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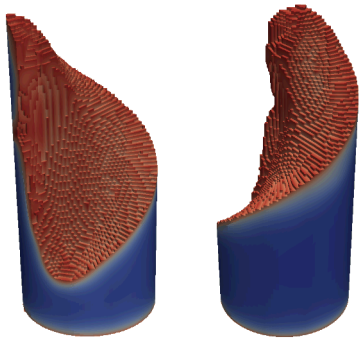

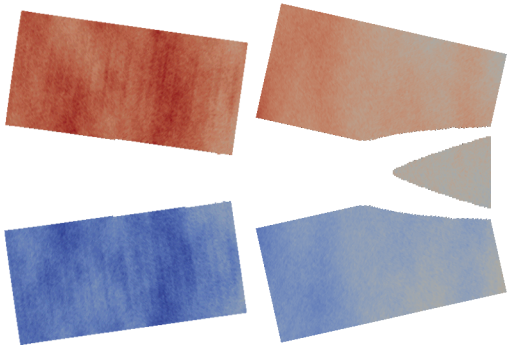

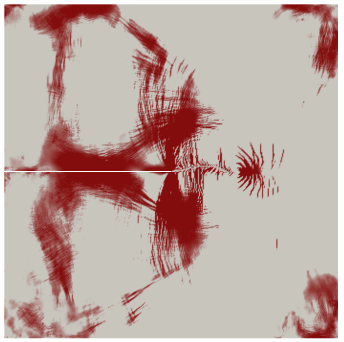

Inverse methods for fracture based on a parabolic regularization of brittle cohesive laws

Michael Tupek

U.S. DEPARTMENT OF ENERGY

National Nuclear Security Administration

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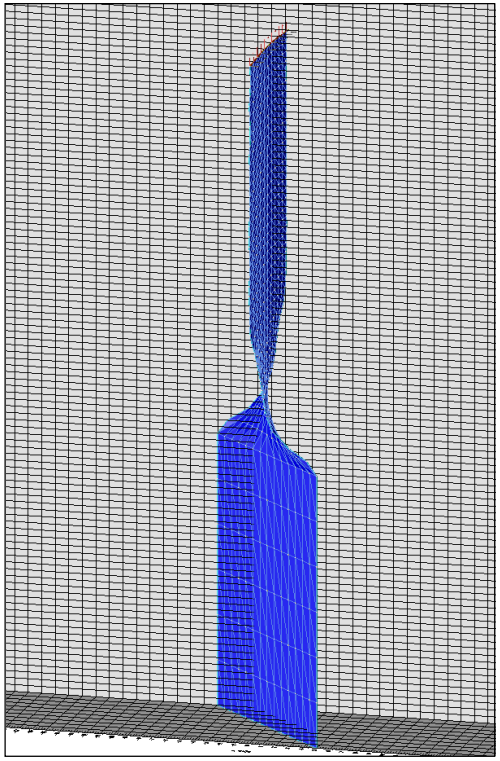
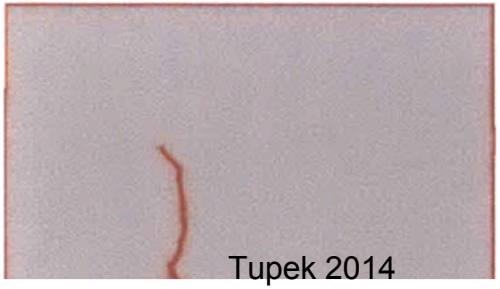
Fracture Renaissance

- Several promising fracture methods have emerged in recent years



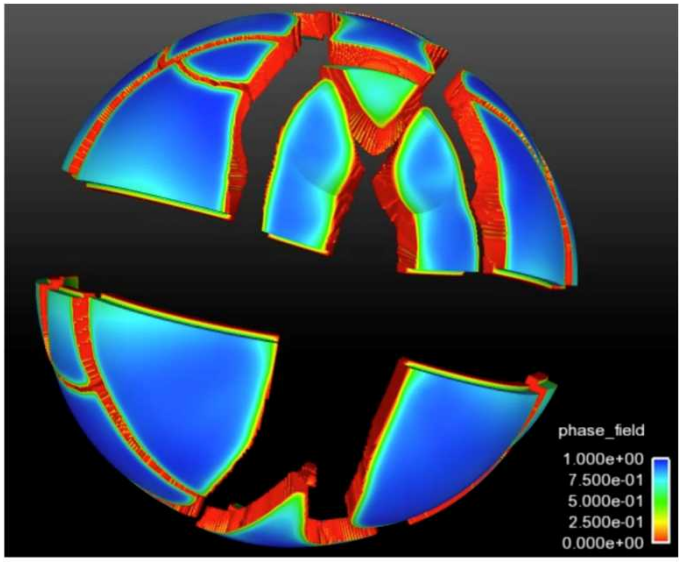
Duarte 2011

GFEM / XFEM



Tupek 2014

Peridynamics



Dolbow, Stevens 2015

Phase field

Inverse methods for fracture

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- Most fracture method focus on predictions
- This remains a significant challenge!
- However, predictions by themselves aren't that interesting

"Computers are useless, they can only give you answers."

-Pablo Picasso

- Predictive simulations should guide decision making

inverse = predictions → decisions

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Forward Problem:

Phase Field Fracture

Classical damage models: ill-posed

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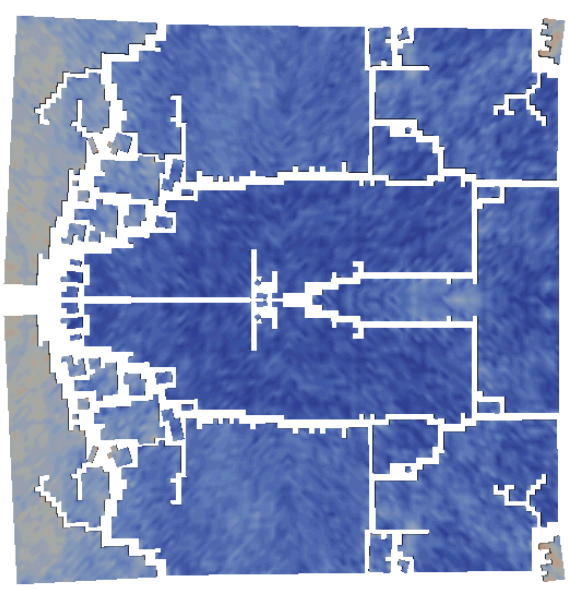
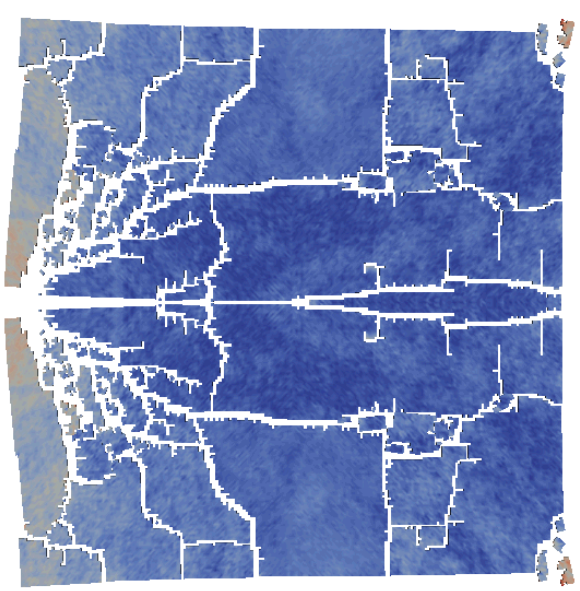
- Fracture modeling is critical to predicting the performance and reliability of many Sandia components and systems
- Many fracture models used at Sandia (and elsewhere) are ill-posed and non-convergent

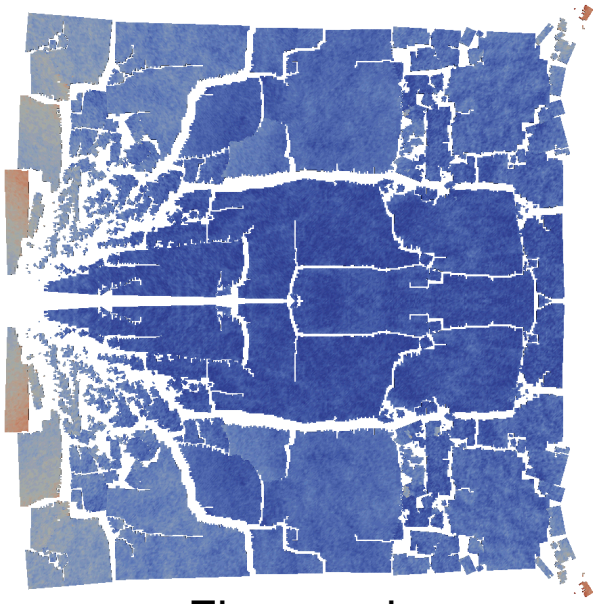
Max principal stress criterion with element death shows significant mesh dependence



Coarse mesh

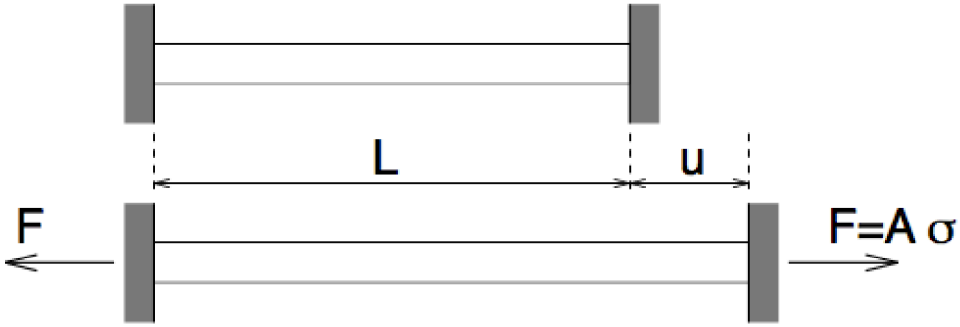


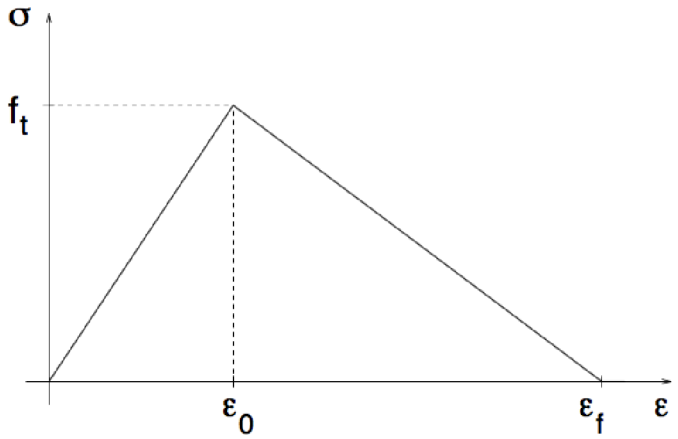
Medium mesh

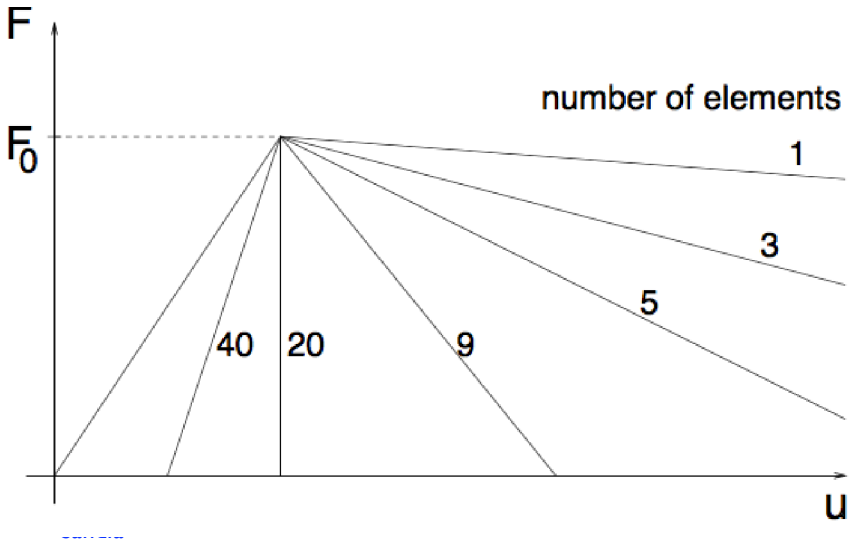


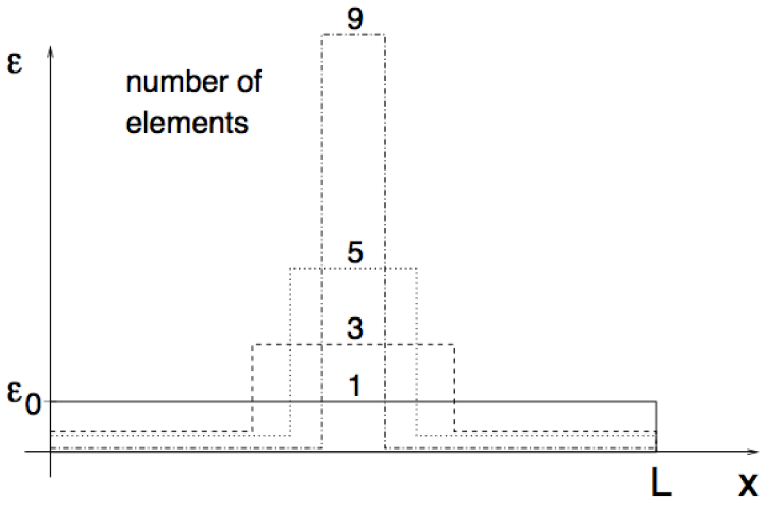
Fine mesh

Classical damage models: non-convergent









Phase field fracture

Standard potential energy:

$$_{pot}(\mathrm{u}, \Gamma) = \int_{\Omega} {}_{e}(r\ {}^{s}\mathrm{u})\, dv + \int_{\Gamma} G_c\, da$$

Approximate surface integral by a volume integral

Elliptically regularized potential energy:

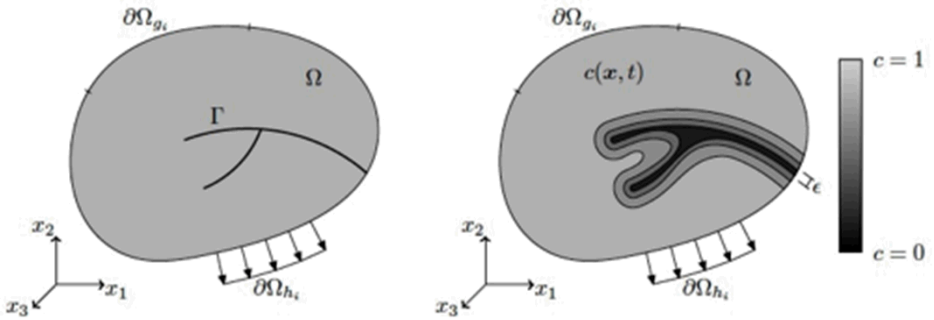
$$_{l}(\mathrm{u}, d) = \int_{\Omega} \left(g(d)\ {}^{+}_{e}(r\ {}^{s}\mathrm{u}) + {}^{-}_{e}(r\ {}^{s}\mathrm{u})\right) dv + \int_{\Omega} G_c \gamma_l\, da$$

Common crack density:

$$\gamma_l = \frac{1}{2l} d^2 + \frac{l}{2} |\nabla d|^2$$

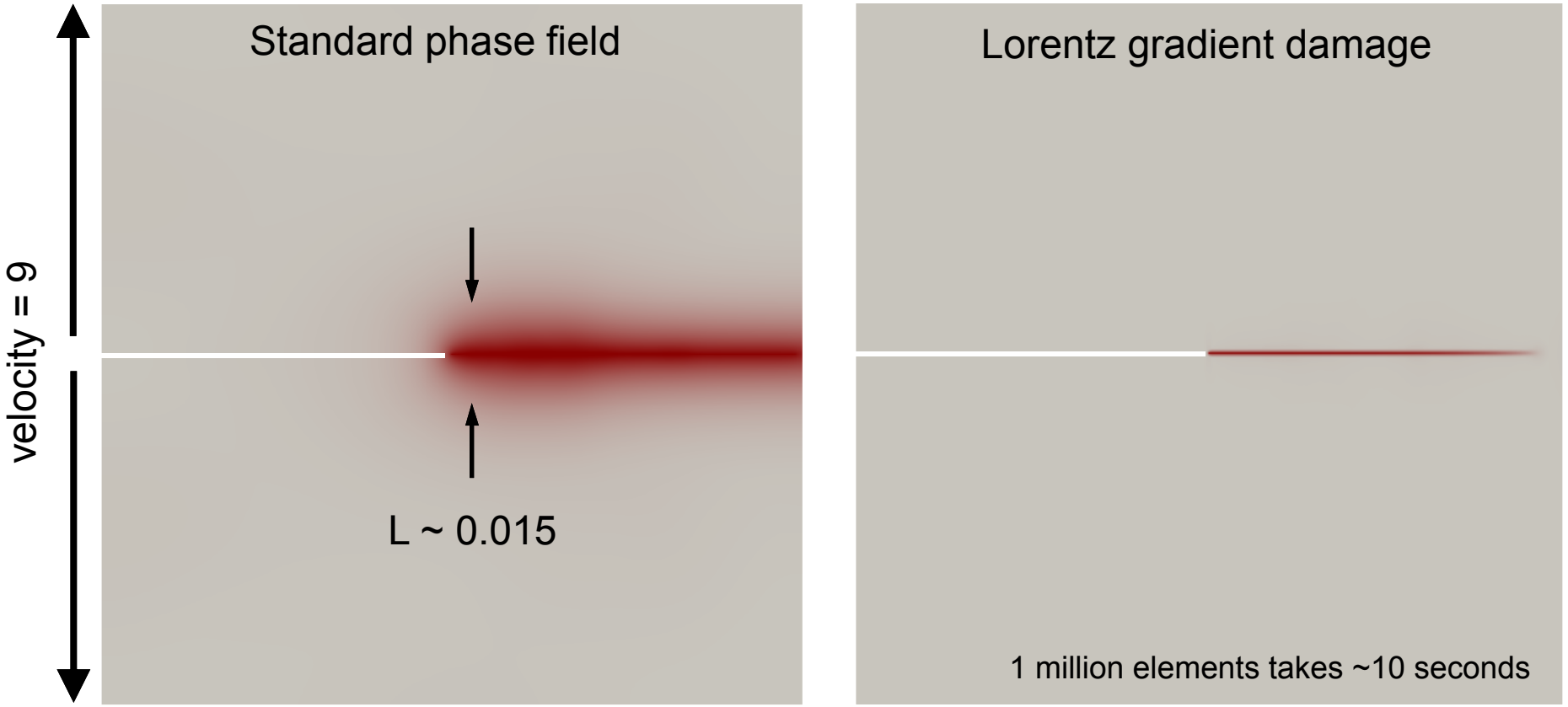
Stress degradation:

$$g = (1 - d)^2$$



Limitations to standard phase field

- No threshold, some damage for any $\varepsilon \neq 0$
- Regularization length scale tied directly to fracture properties $\sigma_{\max} = \frac{9}{16}\sqrt{\frac{EG_c}{6L}}$



$$\sigma_{\max} = 250\text{e}6 \quad G_c = 100e3 \quad E = 200\text{e}9$$

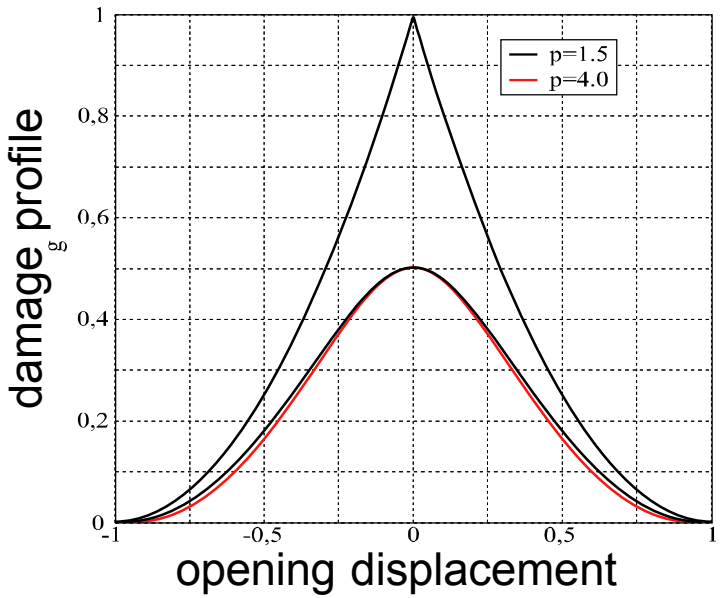
Cohesive gradient damage model

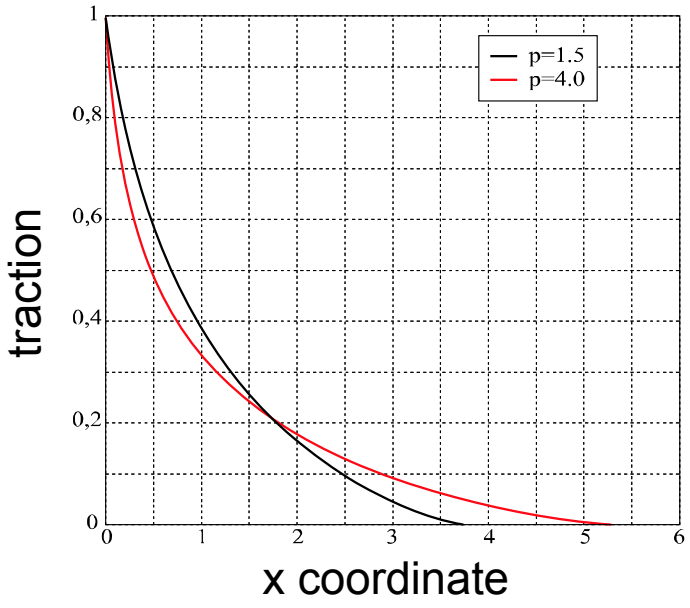
- Regularized gradient damage model by Lorentz, et al. 2011
- Shown to converge to cohesive zone model as $L \to 0$

$$\rho\ddot{\mathbf{u}} = \nabla \cdot \boldsymbol{\sigma}, \qquad \boldsymbol{\sigma} = g(\phi)\frac{\partial \psi_e^+}{\partial \boldsymbol{\varepsilon}} + \frac{\partial \psi_e^-}{\partial \boldsymbol{\varepsilon}}$$ Momentum balance

$$g'(\phi)\psi_e^+ + k = c\nabla^2\phi$$ Phase field equation

$$g(\phi) = \frac{(1-\phi)^2}{1 + (m-2)\phi + (1+pm)\phi^2}$$ Derivable from a phase potential





