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Verification & Validation Study for the Modeling of a Bi-material Composite in a Double Cantilever Beam Test

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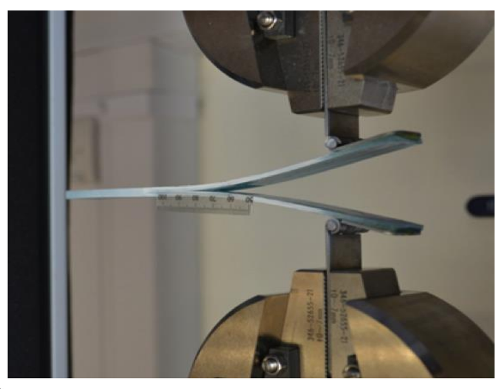
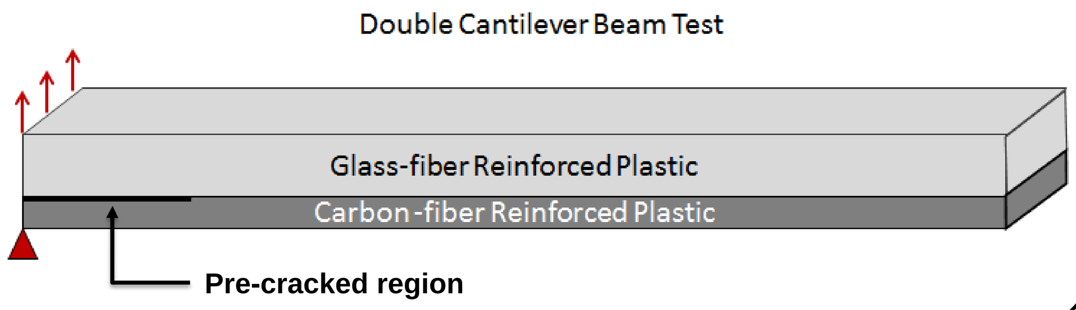
Project Statement

The Verification and Validation process is critical to assessing the accuracy of Finite Element simulations. This study aims at using a Mesh Convergence Study to verify that the Double Cantilever Beam (DCB) model accurately solves for the peak load. A Sensitivity Analysis and Uncertainty Quantification are used to determine which material parameters are most crucial to the fracture toughness of the composite and the uncertainty of the simulated output response from the input parameters.





Model Development

DCB Model Design

- Bi-material interface with an initial crack length
- Thouless-Parmigiani Cohesive Zone model used at the interface of the two composites

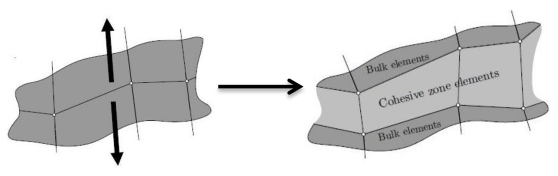
Boundary Conditions

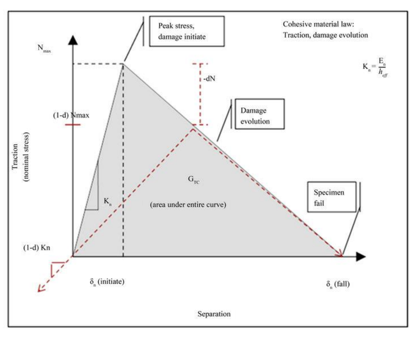
- Initial cooling to form residual stresses
- Prescribed velocity on the top edge of the GFRP
- Fixed displacement on the bottom edge of the CFRP
- Axisymmetric in y-direction

Composite Material Model: Elastic-Orthotropic

Cohesive Zone Material Model

- Utilizes Traction Separation Laws to model the delamination process
- Cohesive elements are inserted once the peak stress is exceeded
  - crack is propagated at the interface





Traction Separation Model

Mesh Convergence Study

Overview

- Mesh refinement can improve the accuracy but severely increase the computational time
  - mesh size selection is a crucial parameter to consider
- Theoretically, as the mesh size reduces to zero, the simulation results should converge to a single continuum value
- Richardson's Extrapolation is used to predict the continuum solution and the resulting error from the selected mesh size

Method: Richardson's Extrapolation

- Uses discrete solutions for the peak load, f_k, and mesh size, h_k
- Order of convergence, p, is approximated based on the behavior of the error coefficients, g_i

