

# A Margin-Based Framework For Setting Product Specification Limits

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## *SUMMARY & CONCLUSIONS*

This document outlines a data-driven probabilistic approach to setting product acceptance testing limits. Product Specification (PS) limits are testing requirements for assuring that the product meets the product requirements. After identifying key manufacturing and performance parameters for acceptance testing, PS limits should be specified for these parameters, with the limits selected to assure that the unit will have a very high likelihood of meeting product requirements (barring any quality defects that would not be detected in acceptance testing).

Because the settings for which the product requirements must be met is typically broader than the production acceptance testing space, PS limits should account for the difference between the acceptance testing setting relative to the worst-case setting. We propose an approach to setting PS limits that is based on demonstrating margin to the product requirement in the worst-case setting in which the requirement must be met. PS limits are then determined by considering the overall margin and uncertainty associated with a component requirement and then balancing this margin and uncertainty between the designer and producer. Specifically, after identifying parameters critical to component performance, we propose setting PS limits using a three step procedure:

1. Specify the acceptance testing and worst-case use-settings, the performance characteristic distributions in these two settings, and the mapping between these distributions.
2. Determine the PS limit in the worst-case use-setting by considering margin to the requirement and additional (epistemic) uncertainties. This step controls designer risk, namely the risk of producing product that violates requirements.
3. Define the PS limit for product acceptance testing by transforming the PS limit from the worst-case setting to the acceptance testing setting using the mapping between these distributions. Following this step, the producer risk is quantified by estimating the product scrap rate based on the projected acceptance testing distribution.

The approach proposed here provides a framework for documenting the procedure and assumptions used to determine PS limits. This transparency in procedure will help inform what actions should occur when a unit violates a PS limit and how limits should change over time.

## *1 INTRODUCTION*

Nuclear weapon (NW) components must meet requirements across a range of use-settings, including different inputs (e.g., electrical and acceleration) as well as different environments (e.g., thermal and mechanical). Production test plans comprise a suite of tests designed to assure that the product is reliable and of high quality across these use-settings. For components that can be tested non-destructively, these production tests can include environments and destructive (E- and D-) testing on a sample of units as well as 100% functional acceptance testing, often at nominal settings. E- and D-testing provides information about how components perform across different settings and can be used to detect quality defects that only manifest in harsh use-settings. Testing at nominal settings does not provide information about such quality defects, but is useful for establishing component functionality as well as assuring margin for key performance characteristics is sufficiently high. This document outlines an approach to setting acceptance testing limits for production test plans using engineering judgment and data to assure margin to requirements across use-settings.

The Product Specification (PS) is a document used to detail how production testing will show that component-level requirements are met.

Product Specification (PS) limits are requirements for component acceptance testing delegated to production facilities from the design agency. If an item has a measured performance characteristic that violates a PS limit, then the item is considered out of specification and in violation of the product quality standards. At the component level, this violation results in the unit being flagged for closer inspection, the production agency contacting the design agency, the potential examination or revision of the testing or production process, the potential scrapping or re-working of the component, and/or a special exemption release authorization from the design agency. Naturally, violations of PS limits at the component-level delay the testing and production process but are key to identifying quality issues. At the sub-component level, items such as commercial off-the-shelf (COTS) parts, piece-parts, and sub-component assemblies in violation of the PS limits are typically scrapped. Hence, while PS limits are critical to establishing quality and functionality

of the manufactured product, PS limits are also critical to product yield, and thus must be set to balance quality and producibility.

PS limits for performance characteristics should be set to provide assurance that components will have margin to requirements. NW components have requirements that are put in place to assure safe and reliable performance of the component. These requirements must be met for all units and settings experienced throughout the operational life of the component. Violations of requirements are typically considered component failures. Production acceptance testing is intended to show functionality of the manufactured product and typically encompasses a narrow range of use-settings. Subsequently, the 'worst-case setting' for which the requirements must be met typically differs from the narrower production acceptance testing space, referred to as the 'acceptance testing setting.' Therefore, determination of PS limits should account for the lack of conservatism in the acceptance testing setting relative to the worst-case setting.

