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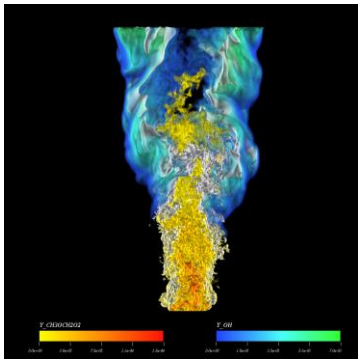

A Convergence Study in Global Sensitivity Analysis

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Introduction

Objective:

Study the convergence properties of a Monte Carlo-based sampling strategy for first-order sensitivity indices.

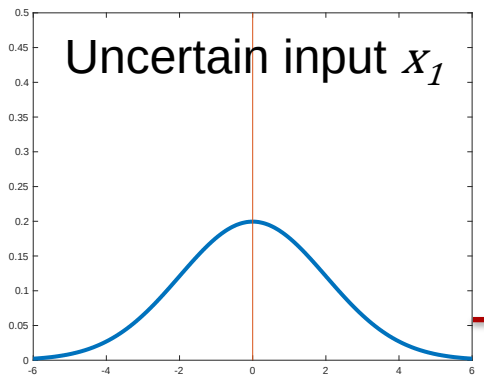
Introduction – Monte Carlo Sampling

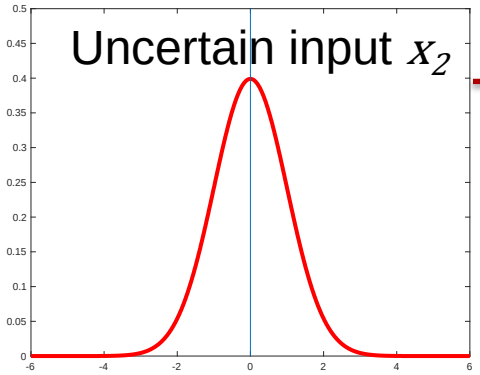
- Use randomly-selected values from a probability distribution to evaluate a model [1]
- $1/\sqrt{\mathrm{N}}$ error convergence as N number of samples increases

Source: Casino Monte Carlo, http://www.casinomontecarlo.com/wp-content/uploads/2015/10/roulette-anglaise-monaco-1100x358.jpg, accessed 14 July, 2016.

Introduction – Global Sensitivity Analysis

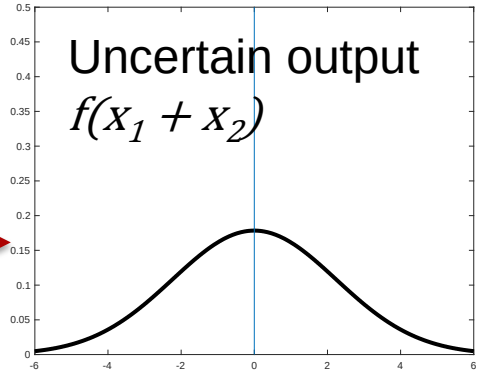
- All parameters are varied simultaneously [2]
- Determine input variable uncertainty contribution to output uncertainty [2]

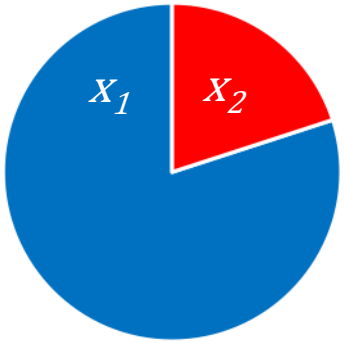




Model

$f(x_1 + x_2) = x_1 + x_2$





Introduction – Sobol' Sensitivity Index

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First-order sensitivity index [3]:

$$S_i^k = \frac{\mathrm{Var}\left[\mathrm{E}\left(y|x_i^k\right)\right]}{\mathrm{Var}(y)}$$

where S_i = first-order sensitivity index,

x_i^k = model input *i* in ensemble *k*

Introduction – Expectations

- Error in average input and output variances will be proportional to 1/N.
- Error in average first-order sensitivity index will be $1/\sqrt{\mathrm{N}}$.