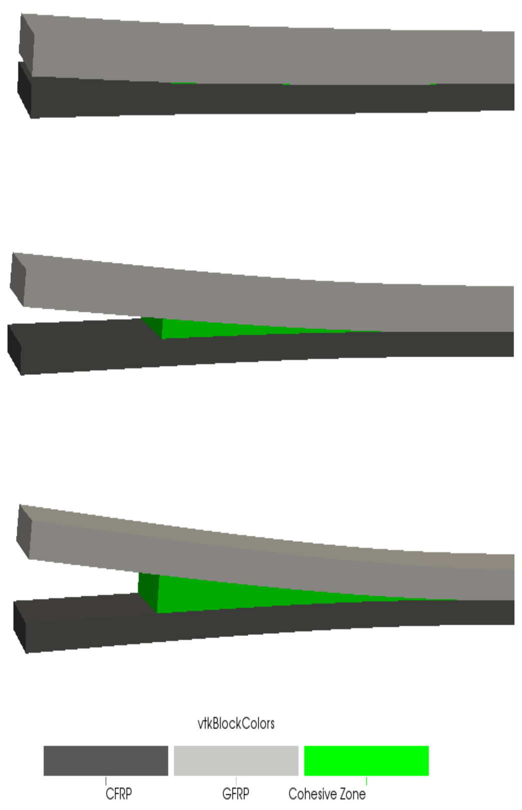
Equations

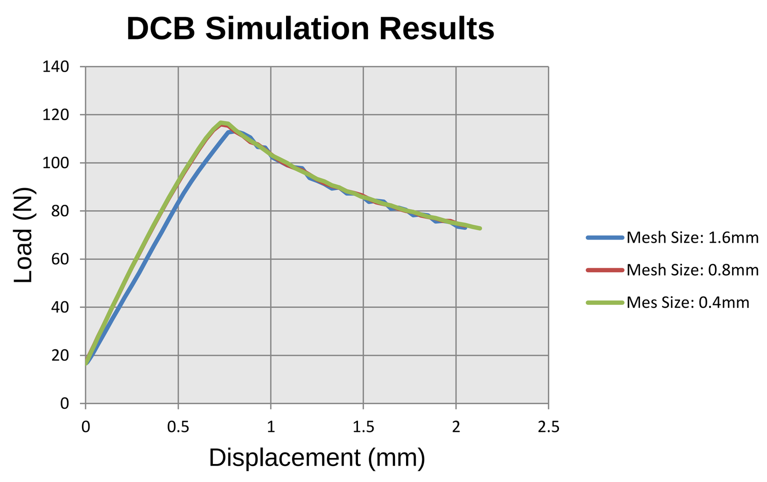
$$f_k = f_{exact} + g_1 h_k + g_2 h_k^2 + g_3 h_k^3 + \cdots$$

$$f_{exact} \approx f_1 + \frac{f_1 - f_2}{r_{12}^p - 1}, \text{ where } r_{12} = h_2/h_1$$

$$\frac{\varepsilon_{23}}{r_{23}^p - 1} = r_{12}^p \left(\frac{\varepsilon_{12}}{r_{12}^p - 1}\right), \text{ where } \varepsilon_{12} = f_1 - f_2$$

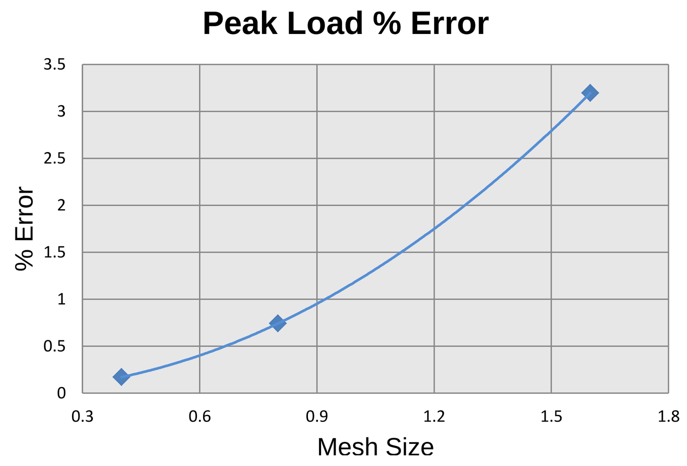
Constant Grid Refinement: $\bar{p} = \frac{\ln(\varepsilon_{23}/\varepsilon_{12})}{\ln(r)}$

