DOE/NV/25946--3270

National Security Technologies, LLC
(NSTec)

Skin Effect Simulation for Area 11 DPF Hot Plate

July 27, 2017

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This work was done by National Security Technologies, LLC.,
under Contract No. DE-AC52-06NA25946 with the U.S. Department of Energy.

Executive Summary

Two arc flashover events occurred at the DPF Area 11 facility. These flashover events happened in the same location on the bank current delivery plates. The damage from one of these events can be seen on the left-hand side of Figure 1. Since the flashovers occurred in the same area of the bank, and the reliability of the bank is important for future DPF experiments, a failure analysis effort was initiated. Part of this failure analysis effort was an effort to understand the physical reasons behind why the flashover happened, and why it happened in the same place twice. This paper summarizes an effort to simulate the current flow in the bank in order to understand the reasons for the flashover.

There are always magnetic fields associated with electric current. When current is flowing in a conductive medium, the magnetic fields influence the current to flow on the exterior (i.e. the skin) of the conductor, and is for this reason known as the *skin effect*. Whenever there are large gradients in current density, there will also be large voltage gradients. The locations of these large voltage gradients then in turn become locations for arc flashovers.

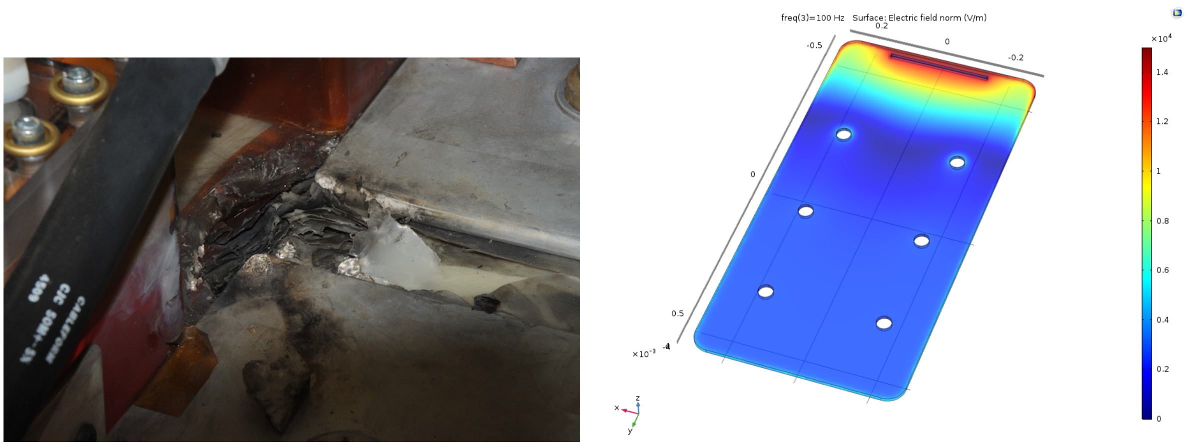

Figure 1: On the left: mylar insulator damage shown between two hot plate segments on the DPF Area 11 bank. Both failures seemed to happen at the corners of the hot plate and burn through the mylar insulator sheets to the ground plate. On the right: surface current density for a hot plate segment. The upper right and left corners of the hot plate are now thought to be regions of high voltage gradient.

Until the reasons for the arc flashovers are known, it is impossible to properly design flashover mitigating features into the bank. From the simulations, it would appear in Figure 1 that the most straightforward way of reducing the probability of arc flashover would be to increase the radius of curvature on the corners of the hot plates, and to increase the thickness of the mylar insulation between the hot and ground plates.

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1 Description of Bank Failure

The Area 11 DPF capacitor bank suffered an arc flashover that damaged two of the hotplate[1] segments, the insulation, and the ground plate. A post-event analysis of the damage lead the DPF team to conclude that an arc may have initiated from the corner of one of the hotplates. The question of why the arc flash initiated at that location was less clear though. The damage is shown in the right-hand side of Figure 2. The left-hand side of Figure 2 is shown to clarify the arrangement of the conductors on the current delivery header. Since this bank must be highly reliable in order to be trusted as a part of a larger experimental plan, all known failure modes should be identified and their probabilities of occurrence minimized.

