Comparison of Nonlinear System Identification Methods for Free Decay Measurements with Application to MEMS Devices

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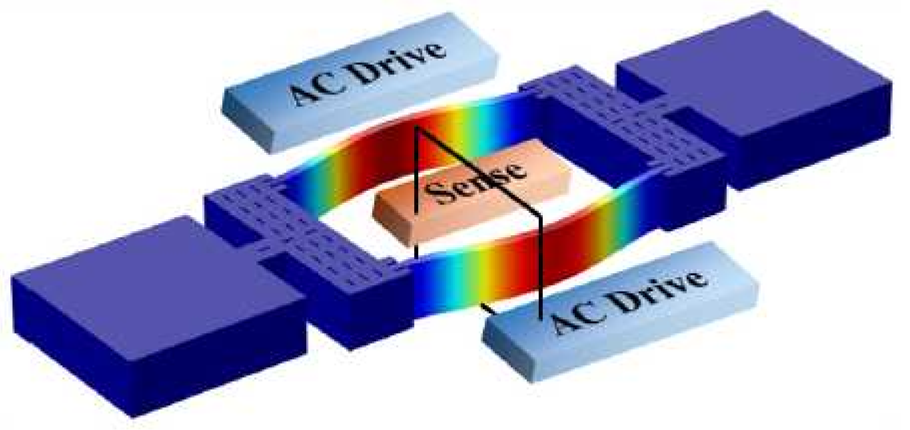
Objective

- Assess efficacy of selected Nonlinear System Identification methods and understand their range of applicability
- Investigation of simulated test cases as well as experimental data

Motivation

- A number of methods for nonlinear system identification in both time and frequency domain has been developed in the past [1]
- These methods have application to many systems, ranging from **microscale devices** to macroscale systems, sometimes with uncertain results
- The goal is to extend existing nonlinear system identification methods to MEMS
- Methods have been selected to deal well with free decay measurements



MEMS Device under study [5]
Double-anchored double-ended-tuning-force resonator

Selected Methods

- Restoring Force Surface Method (RFS)

$$m\ddot{x} + f(x,\dot{x}) = F(t) \quad \rightarrow \quad f(x,\dot{x}) = F(t) - m\ddot{x}$$

- Hilbert Transform (HT)

$$z(t) = x(t) + i\tilde{x}(t) = a(t)e^{\theta(t)}, \omega(t) = \frac{d\theta(t)}{dt}$$

- Zero-Crossing Methods (ZC)

$$a(t_i) = max|x|, \quad \omega(t_i) = 2\pi(t_i - t_{i-1})^{-1} = 2\pi T_i^{-1}$$

- Direct quadrature (DQ)

$$a(t) = spline(max|x|), \quad \omega(t) = \frac{d[arccos(FM)]}{dt}$$

- Short-Time Fourier Transform (STFT)

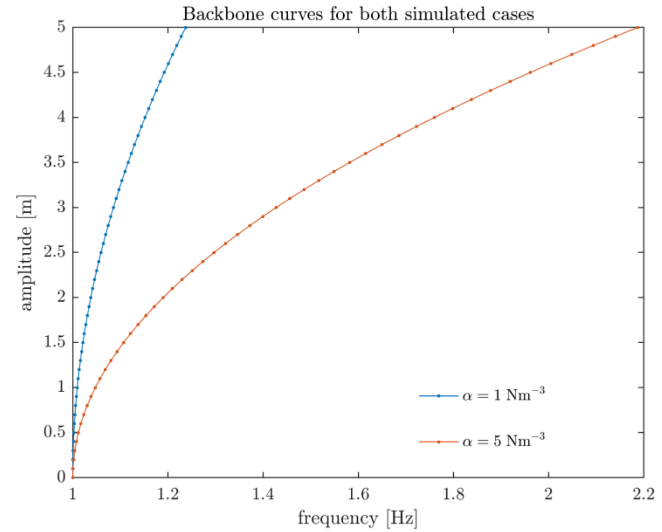
Simulated Case

- The chosen Duffing oscillator is widely used to validate identification methods

