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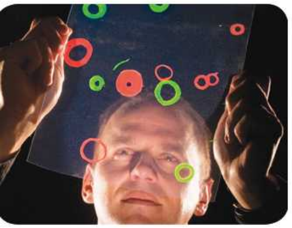

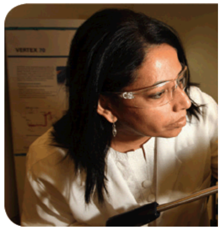

Accelerating and automatic tuning for Progressive Hedging

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International Conference on Stochastic Programming (ICSP)
27 June 2016

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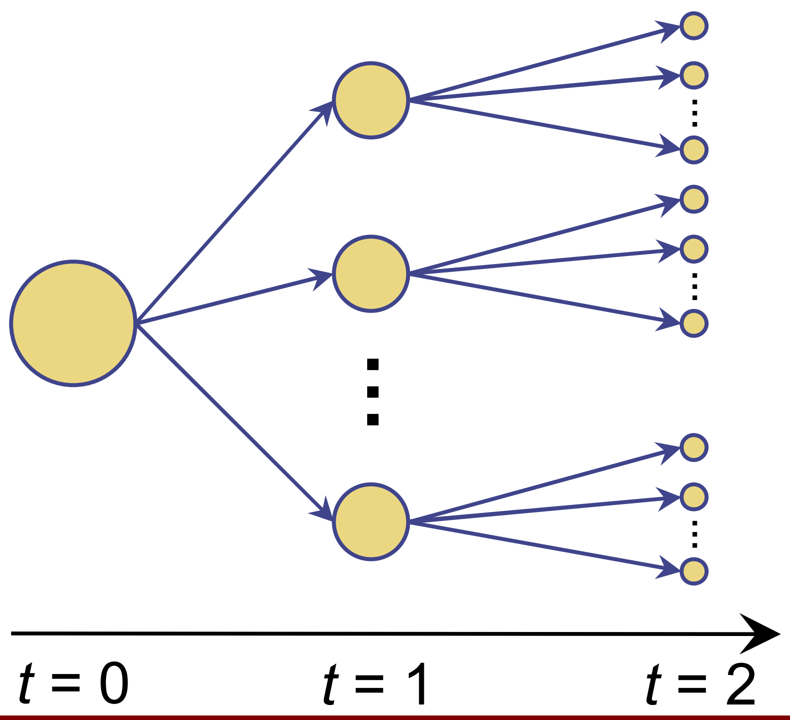
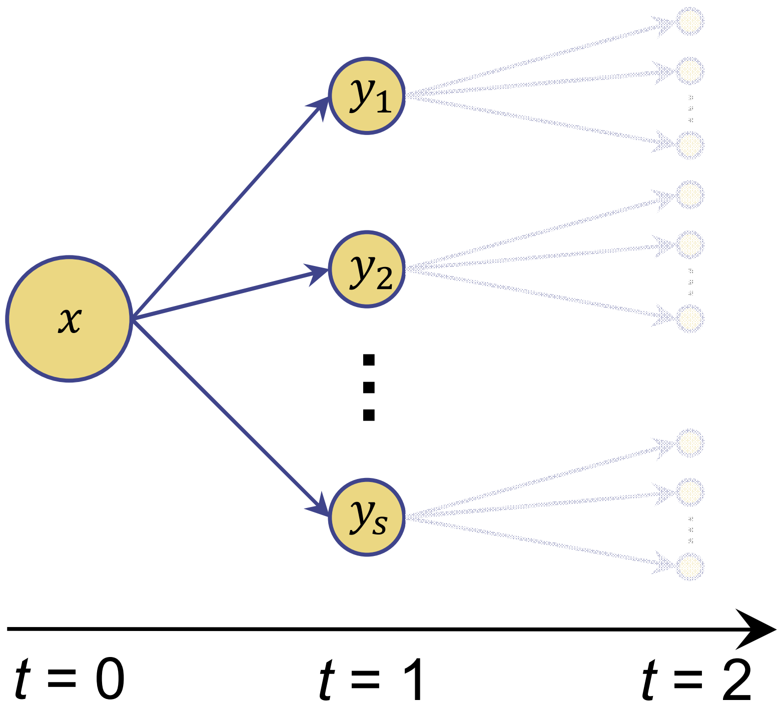
A graphical view of SP decomposition

- Stochastic programming is explicitly built on a tree of scenarios where nodes represent opportunities for decisions
  - The monolithic problem (extensive form) is typically intractable
  - Decomposition and iterative convergence the workhorse of SP



