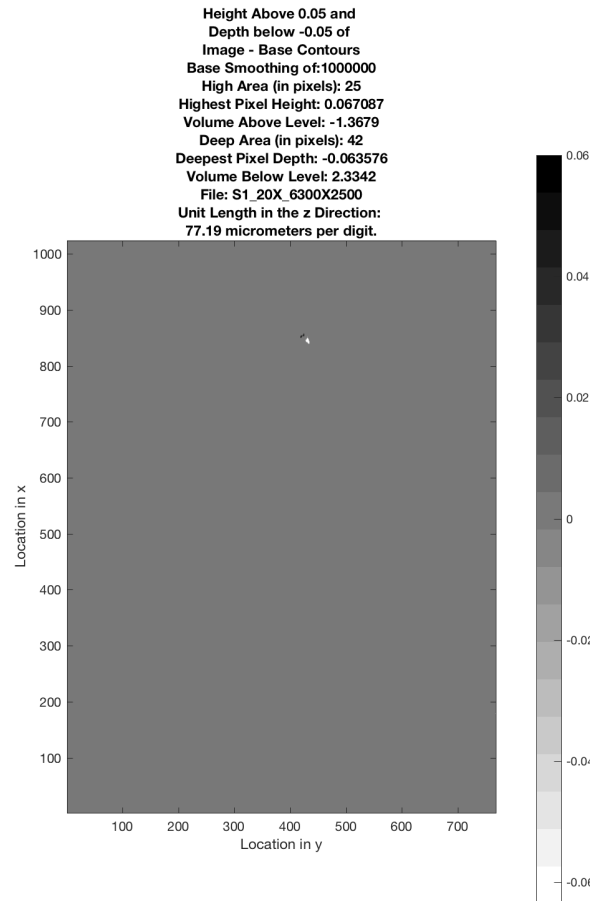
S1_20X_6300X2500



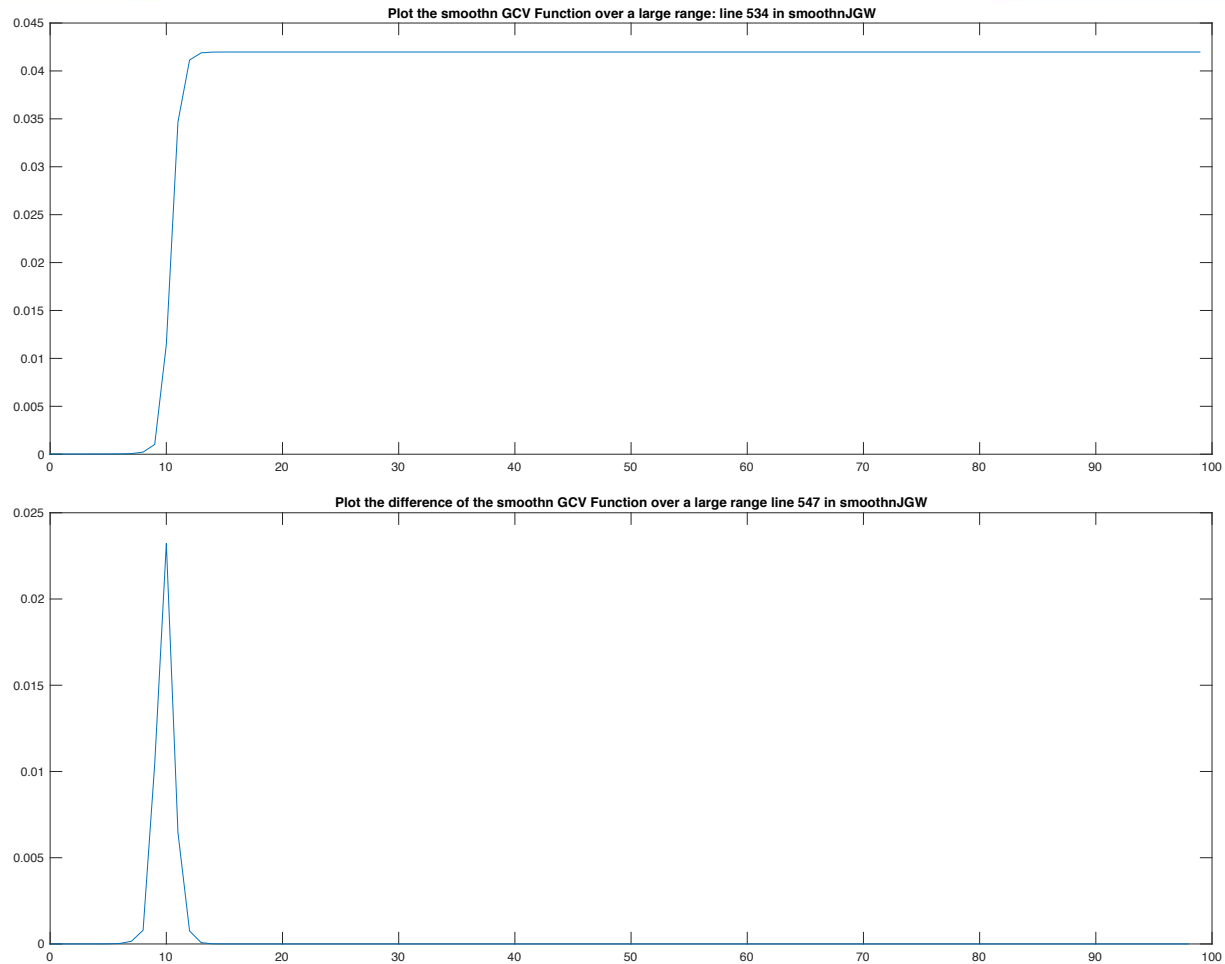
S1_20X_6300X2500



S1_20X_6300X2500