Parameters of damage model

Depends on cohesive fracture parameters:

$$k = \frac{3}{4}\frac{G_c}{L} \quad c = \frac{3}{8}LG_c \quad m = \frac{3}{2}\frac{EG_c}{\sigma_c^2 L}$$

Note: the fracture energy, G_c, is ignored in most damage models

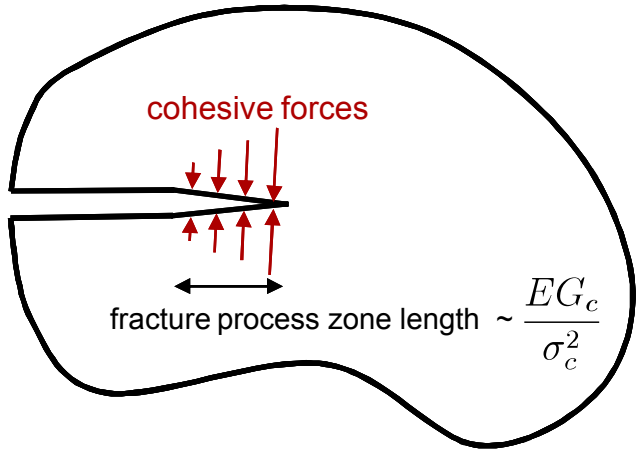
Physically justified constraints on the parameters ensures

- Regularization length scale resolves fracture process zone

$$L < \frac{3}{2(p+2)}\frac{EG_c}{\sigma_c^2}$$

- Monotonic stress decay

$$p \geq 1$$



Parabolic regularization

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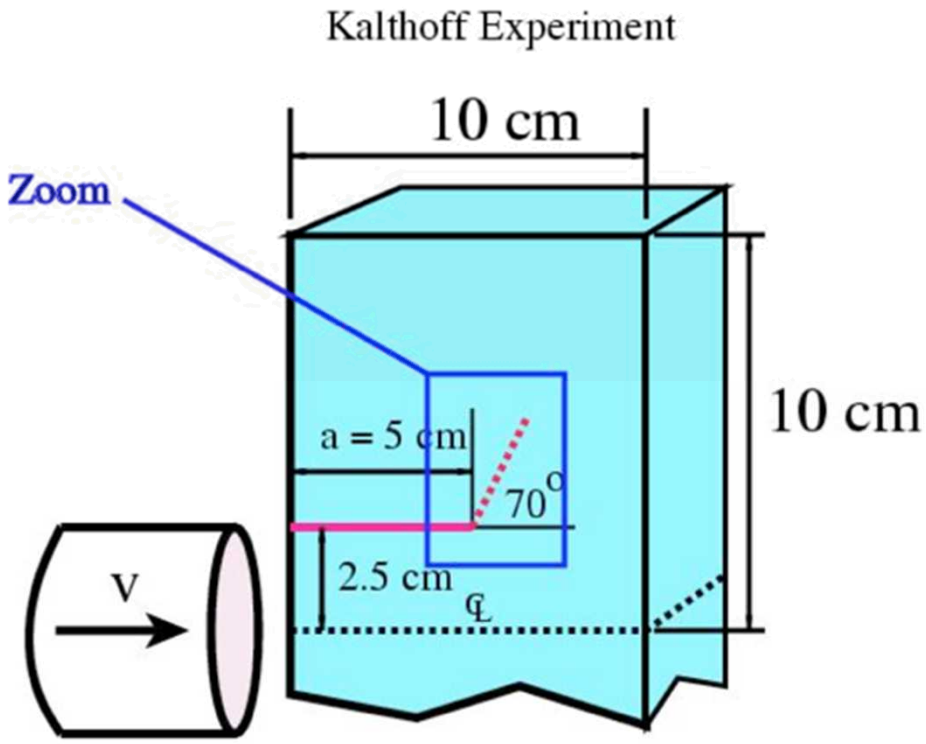
- For explicitly integrated dynamic problems, solving a nonlinear phase field equation is VERY expensive
- Add a viscous regularization (e.g., Miehe 2010)

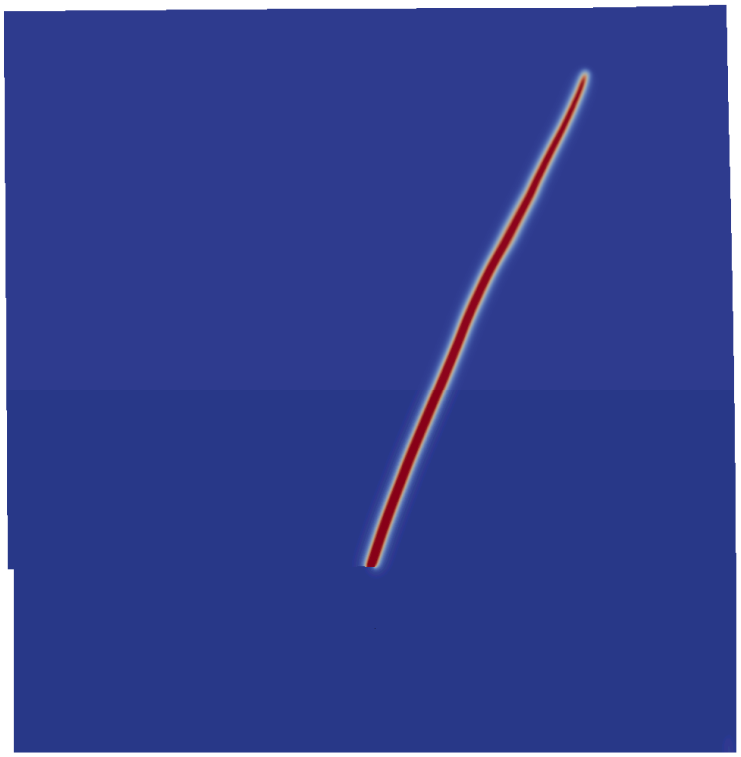
$$\eta\dot{\phi} = -g'(\phi)\psi_e^+ - k + c\nabla^2\phi, \quad \dot{\phi} \geq 0$$

- Integrate parabolic equation explicitly!
- Hyperbolic timestep constraint: $\Delta t \leq \frac{1}{s}\Delta x$
- Parabolic timestep constraint: $\Delta t \leq \frac{\eta}{c}(\Delta x)^2$
- Can use hyperbolic timestep if: $\eta \geq \frac{c}{s\Delta x} = \frac{3}{8}\frac{G_c}{s}\frac{L}{\Delta x}$

Kalthoff validation



Damage contour

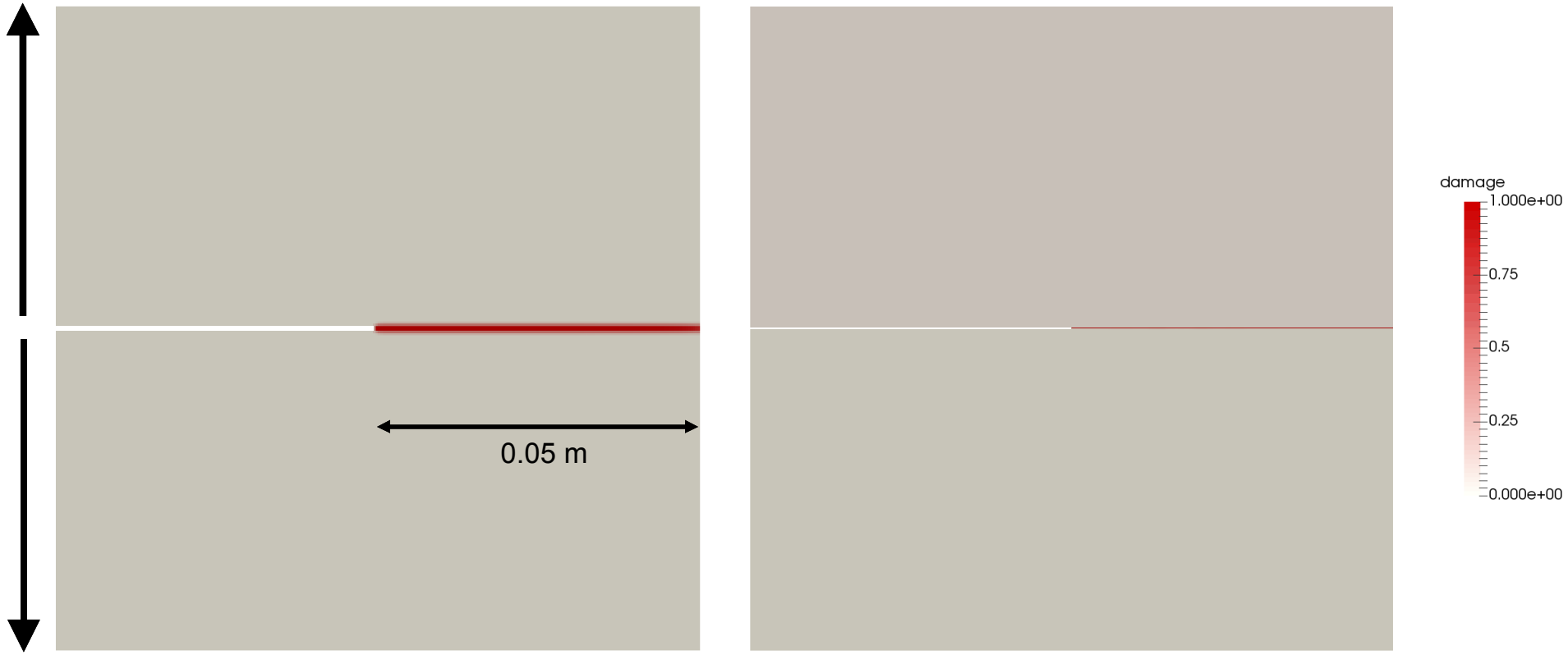


Achieves the expected crack propagation angle: ~70°

Mode-I: Length scale refinement

Length-scale decreases with Δx

v = 25 m/s



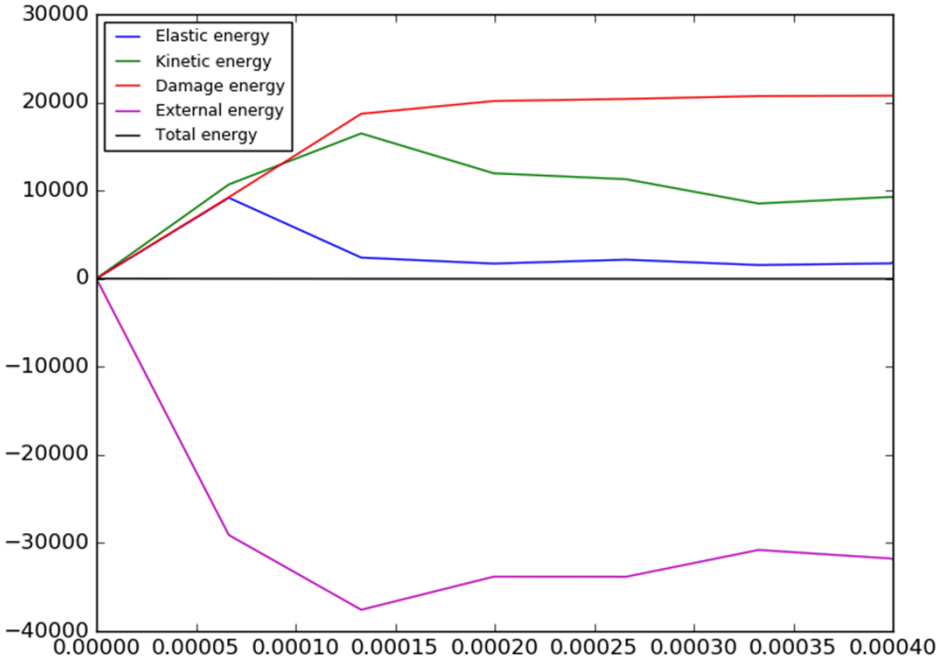
71.5 thousand elements

14.4 million elements

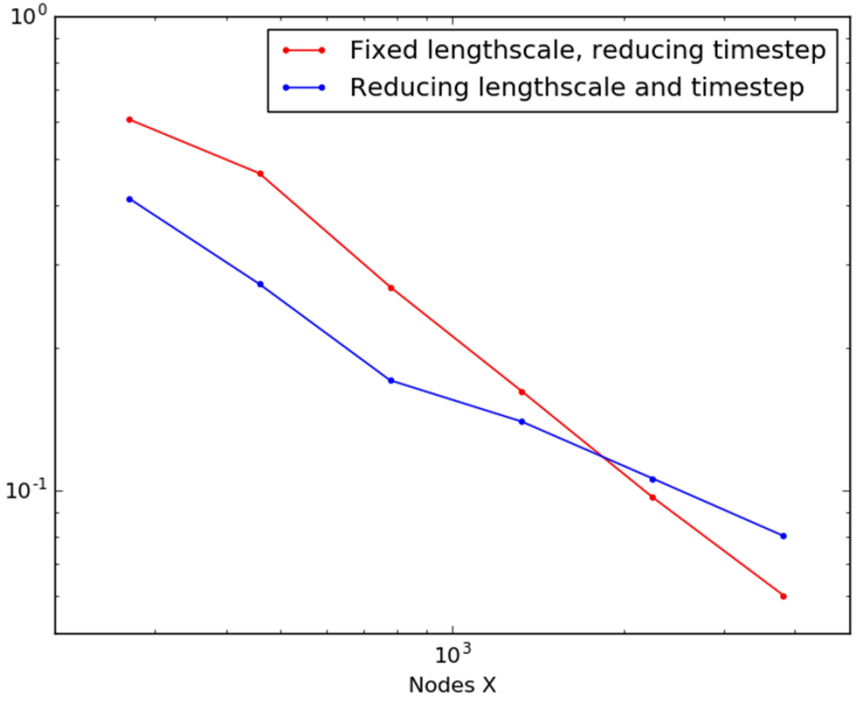
G_c = 4e5

Accuracy

Total numerical energy is preserved



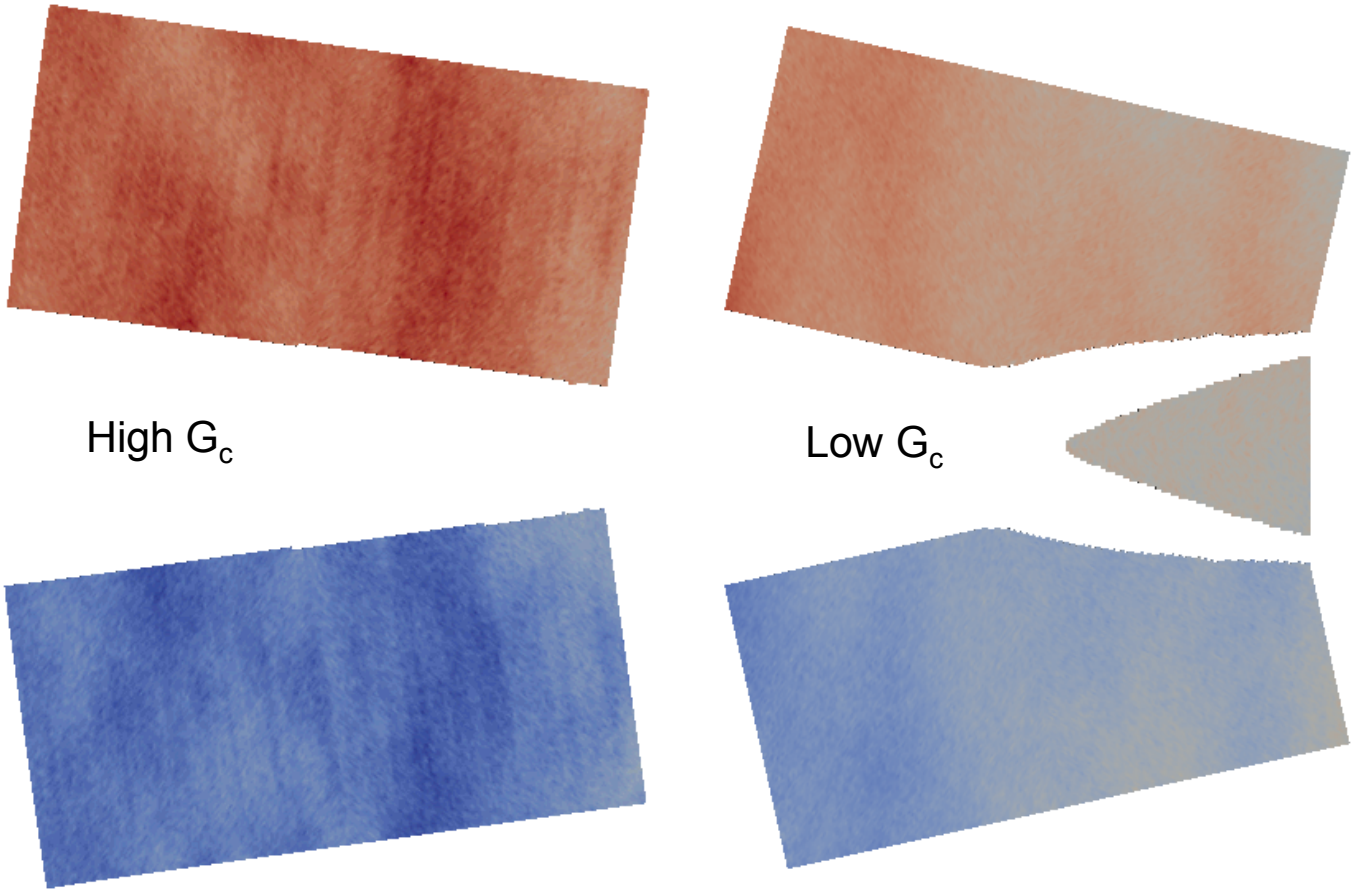
Dissipated fracture energy error



- Convergence rate of ~1 for fixed length scale
- Convergence rate of ~0.5 otherwise
- Dissipation due to viscous regularization is non-negligible

