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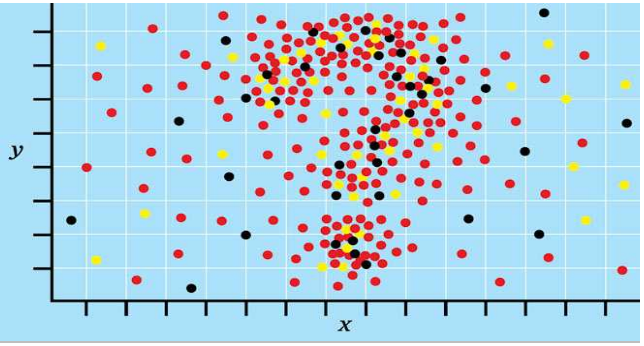

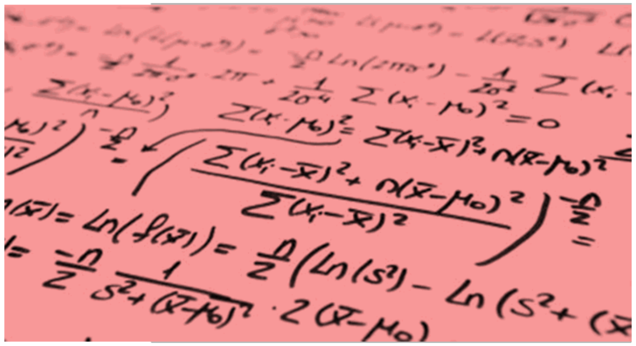

Construction of Tolerance Bounds for a Multivariate Response Associated with a Covariate: A Case Study

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U.S. DEPARTMENT OF ENERGY NNSA National Nuclear Security Administration

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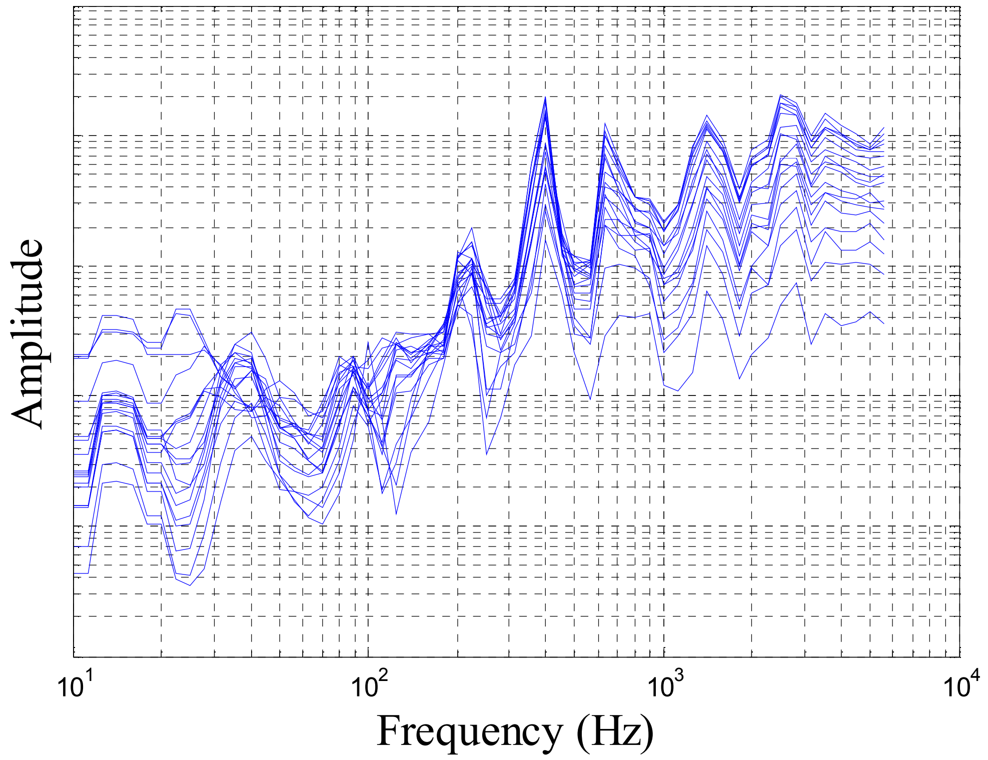
Outline