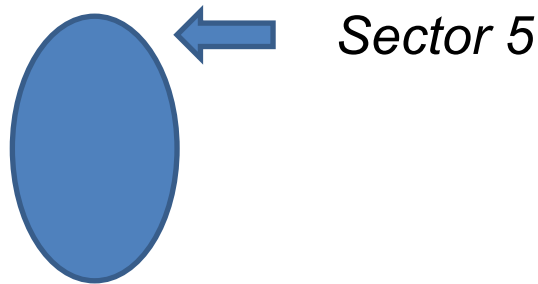


S1_20X_6300X2500

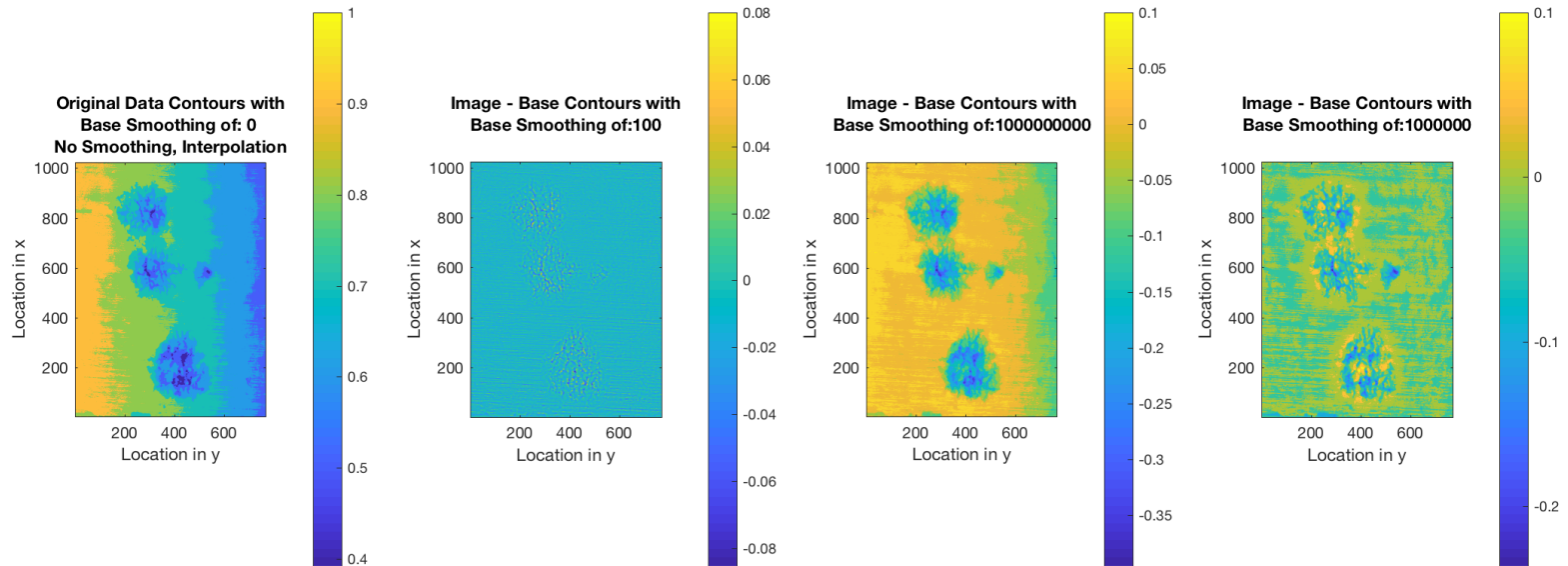


S19_5_3d

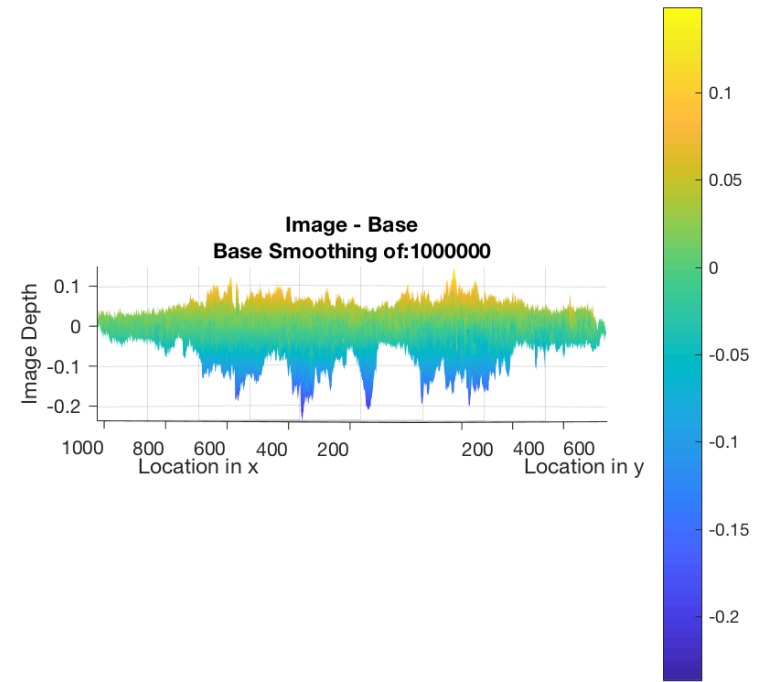