## *2 STEP 1: SPECIFY THE ACCEPTANCE TESTING AND WORST-CASE PERFORMANCE CHARACTERISTIC DISTRIBUTIONS AND THE MAPPING BETWEEN THESE DISTRIBUTIONS.*

Prior to development of production test plans, including PS limit specification, performance characteristics that are critical to evaluating component performance should be identified, on which to collect data. PS limits are only useful insofar as this set of performance characteristics is integral to product quality and potential failure modes and mechanisms.

Once these critical performance characteristics are identified, the first step in setting PS limits is identifying the acceptance testing and worst-case use-settings and specifying the performance characteristic distribution in these settings as well as the mapping between these distributions. Variability in component performance is largely driven by unit-to-unit variability and use-settings (inputs and conditions). Understanding the range of component performance across these sources of variability is key to quantification of margins and uncertainty and subsequently understanding the risk of margin failures. For each possible use-setting, component performance may vary between units; thus, there is an associated distribution of a performance characteristic across units for each use-setting. For specifying PS limits, we are concerned with two specific use-settings: the acceptance testing setting and the worst-case setting. Therefore, the first step in setting PS limits is identifying the acceptance-testing and worst-case use-settings and then estimate the joint distribution across these settings.

### *2.1 Identify the acceptance testing and worst-case use-settings.*

To define the acceptance testing and worst case use-

settings, it is key to understand the range of use-settings within the requirement space, defined as the set of all inputs and conditions in which the component must meet requirements. The inputs and conditions that comprise the use-settings are defined as [1]:

*Inputs:* The stimuli required for the component to operate as intended; types of inputs include environmental (e.g., acceleration profiles) and electrical (e.g., load profiles).

*Conditions:* The settings under which the component is required to operate as intended; types of conditions include lifetime (service life, operational life) and environmental (electrical, mechanical, thermal).

Inputs and conditions can typically take on a range of values within the requirement space that can impact component performance. Identifying the impact of these use-settings on component performance is key to predicting differences between the acceptance testing and worst-case setting in which the component is required to operate (Figure 1). The use-settings reflecting the worst-case setting may differ between performance characteristics, and this process must be repeated for each performance characteristic. Additionally, these settings may differ for performance characteristics with both upper and lower requirements; that is, the worst-case setting for the lower requirement may differ from the upper requirement.

We use probability distributions to characterize uncertainties in component performance. There are two types of uncertainty to consider: aleatory and epistemic. Aleatory uncertainty is irreducible uncertainty. Examples of aleatory uncertainty in component production include: experimental variability (measurement uncertainty), between unit (unit to unit) variability due to tolerance stack-ups, and within unit (run-to-run) variability. Epistemic uncertainty is reducible uncertainty that arises due to knowledge gaps. Examples of epistemic uncertainty in component production include: limited data (i.e., sampling uncertainty), extrapolation from development to production, unknown requirements, unknown effects of use-settings on component performance, limited knowledge of worst-case use-settings, and poorly understood aleatory uncertainties.

We define the acceptance and worst-case distributions for a performance characteristic as follows (Figure 2):

*Acceptance testing distribution:* the performance characteristic distribution across units considering aleatory uncertainty and conditioning on the use-settings specified for production acceptance testing.

*Worst-case distribution:* the performance characteristic distribution across units considering aleatory uncertainty and conditioning on the use-settings specified for the worst-case setting for which the requirement must be met across the component's life. The use-settings for this worst-case scenario should be selected as the setting in which the component is at highest risk of violating the requirement.

These distributions are conditional on use-settings, and the stochasticity associated with these probability distributions arises from unit-to-unit and other sources of

aleatory uncertainty at these use-settings.

Enumerating and considering the magnitude of

aleatory and epistemic uncertainties is key to mapping sources of uncertainty to the risk of requirement violation.