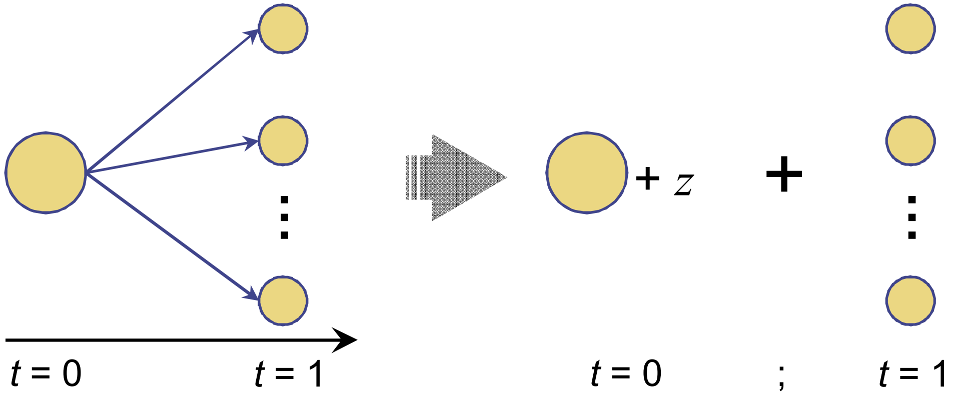
A graphical view of SP decomposition

- Stochastic programming is explicitly built on a tree of scenarios where nodes represent opportunities for decisions
  - The monolithic problem (extensive form) is typically intractable
  - Decomposition and iterative convergence the workhorse of SP
  - For convenience, let us restrict ourselves to 2-stage problems



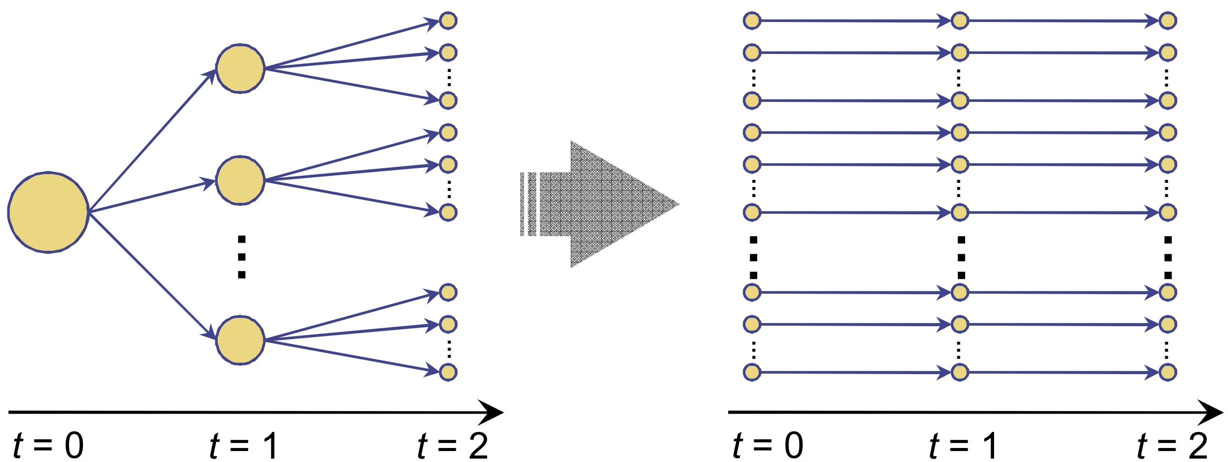
Benders Decomposition: splitting time

- Benders Decomposition splits the problem “in time” into
  - A *master* problem that (initially) has the 1st-stage variables
    - (Plus any discrete variables from the 2nd stage)
    - Plus a proxy variable *z* to capture impact of subproblems
  - Independent *subproblems*, 1 per scenario, for the 2nd-stage variables
    - Must be continuous
- Iteratively solve master to obtain upper bound (incumbent)
  - Solve subproblems to obtain lower bound + cuts for the master
- Challenges
  - Discrete recourse decisions
  - Master problem “bloat”
  - Multistage problems
    - (bookkeeping)



Progressive Hedging: splitting anticipativity

- Progressive Hedging splits the problem by scenarios
  - "No" master problem
  - One subproblem per scenario
  - Relax nonanticipativity constraints
- Iteratively converge the stage nonanticipativity constraints
  - Penalize decision variable value by weight $w^T x$
  - Penalize deviation from average, $\rho \, ||x - \bar{x}||^2$
  - Update w using ρ



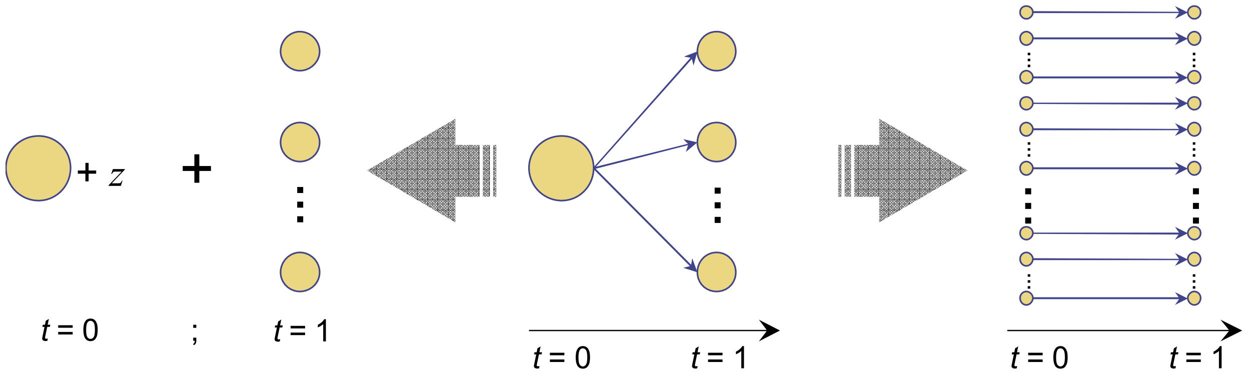
PH: Benefits and open challenges

- PH avoids many of the challenges experienced by Benders
  - No restriction on stage variable domains (discrete 2^{nd} stage OK)
  - "Trivially" extends to multistage problems
  - No master problem, no cuts generated: no problem "bloat"