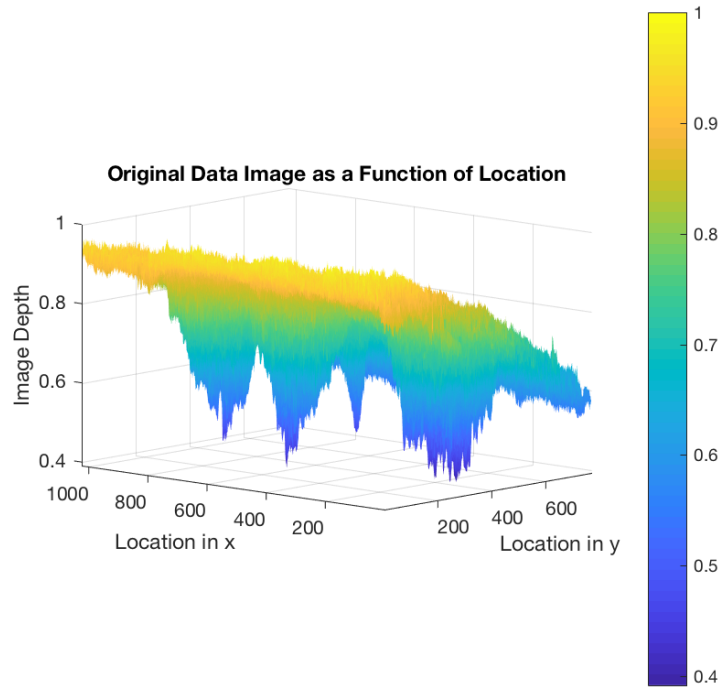
1. 689 nm/pixel
2. Z Calibration .1 nm per digit
3. Width 1024
4. Height 768
5. Sector 5 of Teardrop 19 and 20x magnification



