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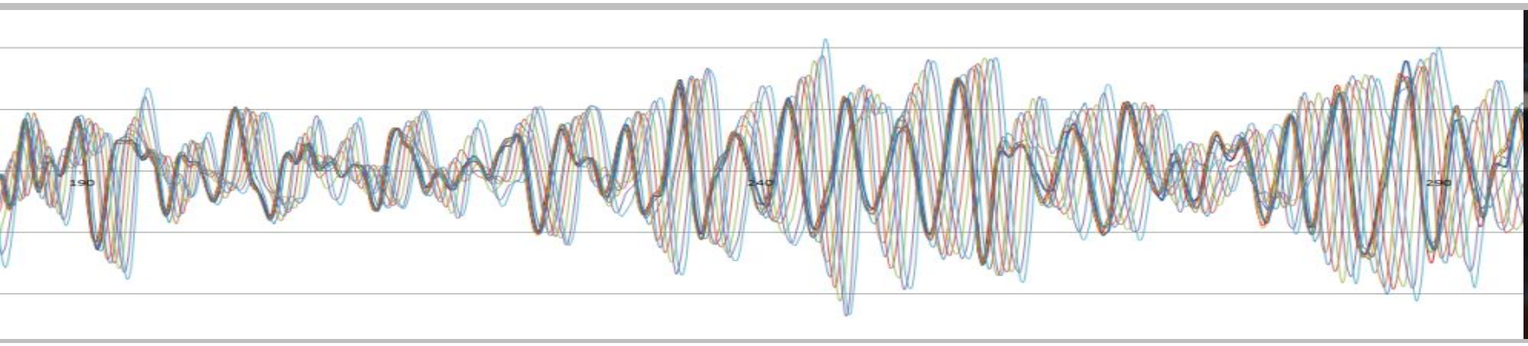

Wave Speed Propagation Measurements on Highly Attenuative Wax at Elevated Temperatures

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U.S. DEPARTMENT OF ENERGY

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Outline

- Ultrasonic Overview (Theory)
- Material Characterization
- Temperature Experiments
  - Wax Experiments (Heating and Cooling)
- Code Development and Refinement
- Additional Instrumented Experiments
- Conclusions

Wave Propagation (Isotropic)

Stress (σ_{ii}) and strain (ε_{ii}) components of material particles when subjected to an ultrasonic wave

$$\sigma_{xx} = (\lambda + 2\mu)\epsilon_{xx} + \lambda(\epsilon_{yy} + \epsilon_{zz})$$

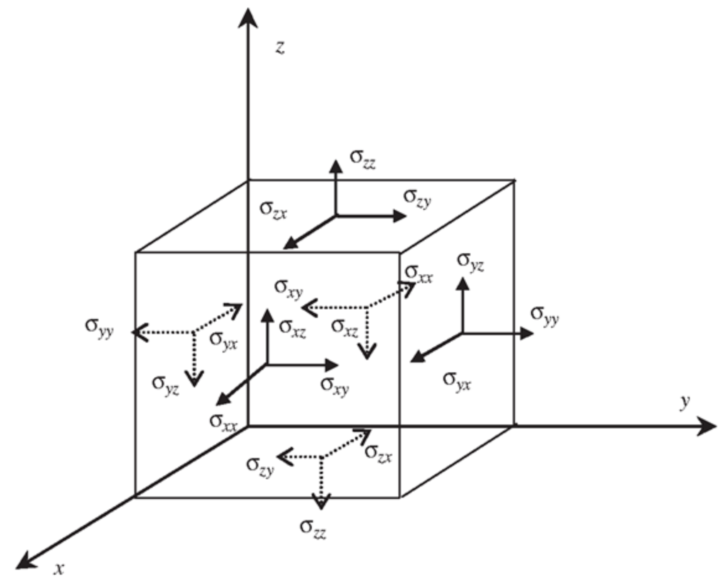
$$\sigma_{yy} = (\lambda + 2\mu)\epsilon_{yy} + \lambda(\epsilon_{xx} + \epsilon_{zz})$$

$$\sigma_{zz} = (\lambda + 2\mu)\epsilon_{zz} + \lambda(\epsilon_{xx} + \epsilon_{yy})$$

$$\sigma_{xy} = \sigma_{yx} = 2\mu\epsilon_{xy} = 2\mu\epsilon_{yx}$$

$$\sigma_{yz} = \sigma_{zy} = 2\mu\epsilon_{yz} = 2\mu\epsilon_{zy}$$

$$\sigma_{zx} = \sigma_{xz} = 2\mu\epsilon_{zx} = 2\mu\epsilon_{xz}$$

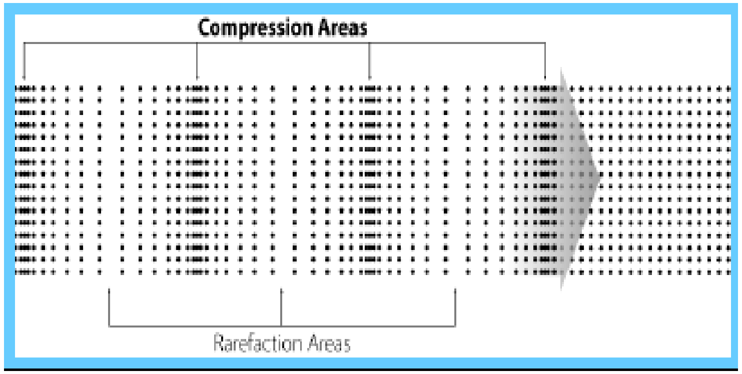


The variables λ and μ are Lamé's constants

Lamé's Constants Wave Motion

The constants are related to the material properties of Young's modulus *E* and Poisson's ratio *v*

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)}$$



The 3D space the governing equations of motion for waves in the x, y and z axis are:

$$\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} + \frac{\partial\sigma_{xz}}{\partial z} = \rho\frac{\partial^2 u_x}{\partial^2 t}$$

$$\frac{\partial\sigma_{xy}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y} + \frac{\partial\sigma_{yz}}{\partial z} = \rho\frac{\partial^2 u_y}{\partial^2 t}$$

$$\frac{\partial\sigma_{xz}}{\partial x} + \frac{\partial\sigma_{yz}}{\partial y} + \frac{\partial\sigma_{zz}}{\partial z} = \rho\frac{\partial^2 u_z}{\partial^2 t}$$

$$u_x = A_x\cos\left(k_x x + k_y y + k_z z - \omega t\right)$$

Particle displacements

$$u_y = A_y\cos\left(k_x x + k_y y + k_z z - \omega t\right)$$

$$u_z = A_z\cos\left(k_x x + k_y y + k_z z - \omega t\right)$$

Birks A. S. and Green Jr., R. E. Nondestructive Testing Handbook 2nd Edition, Volume 7., Ultrasonic Testing, American Society of Nondestructive Testing, Incorporated, 1991.

Anisotropic Materials Properties

$$\begin{Bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ & & C_{33} & C_{34} & C_{35} & C_{36} \\ & & & C_{44} & C_{45} & C_{46} \\ & & & & C_{55} & C_{56} \\ & & & & & C_{66} \end{bmatrix} \begin{Bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{Bmatrix}$$

Crystal Type	No. of Constants
triclinic	21
monoclinic	13
orthorhombic	9
trigonal	7
tetragonal	6
hexagonal	5
cubic	3
isotropic	2

Anisotropic constitutive equations has two orthogonal planes of symmetry. These materials require 9 independent variables (i.e. elastic constants) in their constitutive matrices.

