



# A Convergence Study in Global Sensitivity Analysis

Rebecca Harmon, Iowa State University, B.S. Chemical Engineering, est. May 2017

Habib Najm (8351)\*, Mohammad Khalil (8954)\*

\*Sandia National Laboratories, CA

July 27, 2016

## Abstract

Monte Carlo sampling is a method to span a range of scenarios with a predictable rate of convergence. In this study, a simple model is used to observe the rate of decay in the error of average values for the variance of the model input, output, and ratio of the two. The purpose is to better understand the conventional thoughts on straightforward Monte Carlo sampling. Further efforts will compare the results with classical Monte Carlo estimation.

## Introduction

- Monte Carlo sampling uses randomly-selected values from a probability distribution to evaluate a model. [1]
- Sobol' first-order sensitivity index quantifies how the uncertainty (variance) of an input affects the model output. [2]

$$S_i^k = \frac{\text{Var}[E(y|x_i^k)]}{\text{Var}(y)}$$

where  $S_i$  = first-order sensitivity index,  
 $x_i^k$  = model input  $i$  in ensemble  $k$

- Goal:
  1. Study the convergence properties of a Monte Carlo-based sampling strategy for first-order sensitivity indices.
- Expectation:
  1. The error in average input and output variances will be proportional to  $1/N$ .
  2. The error in average first-order sensitivity index will be  $1/\sqrt{N}$ .