S19_5_3d



S19_5_3d



S19_5_3d

Original Data Image as a Function of Location

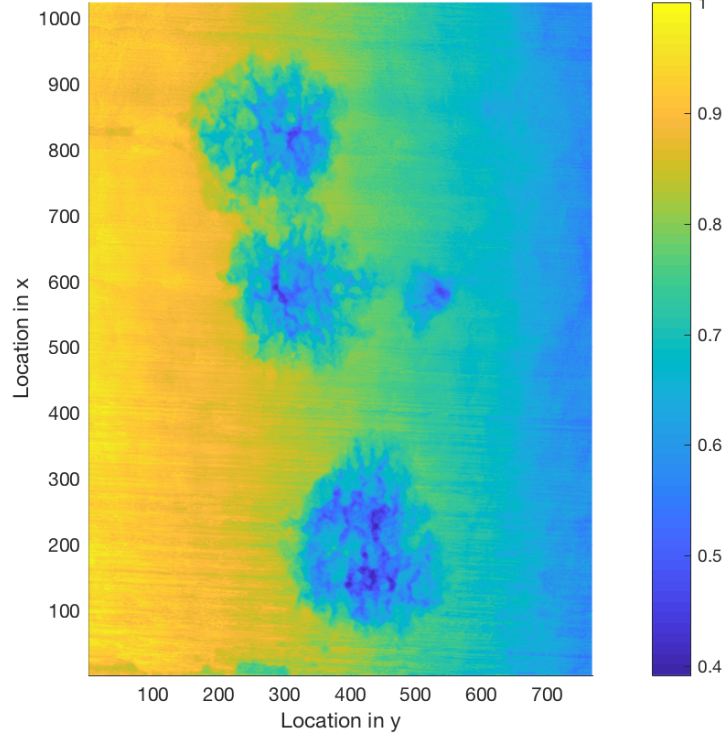
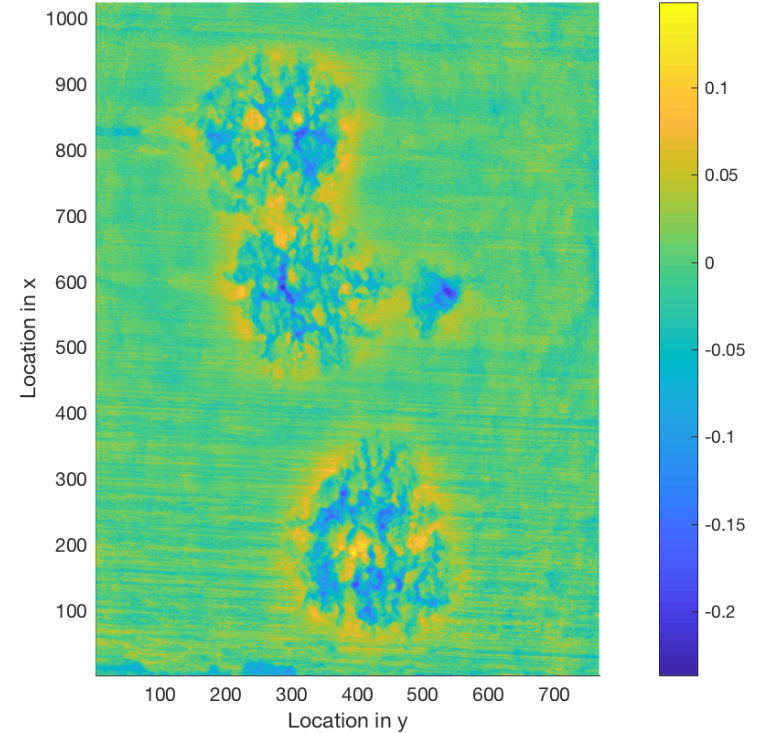
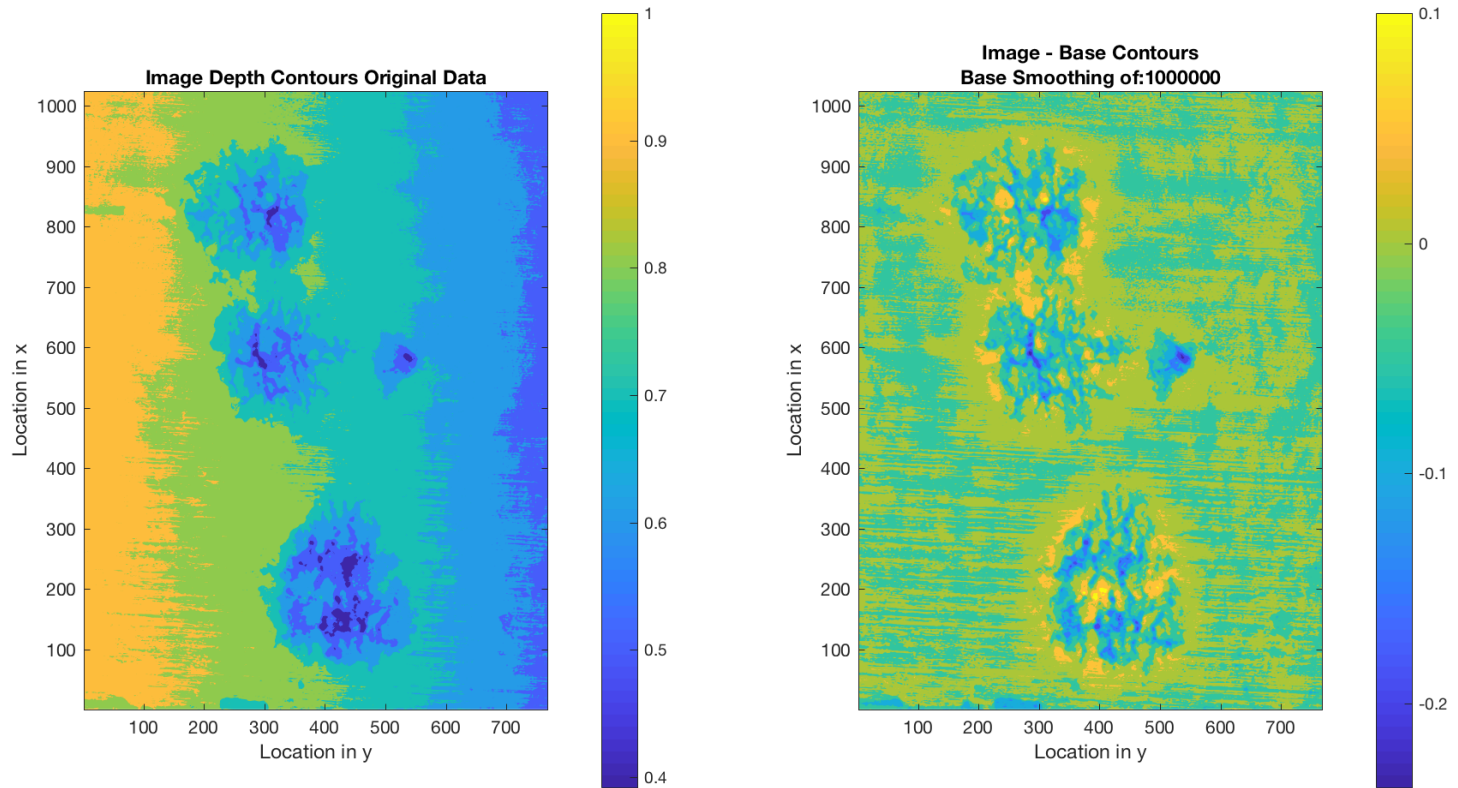