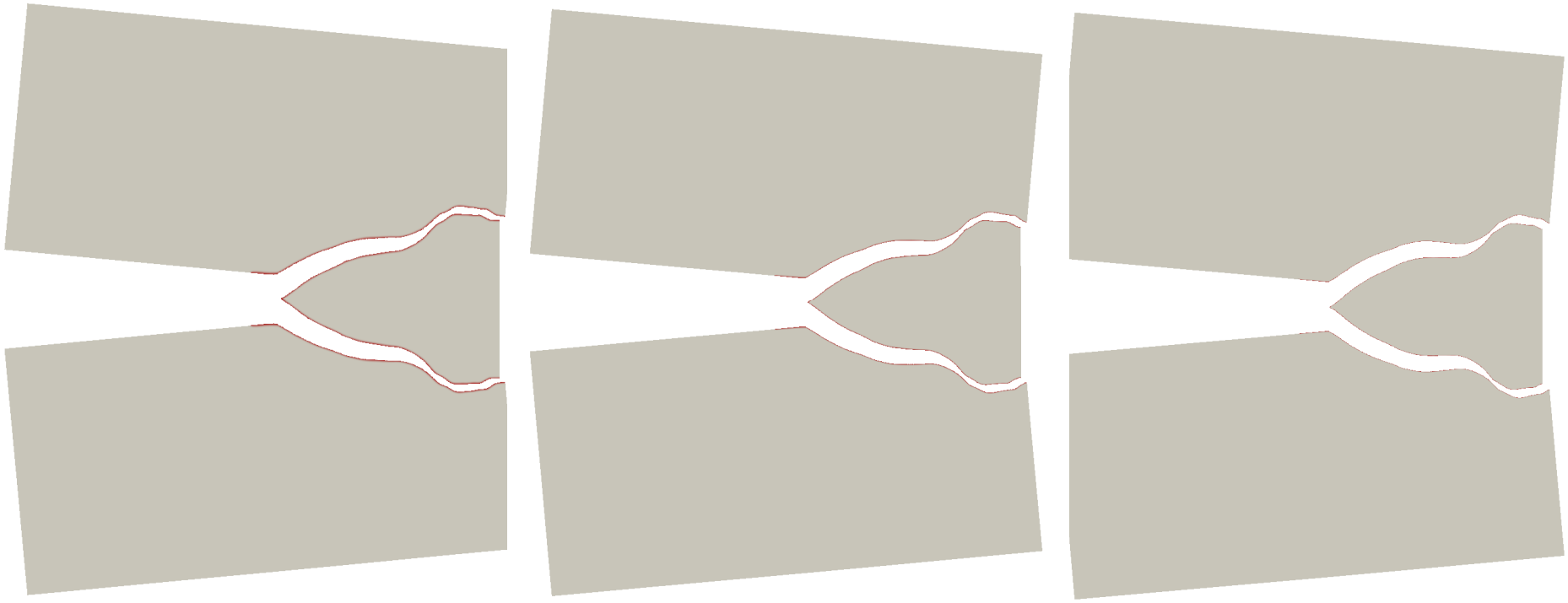
Importance of fracture energy

Mode-I crack transition to branching



Branching is emergent, not prescriptive

Branching: mesh insensitivity



1.4 million elements

4.1 million elements

11.8 million elements

Runs in ~5 hours on 1 GPU

Thermally loaded glass-metal interface

Experimentally observed crack path

1.5 million element simulation runs in 6 hours on 1 GPU

- Loaded by temperature drop
- Metal on top contracts relative to glass below
- Initial vertical crack propagates upward
- Eventually turns to run parallel to the glass-metal interface

Demonstrations in 3D

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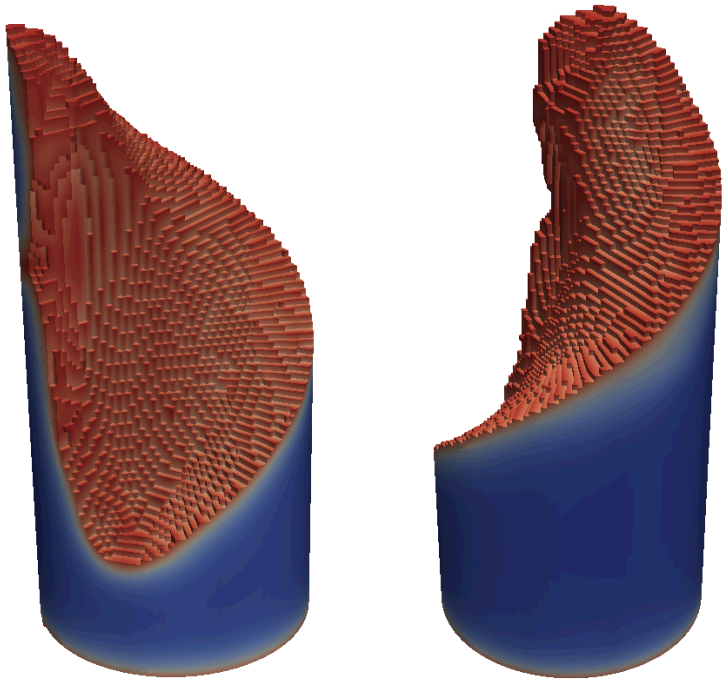
Ceramic impact



Brittle torsion fracture

- Captures complex 3D crack patterns
- No explicit geometry representation
- Naturally predicts initiation, branching and coalescence

Brittle impact demonstration

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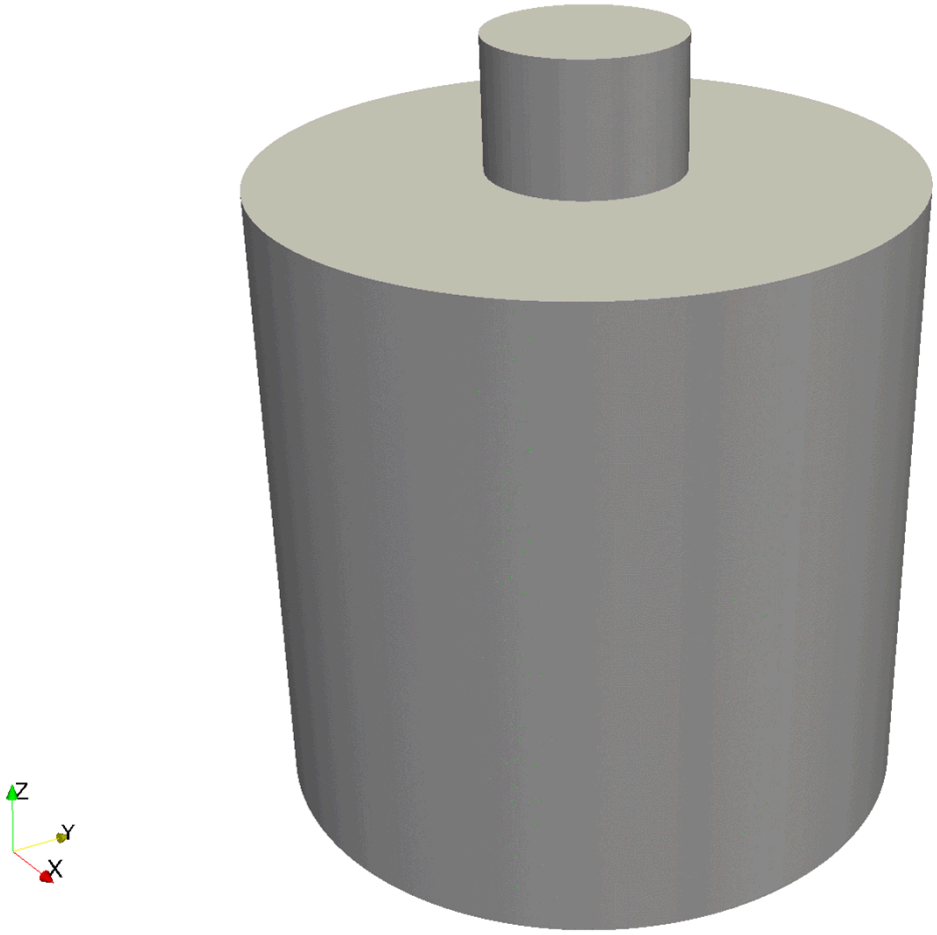

Thread scalable, platform portable

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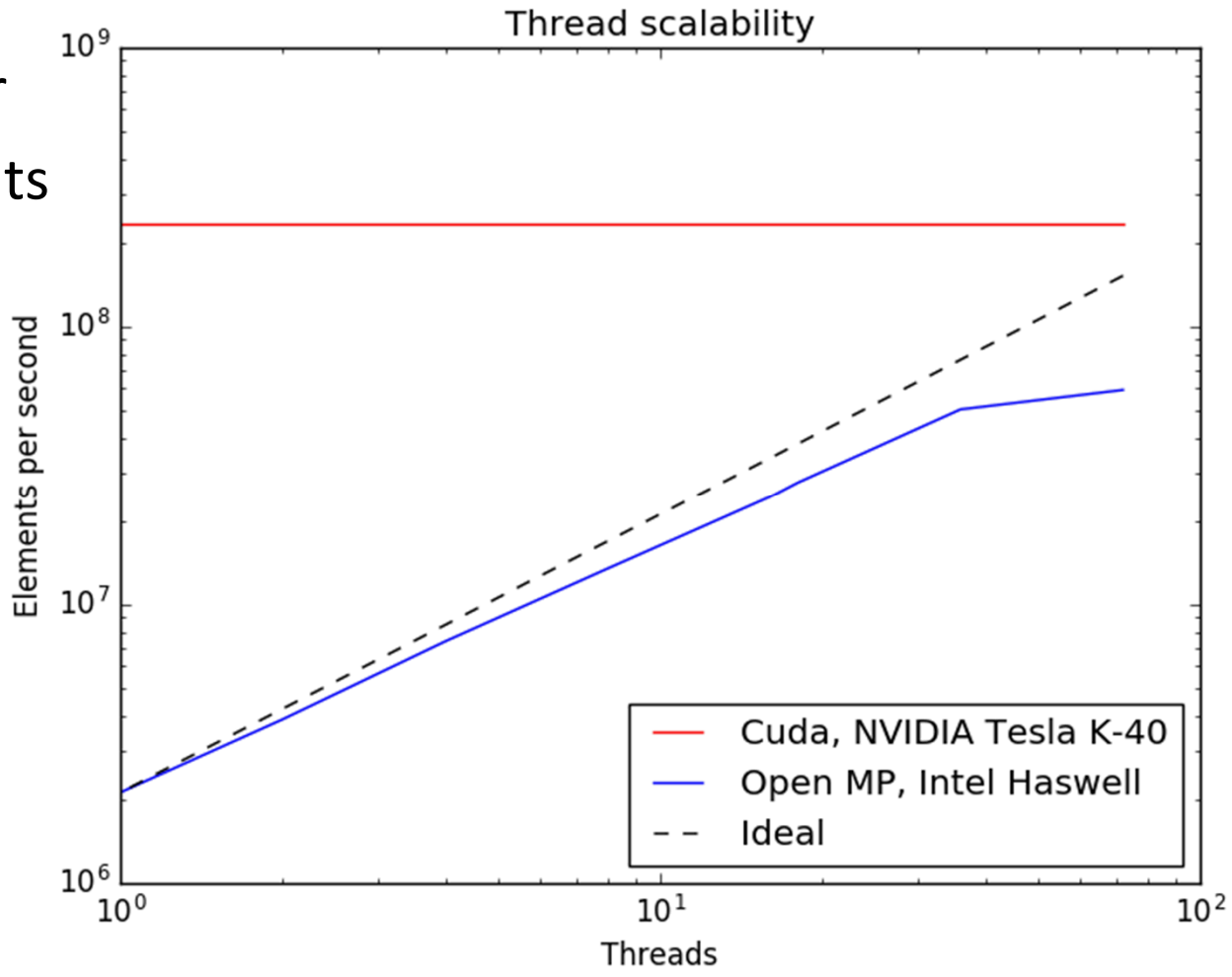
- Sandia's Kokkos library allows cross-platform, thread scalable implementations
- GPU: > 110 X faster
- 250 million elements per second
- OpenMP scales



Conclusions and future work

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- Efficient gradient damage models can be constructed which are cohesive
- Method is massively thread scalable (and MPI scalable)
- Explicit time integration is possible

  (and more than 10 X faster than single linear phase solve)
- Ongoing work with John Dolbow to verify model in fragmentation scaling studies

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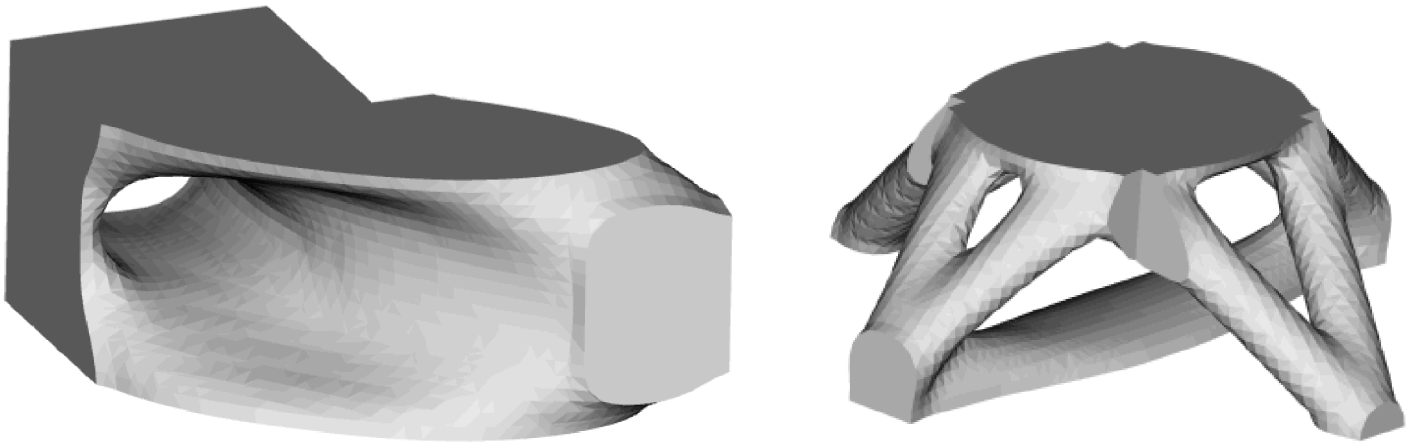
Inverse Problem

Inverse methods:

- Quantities of interest are differentiable functions
- Large # decision variables
- Often, # of decision variables is proportional to mesh resolution
- Require inexpensive gradient evaluations



Example: Topology optimization using

Sandia's Plato code

Inverse methods

Solve multiple “forward” prediction simulations in order to

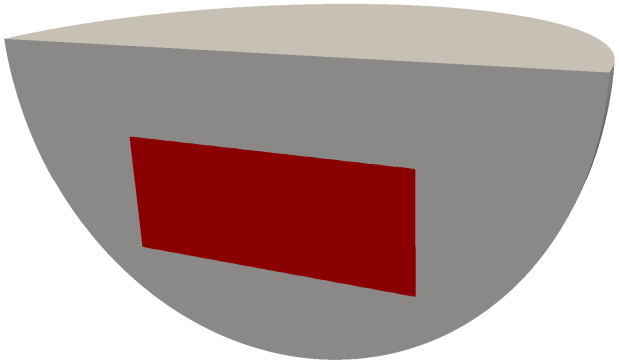
- Design
- Calibrate properties
- Match observations

$$\min_{\boldsymbol{\theta}} \; g(\mathbf{u}, \boldsymbol{\theta})$$

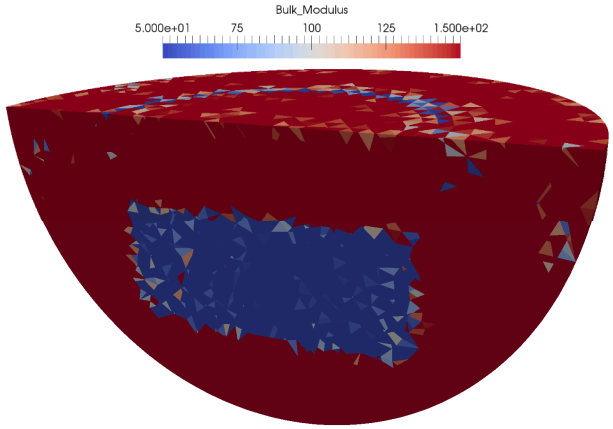
$$\text{s.t.} \;\; \mathcal{L}(\mathbf{u}, \boldsymbol{\theta}) = \mathbf{0}$$

Quantity of interest depends on
- physical solution $\mathbf{u}$
- design parameters $\boldsymbol{\theta}$

Constrained by the physics



Initially unknown hidden tunnel subjected to surface excitation



Reconstructed material properties due to observed surface displacements in Sierra-SD

Fracture Inverse Problems

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- **Non-destructive evaluation**: determine the damage at material points given observations of how waves propagate (unknowns ~ # elements)
- **Crack forensics**: determine what happened to a structure that caused it to damage in an observed way (unknowns ~ # number of BCs time # timesteps)
- **Heterogeneous material design**: how to design composite materials to maximize resistance to crack propagation (unknowns ~ # elements)

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Review of the adjoint method

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- Computing finite difference gradients requires many, many solves: O(#design parameter)
- Adjoint method requires just 1 additional solve!