## Methods

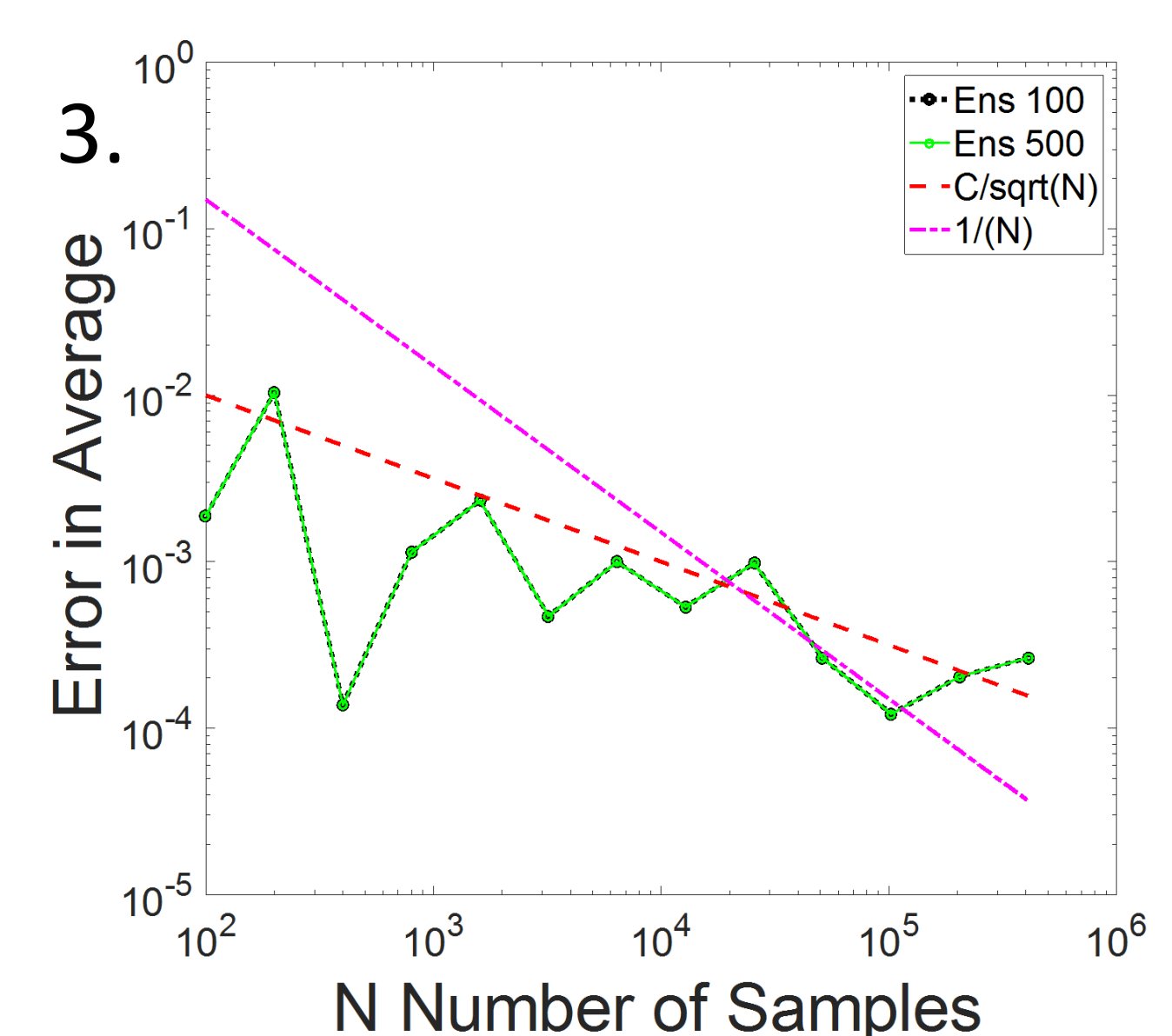
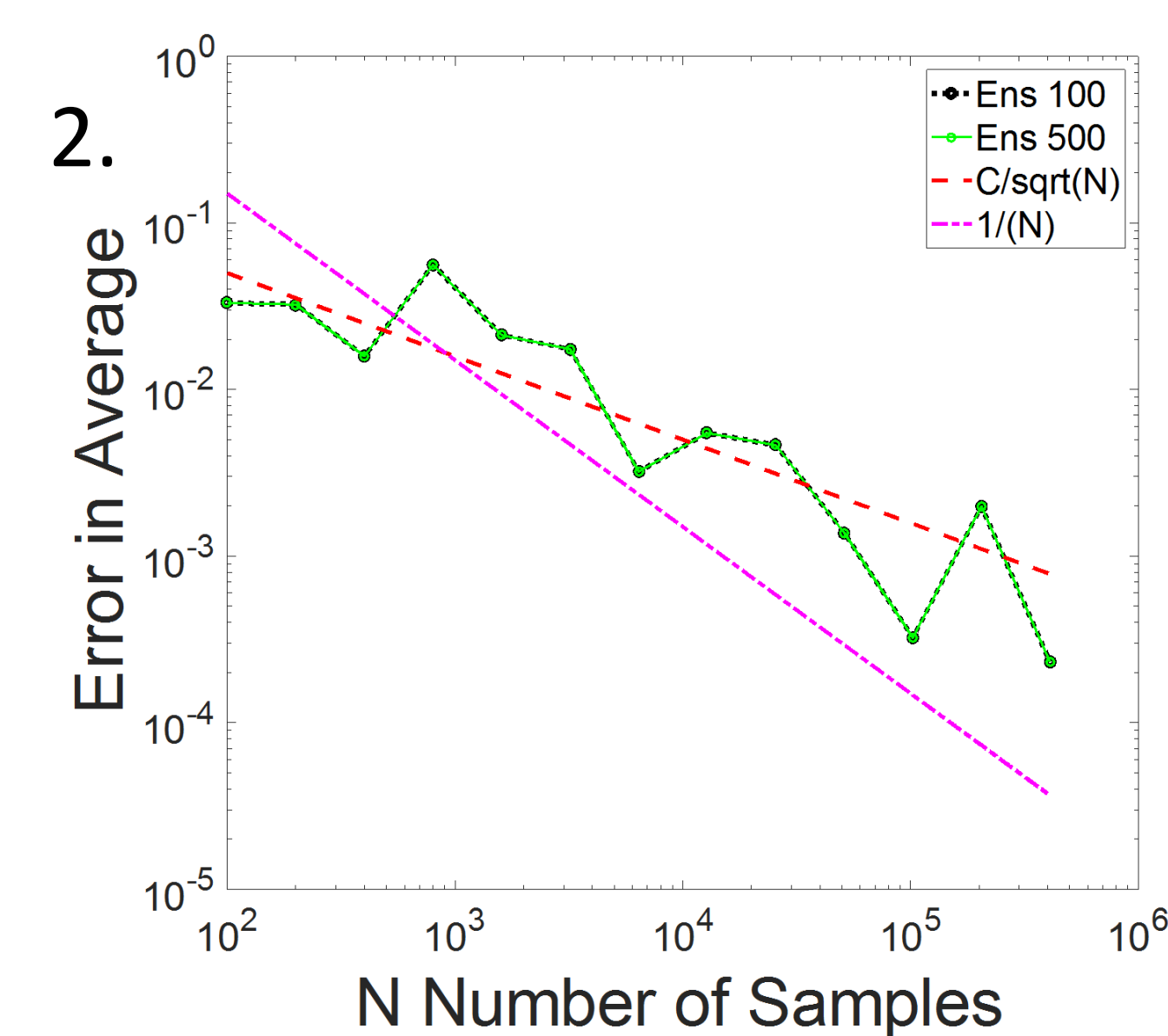
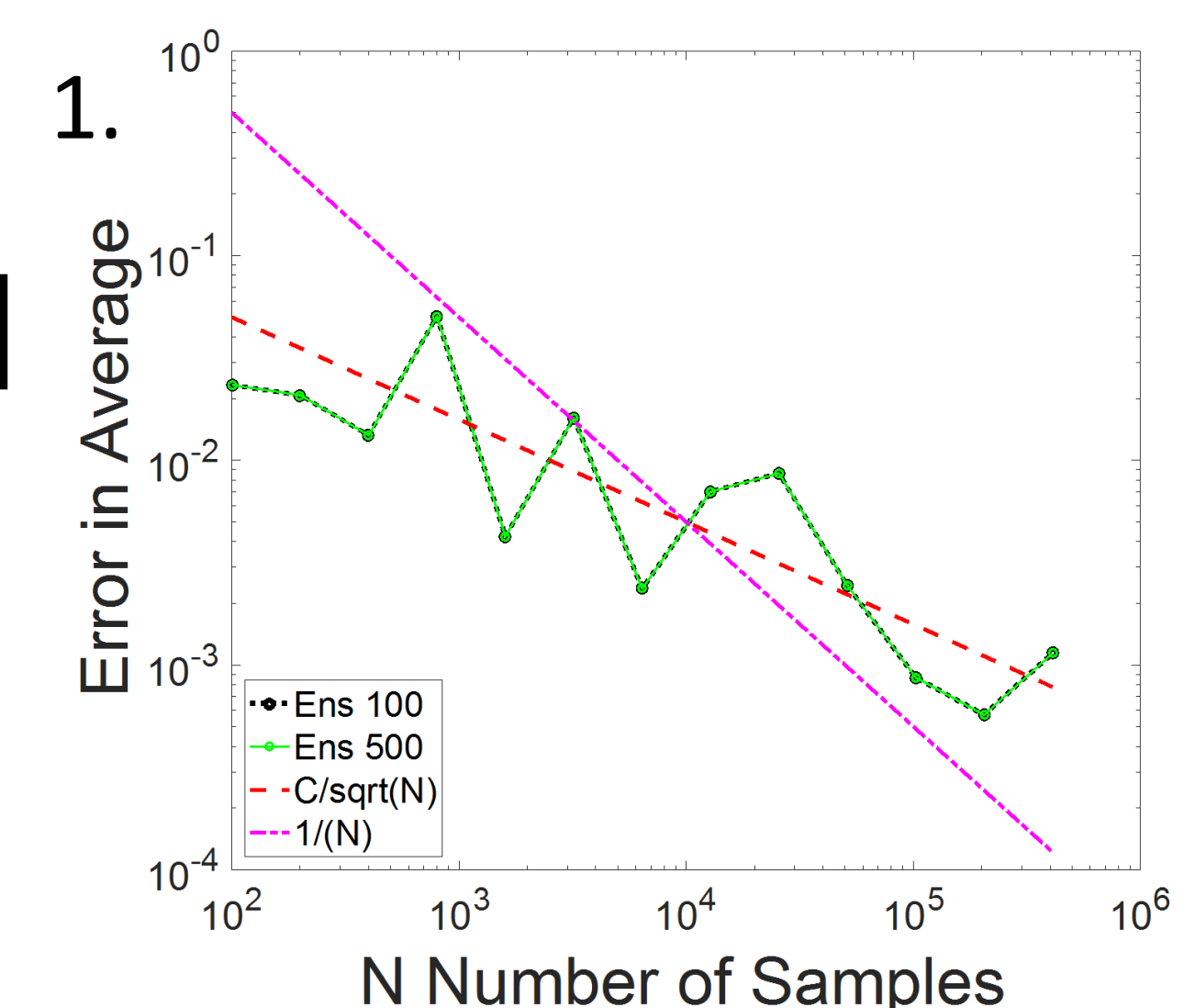
$$y = f(x) = x_1 + x_2 \quad \begin{array}{l} x_1 \sim N(0,2) \\ x_2 \sim N(0,1) \end{array}$$