Methods

$$y = f(x) = x_1 + x_2 \quad x_1 \sim \mathrm{N}(0,2)$$
$$x_2 \sim \mathrm{N}(0,1)$$

For $\mathrm{Var}\left[\mathrm{E}\left(y|x_i^k\right)\right]$, $\mathrm{Var}(y)$, and S_i^k,

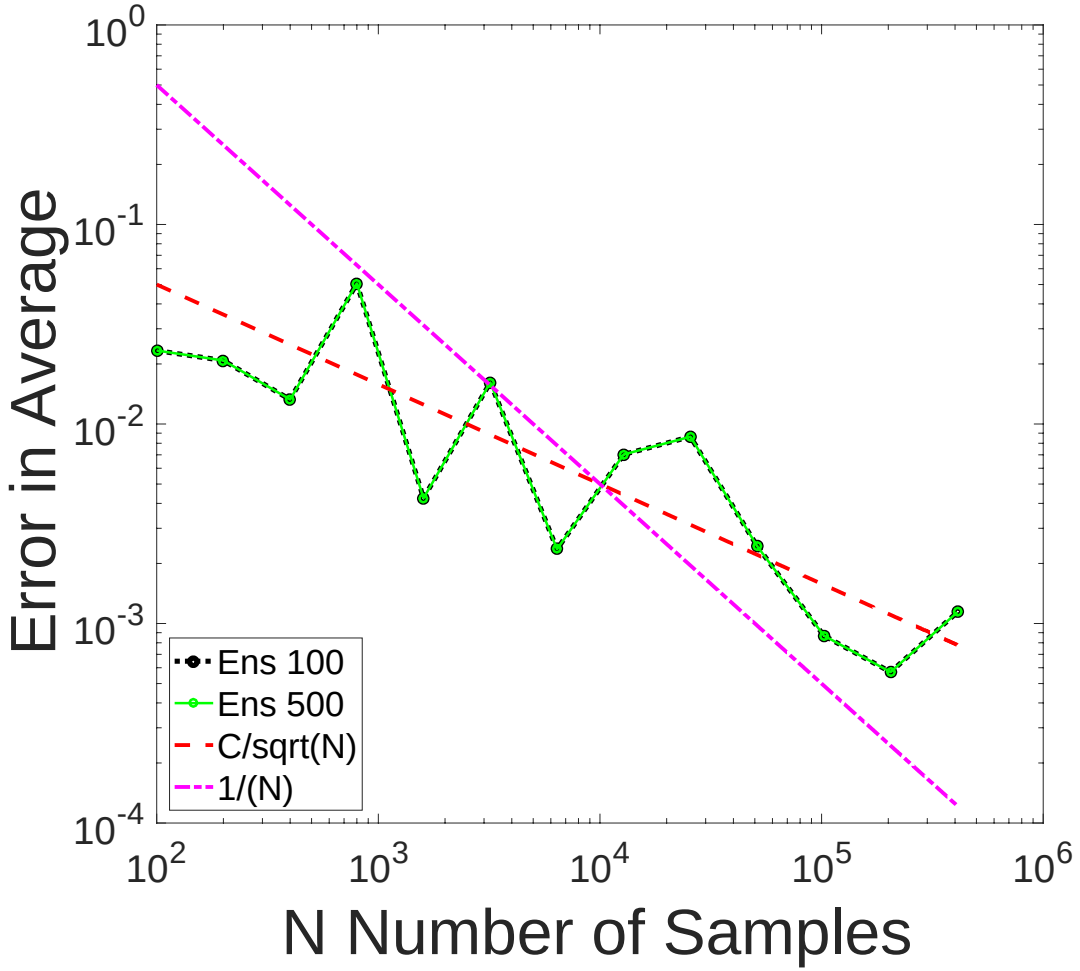
- N number of samples from 100 to 409600 by factor of 2
- *k* ensemble size = 100, 500, (1000)
-