- Motivating Example
  - Captive Carry
- Notation and Definitions
- Parametric Bootstrap Procedure
- Coverage Investigations
- Conclusions and Ongoing Work

Bounding of Captive Carry Environments

- Over the course of a single lifetime, military munition systems are exposed to various sources of mechanical vibrations affecting system performance.
- Significant exposure occurs when munition systems are carried by aircraft, even in “straight and level” flight conditions.
- It is of interest to characterize the distribution of vibration levels using Acceleration Spectral Densities (ASDs) measured by sensors in order to compute appropriate bounds.
- These vibration levels are clearly dependent on environmental factors during flight.

Example of ASD Curves



Motivation of Research

- Recent work by Rathnayake and Choudhary (2015) provides a method for computing tolerance bounds on functional data using a bootstrap procedure.
  - This method does not consider the presence of covariates.
- We propose a parametric bootstrap procedure for computing tolerance bounds with covariates present based on functional PCA.
- The computed tolerance bounds can serve as a basis for creating bounds on vibration behavior.

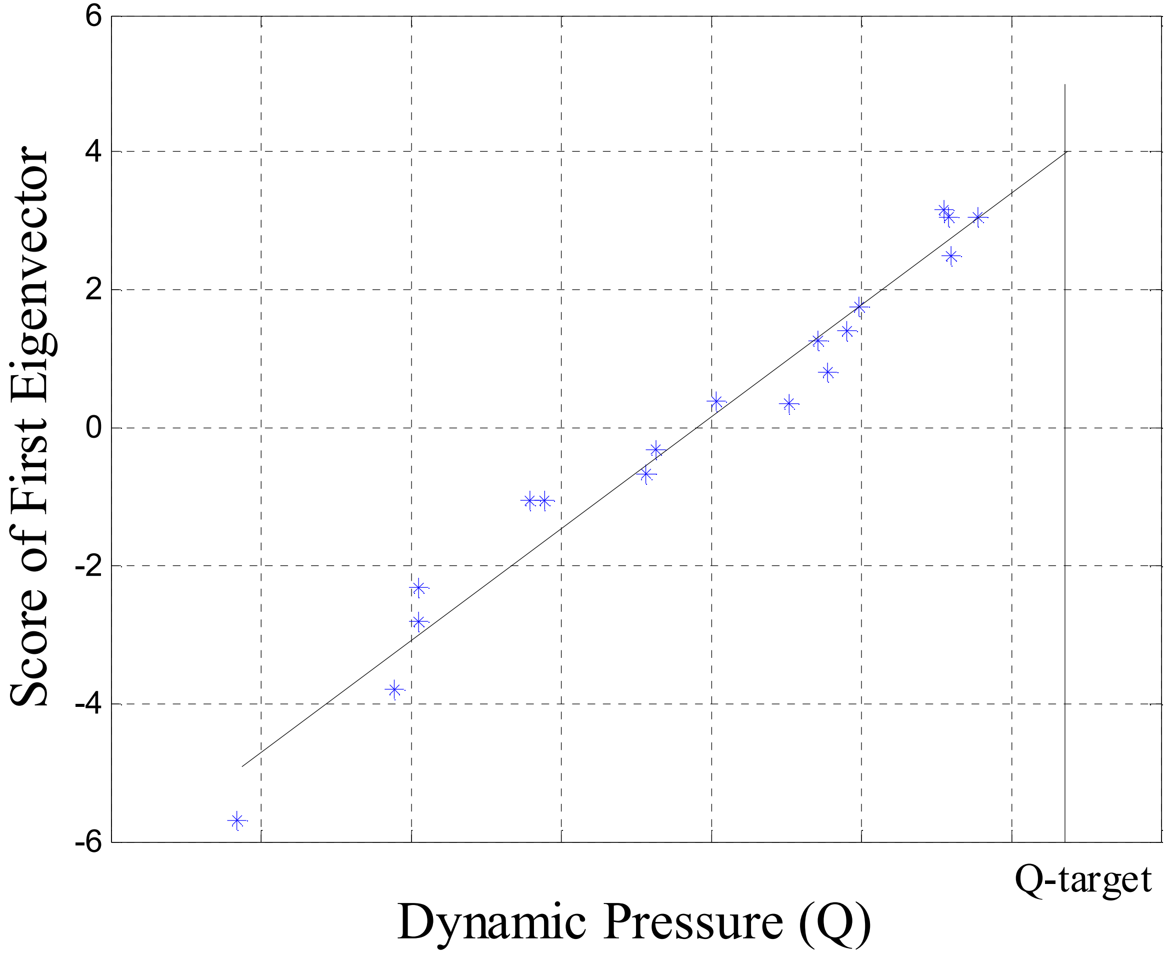
Notation

- Let $Y_i = \{Y_{i1}, Y_{i2}, \dots, Y_{iJ}\}$ be a vector of ASD responses for test flight *i=1,2,…n* across frequency levels *j=1,2,…,J*.
- Let $\boldsymbol{Y}_c$ be the matrix of mean-centered data.
- The matrix $\boldsymbol{E}_{p \times p}$of eigenvectors corresponding to *p* latent components are computed from the principal component decomposition of the covariance matrix of the mean-centered data .
- The matrix $\boldsymbol{S} = \boldsymbol{Y}_c\boldsymbol{E}$ consists of the loadings or *scores* on the eigenvectors.
- Let Q_i be the dynamic pressure measured during the i^{th} test flight.

Model and Extrapolation

- It is assumed that the scores of the first component are related to the covariates through some model.
  - For illustrative purposes, we only use one covariate with a linear model.
  - $S_i = \beta_0 + \beta_1 Q_i + \epsilon_i,\ i = 1,2,\ldots,n;\ \epsilon_i \sim Normal(0, \sigma^2)$