Stored elastic energy F :

$$F(e) = F_0 + g \cdot C_{ij} \cdot e_{ij} + \frac{1}{2} C_{ijkl} \cdot e_{ij} \cdot e_{kl} + \frac{1}{6} C_{ijklmn} \cdot e_{ij} \cdot e_{kl} \cdot e_{mn} + \ldots$$

Energy stored in undeformed condition

Potential energy with 1^{st} order elastic constants

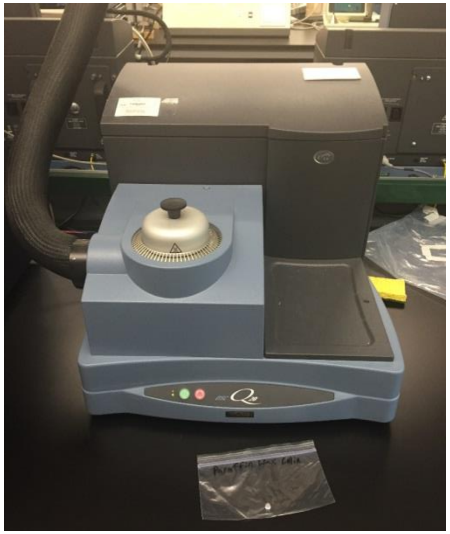
Tensor of 2^{nd} order elastic constants

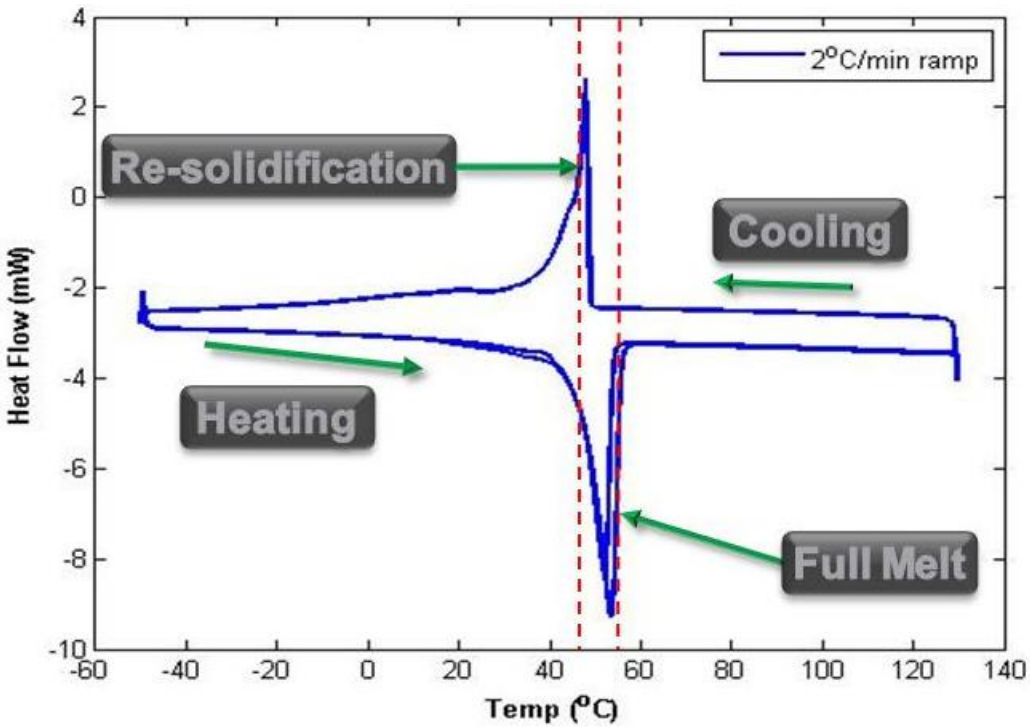
Tensor of 3^{rd} order elastic constants

C. Boller and E. Schneider, Saarland University

DSC Experiments

The wax was analyzed using a differential scanning calorimeter (DSC). This equipment measures the heat flow rate between a sample and inert reference as a direct function of time and temperature. This technique documents the thermodynamic processes and transitions of specific heat capacity.

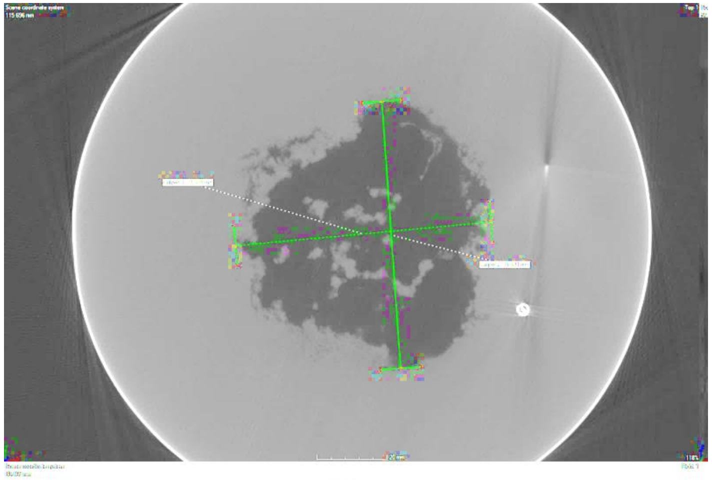


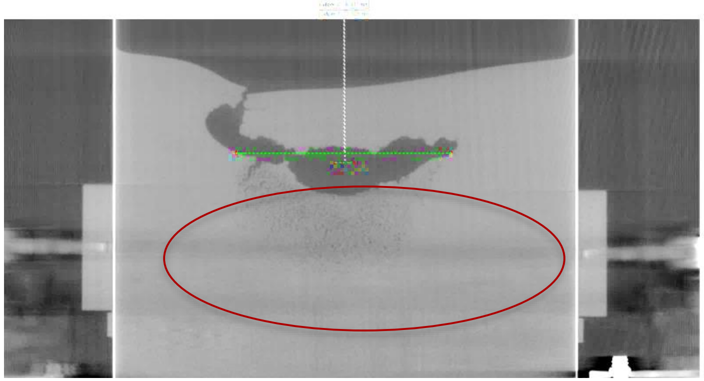


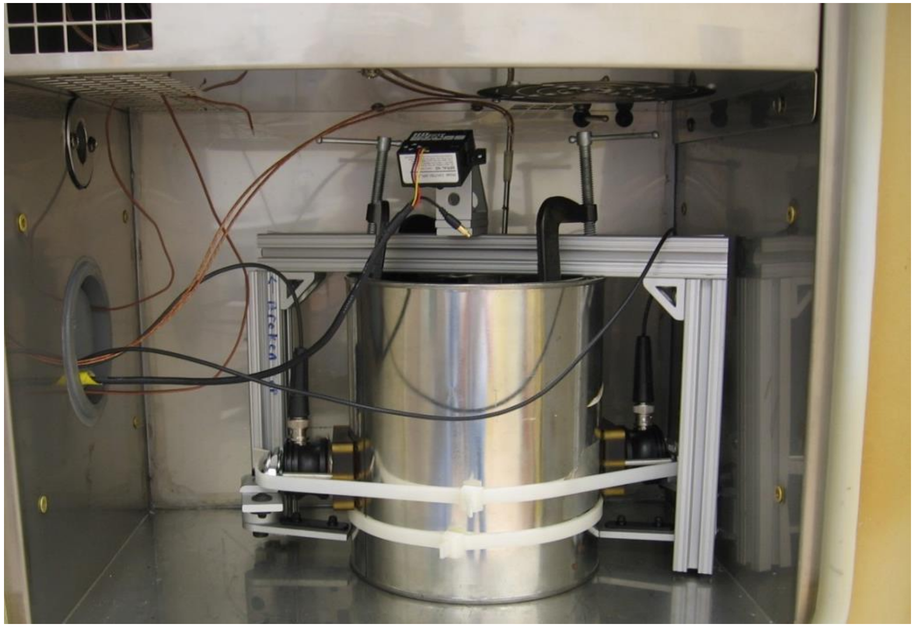
- The first major change is observed at approximately 35°C where the plot bends, this indicates a transition zone.
- This is followed by a steep drop in heat flow until approximately 50-55°C.
- Then there is a sudden spike back to the pre-transition heat flow value, this sudden change is a key indication that a phase change has occurred.
- During the cool down cycle there is a similar spike at approximately 50°C as the wax solidifies. This is followed by a transition region similar to the one in the heating phase. The second heat/cool cycle confirms these events.

Experiment #1

A bucket is filled with wax and placed in an oven at 90 degrees C. Two thermocouples and video camera monitor the wax. Two 0.5 Mhz probes are attached to the bucket (TTU).