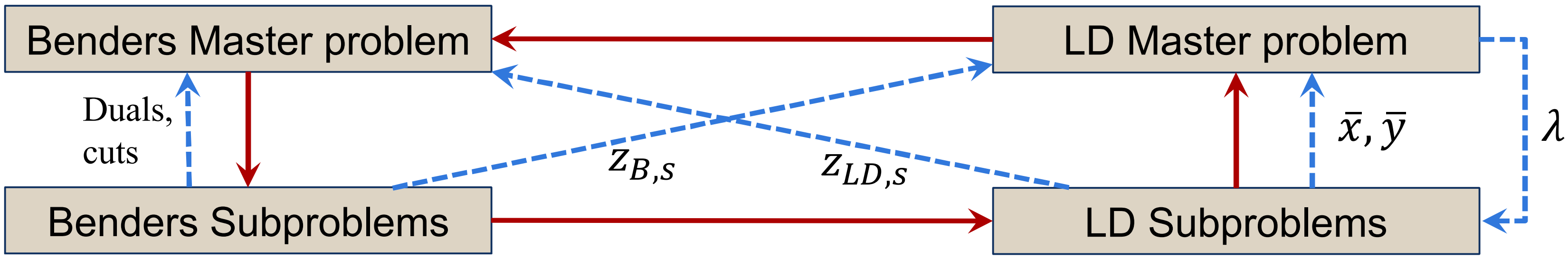
- BUT…
  - Not provably convergent for the discrete case
    - …although bounds exist on the quality of solution you get
  - "Infeasible path" algorithm
  - Discrete variable cycling
  - Slow convergence due to "holdout" scenarios
  - How to choose ρ???

We will attempt to address these challenges

PH + Benders: orthogonal decompositions

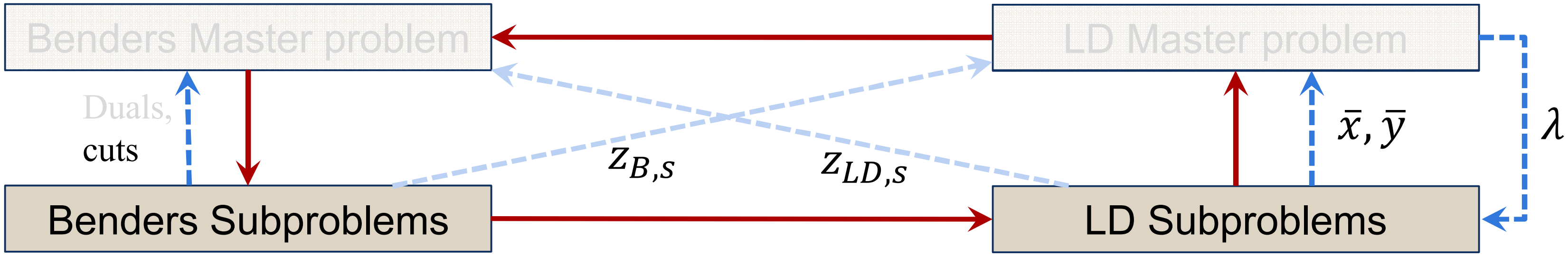


- Can we leverage ideas from each to accelerate the other?
  - Related: Cross Decomposition (Lagrangean Decomposition + Benders)
    - [Van Roy 1983; Holmberg, 1990; Mitra,et al. 2016]