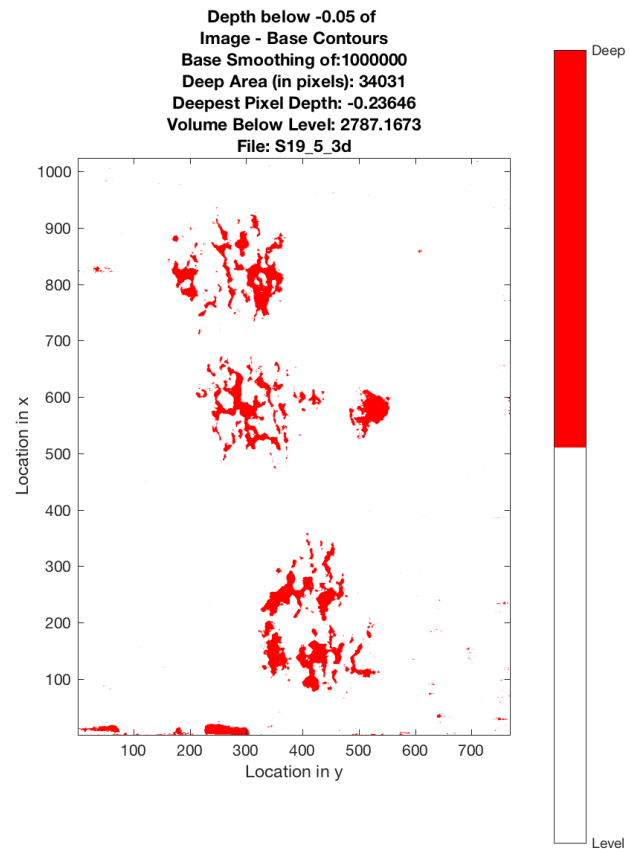
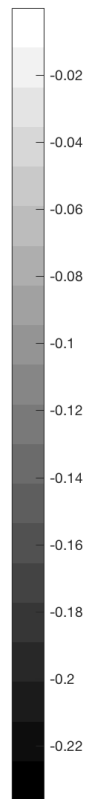
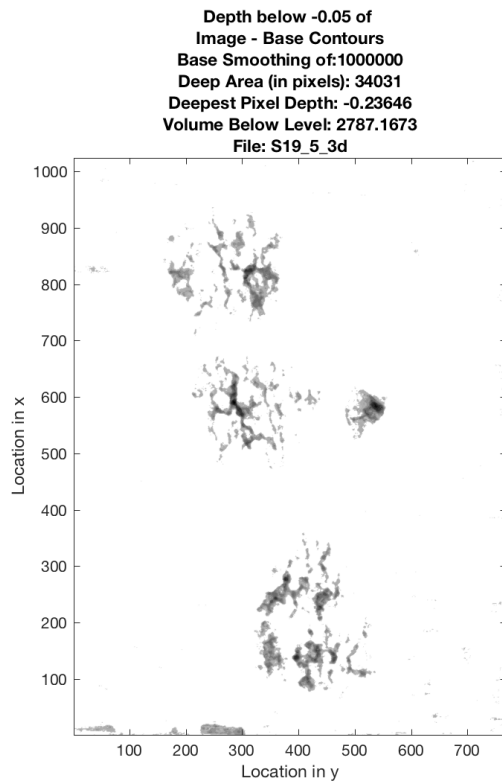
Image - Base
Base Smoothing of:1000000