Explicit update rule: $\mathbf{u}^n = \mathbf{h}^n(\mathbf{u}^{n-1}; \theta)$

Quantity of interest: $g(\mathbf{u}^N; \theta)$

Desired sensitivities: $\dfrac{dg(\mathbf{u}^N; \theta)}{d\theta}$

$$\mathcal{L}(\mathbf{u}, \boldsymbol{\lambda}; \theta) = g\left(\mathbf{u}^N; \theta\right) + \sum_n \langle \boldsymbol{\lambda}^n, \mathbf{h}^n(\mathbf{u}^{n-1}; \theta) - \mathbf{u}^n \rangle$$

$$\frac{d\mathcal{L}}{d\theta} = g_{,\boldsymbol{\theta}} + \langle g_{,\mathbf{u}}, \mathbf{u}^N_{,\theta} \rangle + \sum_n \langle \boldsymbol{\lambda}^n, \mathbf{h}^n_{,\theta} + \mathbf{h}^n_{,\mathbf{u}} \mathbf{u}^{n-1}_{,\theta} - \mathbf{u}^n_{,\theta} \rangle$$

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Adjoint of a recursion relation

Final condition: $$\boldsymbol{\lambda}^N = g_{,\mathbf{u}}(\mathbf{u}^N;\theta)$$

Adjoint update: $$\boldsymbol{\lambda}^n = \left(\mathbf{h}^{n+1}_{,\mathbf{u}}\right)^T \boldsymbol{\lambda}^{n+1}$$

Sensitivity: $$\frac{d\mathcal{L}}{d\theta} = g_{,\theta} + \sum_n \langle \boldsymbol{\lambda}^n, \mathbf{h}^n_{,\theta} \rangle$$

Strang's Interpretation:

$$\mathbf{v}_{N\times 1} = (\mathbf{A}_{N\times N}\ \mathbf{B}_{N\times N})\ \mathbf{u}_{N\times 1}$$

vs

$$\mathbf{v}_{N\times 1} = \mathbf{A}_{N\times N}\ (\mathbf{B}_{N\times N}\ \mathbf{u}_{N\times 1})$$

Example with explicit Newmark

Explicit Newmark time integration

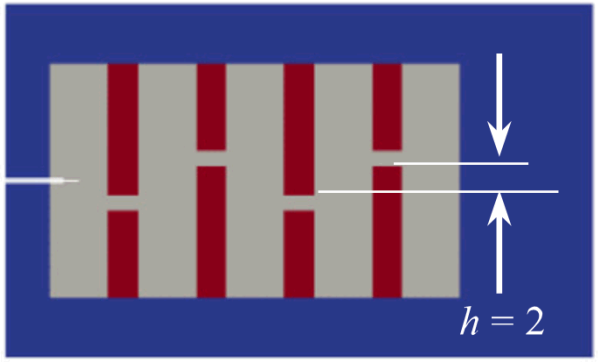
$$\mathbf{u}^{n+1} = \mathbf{u}^n + \Delta t\, \mathbf{v}^n + \frac{\Delta t^2}{2}\mathbf{a}^n$$
$$\mathbf{v}^{n+1/2} = \mathbf{v}^n + \frac{\Delta t}{2}\mathbf{a}^n$$
$$\mathbf{a}^{n+1} = \boldsymbol{\pi}^{n+1}(\mathbf{u}^{n+1})$$
$$\mathbf{v}^{n+1} = \mathbf{v}^{n+1/2} + \frac{\Delta t}{2}\mathbf{a}^{n+1}$$

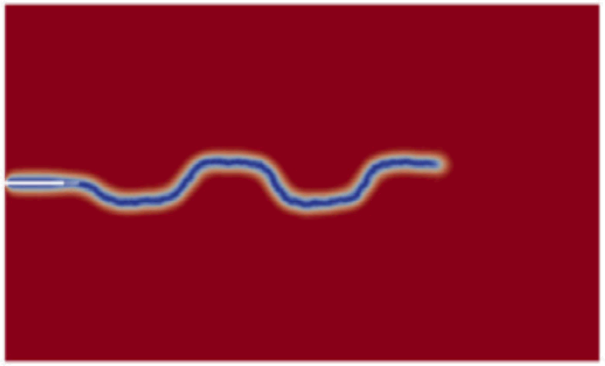
Explicit adjoint

$$\hat{\mathbf{a}}^{n+1/2} = \hat{\mathbf{a}}^{n+1} + \frac{\Delta t}{2}\hat{\mathbf{v}}^{n+1}$$
$$\hat{\mathbf{u}}^{n} = \hat{\mathbf{u}}^{n+1} + \hat{\mathbf{a}}^{n+1/2} \cdot \boldsymbol{\pi}^{n+1}_{,\mathbf{u}}$$
$$\hat{\mathbf{v}}^{n} = \hat{\mathbf{v}}^{n+1} + \Delta t\, \hat{\mathbf{u}}^{n}$$
$$\hat{\mathbf{a}}^{n} = \frac{\Delta t}{2}\hat{\mathbf{v}}^{n}$$

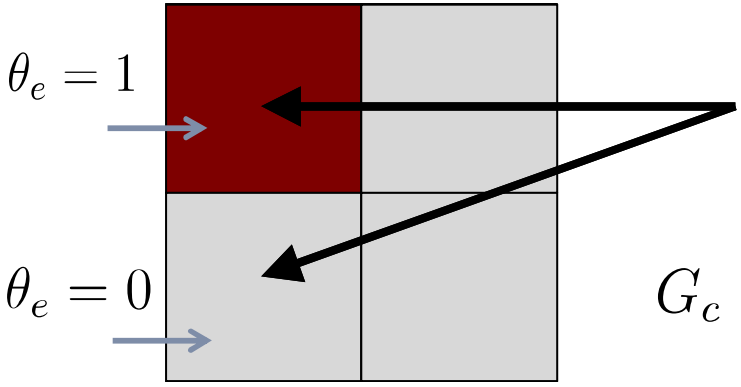
Heterogeneous material design

- Hossain, Hsueh, Bourdin, Bhattacharya 2014 show heterogeneous materials can be tougher than its constituents!





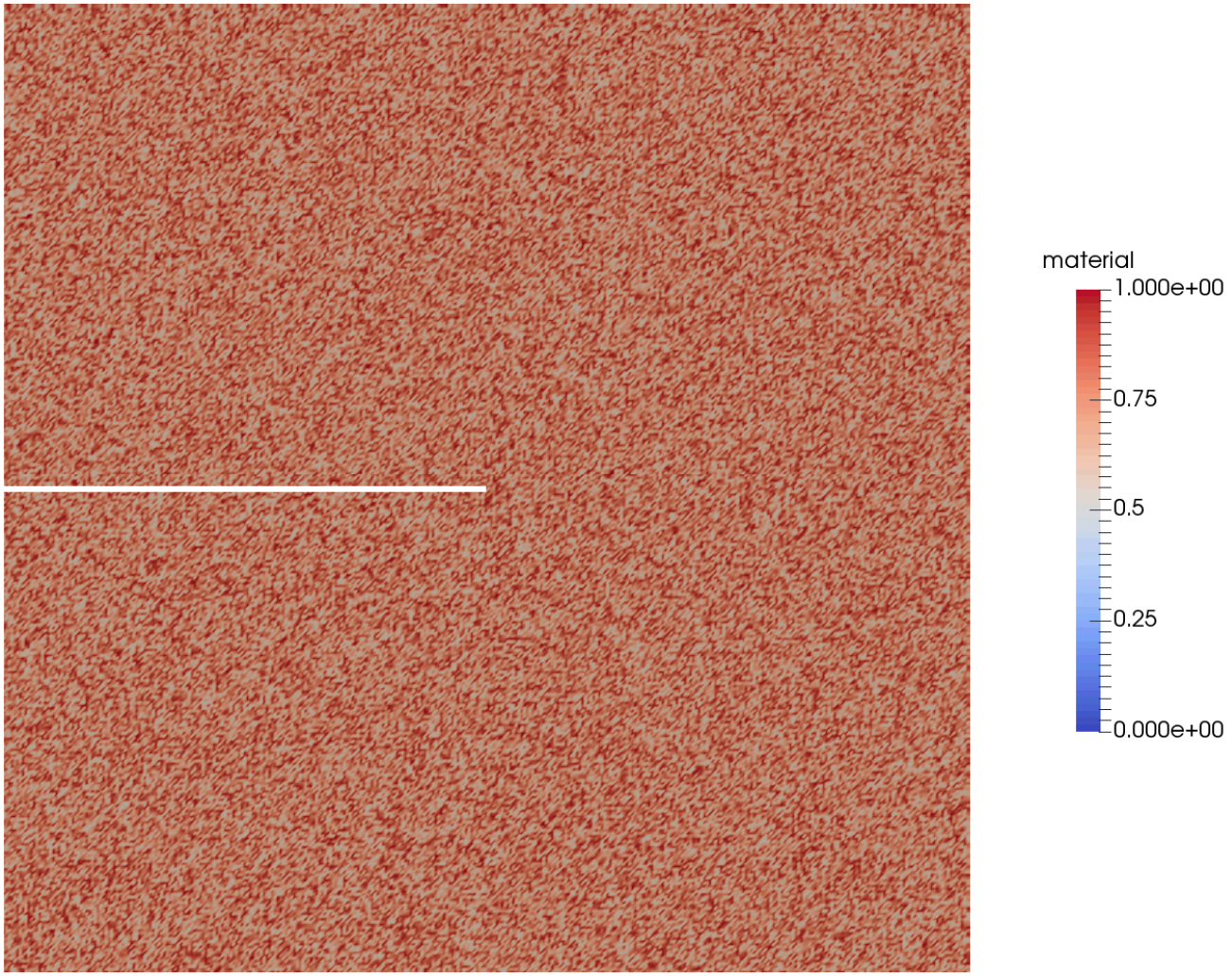
- Use adjoint framework and Sandia's *Rapid Optimization Library* to automatically design tougher material



Each element chooses to be compliant or stiff: $\theta_e \in [0, 1]$

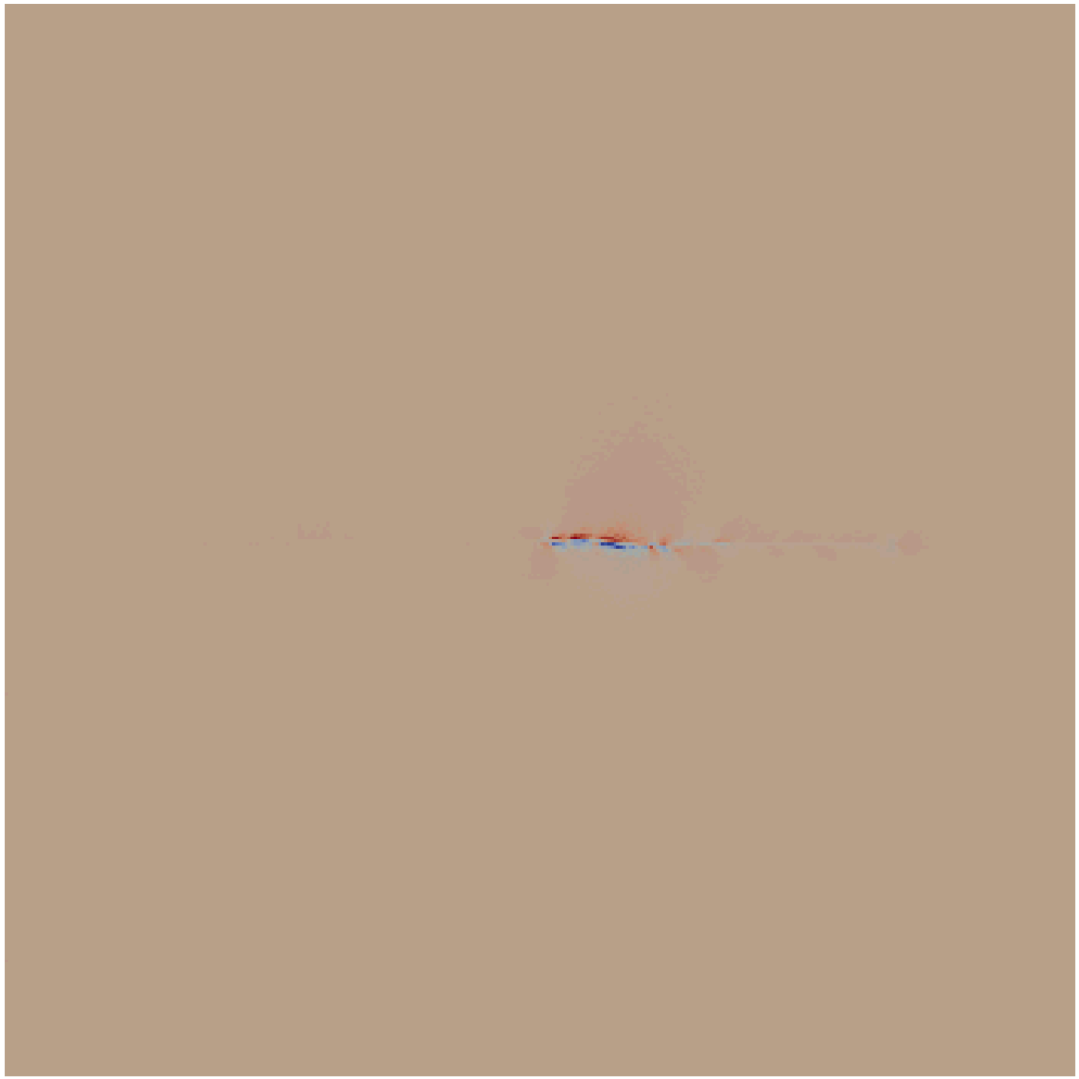
$$G_c = 4\text{e}5 \quad \rho = 8000 \quad E = 47.5\text{e}9 \text{ or } 190\text{e}9$$