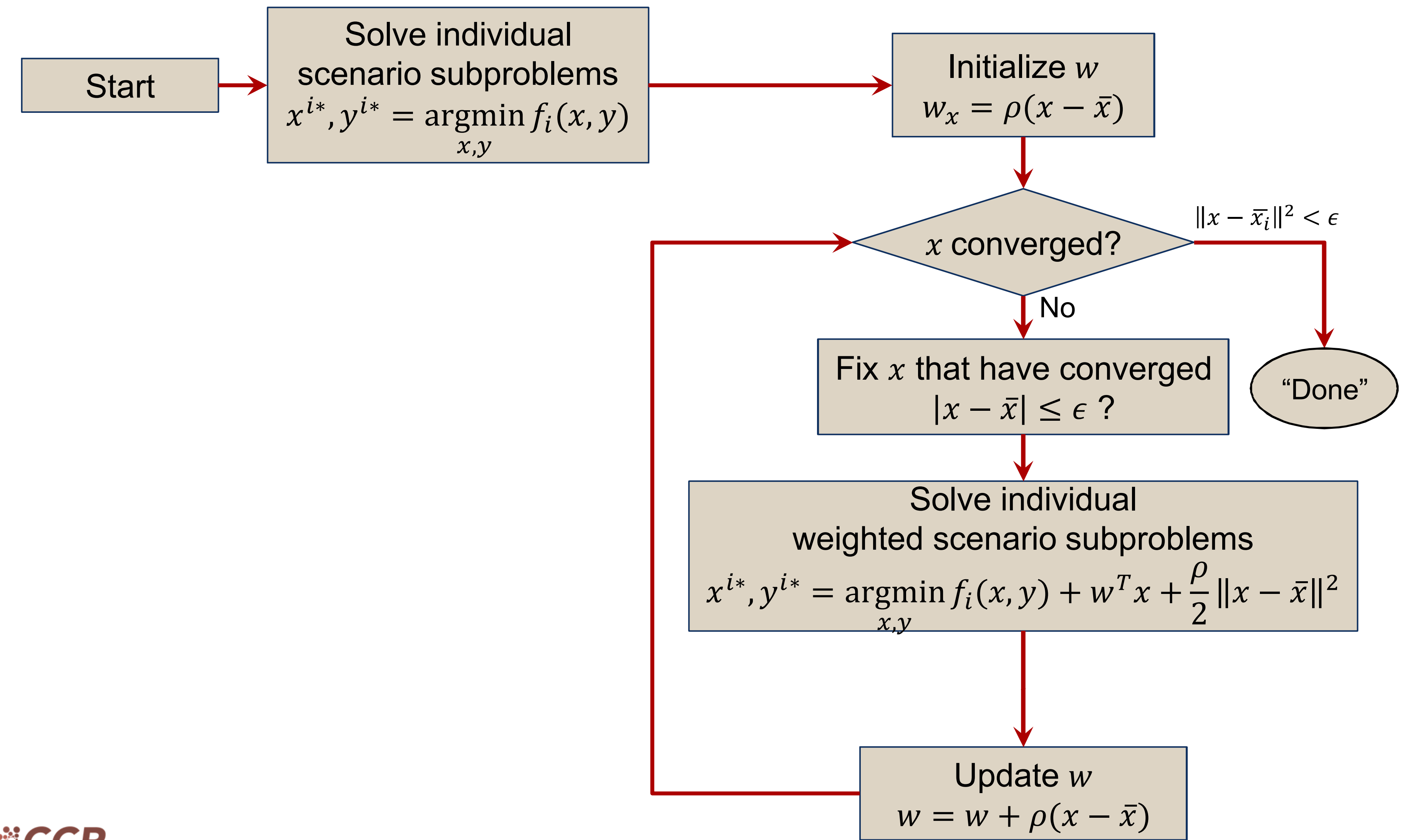


Cross Decomposition for PH?

- Challenges with naïve Cross Decomposition for PH
  - No LD Master problem
  - Discrete second stage variables
  - PH subproblems do not provide proper Lagrangean bounds
    - Except at *iteration 0*



Progressive Hedging: the algorithm



Improving PH: borrowing from Benders

- Consider the Benders "feasibility" cut:
  - Given x^* computed by the RMP, if (dual) subproblem is unbounded, add a cut determined by an extreme ray in the dual space to the RMP
- In PH, a similar operation would be fix the values of x in subproblem f_j to the values computed by subproblem f_i:

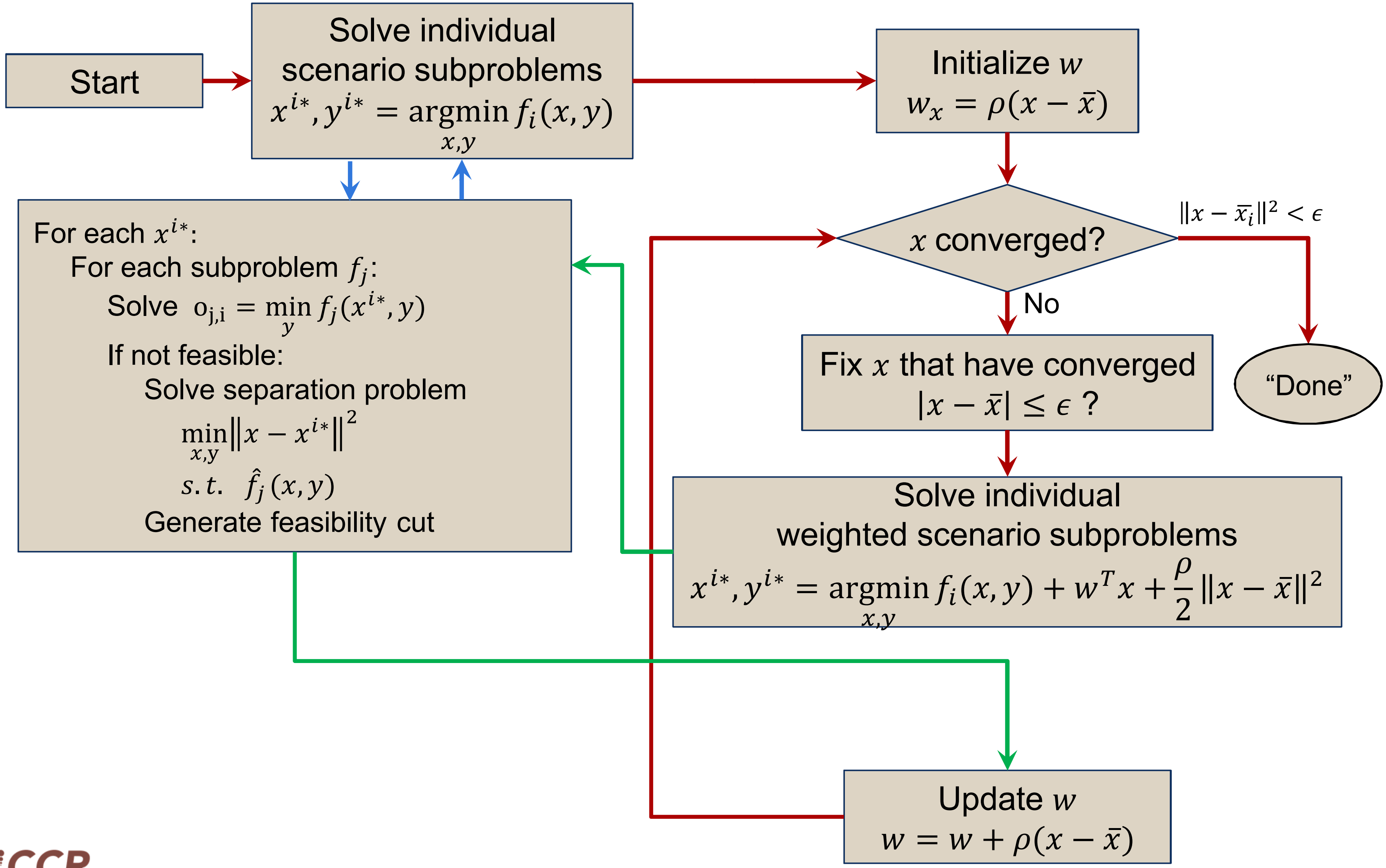
$$\min_{y} f_j(x^{i*}, y)$$

  - If the problem is infeasible, then we can solve a separation problem (in the primal space) to determine a valid cut in the 1st-stage variables:

$$\min_{x,y} \left\| x - x^{i*} \right\|^2$$
$$s.t. \quad \hat{f}_j(x, y)$$

  - Notes:
    - $\hat{f}_j$ is the continuous relaxation of f_j → not guaranteed to generate a cut
    - The resulting cut is valid for *all* scenario subproblems
    - $\sum p_i f_i(x^{i*}, y^{i*})$ for initial scenario solves ($w = 0$) gives Lagrangian bound

PH + feasibility



Case study: stochastic network flow

- 2-stage stochastic network flow from [Watson & Woodruff, 2011]:
  - 1st stage variables:
    - $x_a \; \forall \; a \in Arcs$; $\{x_a | \; x_a \in R, 0 \le x_a \le x_a^{UB}\}$ Capacity of Arc a
    - $b_a^0 \; \forall \; a \in Arcs$; $\mathrm{b}_\mathrm{a}^0 \in \{0,1\}$ Arc a is available
  - 2nd stage variables:
    - $y_a^s \; \forall \; a \in Arcs, s \in Scenarios$; $\{y_a^s \mid y_a^s \in R, 0 \le y_a^s \le x_a\}$ Flow across Arc a in scenario s
    - $b_a^s \; \forall \; a \in Arcs, s \in Scenarios$; $\mathrm{b}_\mathrm{a}^s \in \{0, b_a^s\}$ Arc a is in use for scenario s
- PH ($\rho = 100$, with "Watson-Woodruff" extensions)
  - Significant cycling (no convergence after 1000 iterations)
- PH ($\rho = 100$, with WW extensions, with feasibility cuts)
  - Converges in 42 iterations (objective: 164426, 2726 seconds)
  - 6 preliminary cut passes: raises Lagrangian LB 135085 → 148656

Case study: UC + *N-1* + switching

- 2-stage unit commitment model for the electric power grid
  - 24-hour horizon, 1-hour commitment intervals
  - Explicitly include *N-1* analysis (loss of any 1 generator / non-radial line)
    - Each contingency modeled as a no-cost recourse scenario
    - Line switching (opening /closing a line) in 1st and 2nd stages

- Case 1: 5 busses, 7 generators (13 scenarios):
  - Optimal solution (extensive form): 19.9756
  - Default PH ($\rho = 1$):
    - 17 iterations, objective = 22.9997, total time 123 seconds
  - PH ($\rho = 1$) + Feasibility cuts:
    - 3 feasibility cut iterations at PH iteration 0
      - Improved Lagrangean bound from 19.7 to 19.88
    - 12 iterations, objective = 20.11, total time 568 seconds