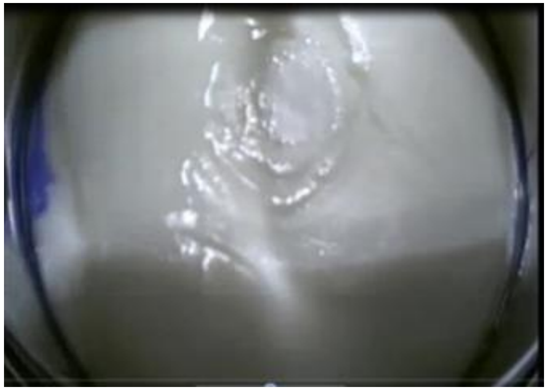






Data is recorded once a minute on each active channel. The sample data is recorded in minutes and depth in microseconds (Y-axis).

Wax Response in Oven 90 °C

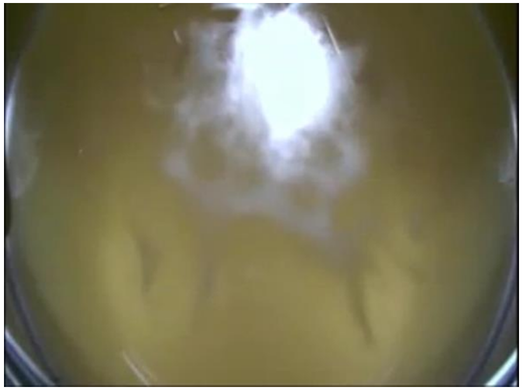




Increasing time

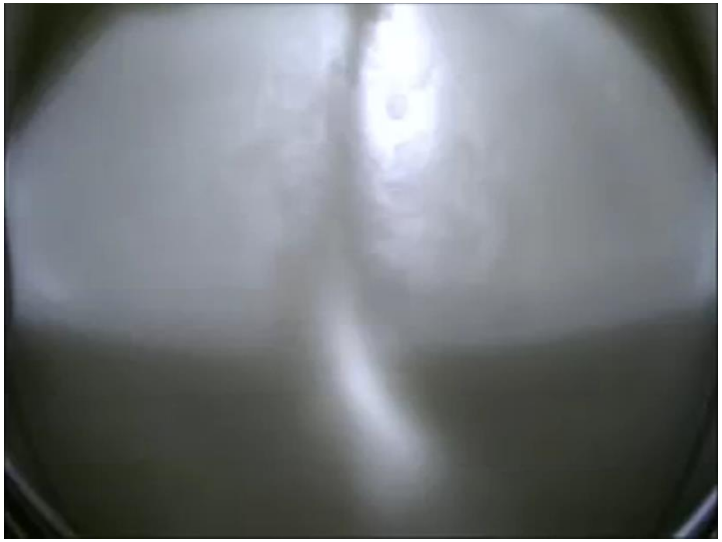


The wax melts from the outside (time = 970 minutes).

Wax Response 90 °C to Ambient



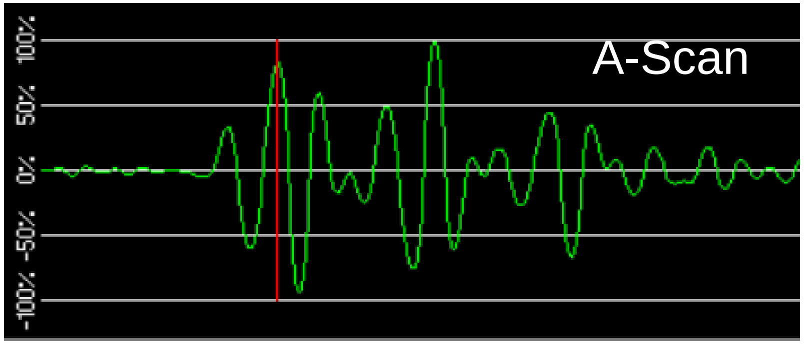
Increasing time

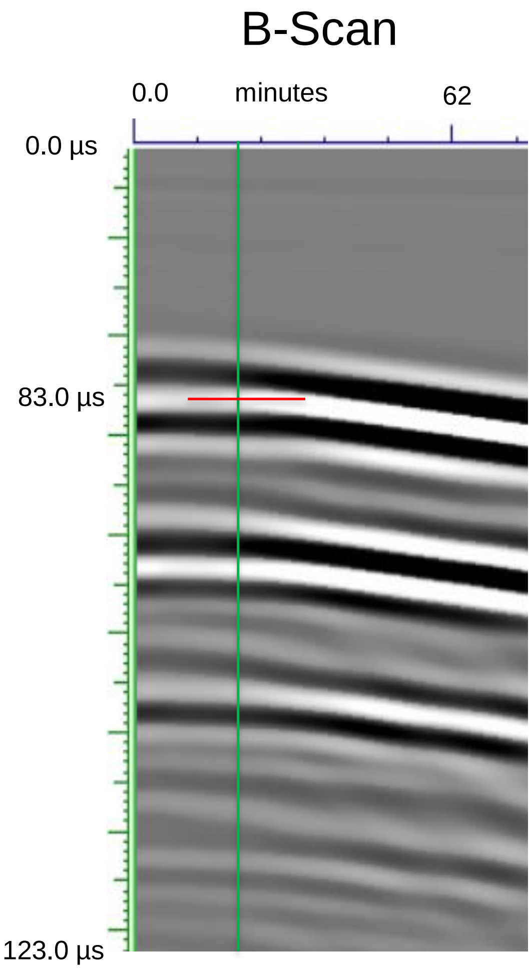


The temperature chamber is turned off and the wax is allowed to cool:

(time = 970 – 1300 minutes).

Data Recording

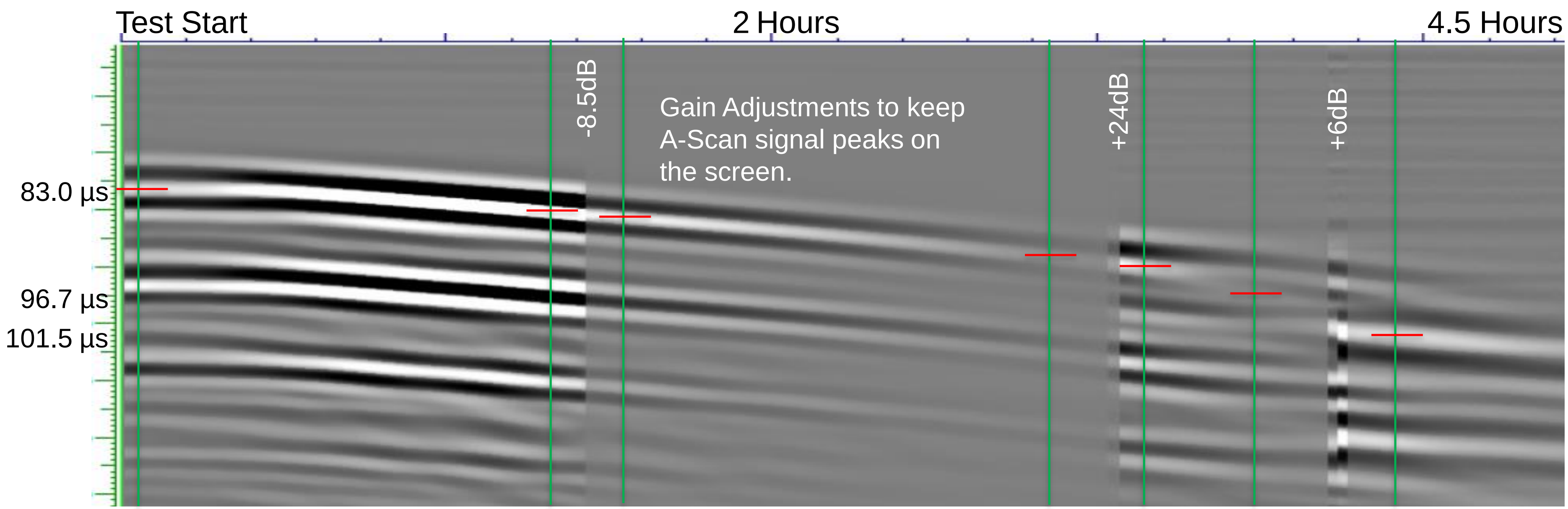


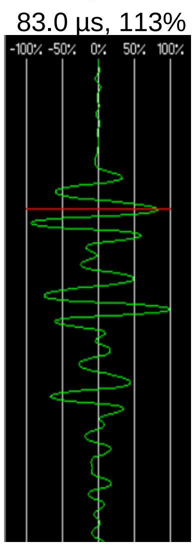


A-Scan images shows the un-rectified signal from the receiving (TTU) probe. These are used to measure the signal peak heights, signal locations in time, and find material velocities. During heating the peak of this signal shifts later in time and loses amplitude.