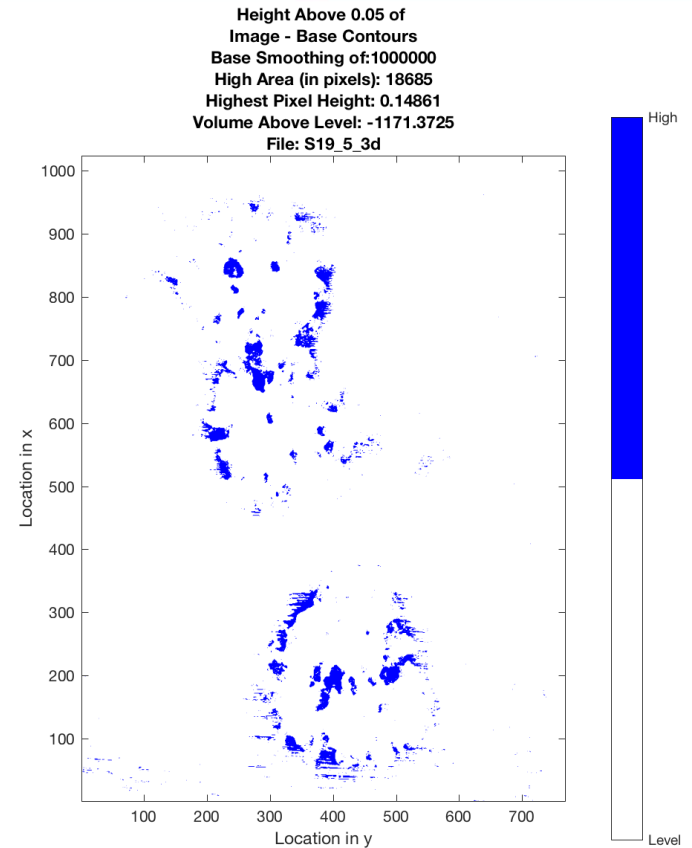
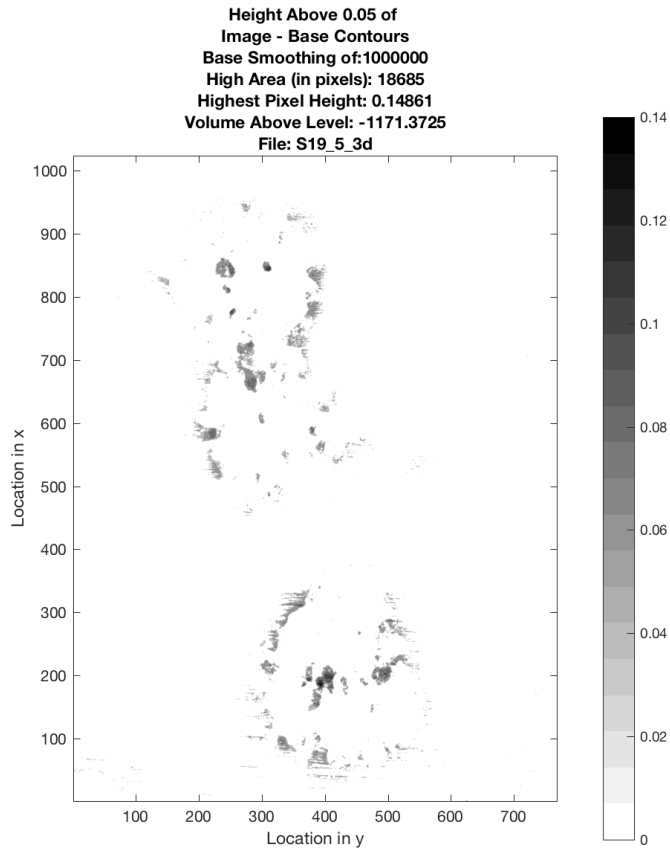
S19_5_3d