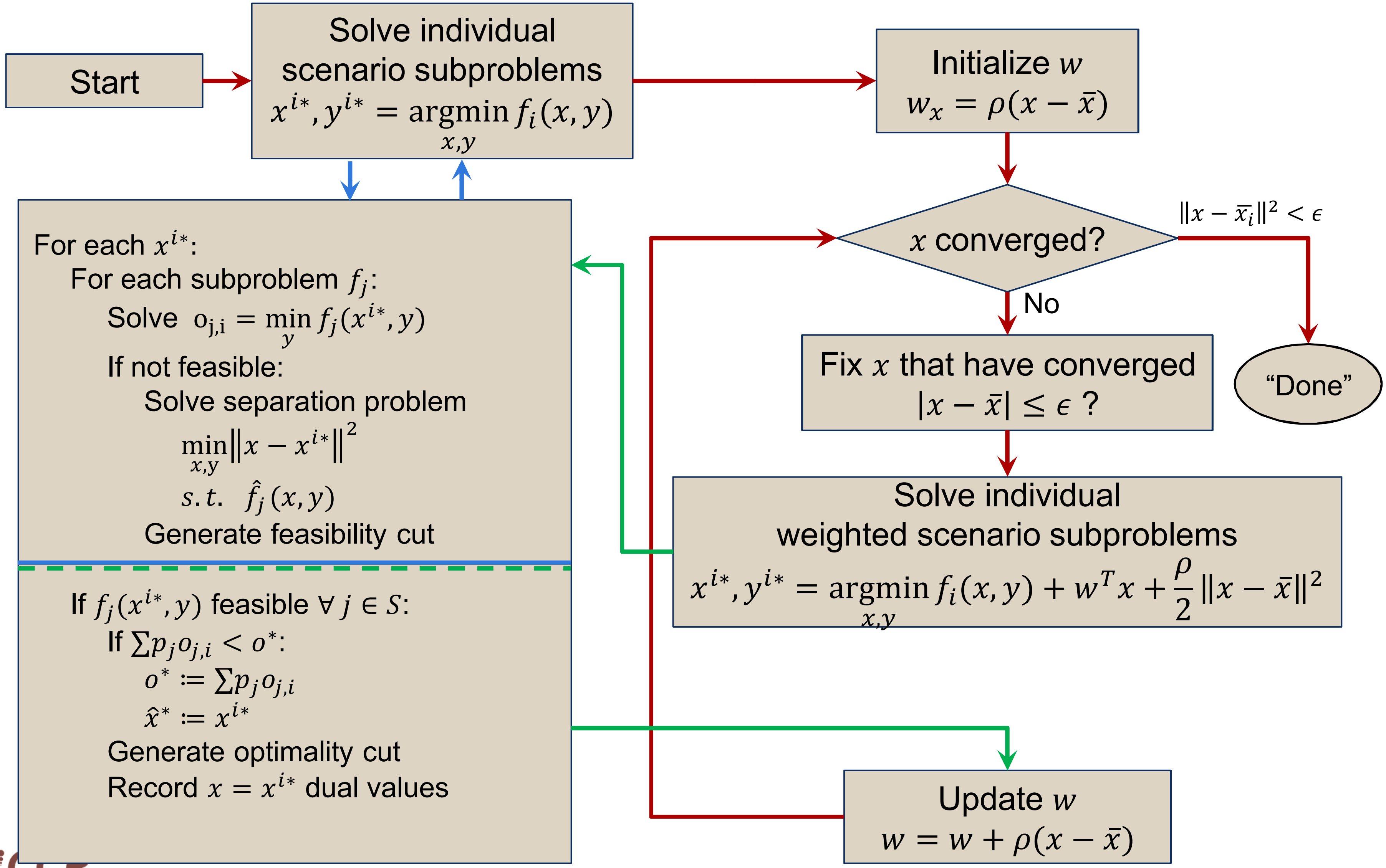
Improving PH: borrowing from Benders

- What happens if the subproblem $\min_{y} f_i(x^{j*}, y)$ is feasible?
  - The 1st stage decisions are valid in this scenario
  - If x^{j*} is valid in ALL scenarios, then x^{j*} satisfies nonanticipativity and the expectation of the subproblems forms a valid upper bound

$$E[f(x^*, y)] \leq E\left[f\left(x^{j*}, y\right)\right]$$

- If the first stage variables are all discrete
  - Repeating this process for all x^{i*} and identify additional nonanticipative solutions, then the upper bound can be used to generate *optimality cuts* to exclude sub-optimal solutions
  - These cuts are also valid on all scenario subproblems

PH + feasibility + optimality cuts



Case Studies: *farmer*

- 3-scenario farmer problem [Birge & Louveaux]
  - Integer acreage allocations
  - Cplex 12.5
    - PH ($\rho = 1$): 297 iterations, 21.55 seconds: objective = -108390
    - PH + optimality: 47 iterations, 8.75 seconds: objective = -108390

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- Very sensitive to the solver!
  - Gurobi 6.0.4
    - PH ($\rho = 1$): 39 iterations, 1.09 seconds: objective = -108390
    - PH + optimality: 154 iterations, 16.4 seconds: objective = -108390