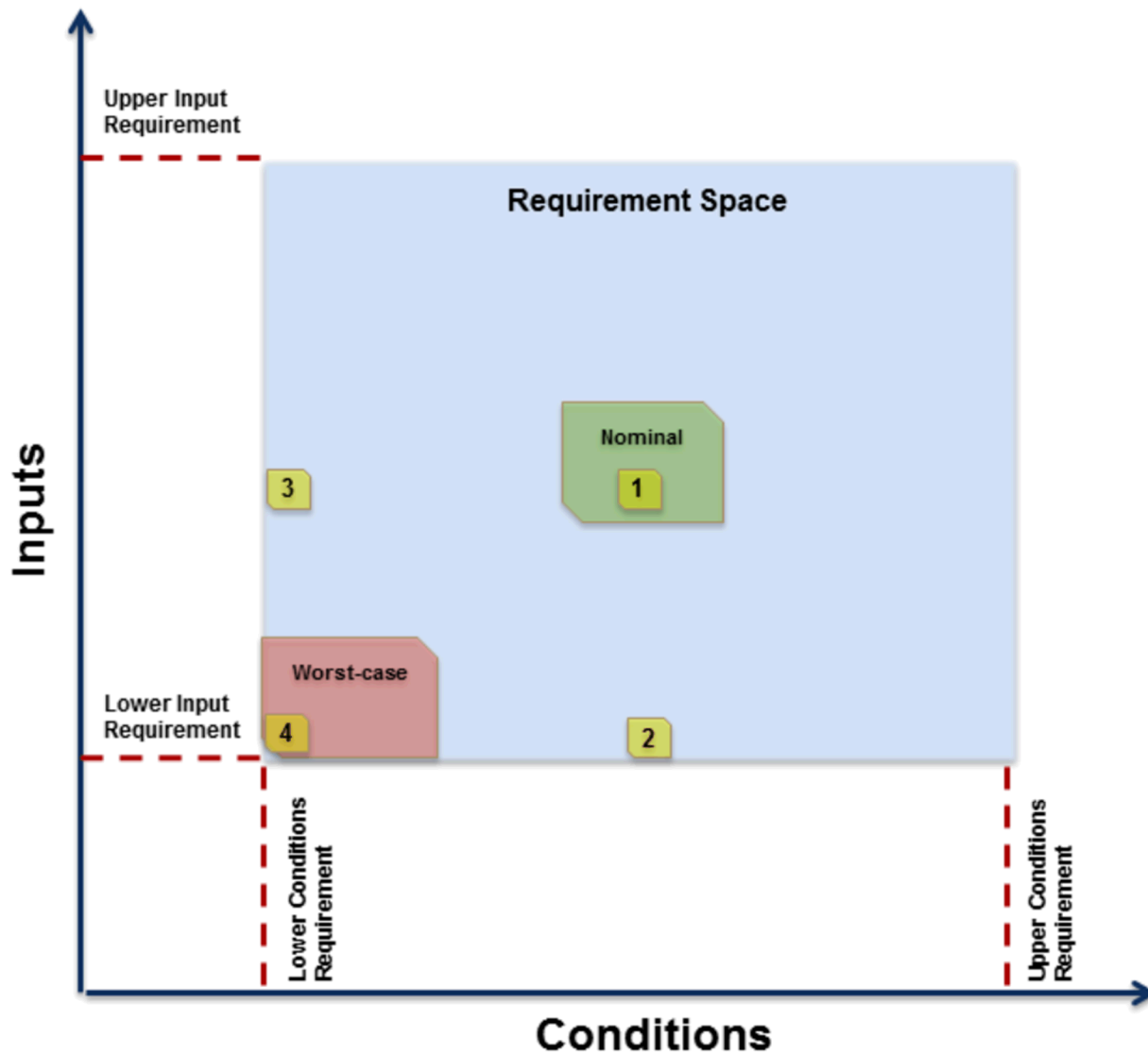


Figure 1- Example acceptance testing settings in relation to the worst-case setting. The requirement space is high-dimensional in practice, but is visualized in two-dimensions for simplicity. The green box depicts nominal use-settings, set at the center of the requirement space. The red box depicts the worst-case use setting, which, in this example, occurs at the lowest inputs and conditions (the worst-case will vary by component and by requirement). The yellow boxes denote different examples of acceptance testing settings: (1) at nominal, far from the worst-case setting; (2) at the lowest input but at nominal conditions; (3) at the lowest conditions but nominal input; and (4) at the worst case setting.