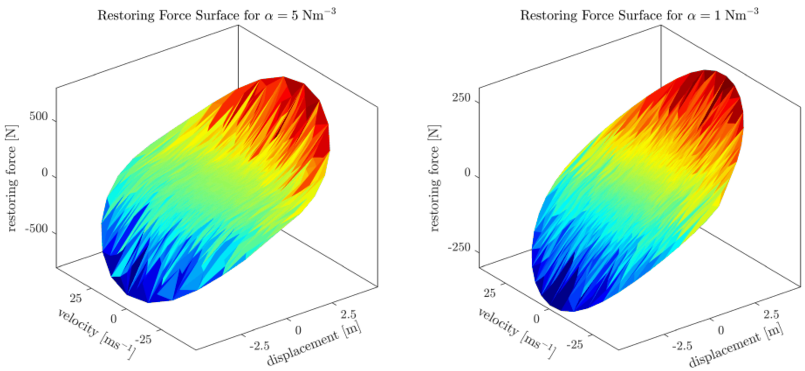
$$m\ddot{x} + c\dot{x} + kx + \alpha x^3 = 0$$

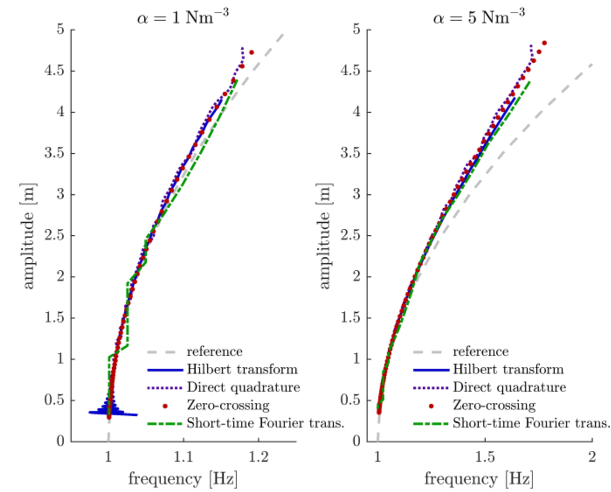
- Parameters:

$$m = 1kg, \quad c = 0.2Nm^{-1}s^{-1}, \quad k = (2\pi)^2 Nm^{-1}$$
$$x_0 = 5m, \quad \dot{x}_0 = 0m, \quad f_s = 50Hz$$
$$\alpha_1 = 5Nm^{-3}, \quad \alpha_2 = 1Nm^{-3}$$