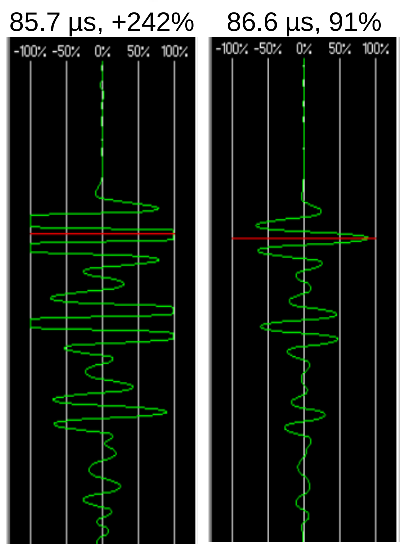
Each vertical data line of a B-Scan image is created by applying a color scale to every A-Scan recorded at a set point in time. On the color scale -100% is the darkest moving toward white at +100%. The Vertical Green line represents the individual A-Scans. The horizontal red line corresponds to the red line in the A-Scan, and represents the length of time it took the signal to get through the material in microseconds.

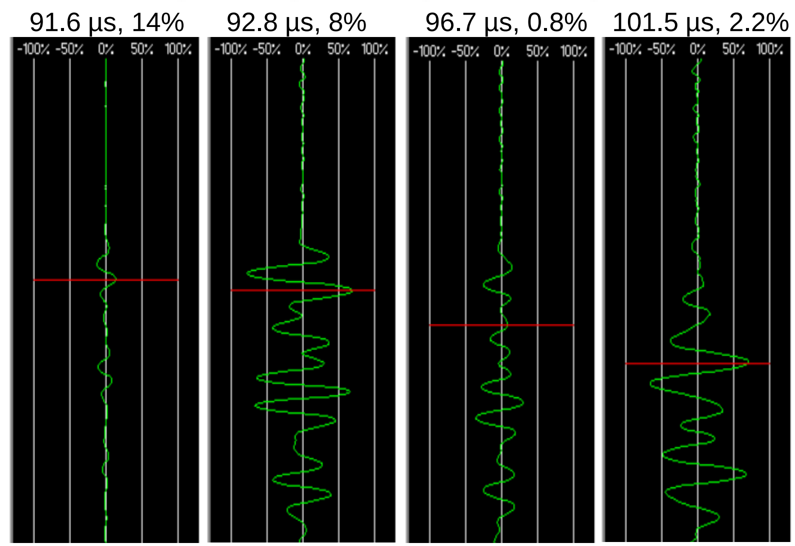
Ultrasonic Response to Temperature



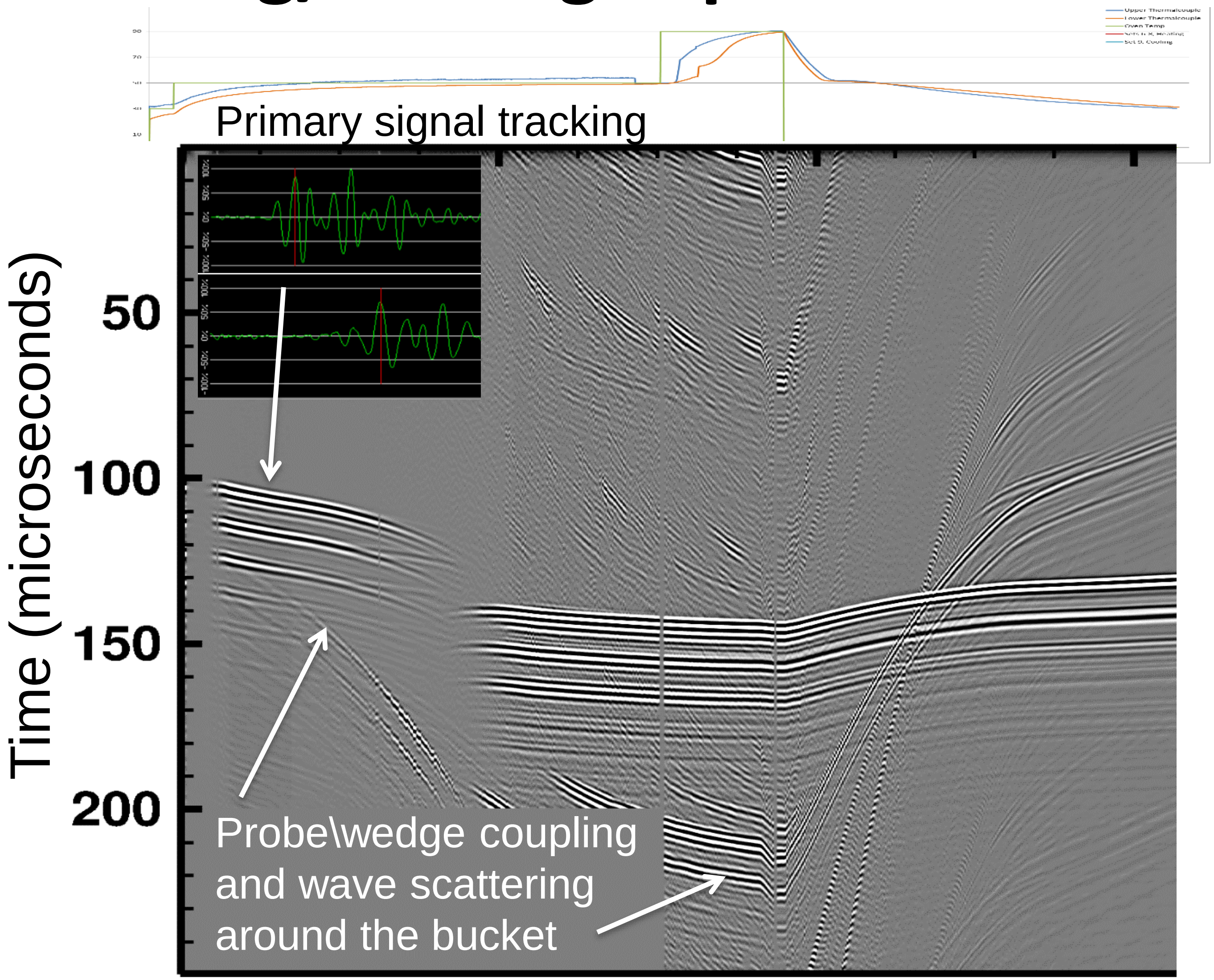


Note: All listed Gain percentages were adjusted to be in the same scale. During the test the gain was raised and lowered to keep the signals on the screen.



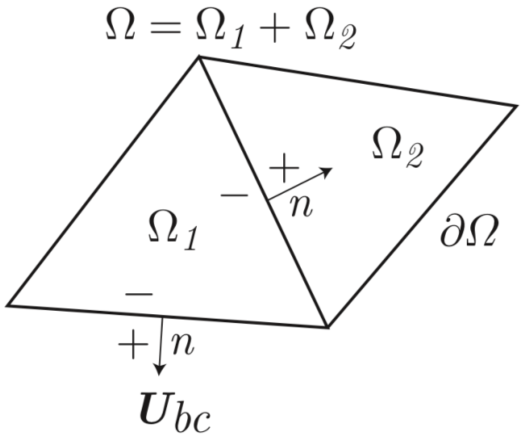


Wax Melting/Cooling Experiment #1



Melting | Cooling

Discontinuous Galerkin



Strong form:

$$\mathbf{U}_{,t} + \mathbf{F}_{i,i} = \mathbf{S}, \quad \text{in } \Omega$$
$$\mathbf{U}(x,0) = \mathbf{U}_0(x) \quad \text{at } t = 0$$

and appropriate boundary conditions on $\partial\Omega$.