- It is desired to compute tolerance bounds at some target level *Q-target* of the dynamic pressure.
  - The scaled spectra at the target value are computed as $\boldsymbol{Y}_{adj} = \boldsymbol{Y} + \beta_1 \left(Q_{target} - \boldsymbol{Q}\right) \boldsymbol{e}_1$, where $\boldsymbol{e}_1$ is the eigenvector associated with the first principal component.

Model Example



Parametric Bootstrap Procedure

- For b=1,2,…,B
  1. Simulate $\tilde{\sigma}_{kb} \sim \hat{\sigma}_k \cdot \sqrt{\frac{df}{\chi^2_{df}}}, k = 1,2, \ldots p$
     - p is the number of principal components
     - *df* is the degrees of freedom *(df=n-2* for component 1 and *df=n-1* otherwise)
     - χ^2_{df} is a chi-squared random variable with *df* degrees of freedom
     - $\hat{\sigma}_k$ is the estimated standard deviation for component *k*
  2. Simulate $\tilde{\mu}_{kb} \sim Normal(\hat{\mu}_k, \tilde{\sigma}^2_{\mu})$
     - $\tilde{\sigma}^2_{\mu} = \tilde{\sigma}^2_1 \cdot \boldsymbol{x}_0^T \cdot (\boldsymbol{X}^T\boldsymbol{X})^{-1} \cdot \boldsymbol{x}_0$ for k=*1* and $\tilde{\sigma}^2_{\mu} = \frac{\tilde{\sigma}^2_k}{n}$ for all other k
     - $\hat{\mu}_1 = \boldsymbol{x}_0^T\hat{\beta}$ for k=*1* and 0 for all other k
     - $\boldsymbol{x}_0^T = \{1\ Q_{target}\}$
  3. Compute $\overline{\mathbf{Y}}_b = \overline{\mathbf{Y}} + [\tilde{\mu}_{1b}, \tilde{\mu}_{2b}, \ldots, \tilde{\mu}_{pb}] \cdot \boldsymbol{E}^T$ and $S_{bj} = \sqrt{\sum_{k=1}^{p} E^2_{jk}\tilde{\sigma}^2_{kb}}$
  4. Compute $P_{bj} = \bar{Y}_{bj} + z_{1-\alpha} \cdot S_{bj}$, where $z_{1-\alpha}$ is the 1-α percentile of the standard normal distribution.
- The pointwise γ-level upper confidence bound is given by the γ-percentile of P_{bj}, *b=1,2,…B.*

Coverage Investigation

- Assessment of coverage using simulation
- Considered both simulated data sets of the same size as original (N<20) and one with 10 replicates of each observation.
- Content/coverage pairs taken from the following: 99/90, 99/50, 99/75, 95/90, and 95/50.
- Data simulation based on simulated scores; original data mean and eigenvectors retained.