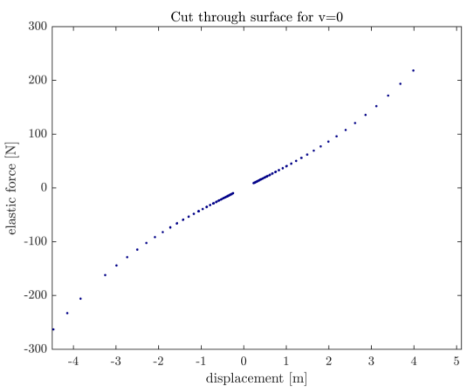


Results for simulated cases



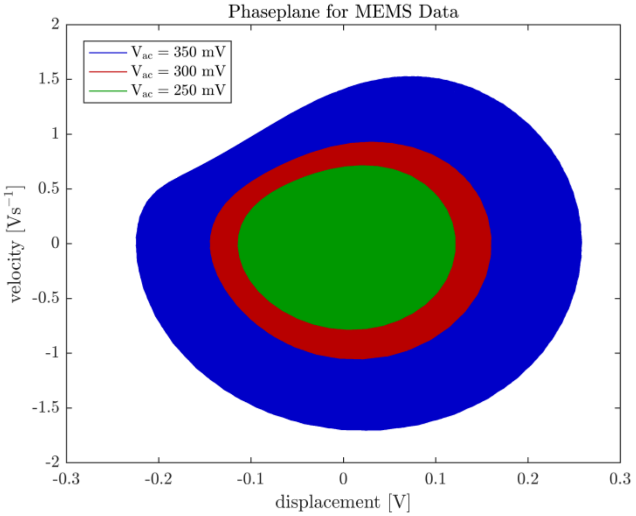


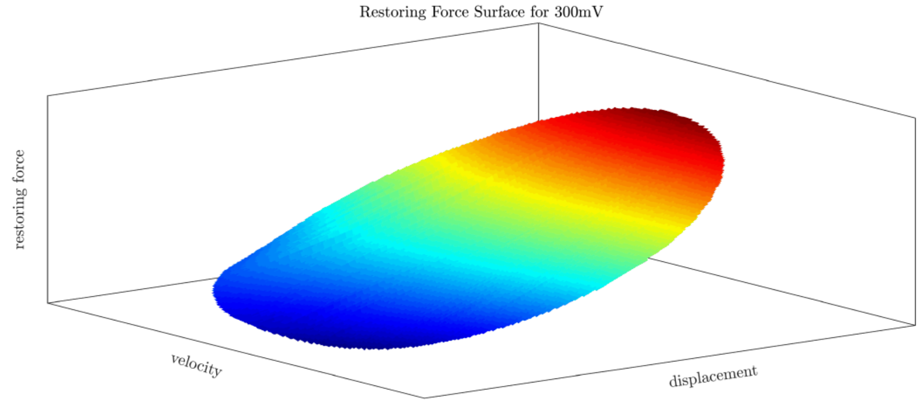
Accuracy of methods

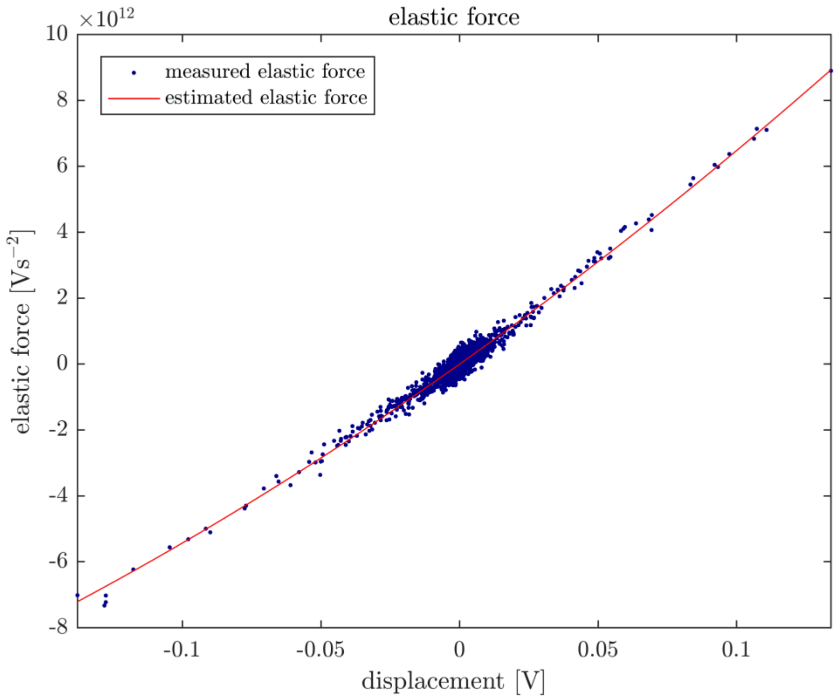


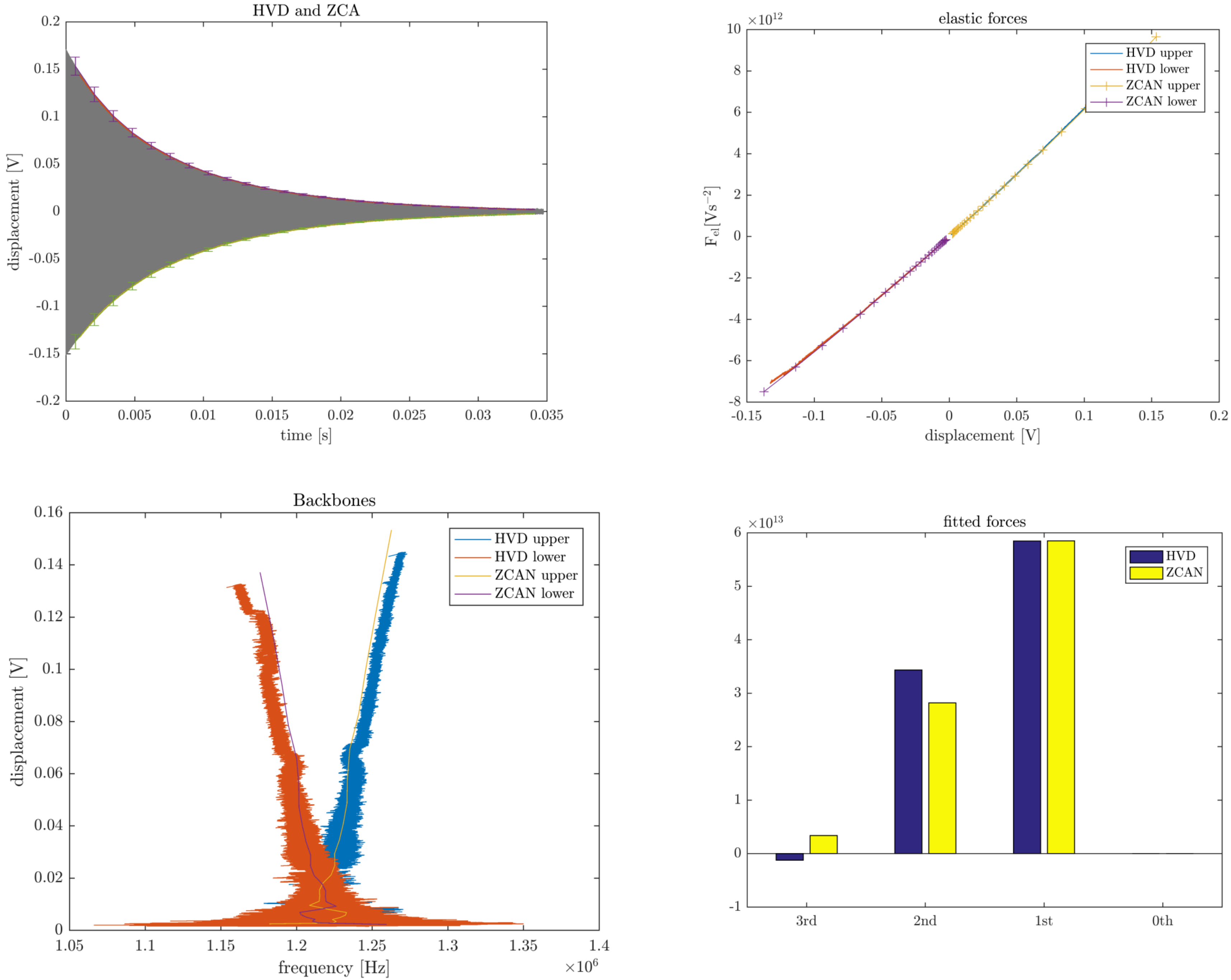
	Stronger non-linearity			Weaker non-linearity		
	k_err [%]	alp_err [%]	c_err [%]	k_err [%]	alp_err [%]	c_err [%]
Restoring Force Surface (RFS)	10.59	-8.04	2.45	0.78	-4	-0.5
Hilbert transform (HT)	-0.93	-23.4	-2.5	-0.09	-26	-0.8
Direct quadrature (DQ)	1.62	-29.8	-2.6	0.10	-28	-1.2
Zero-crossing (ZC)	0.76	-27.2	19	-0.07	-27	7.45
Short-time Fourier transform (STF)	-8.25	-13.4	-2.2	-8.86	10.27	0.35
Damping estimation (DAM)	-----	-----	-2.1	------	-------	-1
Zero-crossing averaged (ZCN)	0.86	-27.4	14.4	0.20	-27	4.75
Zero-crossing asymmetric (ZCA)	-1.46	-22.4	-5	0.05	-25	-3.8
ZCA averaged (ZCAN)	0.81	-27.2	-15	0.15	-26	-18.1
Hilbert Vibration Decomposition (HVD)	0.20	-14.6	----	0.28	-26	-18.1