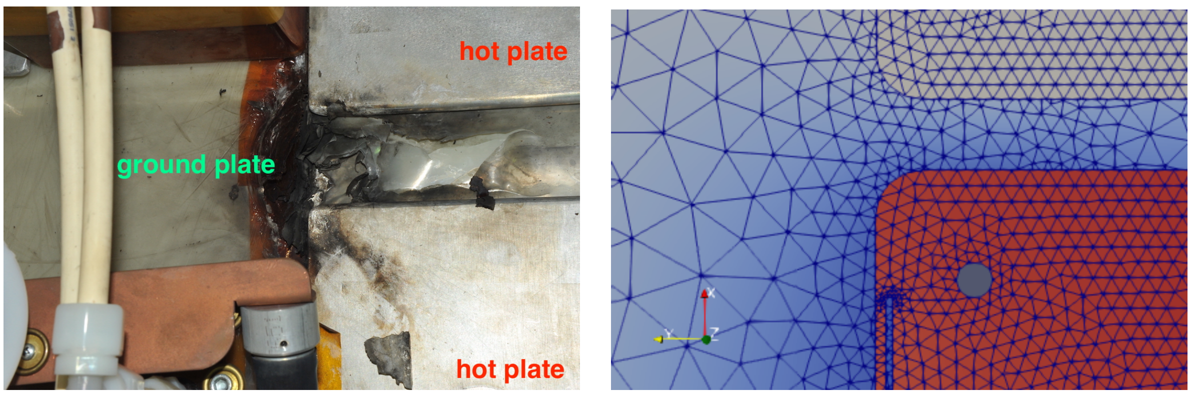

Figure 2: The left image is of the bank failure at the Area 11 facility. It appears as if a flashover happened from the corner of the one of the hotplates burned through the insulation to the ground plate. The right image is of the meshed model used for simulating the Area 11 bank. The blue region is the ground plate, and the red and grey regions are two separate hot plate segments. These are shown in order to help identify features in the photograph to the left.

During the initial investigations, we looked for obvious causes of the flashover, such as: scratches or sharp points on the metal that may have been caused during assembly or maintenance, debris buildup on the insulator, holes in the insulator, and previously unknown ramifications of the effects of self-magnetic field on the current flow in the power delivery conductors. While we could not completely rule out the mechanical causes, the DPF's more than decade long experience with running pulsed power machines has made us extremely careful with assembly and maintenance issues. The other unknown was the current flow in the power delivery conductors, which we will discuss in much greater detail in the next section.

Since the skin effect causes the current in good (but not perfect) conductors to preferentially flow on the outer boundaries (the *skin*) of conductors, it also causes voltage gradients near corners on the conductors. These voltage gradients would not be expected in the case of a perfect conductor, and would not be very noticeable in circumstances of small currents. However, for the power delivery conductors of pulsed power systems, these voltage gradients can be rather large. Ohm's law tells us that for a given resistance, R, that the voltage across this resistor is porpotional

[1]These features of the bank are so named because when the capacitor bank is charged, the *hot* plate is at high potential, and the *ground* plate is at ground potential.

to the current flow through it: $V = IR$. While 0.01Ω is considered a very small resistor, when the 2 million amperes of a typical DPF discharge flows through it, it will generate a 20 kV potential difference. This illustrates how subtle changes in the current flow for the power delivery conductors can have dramatic results. However, predicting where these voltage gradients will be is problematic.

2 Physics of the Skin Effect

In this section, we start out with Maxwell's equations, and then apply all of the simplifying assumptions to arrive at the skin effect PDE that we will use in the analysis. The two major simplifications that are going to used are: neglecting the displacement current, and solving the PDE in the frequency domain. We start with Maxwell's equations, specifically, with Ampere's law and the Maxwell-Faraday equation:

$$\begin{aligned} \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} &= \mu \left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right). \end{aligned} \tag{1}$$

Where $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic induction, $\vec{J}$ is the current density, ε is the permittivity of the medium, and μ is the permeability of the medium. In Ampere's Law (Equation 1), on the right-hand side, the electric field time derivative term is the displacement current term. For the simulation of currents flowing in the power delivery conductors, this term may safely be ignored (see [1], and chapter 14 of [3]). We may also use Ohm's law to rewrite the electric field in terms of the conductivity of the material, σ, and the *free* current density[2] ($\vec{J}_f = \sigma \vec{E}$). We will also use the magnetic field constitutive relationship ($\vec{B} = \mu \vec{H}$) in order to replace $\vec{B}$ with the magnetic field, $\vec{H}$:

$$\begin{aligned} \nabla \times \vec{J}_f &= -\sigma\mu \frac{\partial \vec{H}}{\partial t} \\ \nabla \times \vec{H} &= \vec{J}_f. \end{aligned}$$