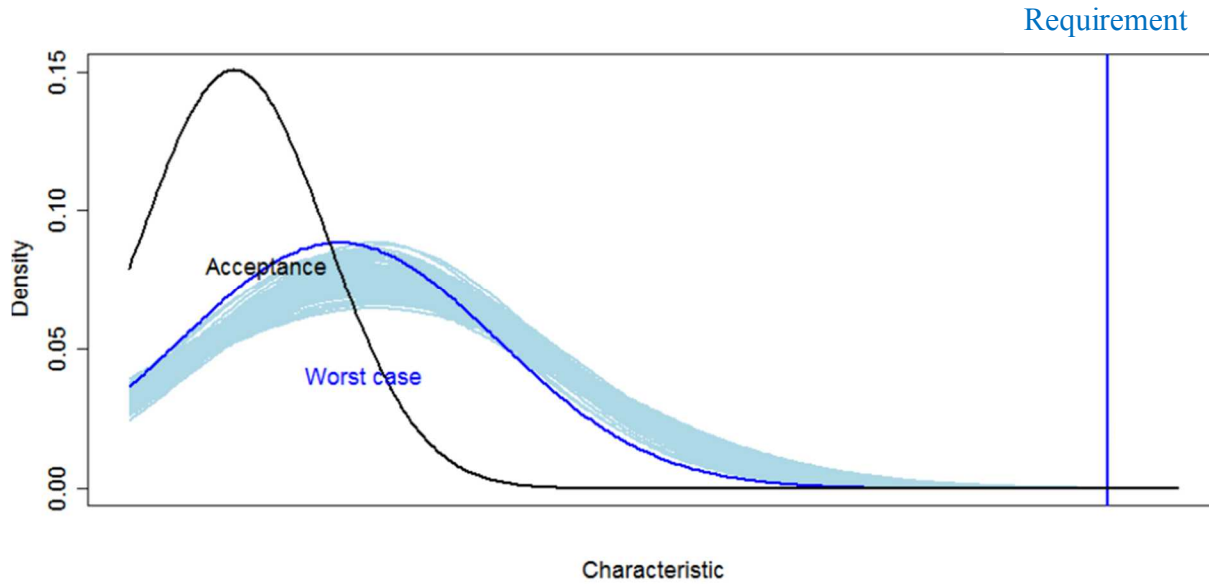
Our proposed approach for PS limit specification requires specification of the worst-case and acceptance testing use-settings along with potential sources of epistemic uncertainty in performance at these use-settings (aleatory uncertainty should be incorporated into the estimates of the acceptance testing and worst-case distributions). If there is not enough data or engineering judgment to identify the worst-case use-setting, then this procedure for setting PS limits

cannot be applied.

## 2.2 Mapping.

Given the acceptance testing distribution, we can generate the worst-case distribution by specifying the mapping from acceptance testing to the worst-case setting. In general, if either the acceptance or worst-case distribution is known and the mapping is defined, then the other distribution can be

determined in simulation or using closed form expressions for case distributions, the mapping will often be defined using



transformations of random variables. Different types of mappings are possible, including: deterministic/stochastic and linear/non-linear. As with the acceptance testing and worst-

stochastic simulation from a validated statistical or physics model.

*Figure 2 - Acceptance and worst-case distributions. The black and blue lines are the acceptance testing and worst-case distributions, which encompass the aleatory uncertainty in the performance characteristic; the light blue lines depict the epistemic uncertainty in the worst-case setting. The PS limits truncate both the acceptance testing and worst-case distributions to ideally prevent failing the requirement in the presence of aleatory and epistemic uncertainty.*

We assume that acceptance testing has been designed such that there is a relationship between passing acceptance testing and the component meeting a requirement in the worst-case setting for which the requirement must be met. That is, we must be able to map from performance in the worst-case setting to the acceptance testing settings; for this to be feasible, components that perform poorly in acceptance testing should also perform poorly in the worst-case setting on average. If it is not possible to relate performance in acceptance testing to such a performance threshold setting, then this margin-based probabilistic approach for setting PS limits should not be used. Instead, approaches based strictly on controlling production variability or based on engineering judgment are more applicable.

Another key assumption of the method is that performance characteristic variability in production is understood and controlled. An example of violating this assumption is an out of control production process. The acceptance testing distribution, worst-case distribution, and mapping between the distributions are not well-defined with an out of control production process, because these distributions change across production. Changes from development to production could also result in changes in the performance characteristic distributions and mapping. It is critical to assess changes in component performance over development and production and update the distributions and

mapping as needed.