Case Studies: *farmer*

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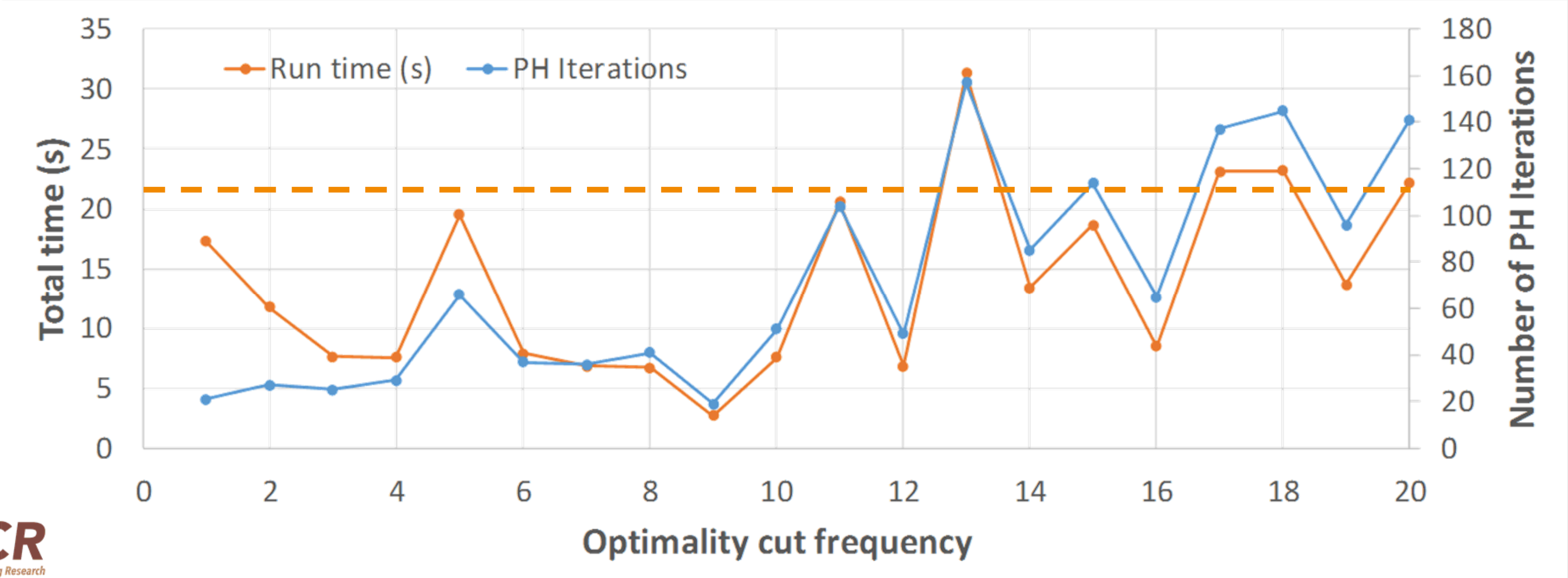


Case study: Stochastic Unit Commitment

- Unit commitment under demand uncertainty
  - 24 hour horizon, 1-hour intervals

- Case 1: 5 busses, 7 generators, 3 load scenarios
  - Optimal solution (extensive form): 348.98
  - Default PH ($\rho = 10$):
    - Significant cycling (no convergence after 100 iterations)
  - PH ($\rho = 10$) + Optimality cuts:
    - 20 iterations, objective = 348.9835

Improving PH: setting ρ

- How do we get "good" values of ρ?
  - Currently: experimentation
  - Challenge: ρ is problem dependent
    - Too low and PH never converges
    - Too high and PH rapidly converges to suboptimal solution
  - Hint: scale relative to cost of each variable [Watson & Woodruff, 2011]

$$\rho_i \propto \frac{C_i}{\left|x_i^{min} - x_i^{max} + 1\right|}$$

- We can get good cost estimates from the subproblem duals
  - When we evaluate $f_j(x^{i*}, y)$, record the duals for $x = x^{i*}$
  - Compute average duals weighted relative to scenario probability

Case Studies: *farmer*

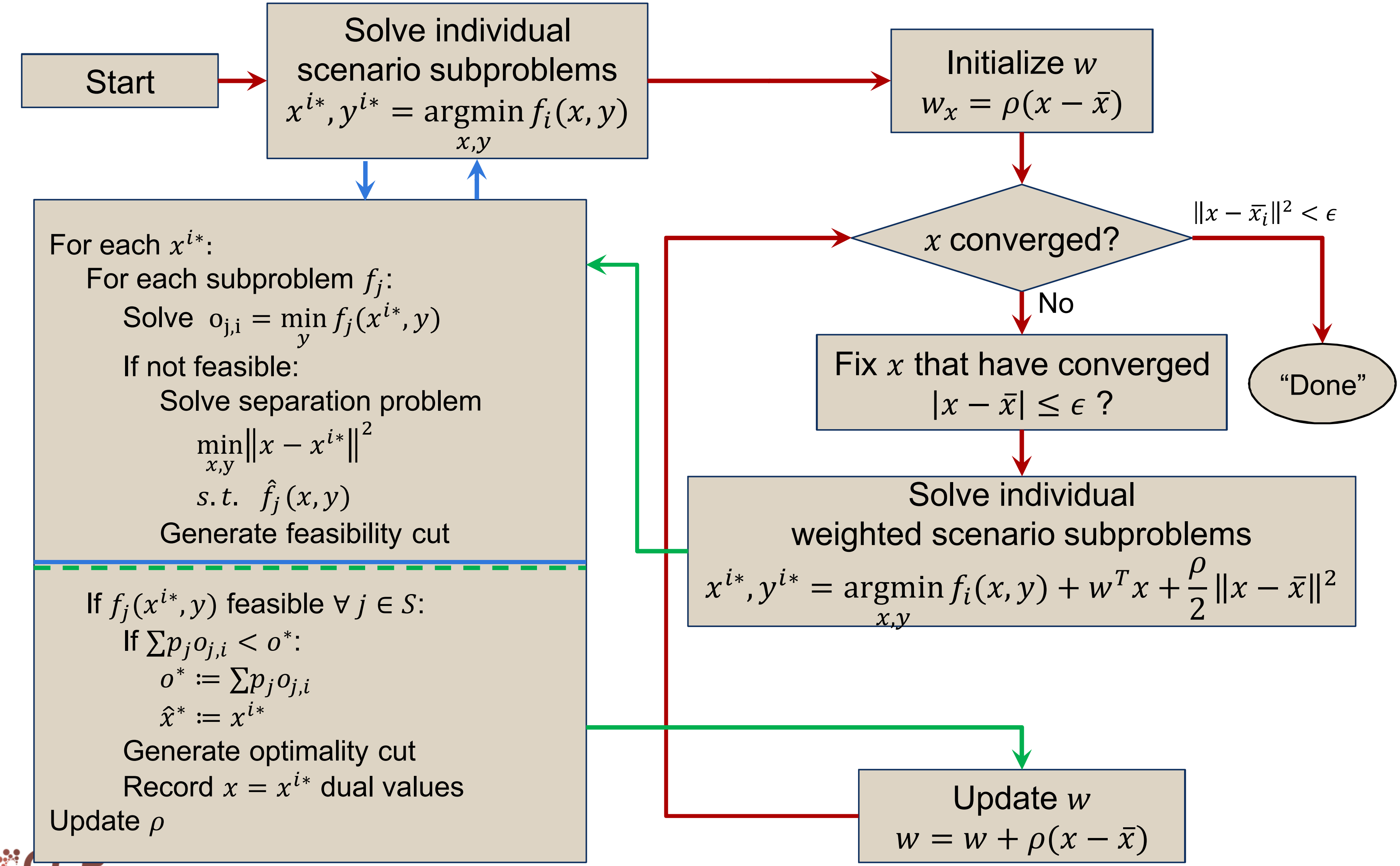
- 3-scenario farmer problem [Birge & Louveaux]
  - Continuous acreage allocations
    - PH ($\rho = 1$):
      - 33 iterations, 1.06 seconds: objective = -108388.7726
    - PH + ρ setter:
      - 34 iterations, 1.34 seconds: objective = -108389.7811
  - Discrete acreage allocations
    - PH ($\rho = 1$):
      - 39 iterations, 1.09 seconds: objective = -108390
    - PH + optimality cuts:
      - 154 iterations, 16.4 seconds: objective = -108390
    - PH + optimality cuts + ρ setter:
      - 40 iterations, 4.42 seconds: objective = -108390