Simulation Results (original N)

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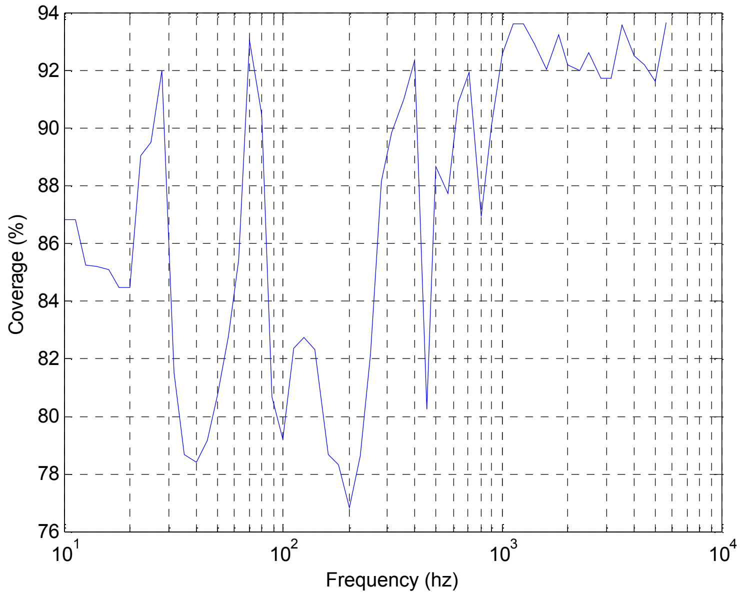
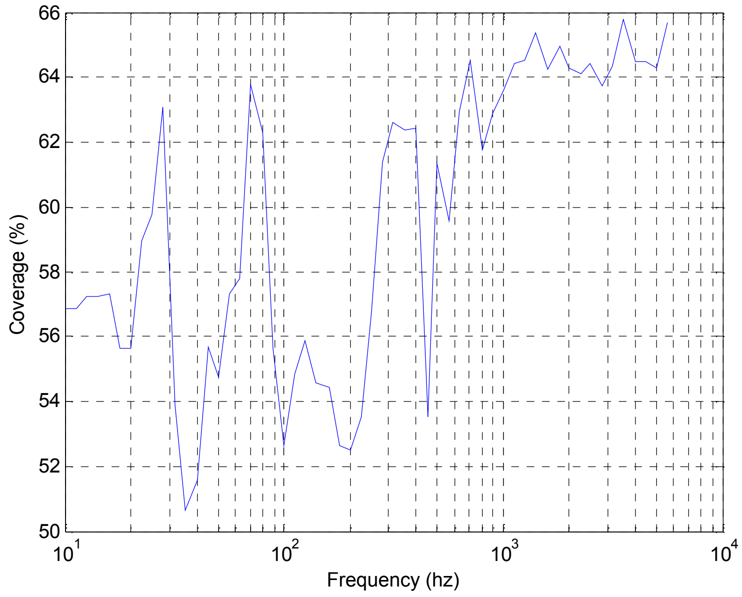
99/90