Initial material property guess



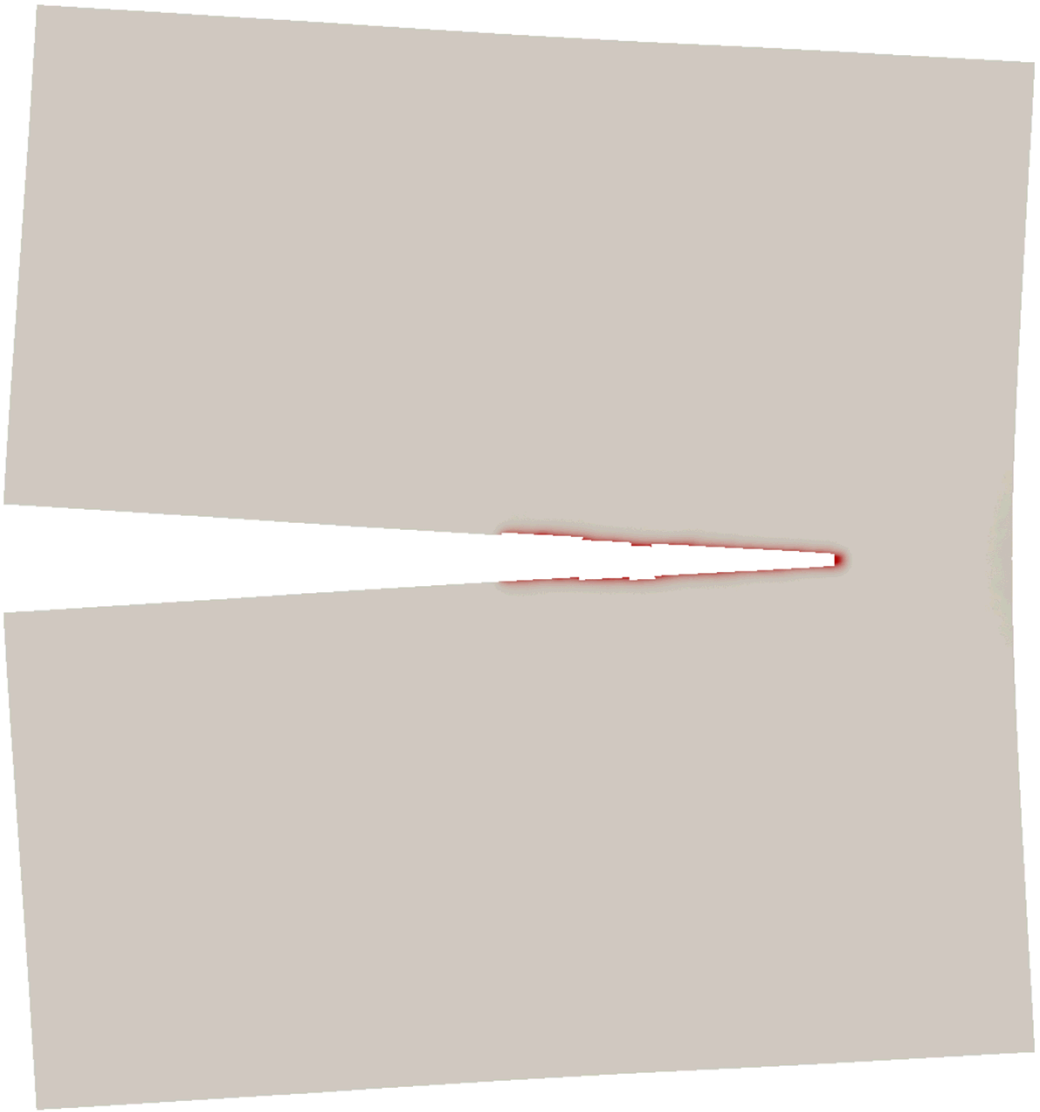
Damage evolution

Adjoint sensitivity to initial props

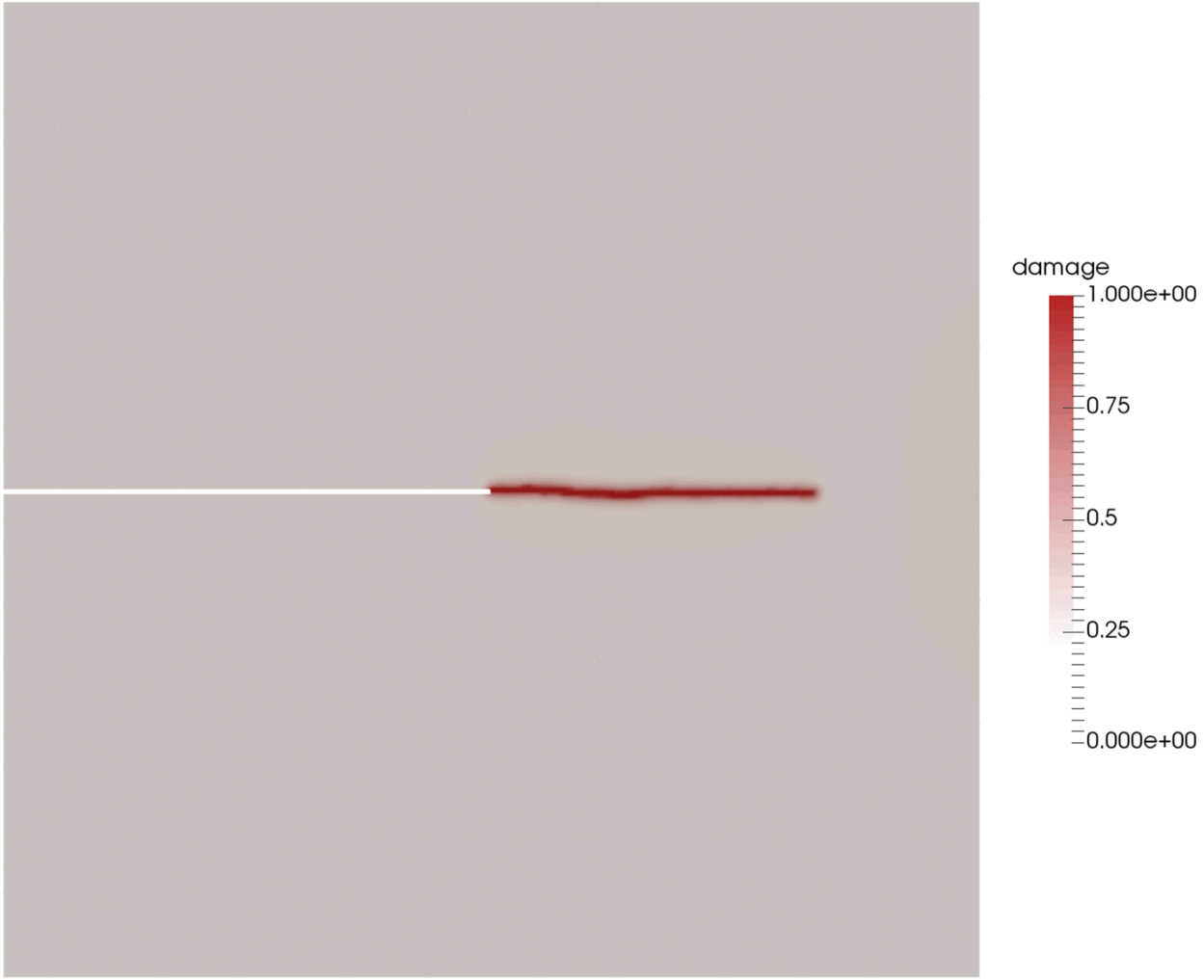


~6 times the cost of a forward simulation

Minimize crack propagaion

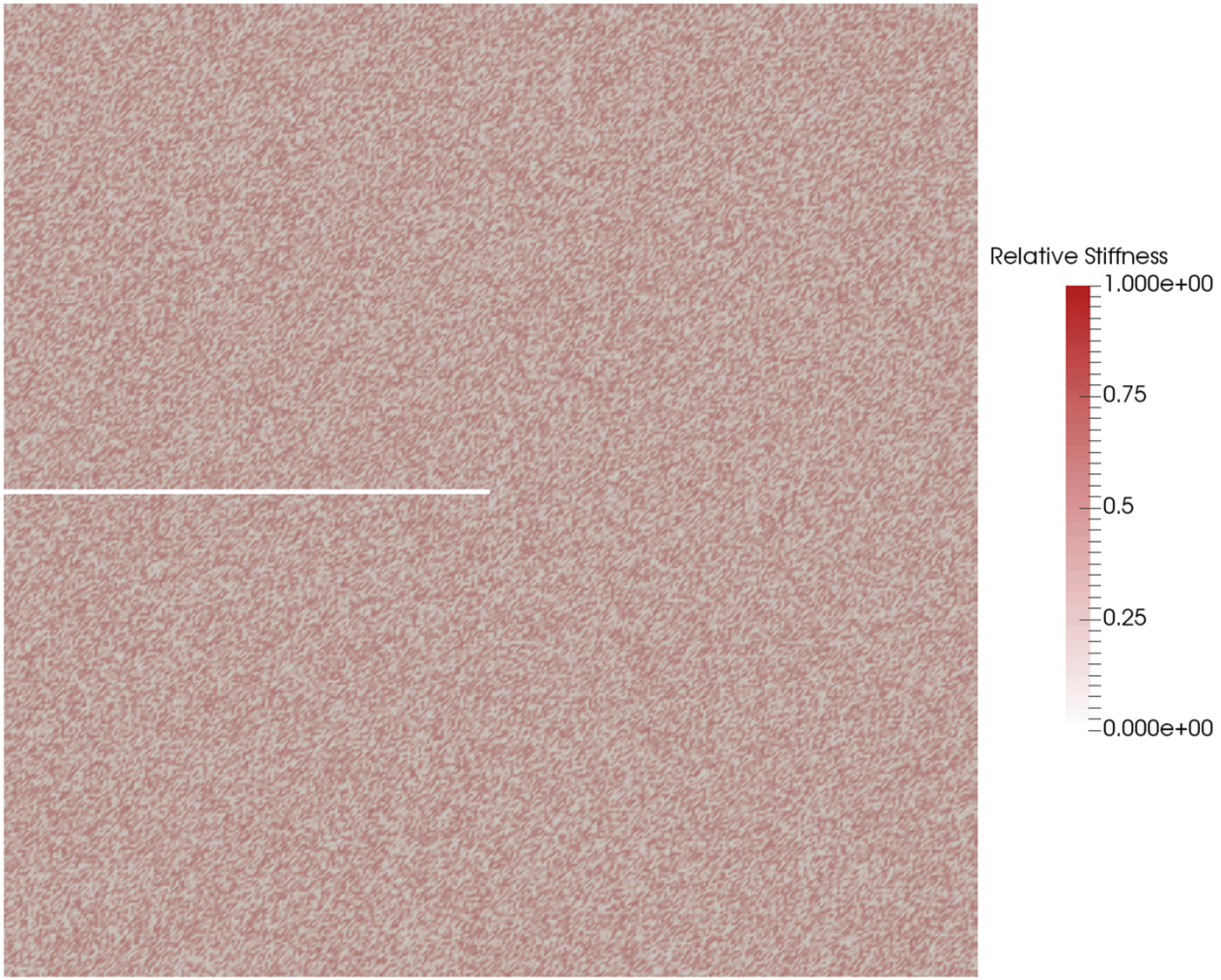


Minimize crack propagation



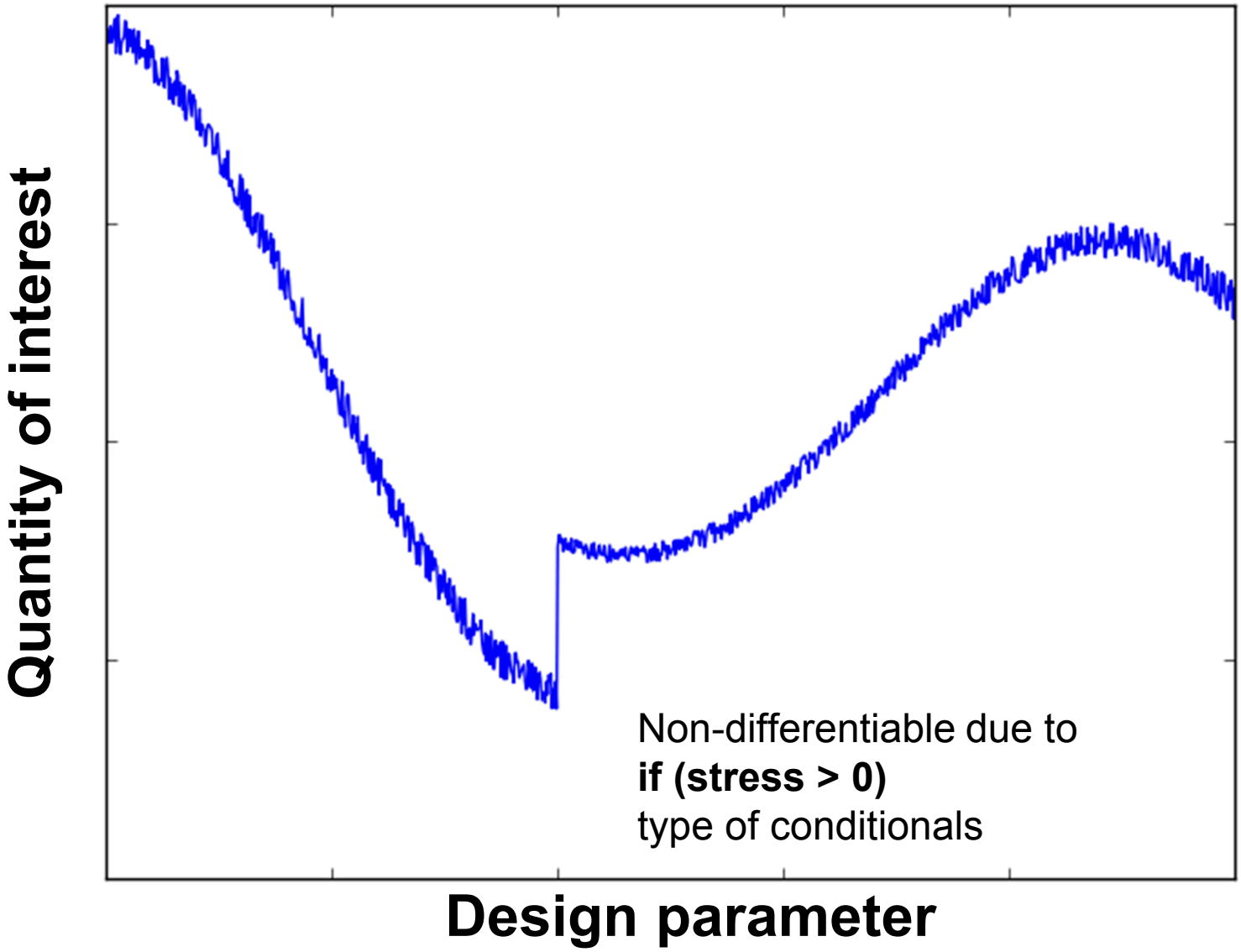
Un-deformed configuration

Find locally optimal design

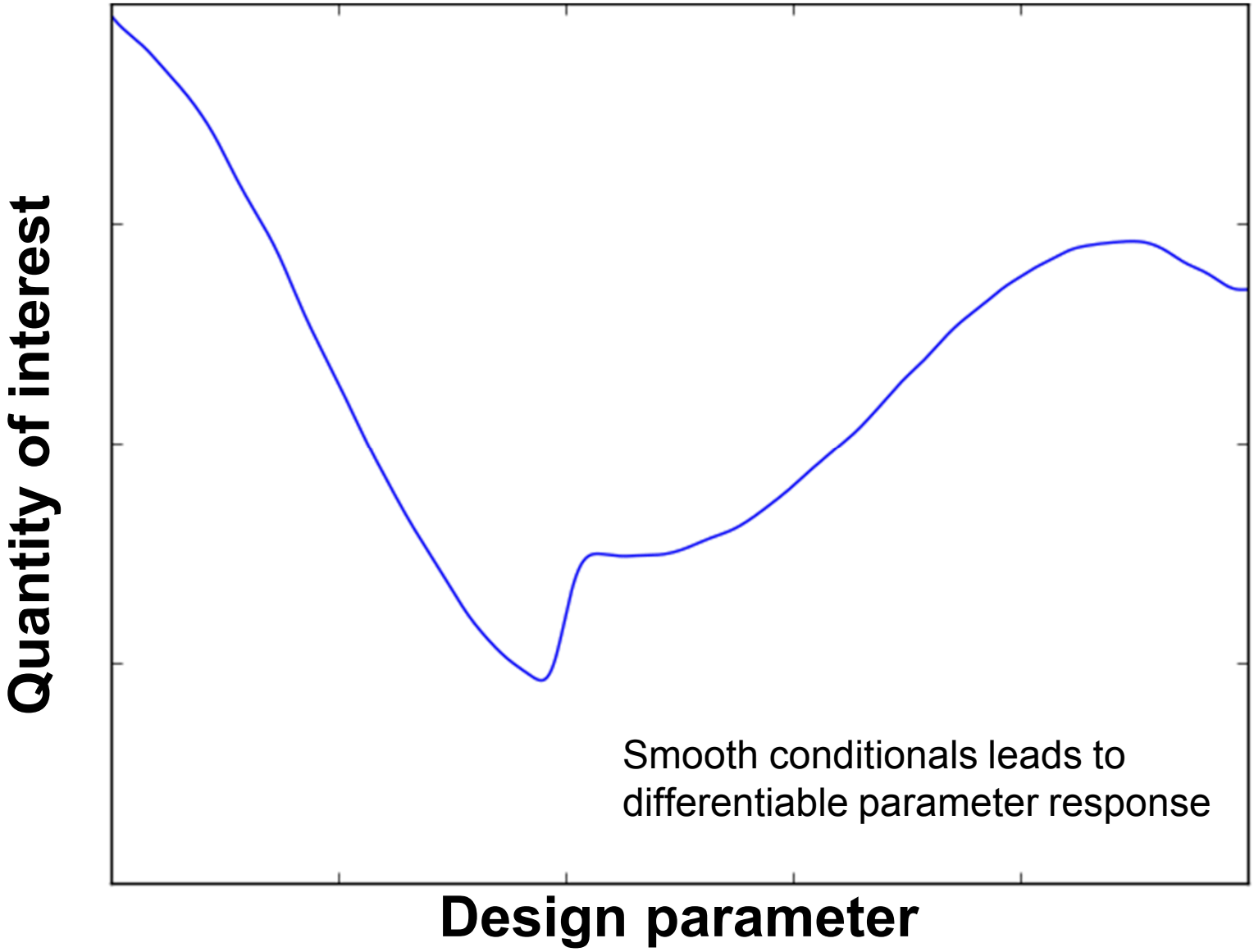


121801 design parameter gradients

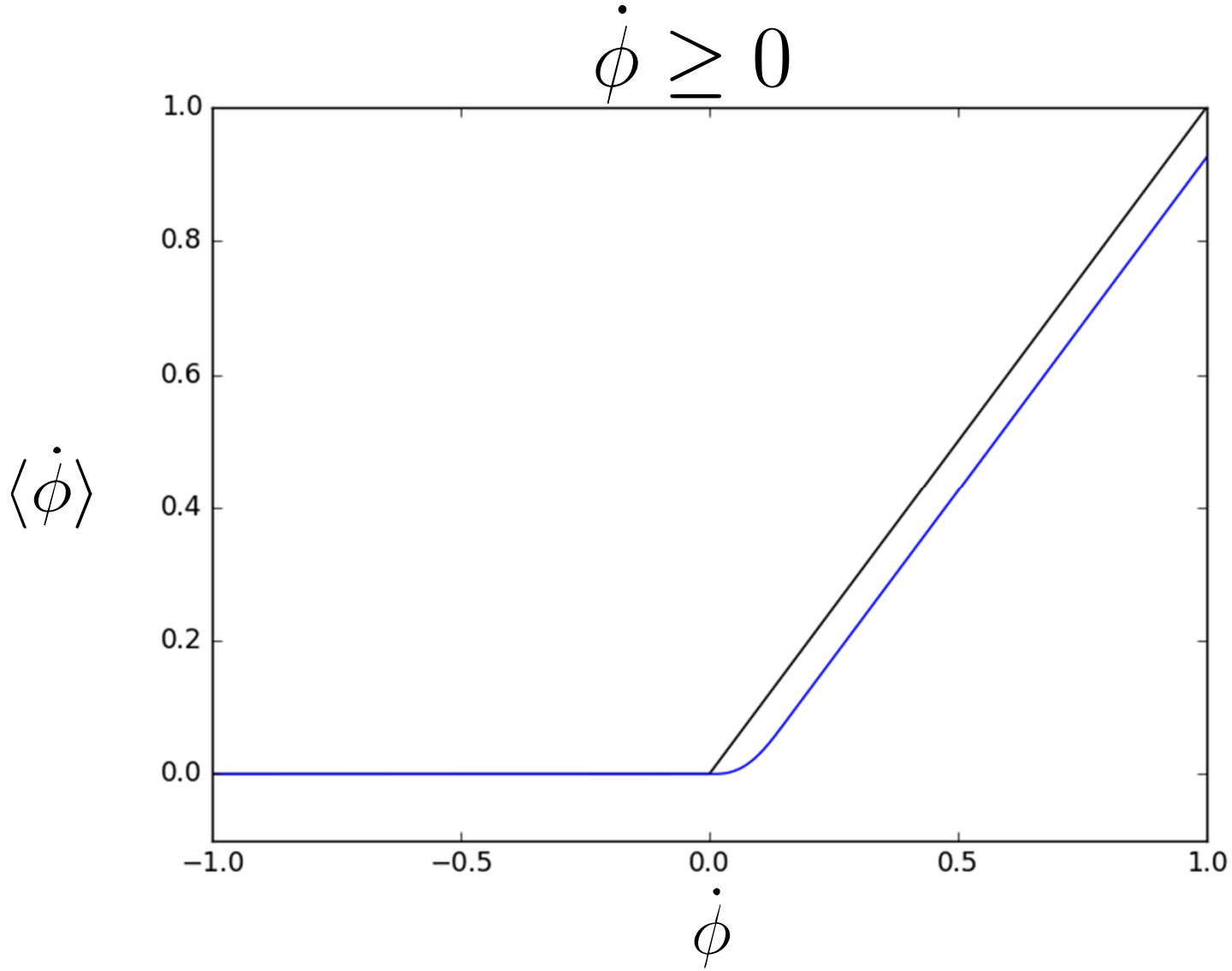
Non-smooth quantity of interest



Smoothed quantity of interest



Differentiable approximation to $\dot{\phi} \geq 0$



Library for efficient computation of parameter sensitivities

Sandia National Laboratories

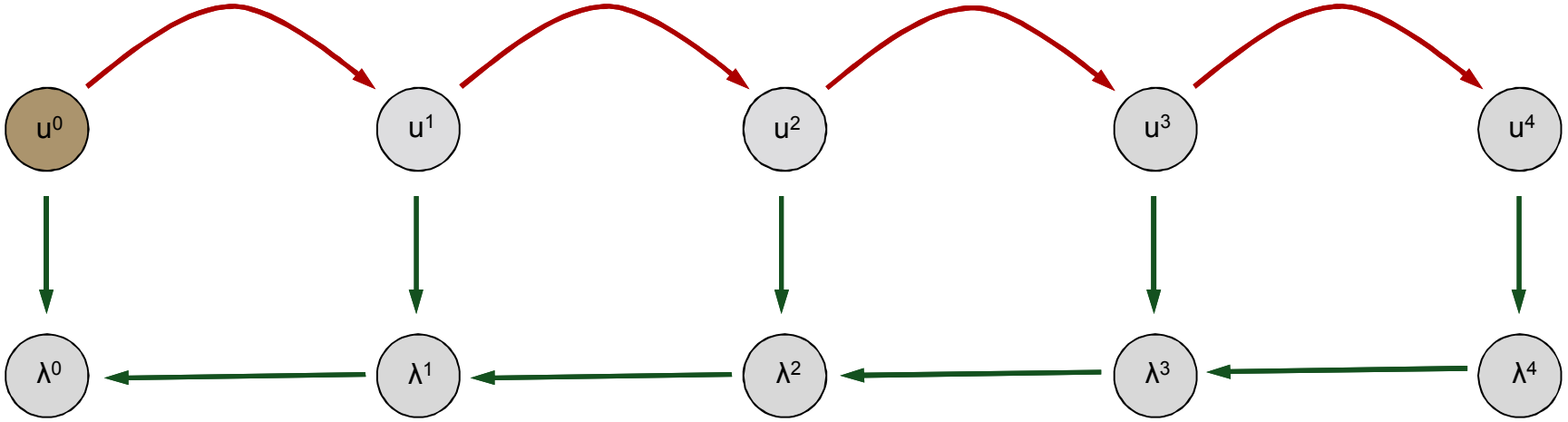
- Developed C++ library: simplifies the implementation and testing of *adjoint* sensitivities for explicit dynamic simulations
- Users define
  - State: **u** (e.g. displacements)
  - Design parameters: **θ** (e.g. initial material properties, BCs, etc.)
  - Operators:

$$\mathbf{u}^{n+1} = \mathbf{f}(\mathbf{u}^n, \boldsymbol{\theta}) \qquad \frac{\partial \mathbf{f}(\mathbf{u}^n, \boldsymbol{\theta}) \cdot \boldsymbol{\mu}}{\partial \mathbf{u}^n} \qquad \frac{\partial \mathbf{f}(\mathbf{u}^n, \boldsymbol{\theta}) \cdot \boldsymbol{\mu}}{\partial \boldsymbol{\theta}}$$

$$\text{q.o.i.} = g(\mathbf{u}^N, \boldsymbol{\theta}) \qquad \frac{\partial g(\mathbf{u}^N, \boldsymbol{\theta})}{\partial \mathbf{u}^N} \qquad \frac{\partial g(\mathbf{u}^N, \boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$$

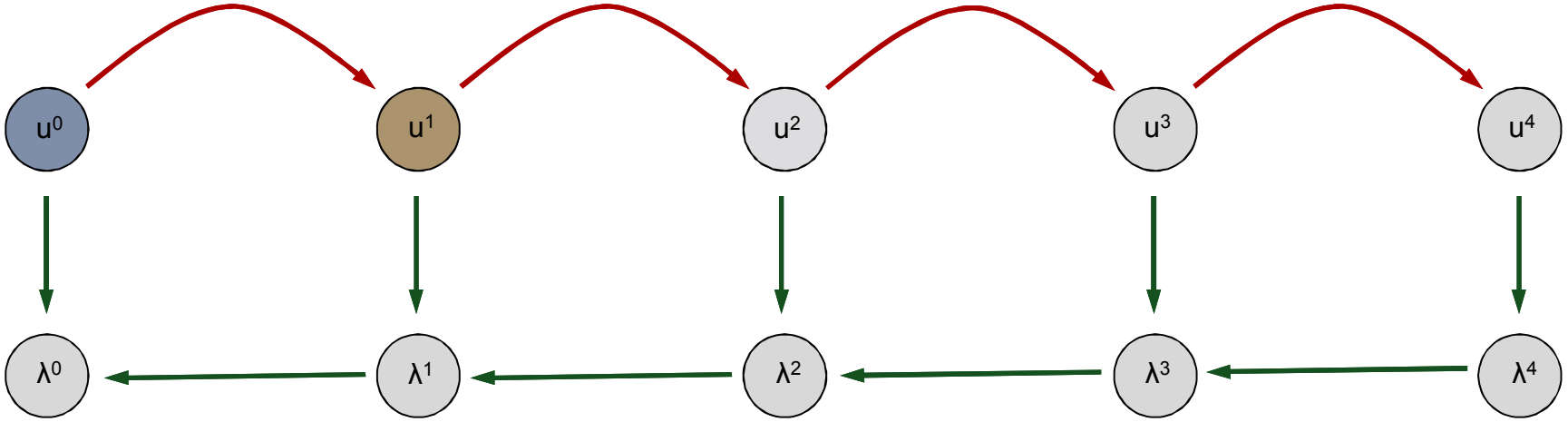
- Library automatically computes $\frac{dg(\mathbf{u}^N, \boldsymbol{\theta})}{d\boldsymbol{\theta}}$
- 1,000 – 1,000,000+ parameter sensitivities take time of ~10 forward solves

Checkpointing for backward integration



Using ***Minimal Repetition Checkpointing Algorithm*** by Q. Wang, et al.