As it would turn out, this PDE is impractical to solve numerically in the time domain. In order to show this, we first transform the PDE into a frequency domain representation:

$$\begin{aligned} \nabla \times \vec{J}_f &= -\mathrm{i}\sigma\omega\mu \vec{H} = \frac{2\mathrm{i}}{\delta^2}\vec{H} \\ \nabla \times \vec{H} &= \vec{J}_f, \end{aligned}$$

where $\delta = \sqrt{2/(\sigma\omega\mu)}$ is the *skin depth*. Let us assume that we have a cube of conductive matter that is three skin depths on a side, and each side is broken up into ten segments. The minimum edge length would then be $\ell = 3\delta/10$. The Courant stability condition (a.k.a. the CFL limit, see [2]) for this PDE says that the time step cannot be larger than the minimum edge-length in the mesh divided by the speed of the waves in the simulation (the speed of light in this case). Thus, the CFL timestep

[2] I am also assuming that there are no bound current densities, therefore $\vec{J} = \vec{J}_f$.

would be $t_{\text{CFL}} = 3\delta/(10c)$. The period of the sinusoidal field would be $t_{\text{source}} = \pi\sigma\mu\delta^2$. Good conductors typically have $\sigma\mu$ $100\text{sec}/\text{m}^2$, thus, the minimum number of steps required to cover a period of the driving field would be $t_{\text{source}}/t_{\text{CFL}} \sim 1.78 \times 10^{10}/\sqrt{f}$, where f is the frequency of the driving field. Thus for a 1 MHz driving field, 10^7 steps would be required![3]

3 Hot Plate Simulation

The 3D hot plate simulation was carried out in COMSOL's *magnetic fields* solver. The simulation could only be carried out in the frequency domain, for reasons discussed in section 2. There is no analytic solution to compare the simulation result with, so we will simply check to see how much the current density profile changes with increasing mesh density. The circuit boundary condition was modeled as a simple voltage source for simplicity, since the plate geometry ultimately determines the fields. All of the capacitor connections were assumed to be one electrical connection, and the rail gap connection was assumed to be the other electrical connection.

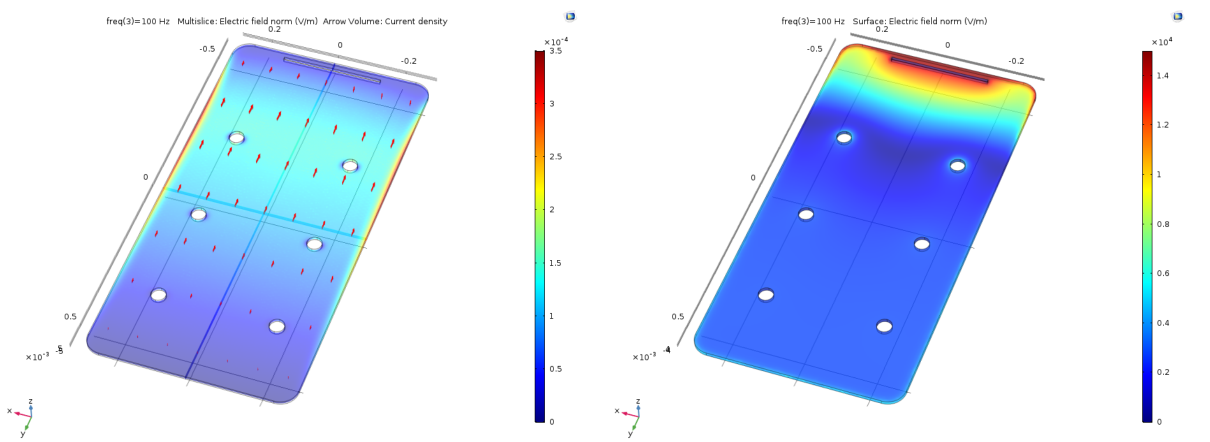

Figure 3: Both of these plots are $||\vec{E}||$ plots for a driving frequency of 100 Hz, because the skin depth was too small to simulate reliably at higher frequencies. On the left, the range of electric field magnitudes ranges from 0 to 3.5×10^{-4} Volts/meter, and is shown for a slice through the *interior* of the hot plate. On the right plot, *surface* electric field magnitudes are shown, and range from 0 to 15 kV/meter. Note that the field strength is very high in the corners near where the failure was seen in the bank.