$$\text{Absolute error} = |\bar{\mu}_k - \mu_{exact}|$$

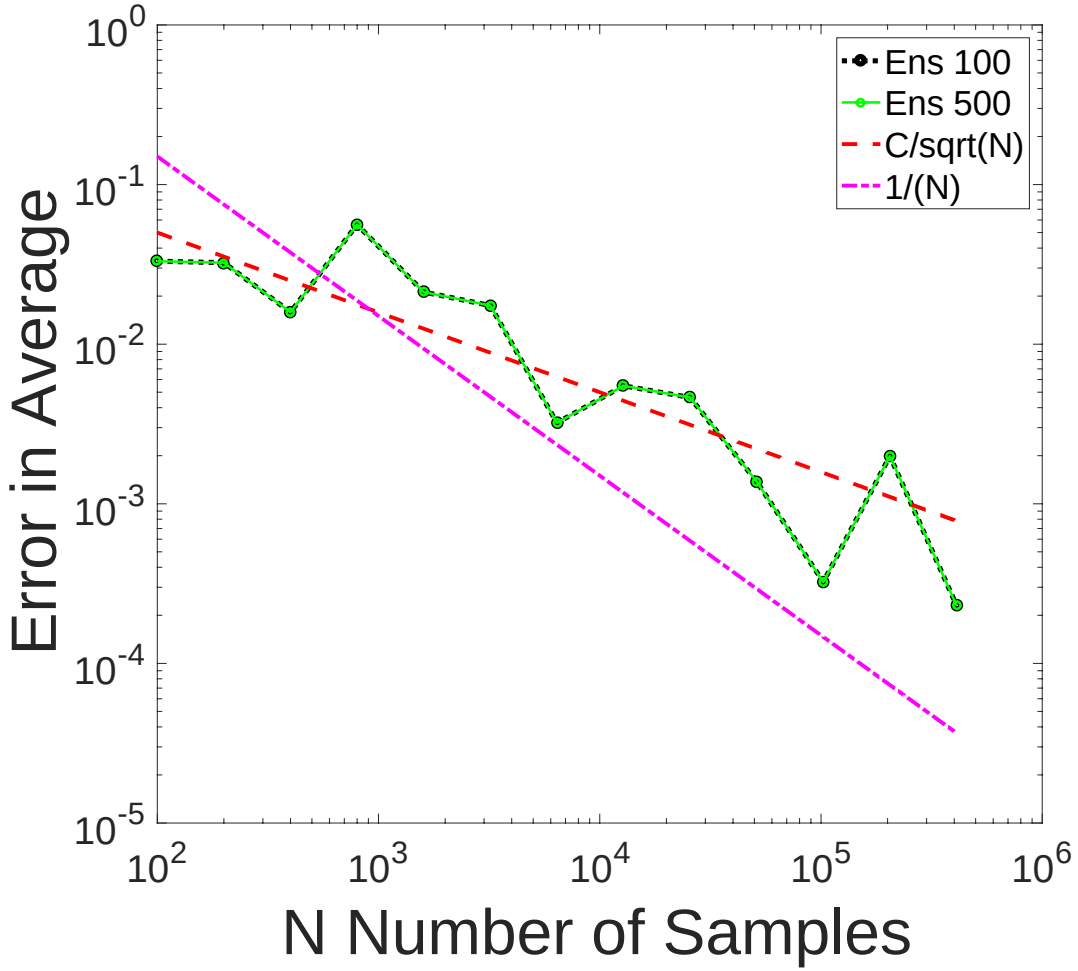
where $\bar{\mu}_k$ = average quantity of interest (QoI)