Case studies: UC + *N-1* analysis

- Recall:
  - Optimal solution (extensive form): 19.9756
  - Default PH ($\rho = 1$):
    - 17 iterations, objective = 22.9997, total time 123 seconds
  - PH ($\rho = 1$) + Feasibility cuts:
    - 3 feasibility cut iterations at PH iteration 0
      - Improved Lagrangian bound from 19.7 to 19.88
    - 12 iterations, objective = 20.11, total time 568 seconds

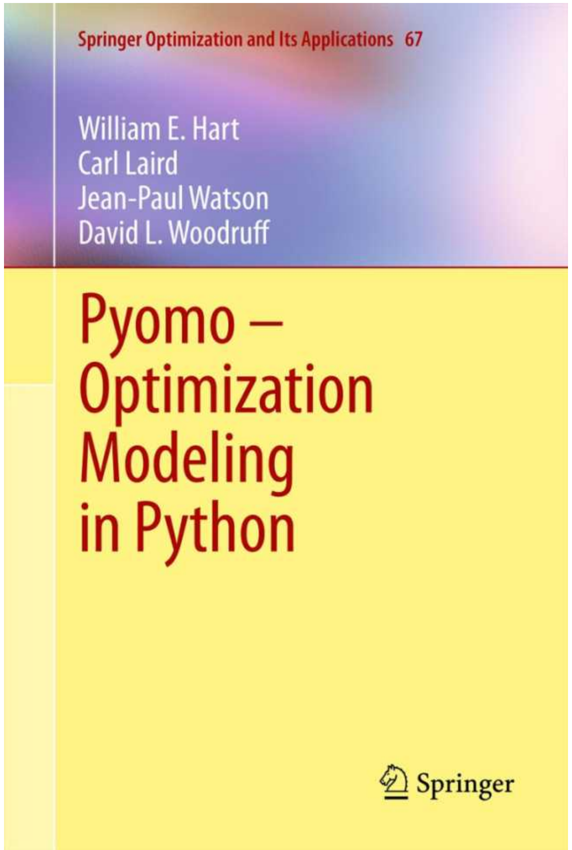
- Now:
  - PH + Feasibility cuts + ρ setter
    - 20 iterations, objective = 19.9808, total time 494 seconds

PH + feasibility + optimality cuts+ ρ



Our software environment: Pyomo

- Open-source optimization modeling environment written in Python
- All experiments performed using Pyomo's PySP PH implementation
  - Cuts and ρ updates implemented using PH callbacks
- Project homepage
  - http://www.pyomo.org
  - Development recently moved to GitHub
- "The Book"
  - Second edition going to press in O(weeks)



- Mathematical Programming Computation papers
  - Pyomo: Modeling and Solving Mathematical Programs in Python (Vol. 3, No. 3, 2011)
  - PySP: Modeling and Solving Stochastic Programs in Python (Vol. 4, No. 2, 2012)

Conclusions and future directions

- Significant benefits from using "Cross-scenario" information
  - Lagrangian bound improvement
  - Cycle breaking
  - Convergence acceleration
  - Automatic tuning
  - "Fewer" problem-specific tuning parameters
- Open questions
  - $|S|^2$ subproblem solves to evaluate solutions is expensive
    - Although it is "trivially" parallelizable
    - Can we gain most of the benefit using only $c|S|$ subproblems?
  - New (scalar) tuning parameters
    - How frequently to evaluate cross-scenario information?
    - How aggressively to update ρ?
    - what is the most robust ρ update formula?