S19_5_3d

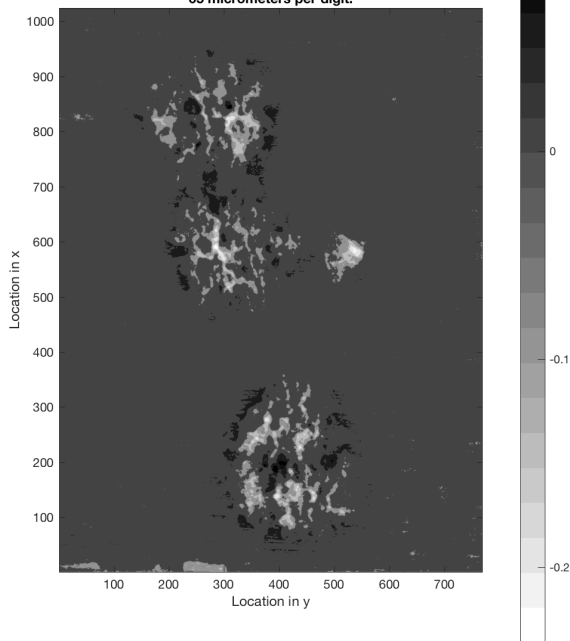


S19_5_3d

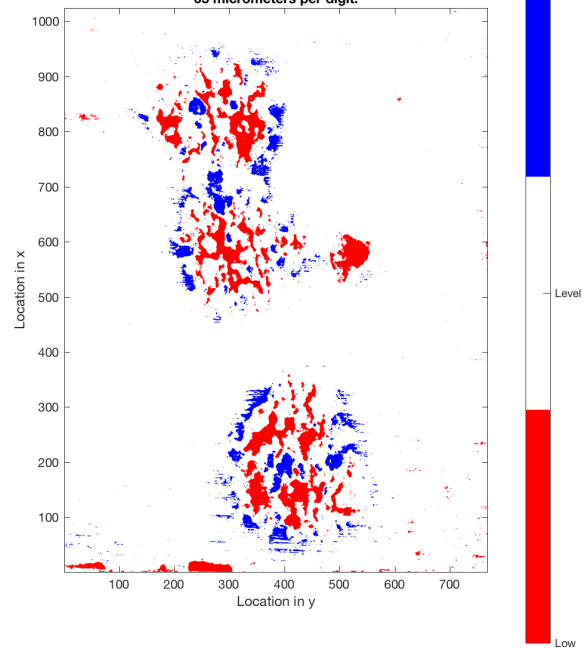


S19_5_3d

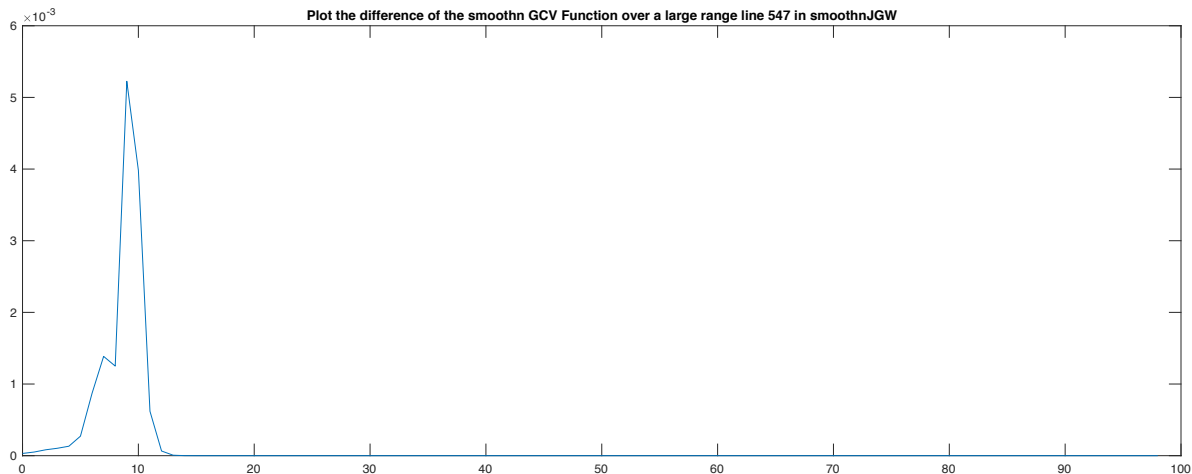
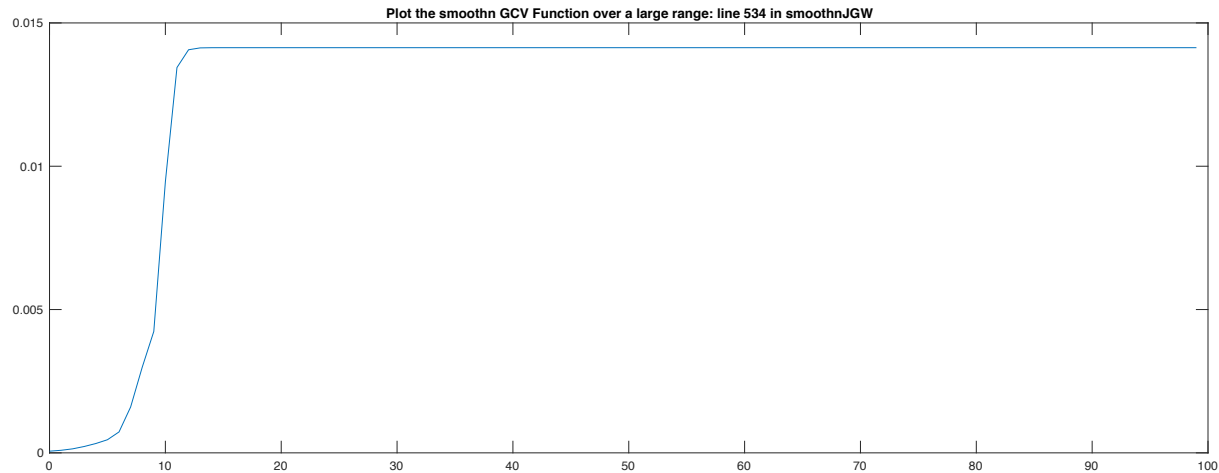
Height Above 0.05 and
Depth below -0.05 of
Image - Base Contours
Base Smoothing of: 1000000
High Area (in pixels): 18685
Highest Pixel Height: 0.14861
Volume Above Level: -1171.3725
Deep Area (in pixels): 34031
Deepest Pixel Depth: -0.23646
Volume Below Level: 2787.1673
File: S19_5_3d
Unit Length in the z Direction:
65 micrometers per digit.



Height Above 0.05 and
Depth below -0.05 of
Image - Base Contours
Base Smoothing of: 1000000
High Area (in pixels): 18685
Highest Pixel Height: 0.14861
Volume Above Level: -1171.3725
Deep Area (in pixels): 34031
Deepest Pixel Depth: -0.23646
Volume Below Level: 2787.1673
File: S19_5_3d
Unit Length in the z Direction:
65 micrometers per digit.



S19_5_3d



Lessons Learned

1. One Smoothing Parameter Works Well to Estimate the Background - 9 Images
2. Generalized Cross Validation
3. Multiple Minima
4. Non Stitched and Stitched Images Are Different
5. Edges Detected and Removed from Analysis
6. Too Small a Feature Should Be Defined
7. Stitched Images Lose Sufficient Resolution
8. Stitched Images Introduced Artifacts
9. Image Post Processing Is Feasible

Future Work

1. Expand to Libraries of Images
 - i. Read Header Information
 - ii. Handle Many Images In One Run
2. Avoid Stitched Images
3. Best Background Estimation
 - i. Edge Details (started working on this)
 - ii. May Result in Non Rectangular Image
4. Individual Feature Definition and Discrimination
 - i. Cracks, Scratches, Elongated Pits, Stitching, Edges
 - ii. Pit Depth, Area, Volume
 - iii. Corrosion Depth, area, Roughness
5. Subsurface Features by Incorporation of Cross Sectional Information/Data

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5. Garcia D, A fast all-in-one method for automated post-processing of PIV data. Exp Fluids, 2011;50:1247-1259.
6. Wood, S.N., Modelling and Smoothing Parameter estimation with Multiple Quadratic Penalties, J. R. Statist. Soc. B (2000), 62, Part 2, pp 413-428.
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