### 3 DETERMINE THE WORST-CASE PS LIMIT BY ASSESSING DESIGNER RISK.

The designer takes on more risk as the PS limits approach the requirements. We introduce the notion of designer margin associated with PS limits to discuss this risk. It is denoted below as  $M_d$ .

The first step in this analysis is calculating a percentile that represents an effective upper/lower bound on the aleatory uncertainty in the worst-case setting, i.e., an extreme percentile on the worst-case distribution. Specifically,  $Q_w$  is the percentile for the worst-case distribution.

We define  $PS_w$  as a hypothetical PS limit associated with the worst-case performance characteristic distribution, reflecting what the PS limit would be if acceptance testing reflected the worst-case setting. Given  $PS_w$ , there are two notions of margin to consider when determining whether the PS limits are sufficiently conservative to assure requirements will be met: overall margin and designer margin (Figure 3).

The designer margin should be selected to assure that the requirement is met in the presence of epistemic uncertainty. Subsequently,  $PS_w$  can be defined as an upper bound for the worst-case distribution accounting for all

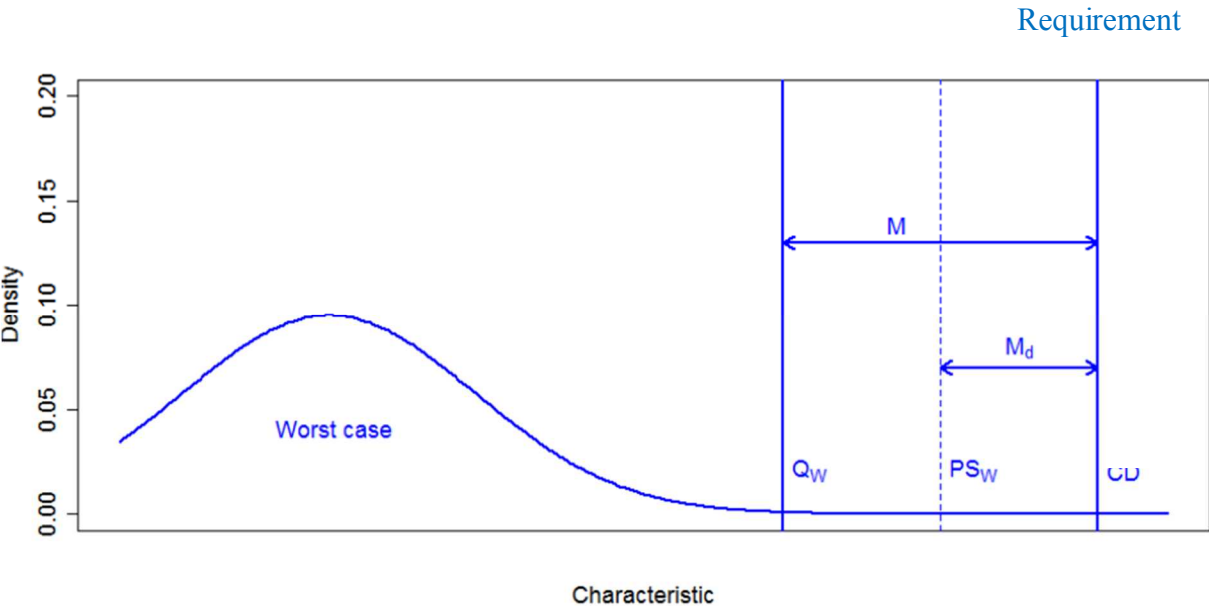
sources of epistemic and aleatory uncertainty. Given  $Q_W$ , selection of  $PS_W$  depends on how much epistemic uncertainty remains in the component performance in the worst-case setting.  $PS_W$  should always be selected such that  $M_d > 0$ . If  $M_d = 0$ , i.e.,  $PS_W = \text{requirement}$ , the production agency is allowed to produce at the requirement, and epistemic uncertainty could result in defective units passing acceptance testing.