Partition Ω into N subdomains Ω_e.

$$\int_{\Omega_e} \left(\mathbf{W}^T\mathbf{U}_{,t} - \mathbf{W}^T_{,i}\mathbf{F}_i\right) d\Omega + \int_{\partial\Omega_e} \mathbf{W}^T\mathbf{F}_n \, d\Gamma = \int_{\Omega_e} \mathbf{W}^T\mathbf{S} \, d\Omega$$

Introduce numerical fluxes $\mathbf{F}_n(\mathbf{U}) \rightarrow \hat{\mathbf{F}}_n(\mathbf{U}^-, \mathbf{U}^+)$ and sum over all elements

$$\sum_{e=1}^{N_e} \int_{\Omega_e} \left(\mathbf{W}^T\mathbf{U}_{,t} - \mathbf{W}^T_{,i}\mathbf{F}_i - \mathbf{W}^T\mathbf{S}\right) d\Omega + \int_{\partial\Omega_e} \mathbf{W}^T\hat{\mathbf{F}}_n \, d\Gamma = 0$$

$$\sum_{e=1}^{N_e} \int_{\Omega_e} \left(\mathbf{W}^T\mathbf{U}_{,t} + \mathbf{W}^T\mathbf{F}_{i,i} - \mathbf{W}^T\mathbf{S}\right) d\Omega + \int_{\partial\Omega_e} \mathbf{W}^T \left(\hat{\mathbf{F}}_n - \mathbf{F}_n\right) \, d\Gamma = 0$$

for all $\boldsymbol{W} \in \mathcal{V}$.

Governing Equations

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Acoustic Physics

$$\frac{\partial p}{\partial t} + \kappa \frac{\partial v_i}{\partial x_i} = -\frac{1}{3}\frac{\partial m_{ii}^{iso}}{\partial t} \quad \text{in } \Omega \times (0,T]$$

$$\rho \frac{\partial v_i}{\partial t} + \frac{\partial p}{\partial x_i} = f_i + \frac{\partial m_{ij}^{dev}}{\partial x_j} \quad \text{in } \Omega \times (0,T]$$

$$p(\mathbf{x},0) = 0 \quad \text{for } \mathbf{x} \in \Omega$$

$$v_i(\mathbf{x},0) = 0 \quad \text{for } \mathbf{x} \in \Omega$$

Point Source

$$m_{ij}(\mathbf{x},t) = -M d_{ij} w(t) \xi(\mathbf{x})$$

$$d_{ij} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$w(t) = \exp(-\pi^2 f_p^2 (t - t_0)^2)$$

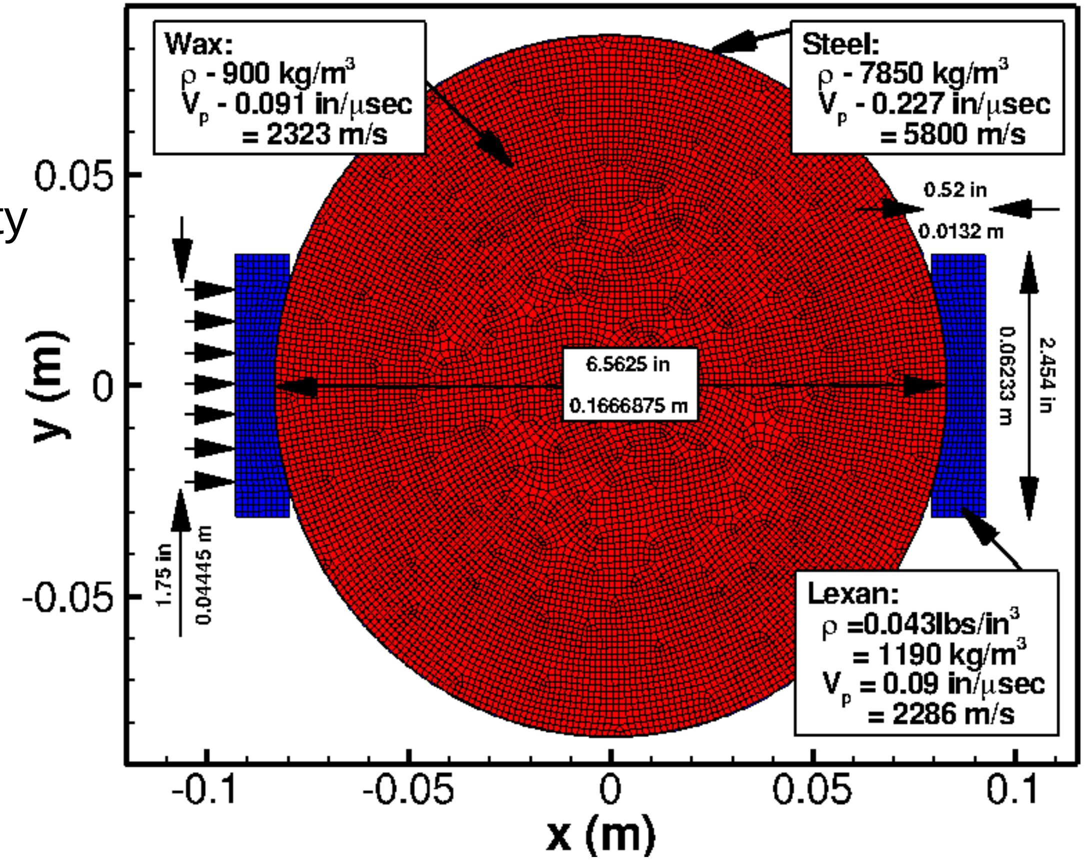
$$\xi(\mathbf{x}) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^N \exp\left(\frac{|\mathbf{x} - \mathbf{x}_s|^2}{2\sigma^2}\right)$$

m_{ij} = moment density tensor
M = moment magnitude
d_{ij} = orientation tensor
$w(t)$ = temporal waveform
$\xi(\mathbf{x})$ = spatial kernel

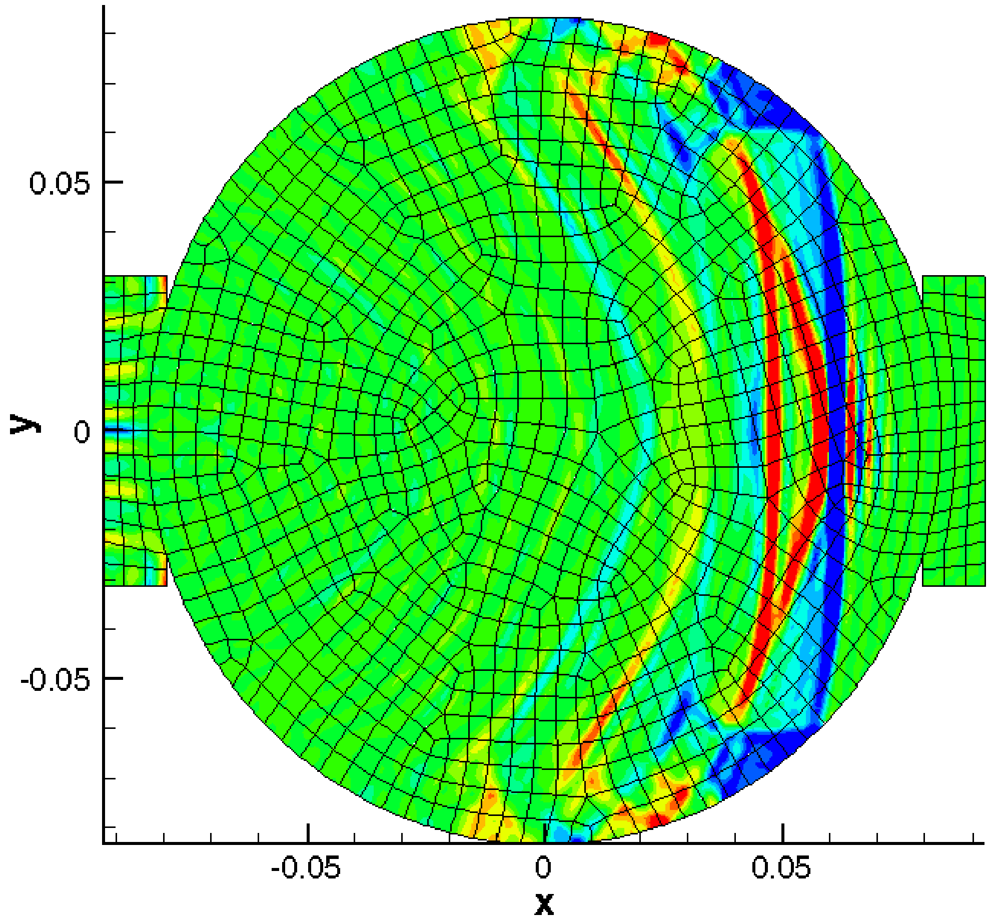
p = pressure
v_i = particle velocity
$\kappa = \rho c^2$ – bulk modulus
ρ = mass density
c = wave speed
f_i = force density
$m_{ij} = m_{ij}^{iso} + m_{ij}^{dev}$ – moment density tensor
$m_{ij}^{iso} \equiv \frac{1}{3} m_{kk} \delta_{ij}$
Ω = computational domain
T = time horizon