The results for the simulation are shown in Figure 3. Two $||\vec{E}||$ plots are shown. The left-hand plot in Figure 3 was a slice taken through the interior of the hot plate, and the right-hand plot is the surface electric field magnitude. The interior electric field magnitudes are about seven orders of magnitude smaller than the surface electric field magnitudes. This is because of the large conductivity of conductors (5.998×10^7 S/m, in this case). Thus, the strongest electric fields are on the surface of the hot plate, and not the interior. We can also observe that the skin effect constrains

[3] These simulations ran in about 3 minutes on a late 2013 Mac Pro, 10^7 simulations taking 3 minutes each would mean almost a year of simulation time on my computer.

the current to flow on the outer edges of the hot plate - so if we are looking to reduce voltage stress on the edges of the conductors, we should make the radius of curvature of the edges as large as possible.

4 2D Benchmark Geometries

As is common in simulation, comparison with known analytical solutions is important to verify the performance of the simulation code. 3D analytical solutions might include a conducting sphere in an infinite, oscillating magnetic field – but no 3D problems with terminal connections were known to me (i.e. those that did not depend on infinite oscillating fields). In 2D, we use a slightly different form of the PDE used in the hot plate simulation, we do not use Ohm's law to replace the $\vec{E}$ field:

$$\begin{aligned} \nabla \times \vec{E} &= -\sigma\mu \frac{\partial \vec{H}}{\partial t} \\ \nabla \times \vec{H} &= \sigma \vec{E}. \end{aligned}$$

We then exploit the 2D symmetry to rewrite the PDE in terms of the z-component of the electric field:

$$\nabla^2 E_z = \sigma\mu \frac{\partial E_z}{\partial t}.$$

Then, for reasons explained in section 2, we assume sinusoidally varying fields to obtain a Helmholtz equation with complex coefficients:

$$\nabla^2 E_z = \mathrm{i}\,\sigma\omega\mu\, E_z. \tag{2}$$

4.1 Round Wire

The solution of equation 2 in polar coordinates is:

$$\boxed{\begin{aligned} E_z(r,t) &= \frac{I J_0(\kappa r)\exp(\mathrm{i}\,\omega t)}{2\pi\sigma \int_0^R J_0(\kappa r)\, r\,\mathrm{d}r} \\ \kappa &= \frac{1-\mathrm{i}}{\delta}. \end{aligned}}$$

Where I is the current magnitude in the wire. The circular wire used for the test case had a radius of 0.075 m, and had a conductivity of 5.998×10^7 S/m (i.e. copper). The current was assumed to be 1 A, and the frequency used in the simulation was 10 kHz. The simulation was performed in COMSOL's *magnetic fields* solver. Figure 4 shows the simulation results. Because the current flows in a shallow region of conductor near the outer boundary of the wire, an extremely fine mesh is needed in order to obtain convergence.

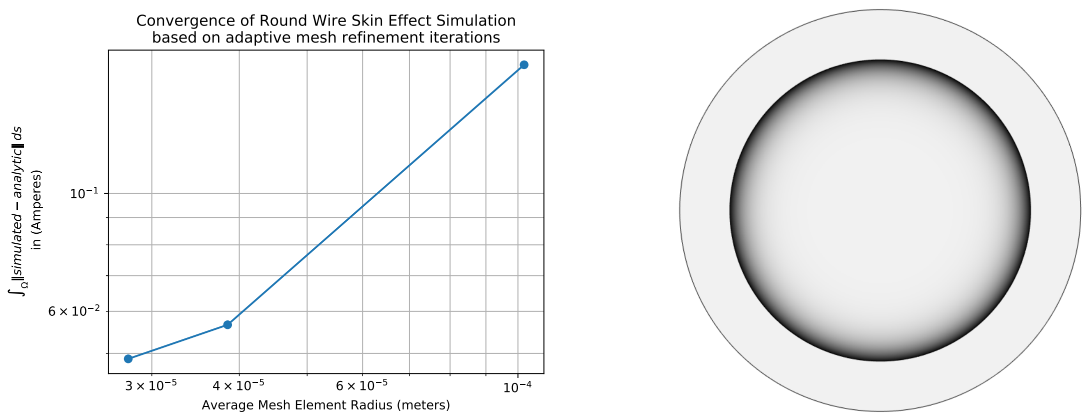

Figure 4: The left image shows the convergence for decreasing average mesh element radius. The right image shows the current density in the round wire of radius 0.075 m at 10 kHz (dark is higher current density). The annular region surrounding the dark ring is the air region around the wire, needed for the boundary condition. The mesh for the problem is not shown, because it requires such a high mesh density that the individual elements would not be visible in the image. (The average mesh element radius for good results is three orders of magnitude smaller than the radius.)