μ_{exact} = exact value of QoI

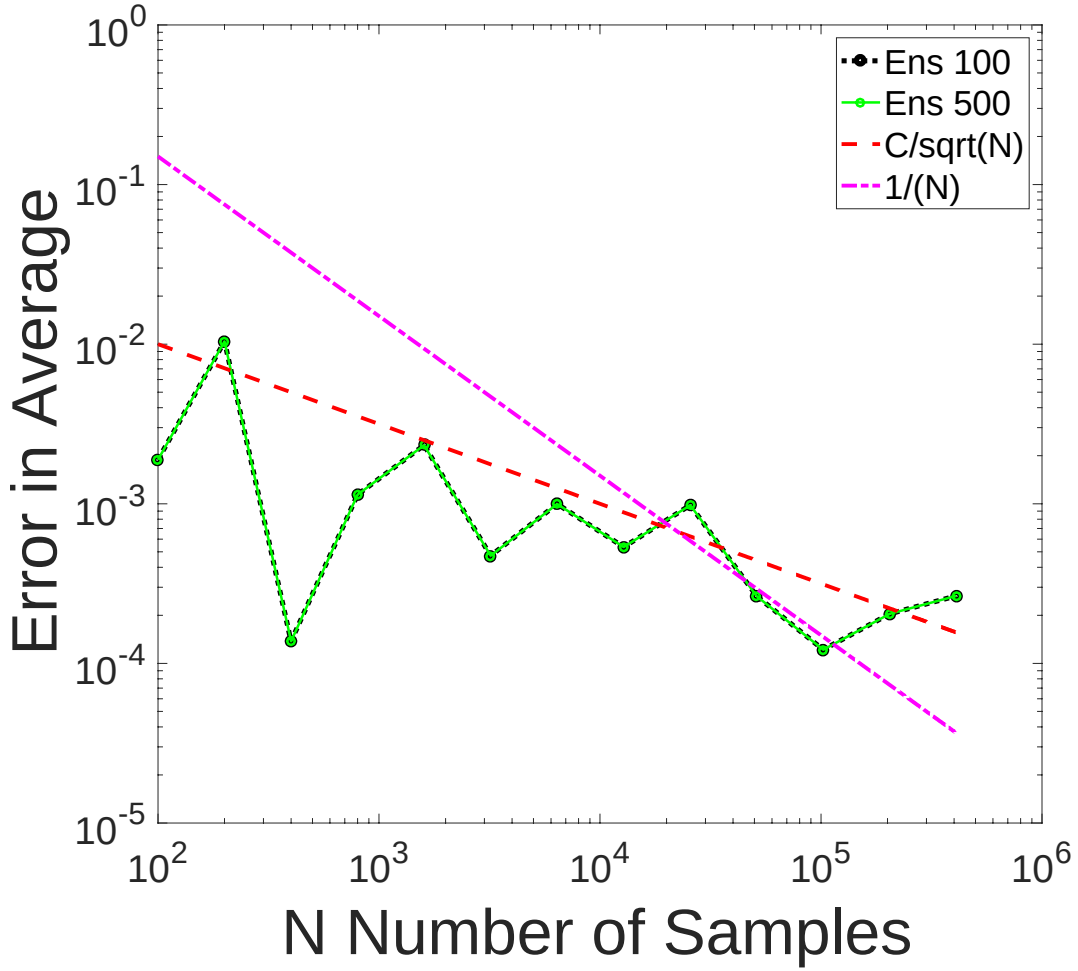
Results – $\mathrm{Var}\left[\mathrm{E}\left(y|x_i^k\right)\right]$



Results – $\mathrm{Var}(y)$



Results – S_i^k


Ens 100
Ens 500
C/sqrt(N)
1/(N)
Error in Average
N Number of Samples
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^2
10^3
10^4
10^5
10^6

Conclusions and Future Work

- Expectation of 1/N for the variances may be misunderstood
- Compare the results with classical Monte Carlo estimation of the same first-order Sobol' sensitivity indices.

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Thank you!

Habib Najm Mohammad Khalil Chris Shaddix

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Questions?

References

1. Saltelli A, Chan K, Scott EM. Hitchhiker's guide to sensitivity analysis. In: *Sensitivity Analysis.* West Sussex, England: Wiley, 2000, sect.4, chapt. 2, p. 21.
2. Saltelli A, Chan K, Scott EM. Hitchhiker's guide to sensitivity analysis. In: *Sensitivity Analysis.* West Sussex, England: Wiley, 2000, sect. 1, pt. 3, chapt. 2, p. 17.
3. Saltelli A, Chan K, Scott EM. Method of Sobol'. In: *Sensitivity Analysis.* West Sussex, England: Wiley, 2000, sect. 3, chapt. 8, p. 174-5.