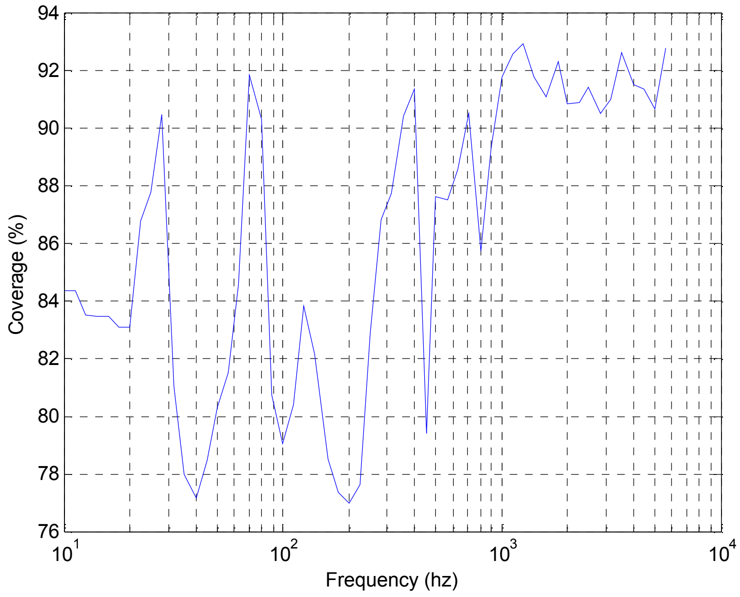


99/50

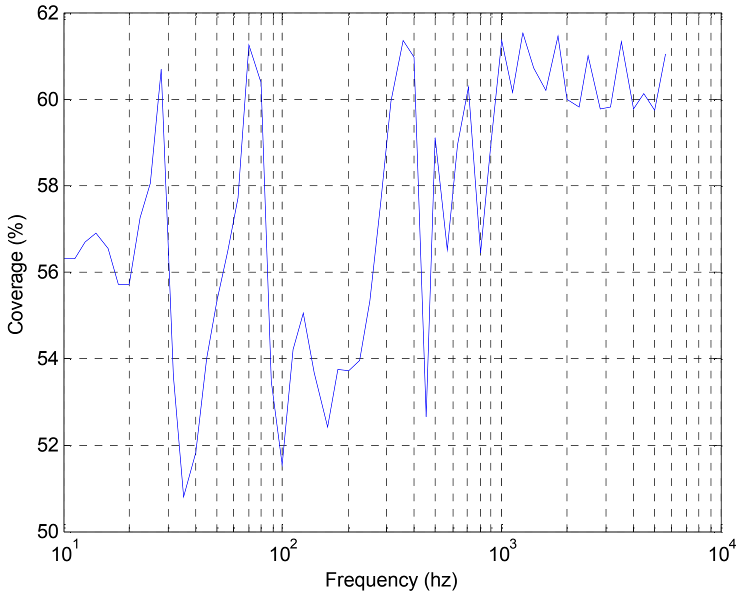


Simulation Results (original N)