Simulation Setup

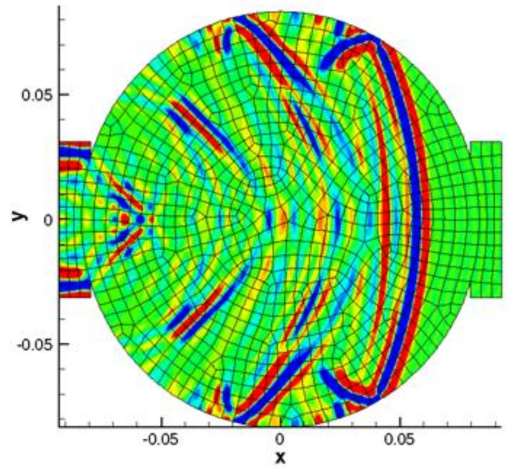
- Specimen boundaries (geometric dispersion)
- Material dispersion (the frequency dependence of the material constants mass density ρ, elastic moduli E, and dielectric constants
- Scattering dispersion (scattering of the waves by densely distributed fine inhomogeneities in the materials.
- Dissipative dispersion (the absorption of wave energy into heat)
- Nonlinear dispersion (dependence of the wave speed on the wave amplitude



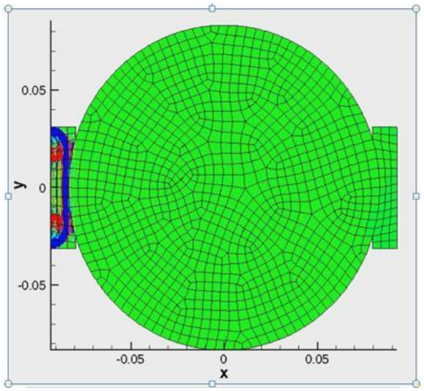
Probe Assembly Coupling with Metal Can

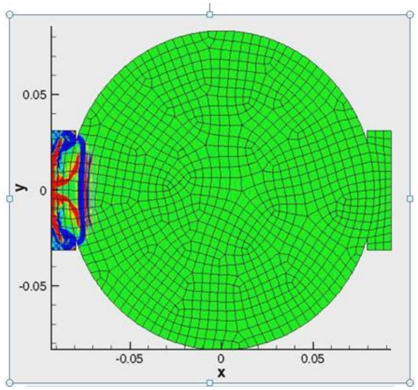


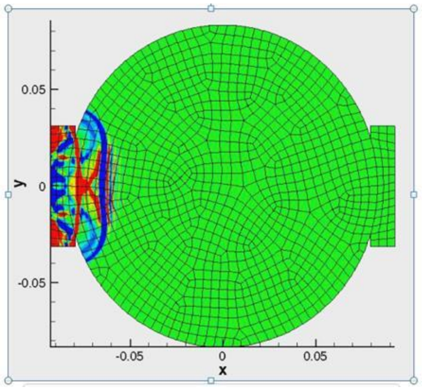
- Recorded Source
  - 25 MHz sampling
- Simulation : Time 150 microseconds
- “Planar” Expansion
- Wedge side ringing
- “Out of plane” wave
  - Pseudo-source somewhat masks echo signal