Checkpointing for backward integration



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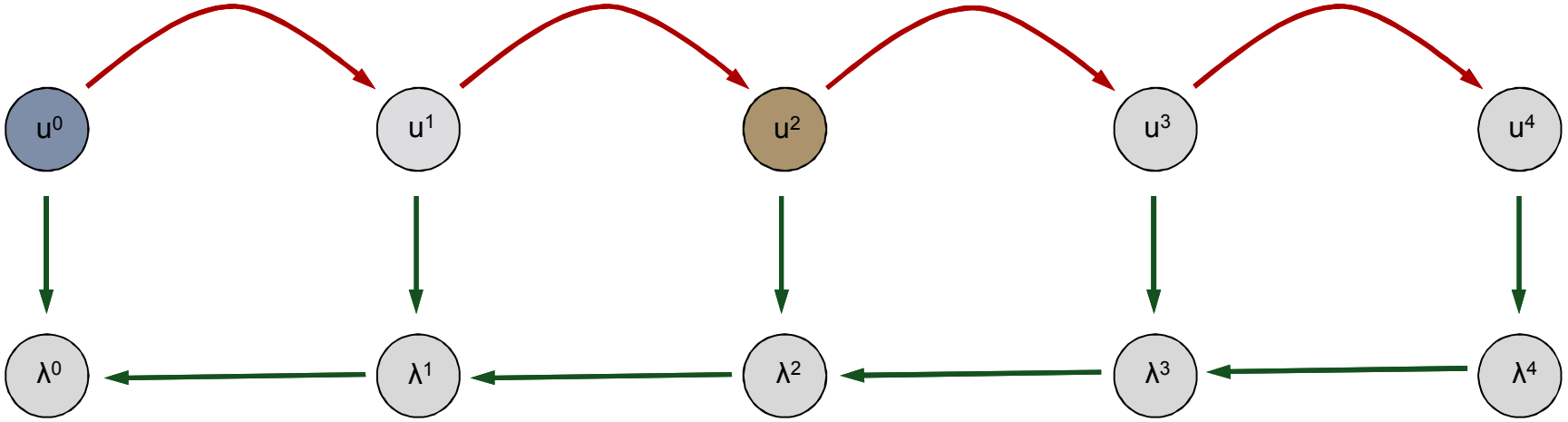

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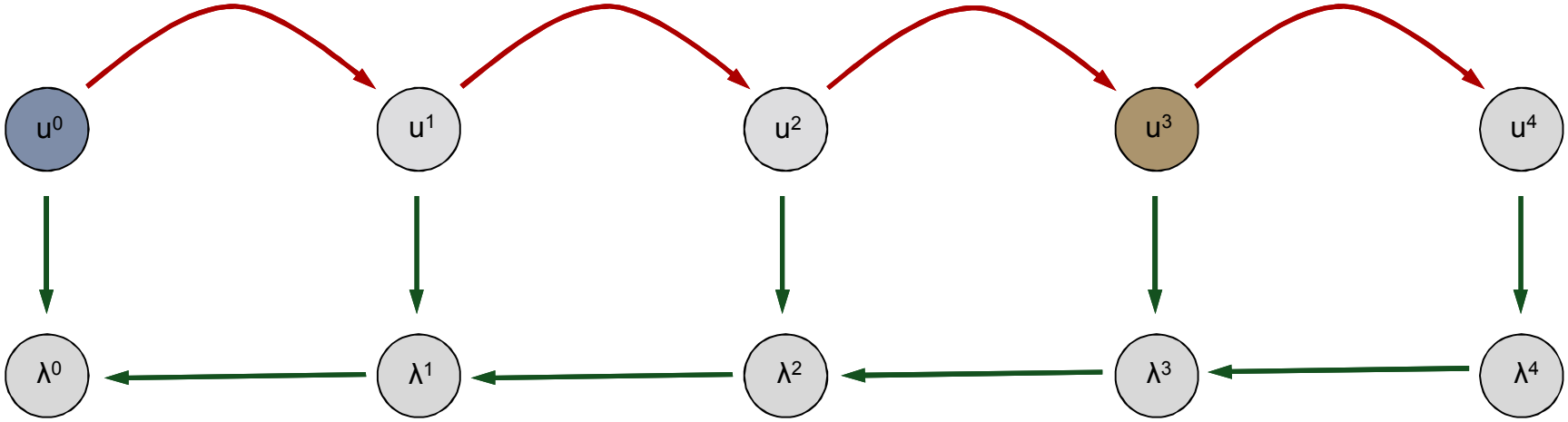

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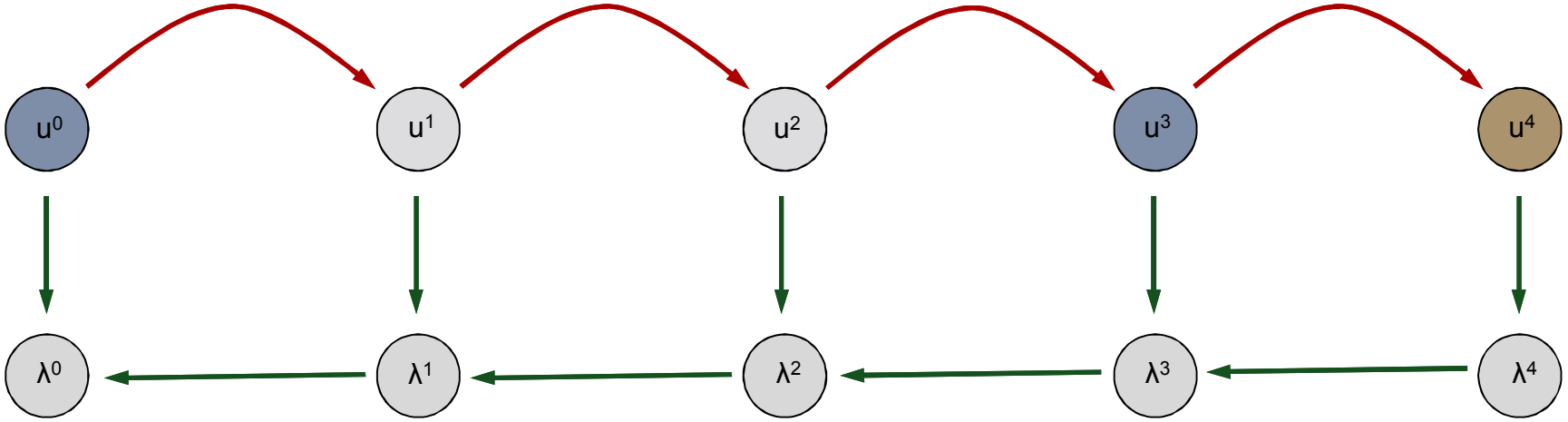

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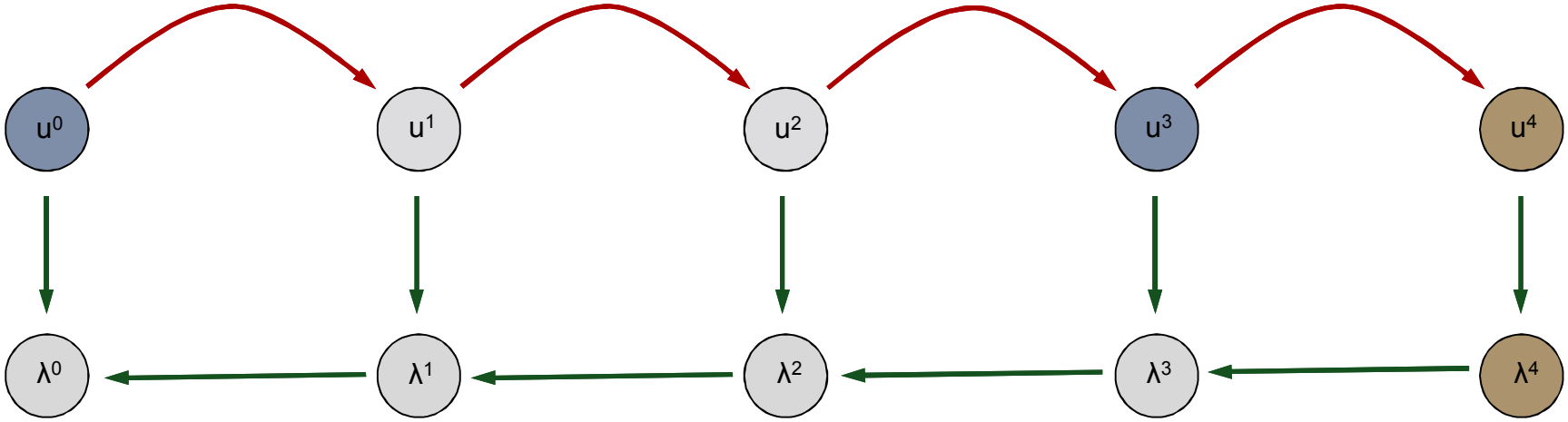

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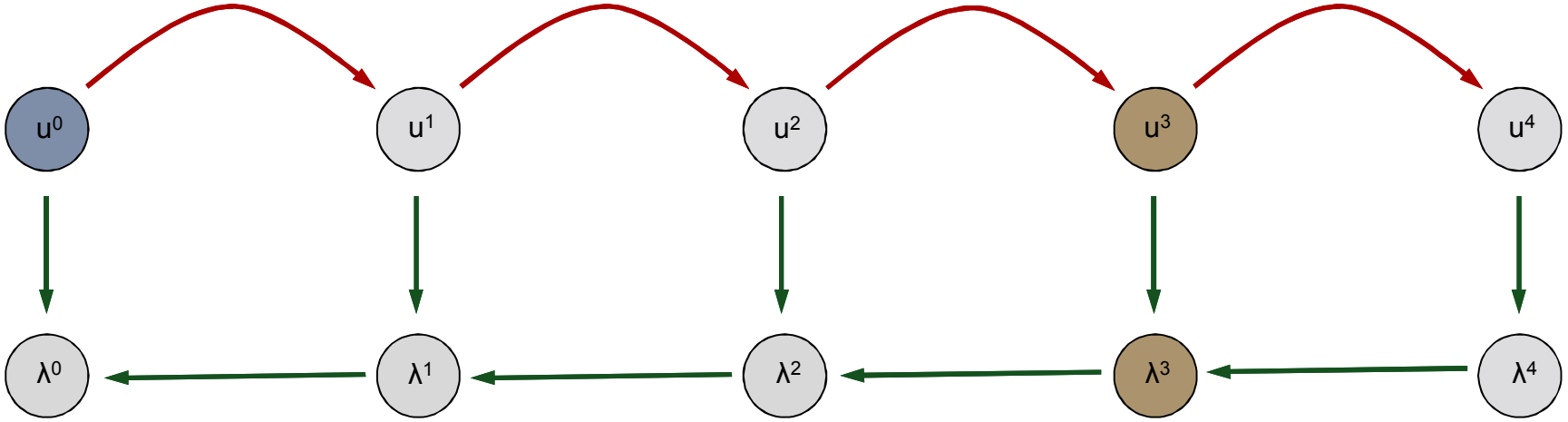

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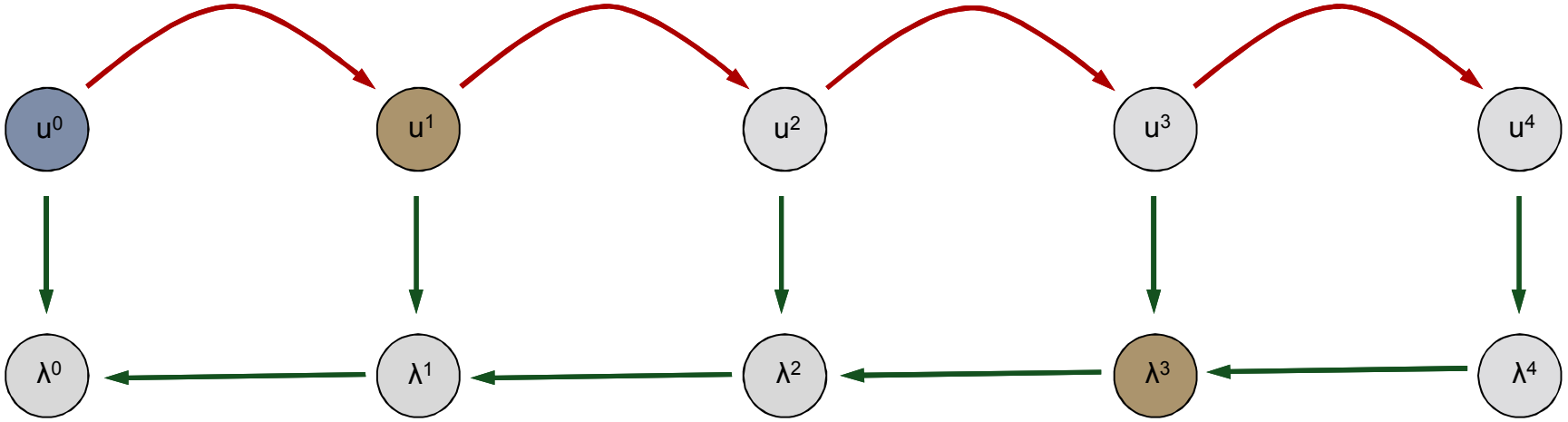

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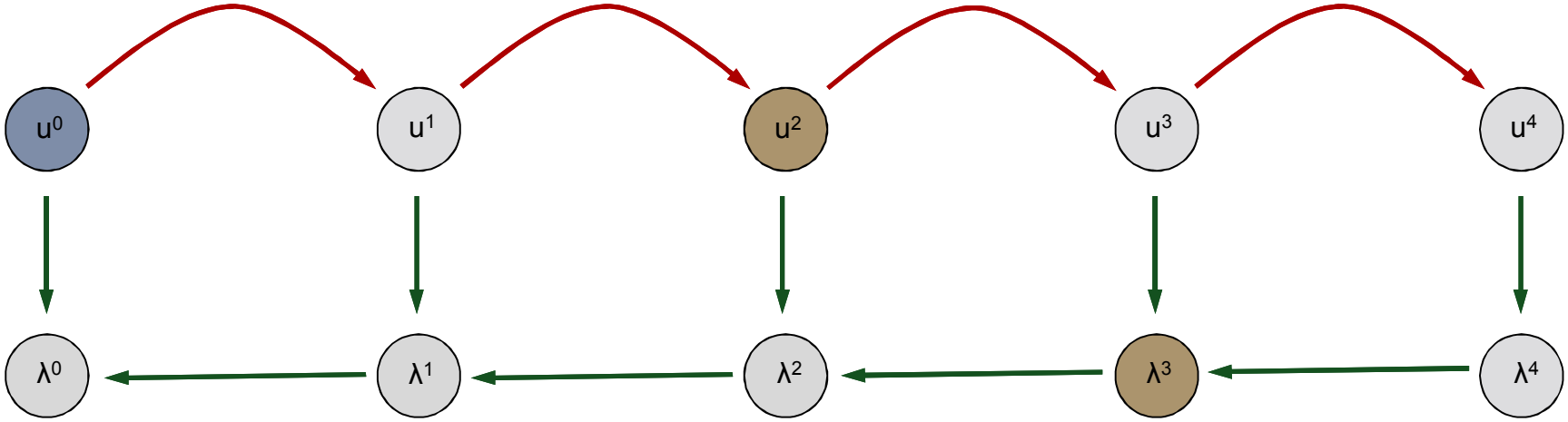

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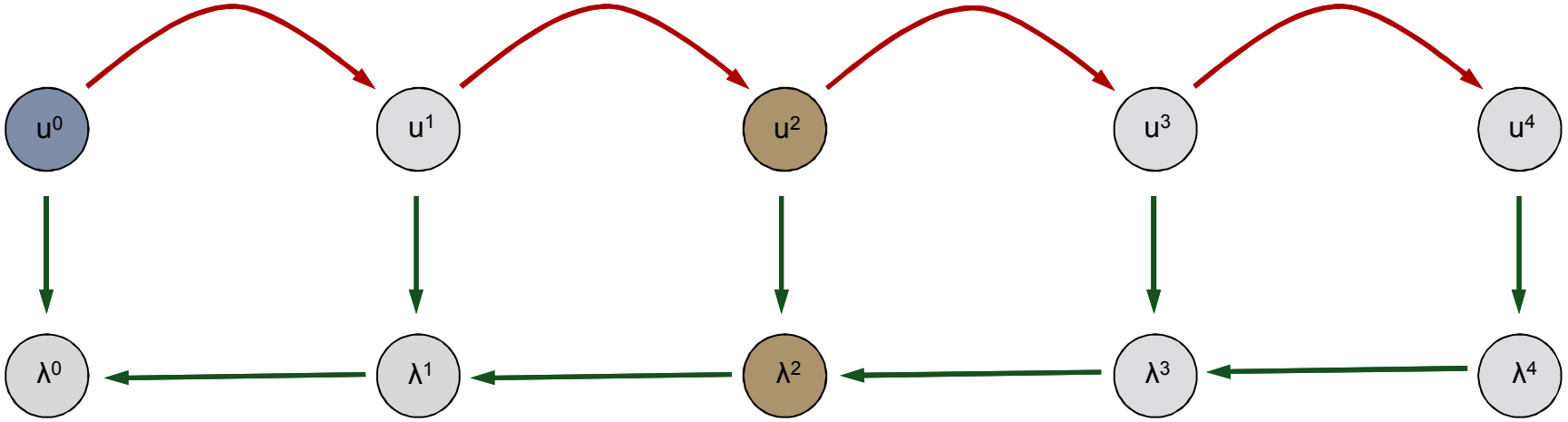

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Adding up the savings

Sandia National Laboratories

- Explicitly integrating damage: > 10 x
- Structured grid, linear kinematics: > 20 x
- GPU vs. CPU: > 100 x
- Adjoint sensitivities: > (#Elements / 10) x

- For 100,000 element simulation: > 2e8 times speedup
- From average human lifetime to ~10 seconds

Conclusions

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- Adjoint method is too important to be ignored
  - Non-trivial to implement and maintain
  - Requires differentiability
- Thread scalability is too important to be ignored
  - Data movement kills performance
  - May need to rethink the algorithms we use
  - Hardware trends are likely to continue
- Next steps:
  - Adjoint method for dynamic contact, plasticity, etc.
  - Improve usability and robustness of inverse framework

Thanks

Sandia National Laboratories

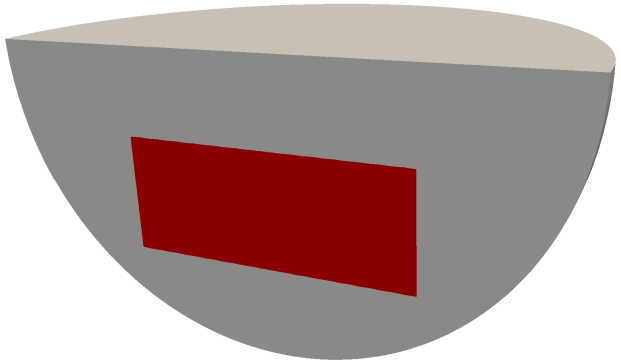
- Solid Mechanics Team
- John Dolbow
- Tim Walsh
- Kyle Starkey
- ROL team
- Kokkos team
- Sierra DevOps

Extra slides

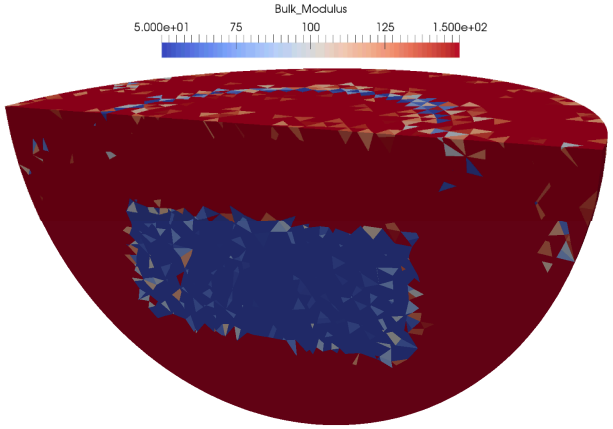
Sandia National Laboratories

Phase-field inversion examples

- Minimize discrepancy between observed displacements and simulated displacements by varying phase (volume fraction) element by element
- Sandia's ROL library used to perform the optimization