95/90

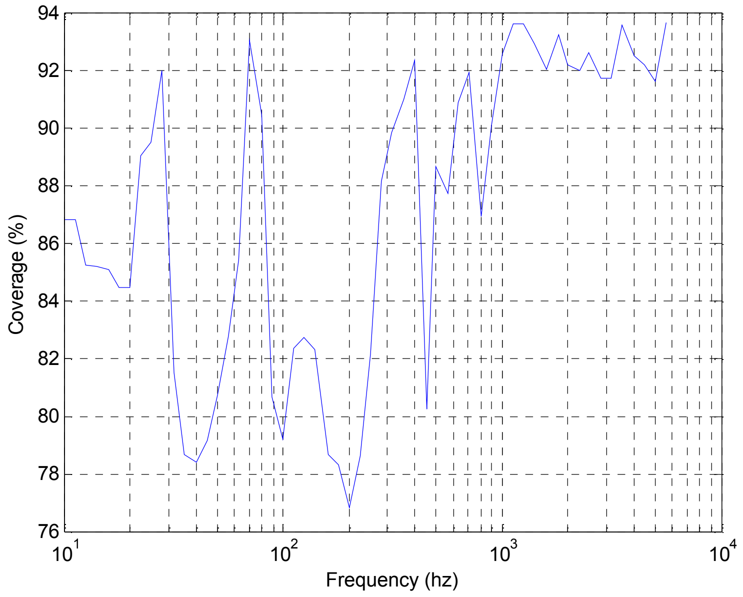


95/50

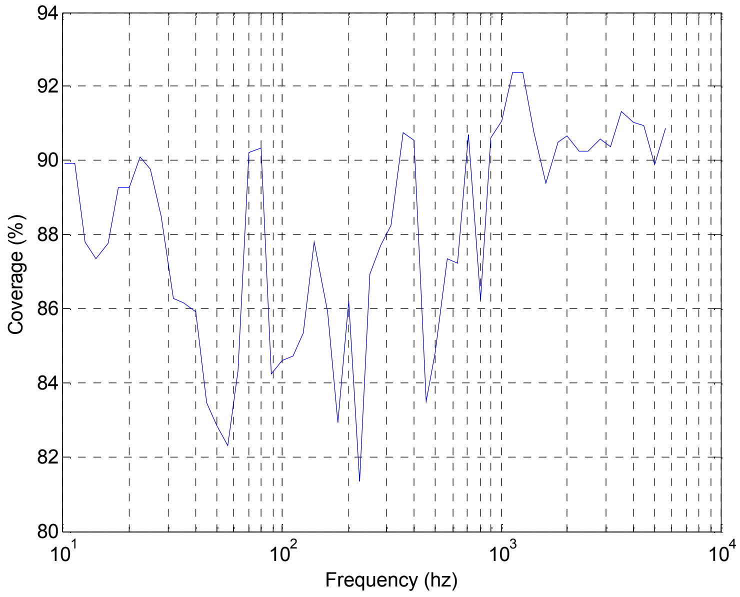


Simulation Results

99/90 – original N

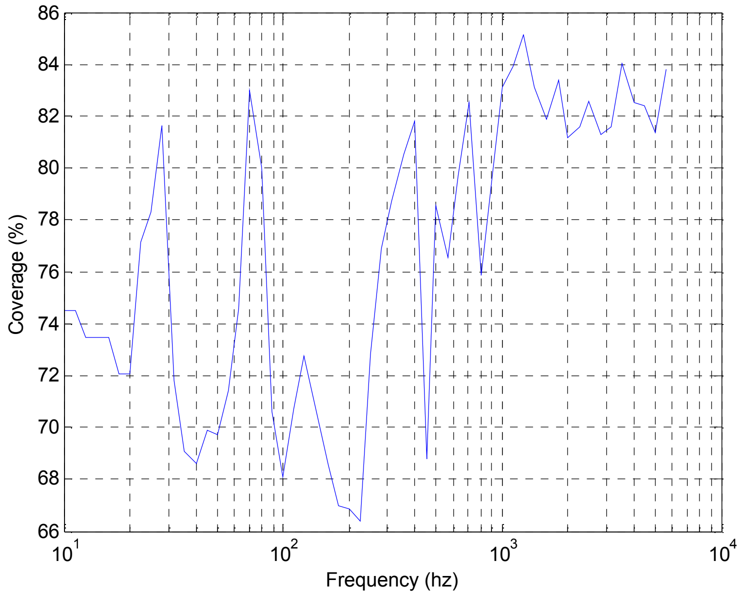


99/90 – 10 replicates

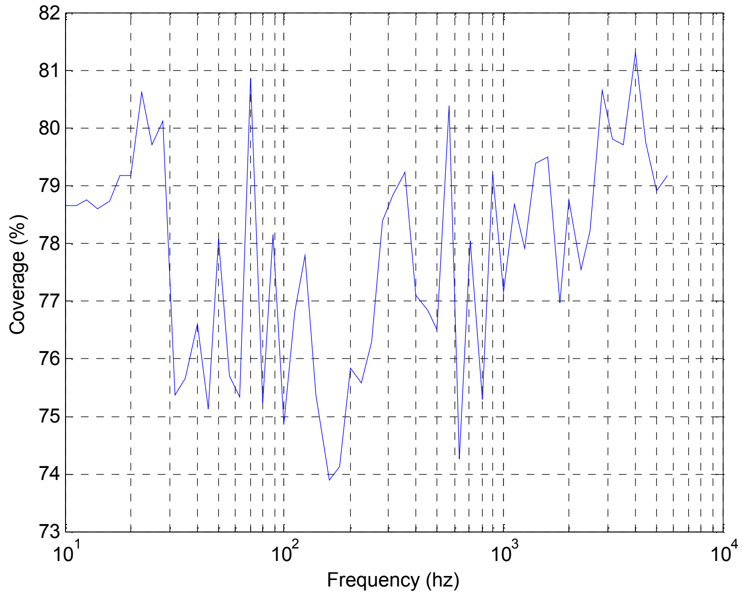


Simulation Results

99/75 – original N

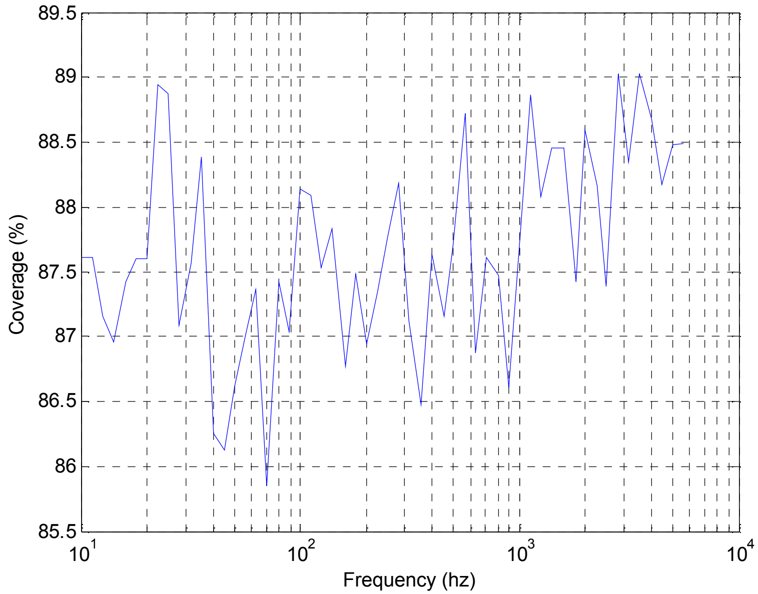


99/75 – 10 replicates

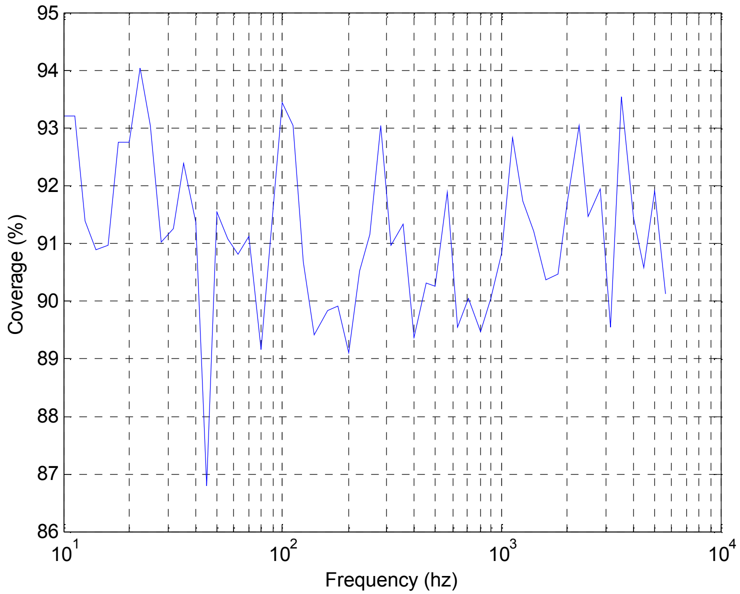


Simulation Results

99/90 – original N – w/o 1st PC



99/90 – 10 replicates – w/o 1st PC



Potential Reasons for Results

- Bootstrap simulation only accounts for random variation in score values
  - Does not account for variation in loadings (eigenvector definition).
- When eigenvectors are well defined (e.g., 10 reps), this variation is small
  - The bootstrap simulations now provide more accurate coverage

Summary and Continued Work

- Current work focused on alternative tolerance bound computation methods.
- Also considering methods to separate and account for phase variation.