4 TRANSFORM THE PS LIMIT FROM THE WORST-CASE SETTING TO THE ACCEPTANCE TESTING SETTING.

To calculate the final PS limits for production,  $PS_W$  (the hypothetical PS limit for the worst-case distribution) is transformed to the acceptance testing setting using the previously defined mapping. We use ‘A’ subscripts to distinguish quantities associated with the acceptance testing distributions, such that  $PS_A$  is the final acceptance testing PS limit.

To map  $PS_W$  to  $PS_A$  (Figure 4), note that PS limits control what percentage of units are flagged as out of

Figure 3 - Overall margin  $M$  is the distance between the estimated percentile  $Q_W$  and the requirement. Designer margin  $M_d$  should be sufficiently large to assure that epistemic uncertainty in the worst-case distribution will not result in a component failure.



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specification and therefore correspond to distribution percentiles. With a deterministic model, there is a 1-1 mapping between percentiles of the acceptance testing and worst-case performance characteristic distributions. As an example, if we select  $PS_W$  to correspond to the 99.95<sup>th</sup> percentile of the worst-case distribution (flagging .005% of units), then  $PS_A$  would correspond to the 99.95<sup>th</sup> percentile of the acceptance testing distribution (again flagging .005% of units).

The rigor that goes into the process of selecting PS limits should be a function of the confidence that the component will meet the requirement, and subsequently is a function of margin, uncertainty, and robustness to use settings. If margin is high, uncertainty is low, and the component is robust to use-settings, then acceptance testing will be a good marker of component performance and PS limit specification will require less rigor. If margin is low, PS limits must be selected with precision to assure the component will meet requirements and is producible. If uncertainty is high, selecting PS limits with precision is challenging.

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BIOGRAPHIES

Name1  
Address 1  
Address 2

Address 3  
City, State or Province, Postal Code, Country  
e-mail:  
Text.  
Name2  
Address 1  
Address 2  
Address 3  
City, State or Province, Postal Code, Country  
e-mail:  
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Name3

Address 1  
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Requirement

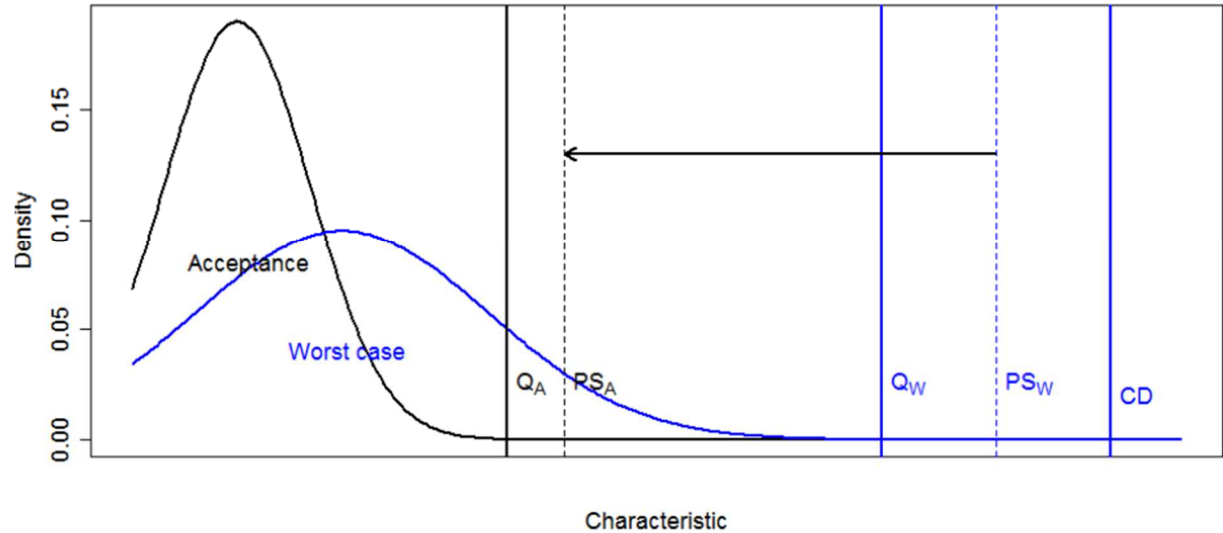


Figure 4 – Shift from worst-case to acceptance testing distribution.