Results for MEMS Data








HVD and ZCA
displacement [V]
time [s]
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
elastic forces
$\times 10^{12}$
F_{el} [Vs^{-2}]
displacement [V]
HVD upper
HVD lower
ZCAN upper
ZCAN lower
10
8
6
4
2
0
-2
-4
-6
-8
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Backbones
displacement [V]
frequency [Hz]
$\times 10^{6}$
HVD upper
HVD lower
ZCAN upper
ZCAN lower
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
fitted forces
$\times 10^{13}$
HVD
ZCAN
6
5
4
3
2
1
0
-1
3rd
2nd
1st
0th

Conclusions

- Simulation
  - Detection as well as characterization of non-linearity and parameter estimation work with all discussed methods for weak non-linearities
  - Differences in coefficients for RFS investigated for the strongly non-linear case occur due to the need for differentiation and lacking robustness of estimation algorithm
  - Differences for the other methods are caused by the fact that they only work for ‘weak’ non-linearities and some signal processing issues occur
  - In presence of measuring noise with low SNR additional smoothing has to be applied to the data
- Experiment
  - In contrary to [6] the system appears to be asymmetric (based on phase plane investigation)
  - Asymmetry has not been investigated for MEMS Devices
  - Time-frequency methods fail due to the asymmetry
  - RFS displays asymmetry in displacement
  - HVD and modified ZC can detect the asymmetry

References

1. Kerschen, G., Worden, K., Vakakis, A.F. and Golinval, J.C., *Past, present and future of nonlinear system identification in structural dynamics*. *Mechanical Systems and Signal Processing*. 2006. 20:p: 505-592.
2. Masri, S.F., and Caughey, T.K., *A Nonparametric Identification Technique for Nonlinear Dynamic Problems.* Journal of Applied Mechanics, 1979. 46: p. 433-447.
3. Worden, K. and Tomlinson, G.R., *Nonlinearity in Structural Dynamics: Detection, Identification and Modelling, Institute of Physics Publishing, Bristol and Philadelphia, 2001*
4. Feldman, M., *Non-Linear System Vibration Analysis using Hilbert Transform – I. Free Vibration Analysis Method 'FREEVIB'*, Mechanical Systems and Signal Processing. 1994. 8:p: 119-127.
4. Polunin, P., Yang, Y., Dykman, M.I., Kenny, T.K. and Shaw, S.W., *Characterization of MEMS Resonator Nonlinearities Using the Ringdown Response*, Journal Of Microelectromechanical Systems. 2016. 2:p: 297-303.

Suggested Layout of slides:

Title (slide 1) Objective (slide 3)	Authors (Slide 2)
Motivation (slide 4)	Experiment (slide 5)
Simulated Case (slide 6)	Result for Simulated Cases (slide 7)
Result for MEMS (slide 8&9)	Conclusions (slide 10)

Please fit references (Slide 11) into on of the bottom corners may need to shrink them to fit. We are unsure, if this layout fits for the poster, just adjust in case it doesn't