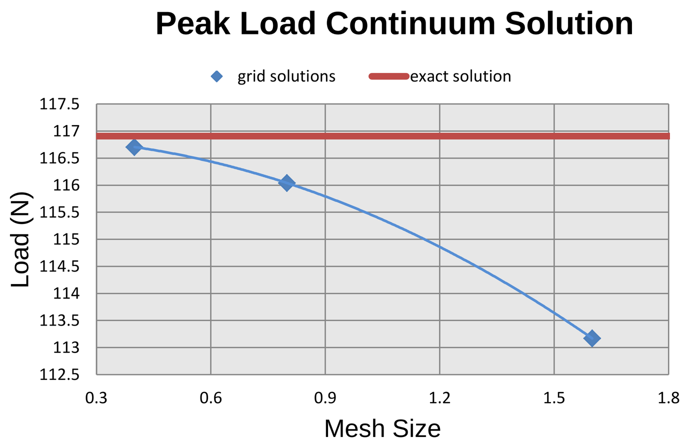




Mesh Size	Peak Load (N)	Percent Error
1.6 mm	113.17	3.196%
0.8 mm	116.04	0.742%
0.4 mm	116.71	0.172%

Order of Convergence (p)	Continuum Value for Peak Load (N)
2.106	116.91





Sensitivity Analysis

Overview

- Each orthotropic composite possess 9 independent material properties
  - Difficult and expensive to experimentally determine
- Generates a quantitative measure of the influence each material parameter has to the overall performance of the composite

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \varepsilon_{yz}\\ \varepsilon_{zx}\\ \varepsilon_{xy}\end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G_{yz}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G_{zx}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G_{xy}} \end{bmatrix} \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{zx}\\ \sigma_{xy}\end{bmatrix}$$

Orthotropic Stiffness Matrix

Method: Box Behnken Design (BBD)

- Sampling method of a k-dimensional design space to recognize trends between input parameters and simulated outputs
  - N: number of required simulations
  - k : number of parameters

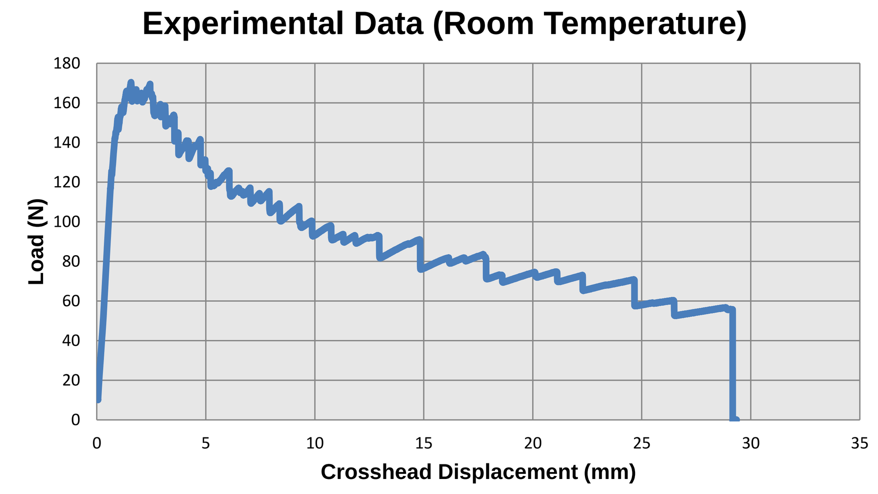
$$N = 2k(k-1) + 1$$

Material Parameters	
CFRP, GFRP	Cohesive Zone
E_{11}, E_{22}, E_{33}	Peak Normal Traction
G_{12}, G_{13}, G_{23}	Peak Tangential Traction
ν_{12}, ν_{13}, ν_{23}	Normal Separation Failure
α_{11}, α_{22}, α_{33}	Tangential Separation Failure

Statistical Analysis & Uncertainty Quantification

Multi-way ANOVA

- Analyzes the individual parameter sensitivity
- Determines which input parameters have a statistically significant effect on the output response



Output Response Uncertainty

- Range of input parameters produced from literature or numerical analyses
- Simulate peak loads from varying input parameter range
- Anderson-Darling test to determine distribution type
- Comparison with experimental results to quantify the error

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