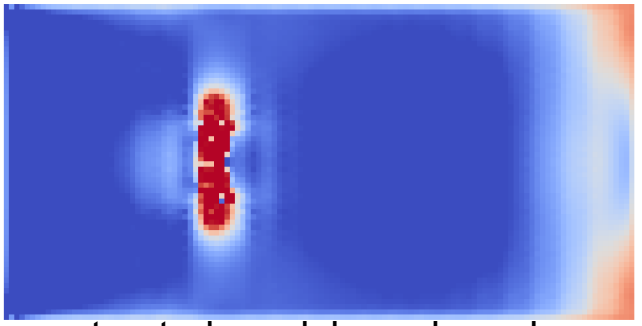


Initially unknown hidden tunnel subjected to surface excitation



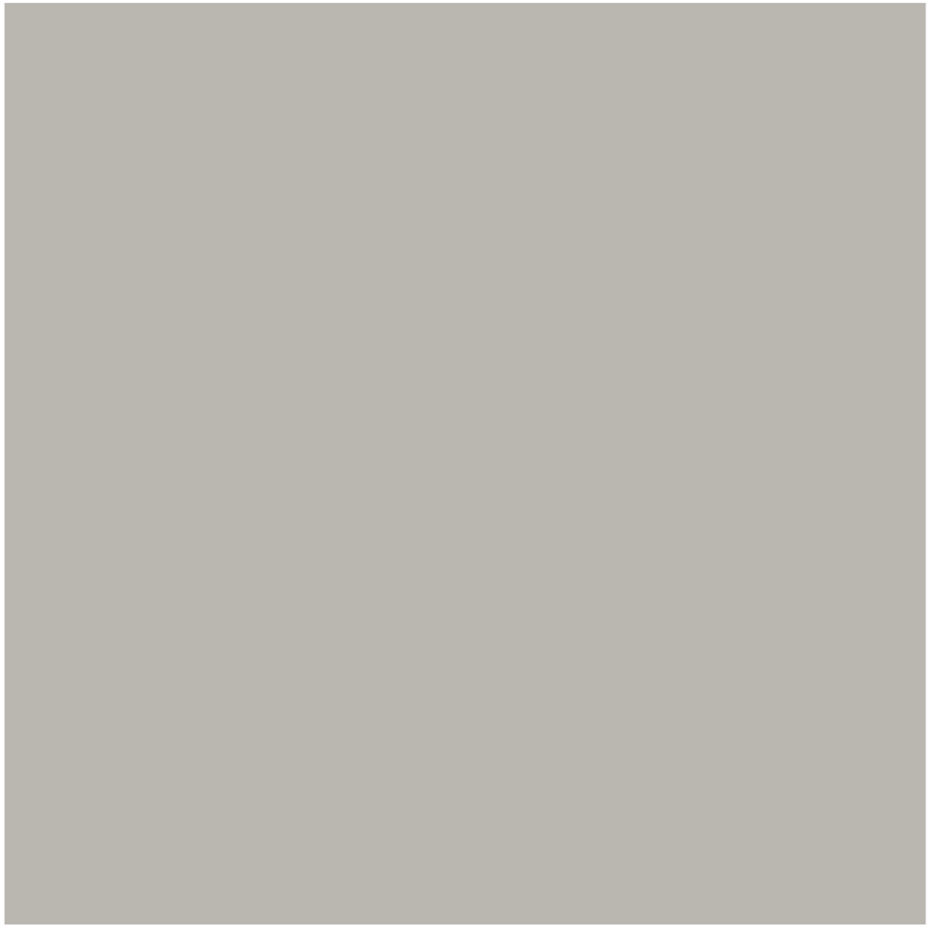
Reconstructed material properties due to observed surface displacements in Sierra-SD

Initial crack/damage in specimen

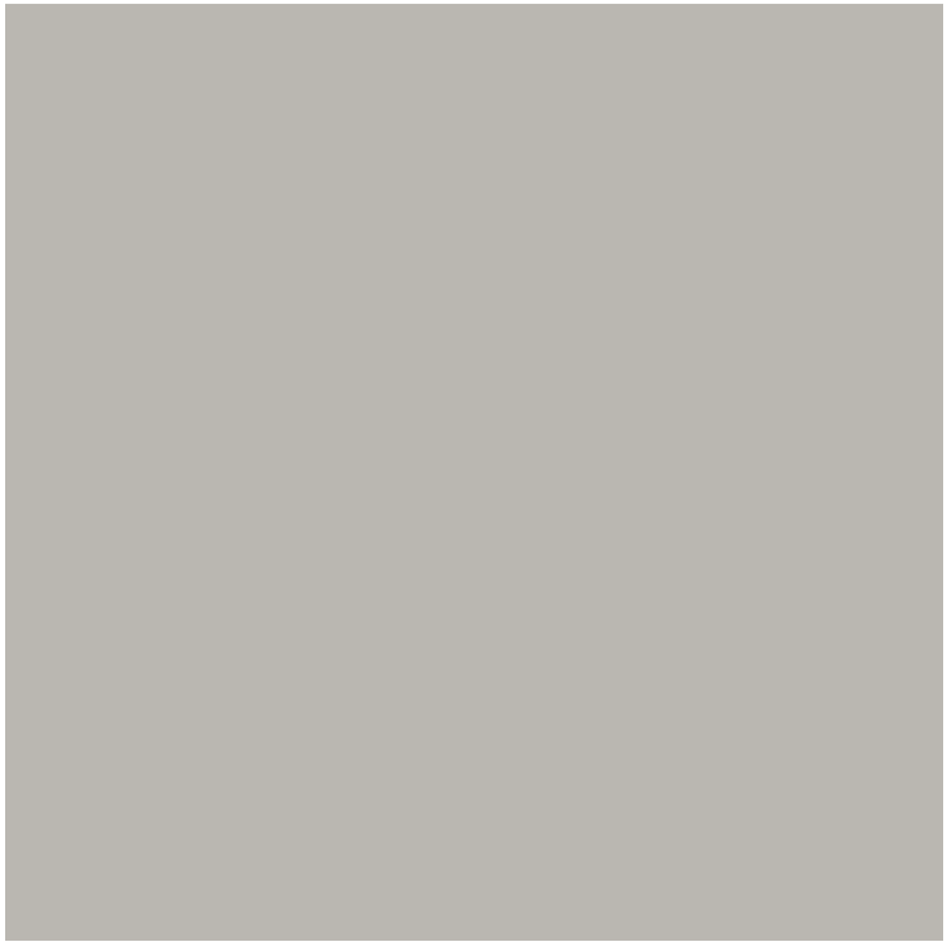


Reconstructed crack based on observed displacements due to acoustic excitation

High fracture energy release rate



Low fracture energy release rate



Mesh insensitivity

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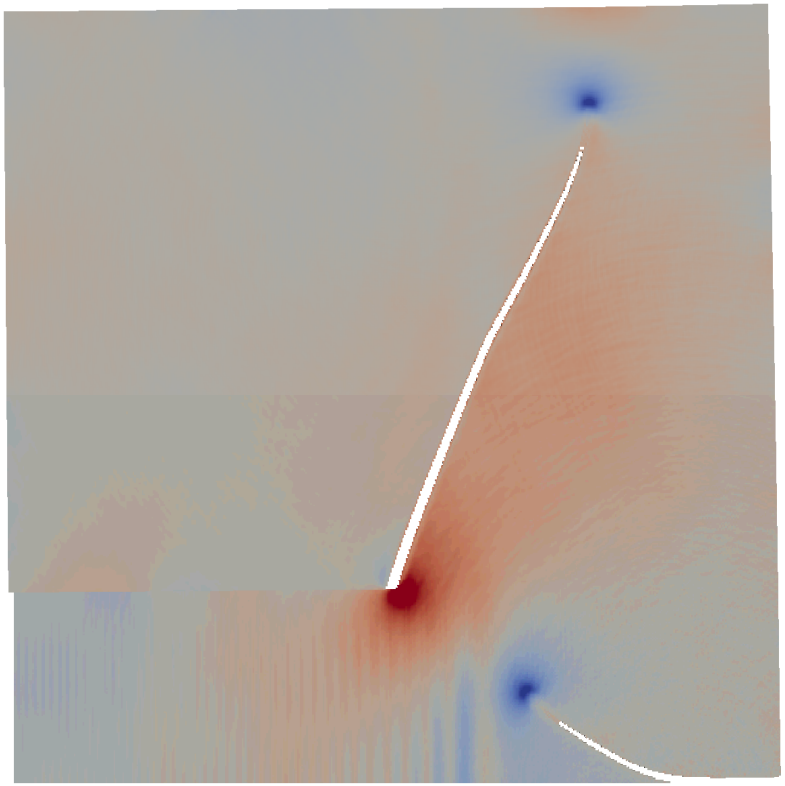
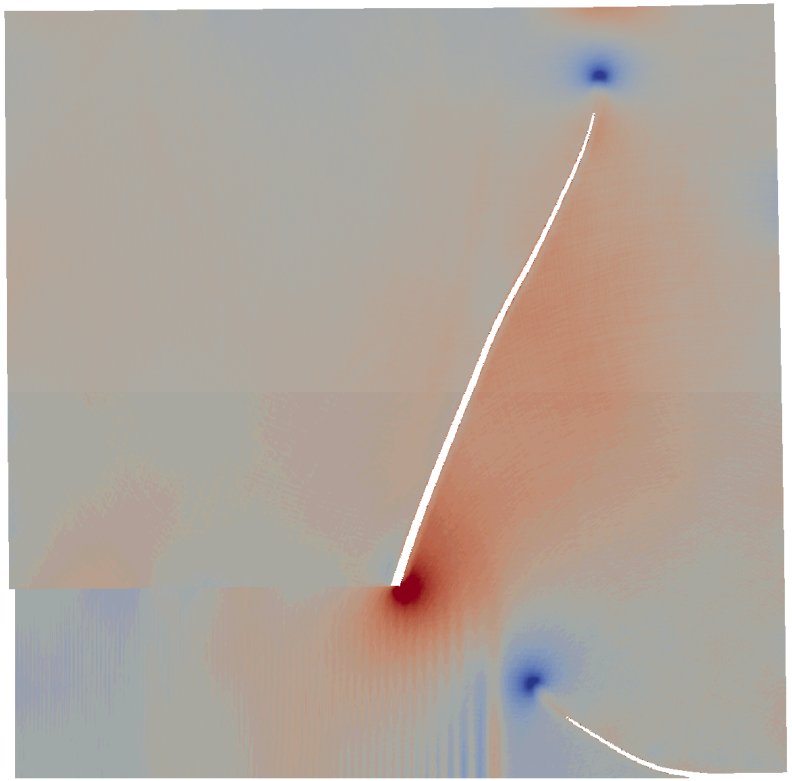
Pressure contours for dynamic crack propagation

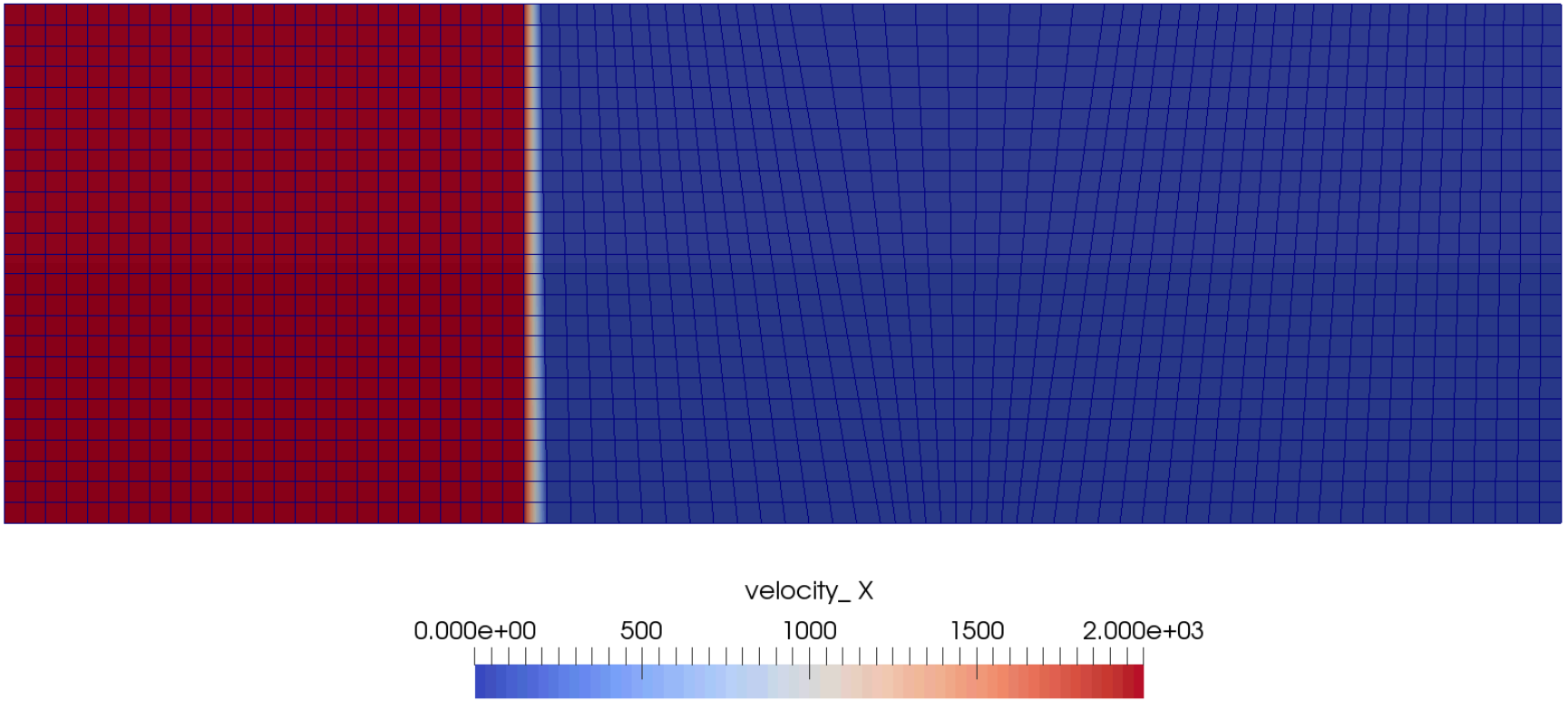


Medium mesh: 250k elements

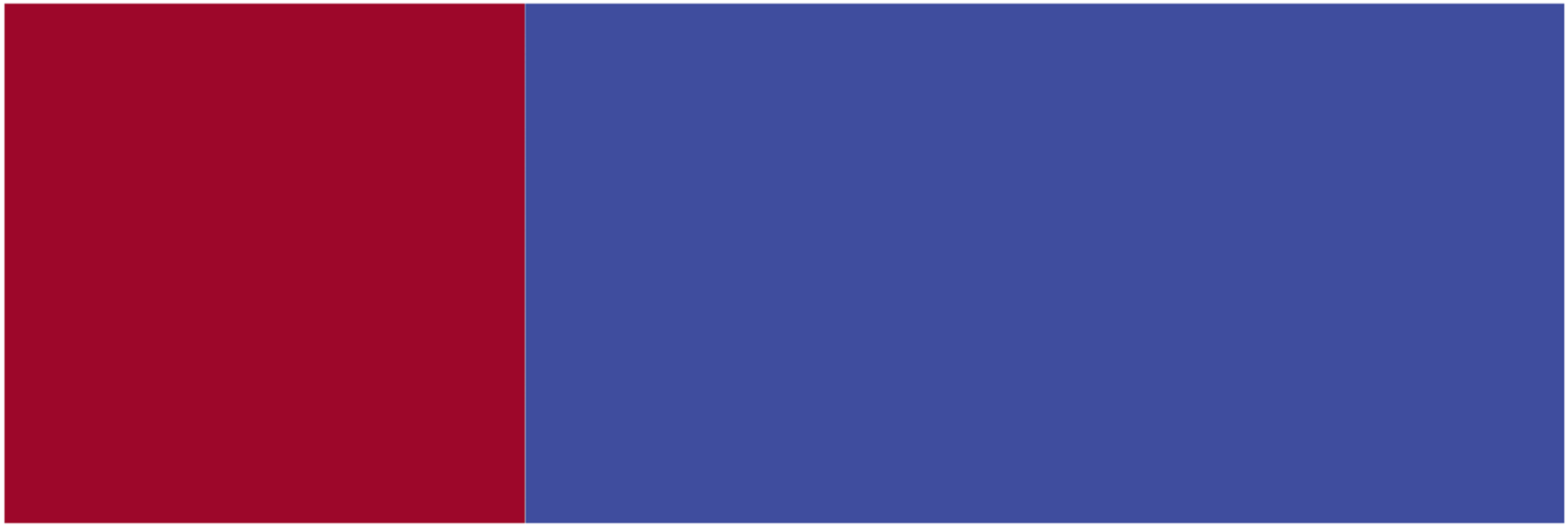


Fine mesh: 1,000k elements

Spall example: Initial velocity

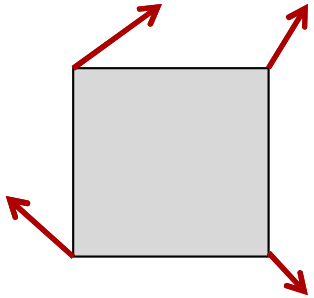


Spall example:

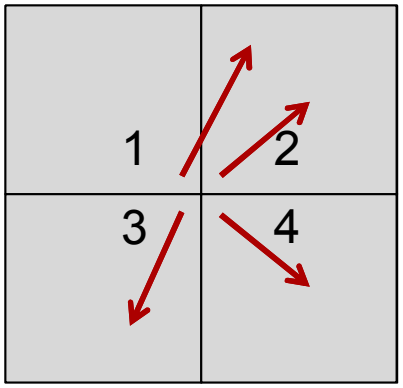


Determinacy in a threaded world

- Computing adjoint for explicit dynamics requires checkpointing and re-computing solutions
- This makes run-to-run determinacy necessary
- Use data duplication:



Loop over element:
store all element forces



Loop over nodes:
sum forces in predetermined order

Reintroducing the Lorentz model

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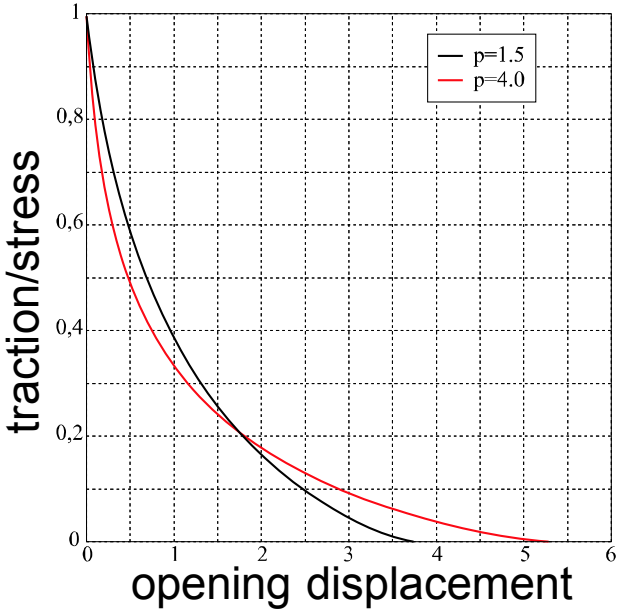
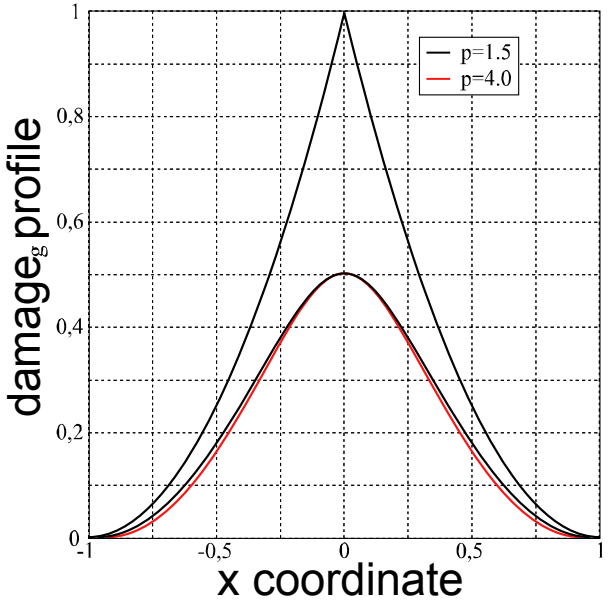
- An elliptic regularization of a cohesive zone model.
- Has a natural critical strain energy criterion.
- Decouples the regularization length scale from the process zone size (though Lorentz argues on physical grounds that the regularization length scale must be smaller than the process zone size).
- Added the now standard decomposition into positive and negative strain energies.
- Irreversibility introduces via $\dot{\phi} \geq 0$.
- However, VERY expensive to solve a nonlinear inequality constrained elliptic PDE, especially for explicitly integrated transient dynamics.