For  $\text{Var}[E(y|x_i^k)]$ ,  $\text{Var}(y)$ , and  $S_i^k$ ,

1. N number of samples from 100 to 409600 by factor of 2
2.  $k$  ensemble size = 100, 500, (1000)
3. Absolute error =  $|\bar{\mu}_k - \mu_{exact}|$   
 where  $\bar{\mu}_k$  = average quantity of interest (QoI)  
 $\mu_{exact}$  = exact value of QoI

## Results

1. Error of average  $\text{Var}[E(y|x_i^k)]$
2. Error of average  $\text{Var}(y)$
3. Error of average  $S_i^k$



## Conclusions

- Expectation of  $1/N$  for the variances may be misunderstood
- Further studies will compare these results with classical Monte Carlo estimation of the same first-order Sobol' sensitivity indices.

## Acknowledgement

Funding was provided by the DOE Office of Science Student Undergraduate Laboratory Internship (SULI) program.

## References

1. Saltelli A, Chan K, Scott EM. Hitchhiker's guide to sensitivity analysis. In: *Sensitivity Analysis*. West Sussex, England: Wiley, 2000, sect.4, chapt. 2, p. 21.
2. Saltelli A, Chan K, Scott EM. Method of Sobol'. In: *Sensitivity Analysis*. West Sussex, England: Wiley, 2000, sect. 3, chapt. 8, p. 174-5.

*Exceptional service in the national interest*



Photos placed in horizontal position  
with even amount of white space  
between photos and header

Box size can be altered to fit your  
images. Keep even white space  
between photos.

# Poster Title

Area for content

All components of this template are movable and can be resized if necessary to fit your content. However in order to maintain brand strength, please do not change the color pallet.

Please refer to the Corporate Graphic Style Guide for questions regarding design attributes.

<http://scg.sandia.gov/resources/common-look-and-feel>

Photos placed in horizontal position  
with even amount of white space  
between photos and header

Photos placed in horizontal position  
with even amount of white space  
between photos and header



*Exceptional  
service  
in the  
national  
interest*