4.2 Rectangular Wire

The solution of equation 2 in Cartesian coordinates is:

$$\boxed{\begin{aligned} E_z(x,y,t) &= \frac{I\kappa_x\kappa_y}{4\sigma}\frac{\cosh(\kappa_x x)\cosh(\kappa_y y)}{\sinh(\kappa_x a)\sinh(\kappa_y b)}\exp(\mathrm{i}\,\omega t) \\ \kappa_x^2+\kappa_y^2 &= \mathrm{i}\,\sigma\omega\mu \\ \kappa_x\sinh(\kappa_x a) &= \kappa_y\sinh(\kappa_y b). \end{aligned}}$$

Where I is the current magnitude in the wire. κ_x and κ_y are spatial frequencies (and are complex numbers). This analysis is discussed in more detail in [4]. The rectangular wire used for the test case had a width, a = 0.01 m, and a height, b = 0.004 m, and had a conductivity of 5.998×10^7 S/m (i.e. copper). The current was assumed to be 1 A, and the simulation was run at 10 kHz. Like the round wire case above, this was performed with COMSOL's *magnetic fields* solver.

The problem with this simulation is that it won't converge to the analytic answer. The simulation over-estimates the current density in the centers of the edges of the wires. This problem does not go away with increasing element density or solver tolerance. This can be seen in figure 5, and shows how difficult it is to implement this boundary condition correctly in arbitrary geometries.

Figure 5: The left image is the COMSOL simulation of the rectangular wire problem. The analytic solution is shown on the right, and represents the formula derived in the box above. The current density on the bottom center and top center is too large in comparison to the analytic solution. The current on the center of the vertical sides is also too large.

5 Conclusion

Until the reasons for the arc flashovers are known, it is impossible to properly design flashover mitigating features into the bank. From the simulations, it would appear in Figure 3 that the most straightforward way of reducing the probability of arc flashover would be to increase the radius of curvature on the corners of the hot plates, and to increase the thickness of the mylar insulation between the hot and ground plates.

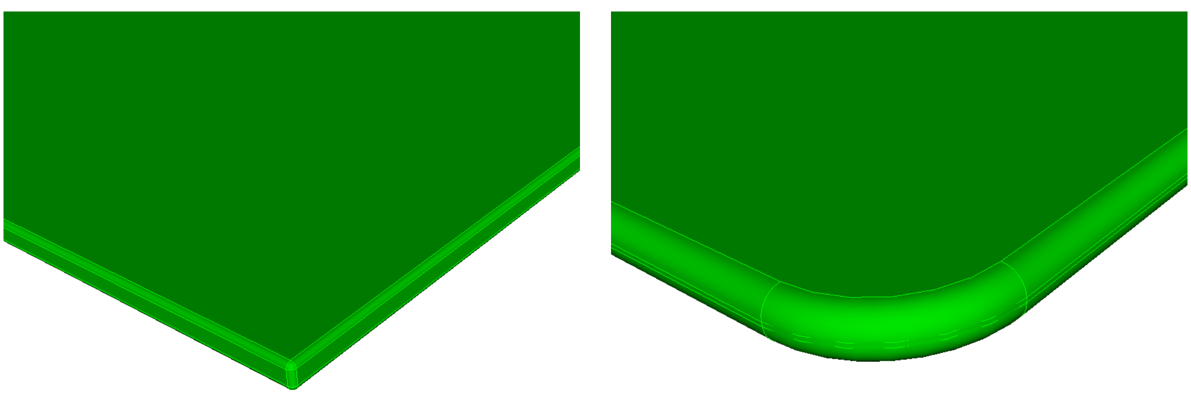

Figure 6: Some of the suggested design changes are shown in these two images. The left-hand image is the existing design. The right-hand image has a thicker plate, significantly increased chamfer radius on the edges, and a significantly increased fillet radius on the corners. All of these changes would lead to lower voltage gradients at the corners of the hot plate.

References

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[4] IEEE, *Approximate Analytical Calculation of the Skin Effect in Rectangular Wires*, 2009.