Improved forward modeling of crack propagation in brittle materials

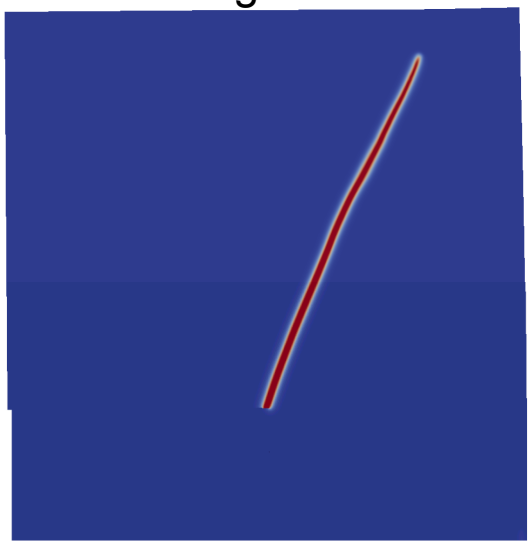
- Developed an explicitly integrated phase-field/gradient-damage model which avoids the usual nonlinear equation solve at each time step (>10 x better performance)
- Extension of the damage model by Lorentz, et al. 2011, which was shown to converge to a cohesive zone model as the length scale approaches 0

$$\eta\dot{d} = h(d)\psi(\boldsymbol{\sigma}) - \frac{3}{4}\frac{G_c}{l} + \frac{3}{8}G_c l\nabla\cdot\nabla d \qquad \rho\ddot{\mathbf{u}} = \nabla\cdot\boldsymbol{\sigma}(\boldsymbol{\epsilon}, d)$$





Damage contour



Cohesive phase-field traction separation law (left) and through 'crack' damage profile (right), p is a parameter of the model. Adapted from Lorentz, et al. 2011.

Kalthoff validation problem achieves the expected crack propagation angle: ~70°

Fracture inverse problems

Sandia National Laboratories

- Most analyses today are forward problem solves: simulate what happens in a given scenario
- However, what is often truly desired is the solution to the corresponding inverse problem
- Example fracture inverse problems:
  - Crack detection/non-destructive evaluation: determine the existence and locations of cracks in a structure
  - Crack forensics: find the loadings applied to a structure which result in the observed failure pattern
  - Material design: optimize the properties and geometry of a structure to resist crack initiation and propagation
- These last examples require accurate fracture prediction

General inverse problem statement as a PDE constrained optimization problem

$$\min_{\boldsymbol{\theta}} \; g(\mathbf{u}, \boldsymbol{\theta})$$

$$\text{s.t.} \;\; \mathcal{L}(\mathbf{u}, \boldsymbol{\theta}) = \mathbf{0}$$

$\mathbf{u}$: state variables
$\boldsymbol{\theta}$: design parameters
$\mathcal{L}$: differential operator
g: quantify of interest

Work in progress

- Collaborating with Sandia analysts to validate phase-field models for lab relevant 3D problems
- Applying the new inverse framework to the new phase-field crack propagation model in order to solve unprecedented crack inverse problems
- SANDIA reports and journal articles:
  - Brittle fracture phase-field modeling of a short-rod specimen (published SANDIA report)
  - A parabolic regularization of cohesive fracture for explicit dynamic crack propagation (to be submitted for peer review)
  - A C++ library for efficient adjoint sensitivities and automatic checkpointing in nonlinear explicit dynamic simulations (in progress SANDIA report)
- References:
  - Convergence of a gradient damage model toward a cohesive zone model, E. Lorentz, et al.
  - Minimal repetition dynamic checkpointing algorithm for unsteady adjoint calculation, Q. Wang, et al.
  - Crack identification by ‘arrival time’ using XFEM and a genetic algorithm, D. Rabinovich, et al.

- Consider a purely local model of damage
- Stress-strain relationship

$$\sigma(x,t) = (1 - d(x,t))C\varepsilon(x,t)$$

- Strain-displacement

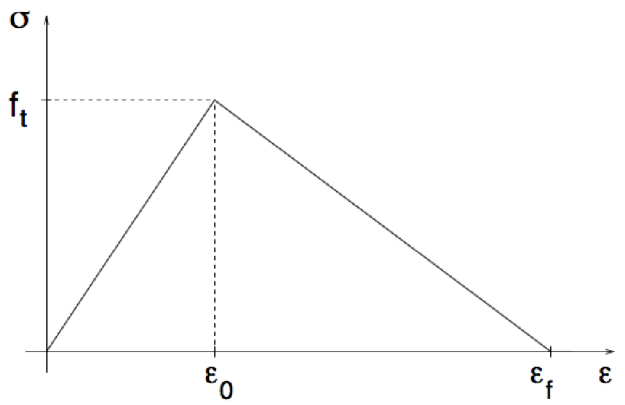
$$\varepsilon(x,t) = r^{s}u(x,t)$$

- Local state variable

$$Y(x,t) = Y(\varepsilon(x,t))$$

- Damage evolution

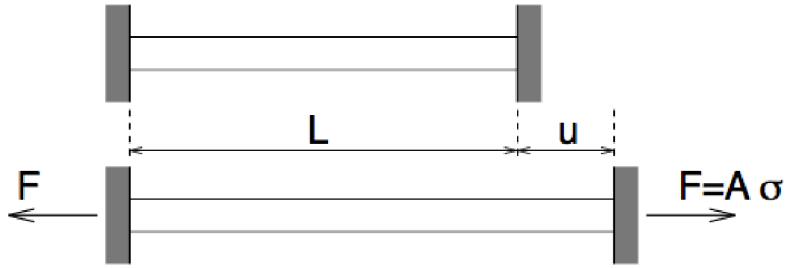
$$d(x,t)) = D\left(\max_{\tau \le t} Y(x,\tau)\right)$$

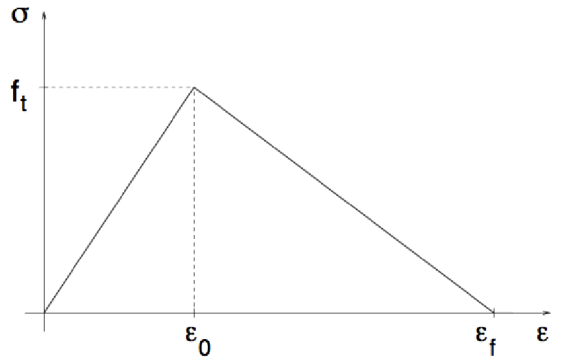


$$d(x,t)) = \frac{Y_f}{Y_f - Y_0}\left(1 - \frac{Y_0}{Y}\right)$$

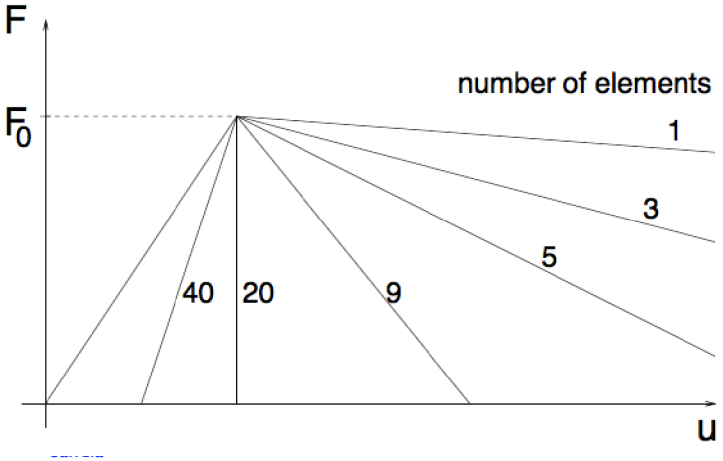
Fundamental Difficulty (cont'd)

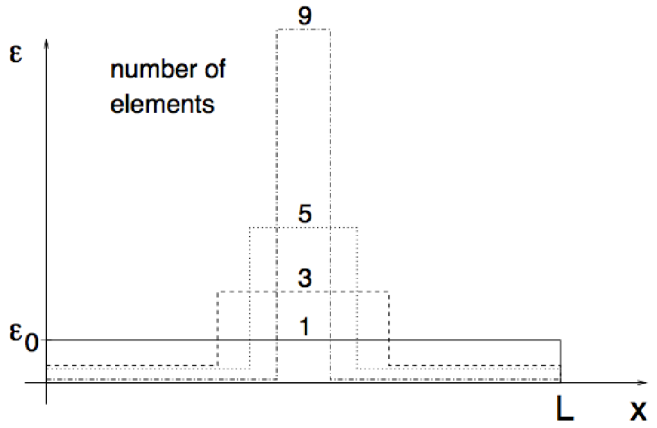
- Consider a 1D bar in tension (quasi-statics)





- Discretized with finite elements (from Jirasek, 2002):





- Basic idea: replace Y with a non-local version:

$$d(x,t)) = D \max_{\bar{x} \le t} \tilde{Y}(x,\bar{x})$$

- Integral-type models (Pijaudier-Cabot and Bazant, 1987) employ a non-local state variable, e.g.

$$\tilde{Y}(x,t) = \int_{V_x} \alpha(x,z)\, Y(z,t)\, dz$$

- Gradient-type models (de Borst et. al, 1995)

$$\tilde{Y}(x,t) - c\Delta \tilde{Y}(x,t) = Y(x,t) \quad \text{in } \Omega \qquad n \cdot \nabla \tilde{Y} = 0 \quad \text{on } \partial\Omega$$

- These two approaches are closely related (Huerta and Pijaudier-Cabot,

- Introduced by Rodriguez-Ferran et al. (2005)
- Stress-strain and strain-displacement:

$$\sigma(x,t) = (1 - d(x,t))C\varepsilon(x,t) \qquad \varepsilon(x,t) = r^s u(x,t)$$

- Non-local displacement:

$$\tilde{u}(x,t) - c\Delta\tilde{u}(x,t) = u(x,t) \quad \text{in } \Omega \qquad \tilde{u} = u \quad \text{on } \partial\Omega$$

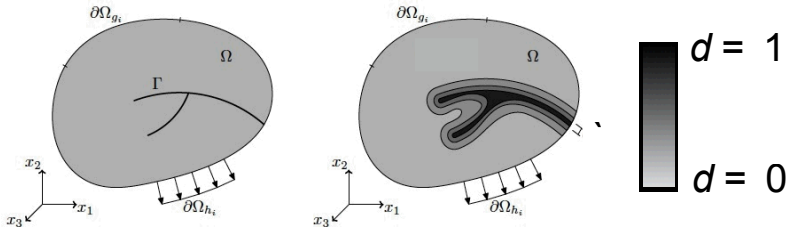
- Non-local strains and state variable:

$$\varepsilon_{NL}(x,t) = r^s \tilde{u}(x,t) \qquad Y_{NL}(x,t) = Y(\varepsilon_{NL}(x,t))$$

- Damage evolution:

$$d(x,t)) = D \max_{\tau \le t} Y_{NL}(x,\tau)$$

Phase-Field Methods for Fracture



- Motivated differently from gradient damage
- Basic idea: replace the sharp discontinuity by a small, but finite zone with sharp gradients in a mathematically consistent manner with a crack-density functional

$$\Gamma \approx \Gamma_l(d) = \int_\Omega \gamma_l(d, \nabla d)\, dV$$

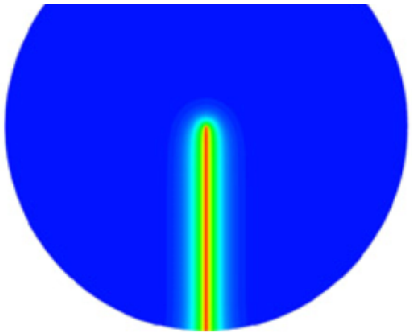
- Fracture energy (for brittle materials):

$$G_c\, dA \approx \int_\Omega G_c\, \gamma_l\, dV$$

Γ

- Common crack-density functional:

$$\gamma_l = \frac{1}{2l} d^2 + \frac{l}{2} |r\ d|^2$$



- Coincides with forms obtained in gamma-convergent regularizations of free-discontinuity problems (Ambrosio and Tortorelli, 1990).

- Minimization principle of diffusive crack topology

$$d(x) = \arg \left\{ \inf_{d \in W} \Gamma_l(d) \right\} \qquad W = \{ d | d(x) = 1 \text{ at } x \in \Gamma \}$$

- Gives rise to Euler-Lagrange equation:

$$d - l^2 \Delta d = 0 \quad \text{in } \Omega \qquad r\ d \cdot n = 0 \quad \text{on } \partial\Omega$$

- Standard total potential energy (elastostatics)

$$\Pi_{pot}(u, \Gamma) = \int_{\Omega} \psi_e(\varepsilon(u))\, dv + \int_{\Gamma} G_c\, da$$

- Phase field regularization

$$\Pi_l(u, d) = \int_{\Omega} \left(g(d)\, \psi_e^+(\varepsilon(u)) + \psi_e^-(\varepsilon(u)) \right) dv + \int_{\Gamma} G_c \gamma_l\, da$$

- Degradation function requirements

$$g : [0, 1] \to [0, 1]$$
$$g'(d) < 0 \quad d \in [0, 1]$$
$$g'(1) = 0$$

$$g = (1 - d)^2$$

- Stress-strain response

$$\sigma(x,t) = (1 - d(x,t))C\varepsilon(x,t)$$

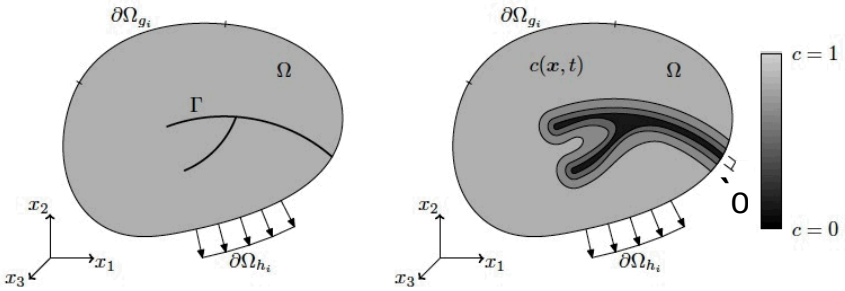
$$\sigma(x,t) = [(1 - d(x,t))^2 + k]C\varepsilon^+(x,t) + C\varepsilon^-(x,t)$$

- Non-local equations

$$\tilde{Y}(x,t) - c\Delta\tilde{Y}(x,t) = Y(x,t) \quad \text{in } \Omega$$

$$d\left[\frac{2lH(\varepsilon^+)}{G_c} + 1\right] - l^2\Delta d = \frac{2lH(\varepsilon^+)}{G_c}$$

- Total potential energy



$$\Psi_{pot}(u,\Gamma) = \int_{\Omega} \psi_e(\varepsilon^s u)\, dv + \int_{\Gamma} G_c\, da$$

- Approximate the fracture energy with a crack density functional (Miehe et al. 2010)

$$\int_{\Gamma} G_c\, da \approx \int_{\Omega} G_c \gamma_{c,n}\, dv \qquad \gamma_{c,n} = \frac{1}{4\ell_0}\left[(c-1)^2 + 4\ell_0^2 |\nabla c|^2\right]$$

- Implementation based on Borden et al. (2012)

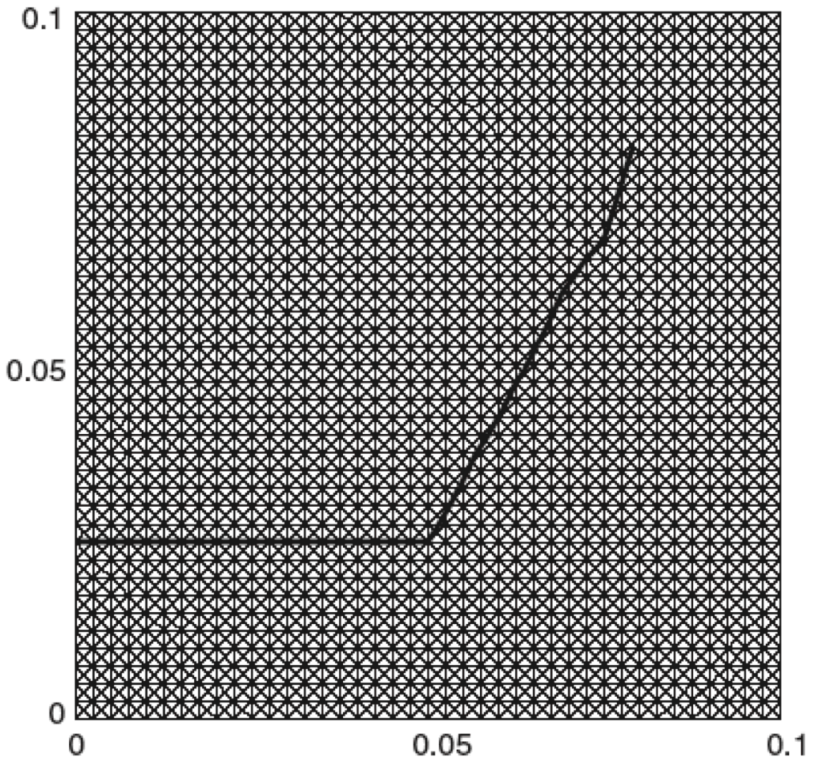
$$c = 1 - d$$

- Strong form for dynamics:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \ddot{u}_i \quad \text{on } \Omega \times]0, T[$$

$$\left(\frac{4 \ell_0 (1-k) \mathcal{H}^+}{G_c} + 1 \right) c - 4 \ell_0^2 \frac{\partial^2 c}{\partial x_i^2} = 1 \quad \text{on } \Omega \times]0, T[$$

$$\sigma_{ij} = [(1-k)c^2 + k] \frac{\partial \psi_e^+}{\partial \varepsilon_{ij}} + \frac{\partial \psi_e^-}{\partial \varepsilon_{ij}}$$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0
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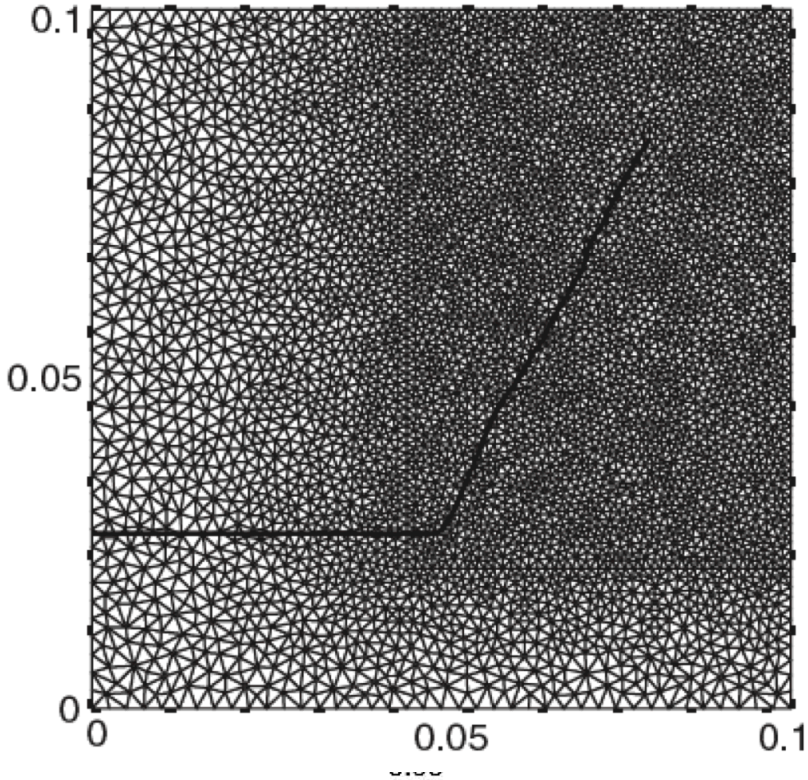
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Kalthoff Problem