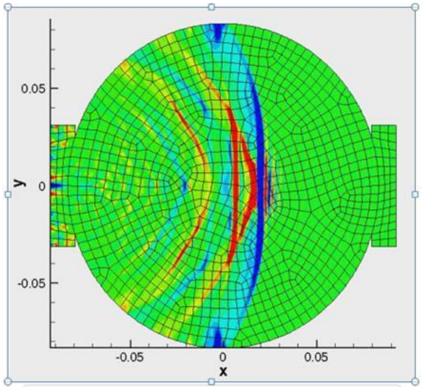


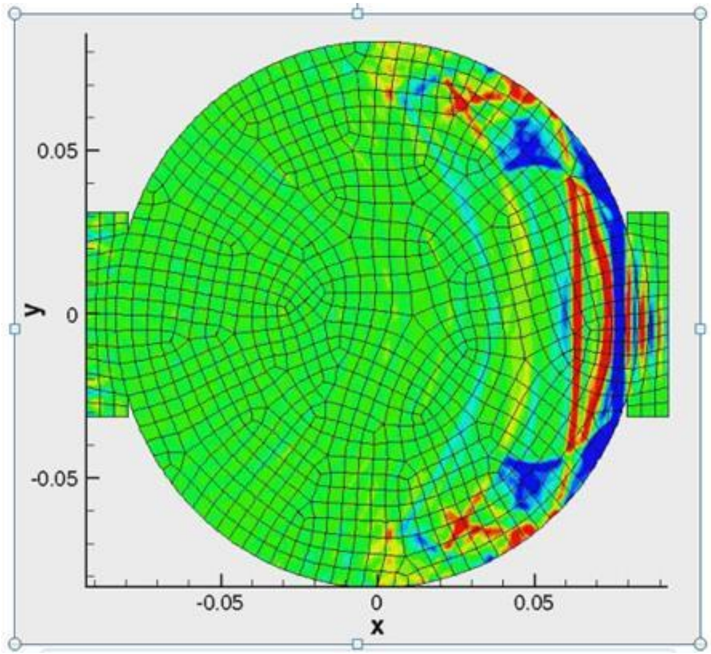
Interface Reactions

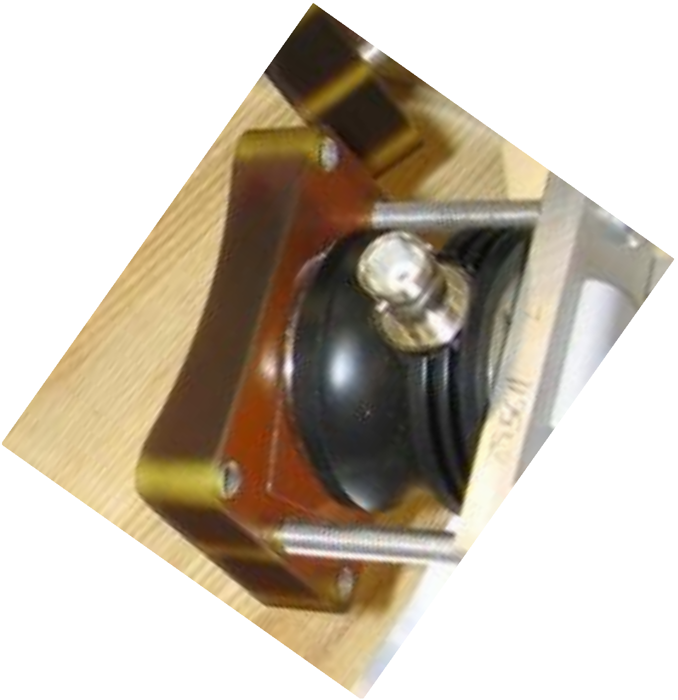


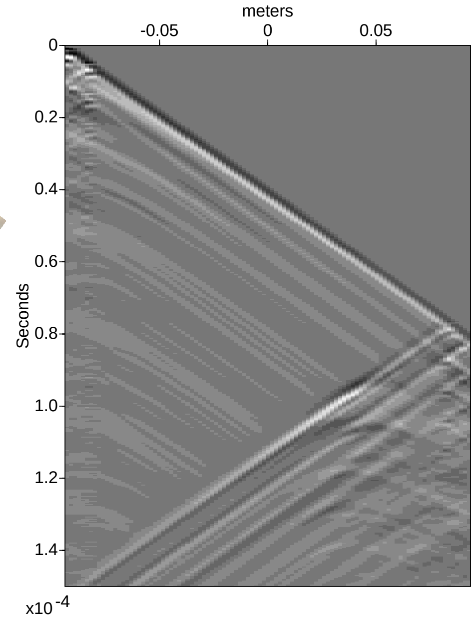




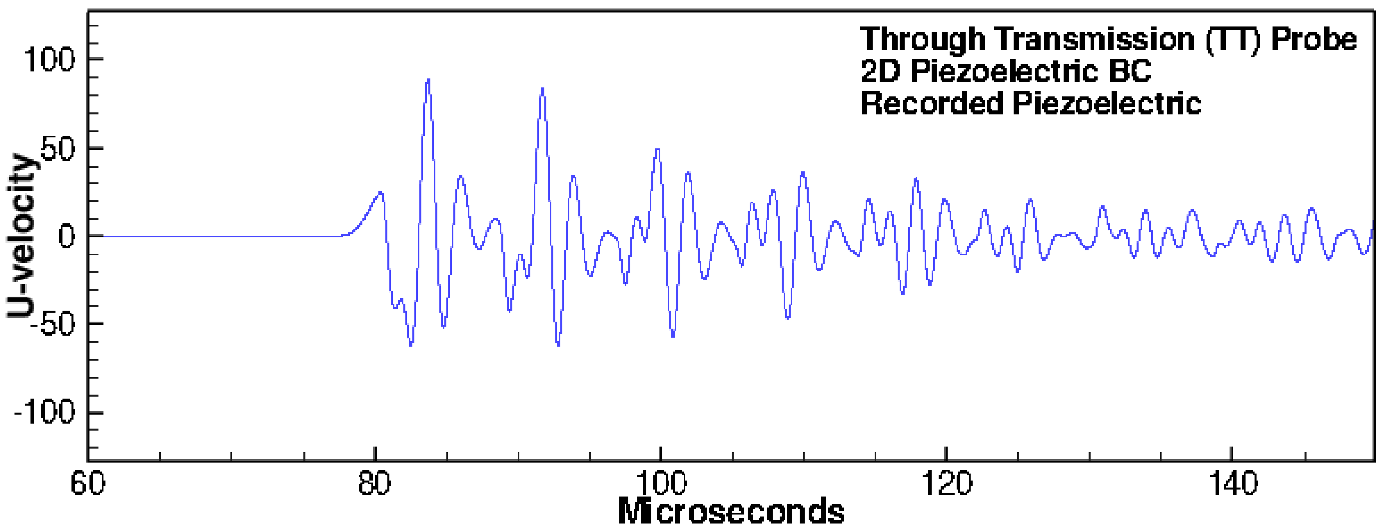


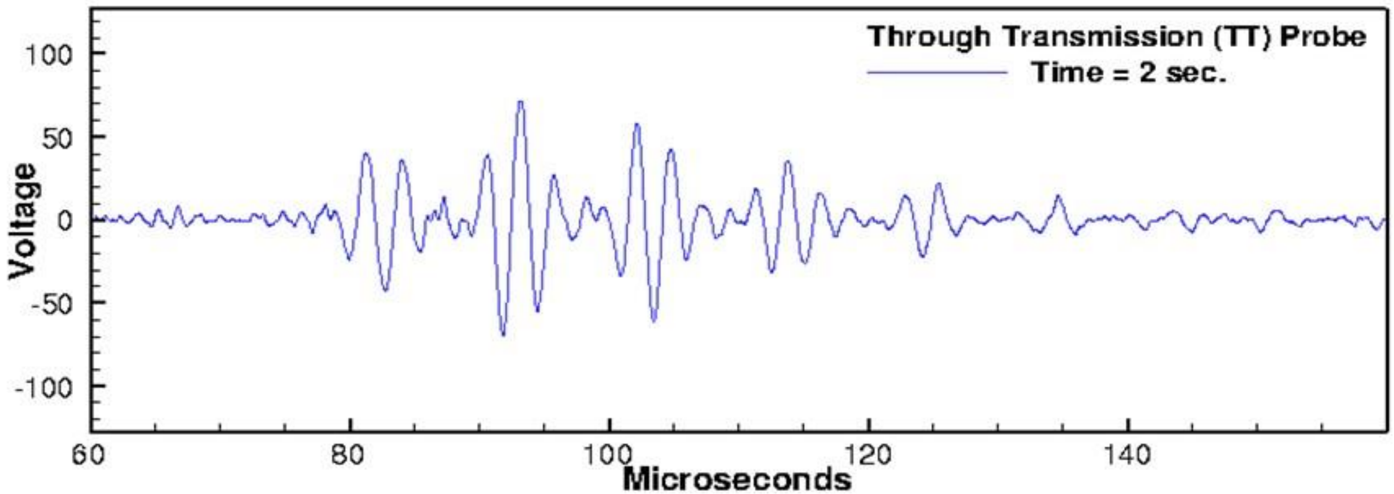




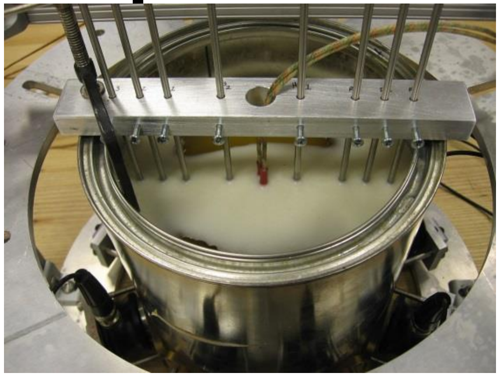


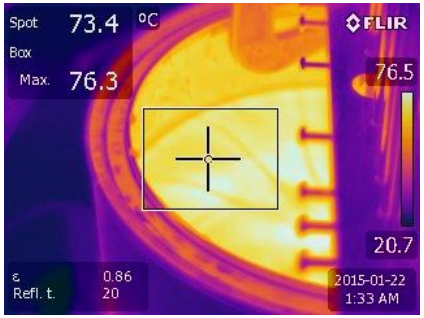
Signal Comparisons

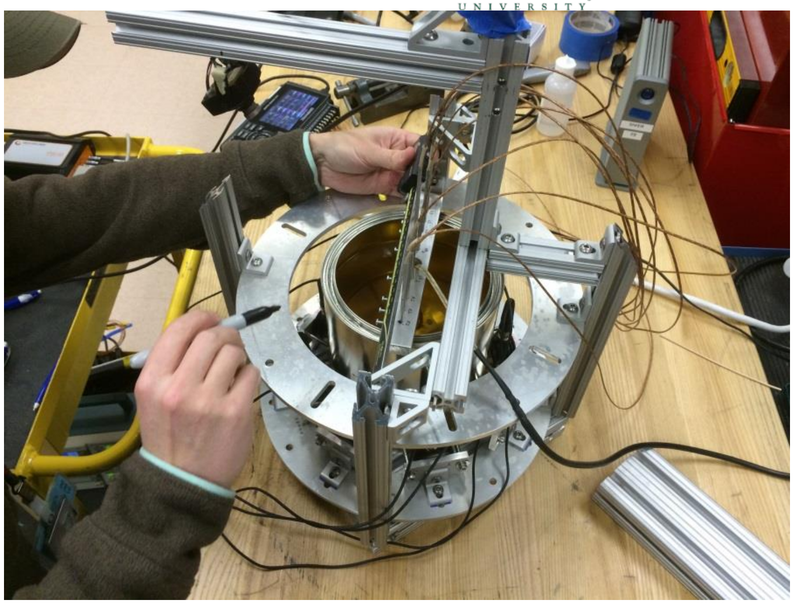




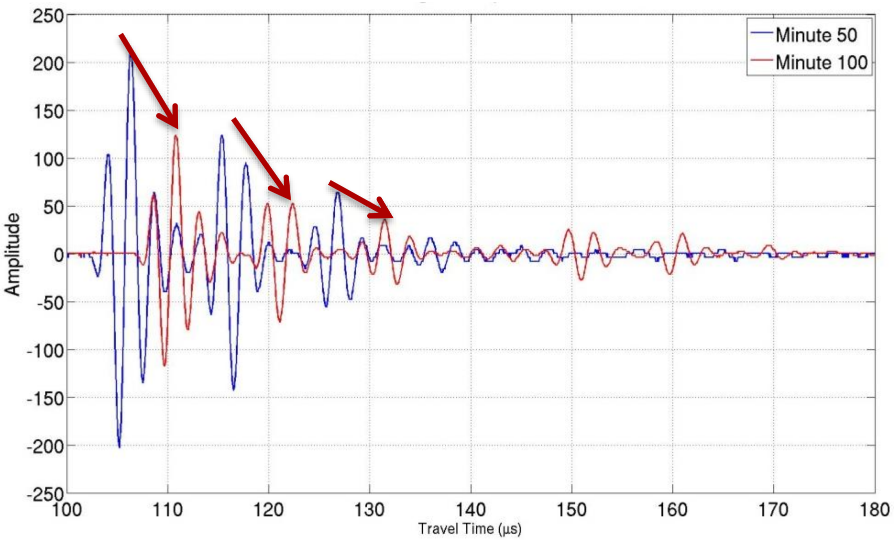
Experiment #2



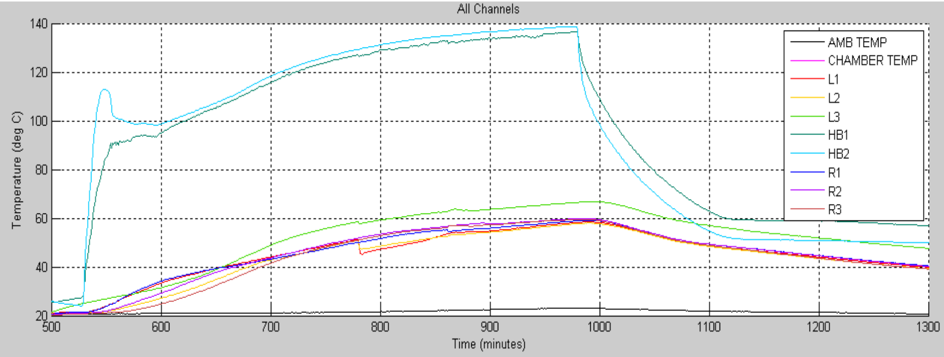


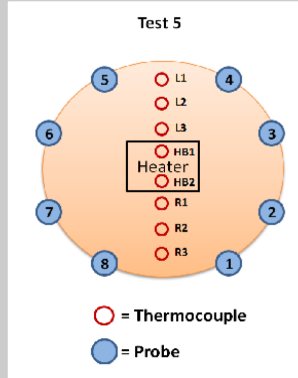


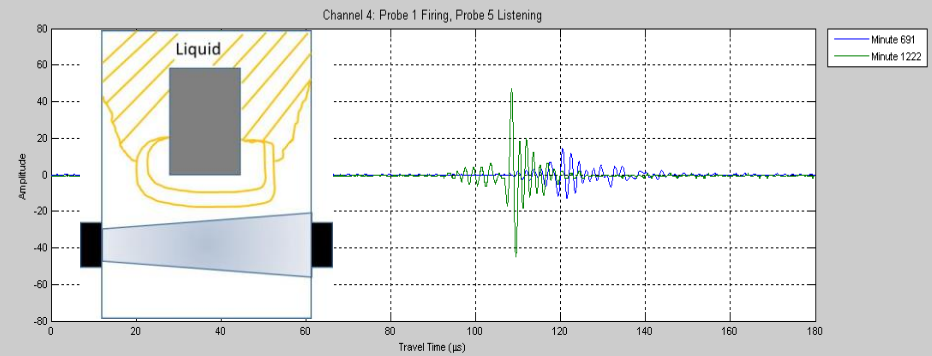
Additional thermal couples and 64 channels of through transmission ultrasonic data.

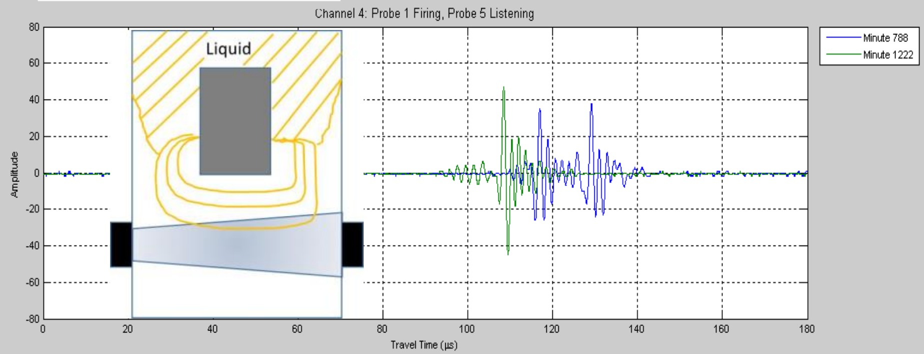


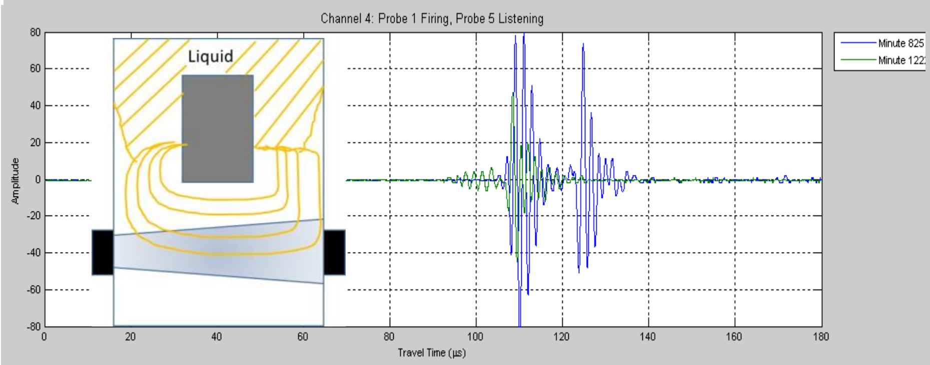
Experiment #2 Results

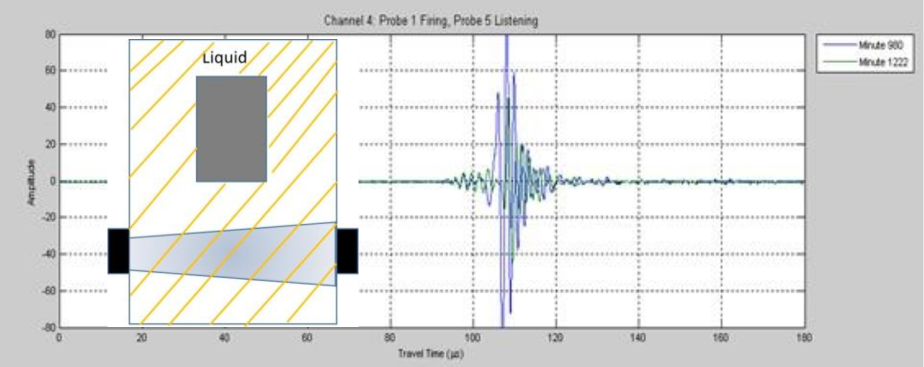












Conclusions

Elevated temperature and long exposure times will affect the sound coupling. Gel super clue works for now. Changes appear to be quite large 17.3% drop in velocity 112% drop in amplitude.

As the material is heated the “solid” signal response increases then is moves toward the right and lowers. Coupling through the outside with a wedge can be an issue. This thinner the wedge the better. Internal voids are problematic. Velocity and attenuation values are averages between the distance between probes.

The experimental results show the distortion experienced by the ultrasonic wave in the wax material. The wax is dispersive in nature and has a varying group velocity as the internal temperature changes. It is also found that the wave contains frequency components within the bandwidth of the carrier frequency.

Full waveform acoustic/elastic inversion is a flexible numerical method that can accurately handle strong inhomogeneities in high attenuative dispersive materials. The partial differential equations governing acoustic wave propagation, scatter and attenuation properties must be written in a conservation form to allow arbitrary variation within each element.

Acknowledgements

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- Baylor University

Extra Slides

Algorithm Options - Background

- Coherence: measure of the strength of a linear relationship between two signals (> 0.7 is good)

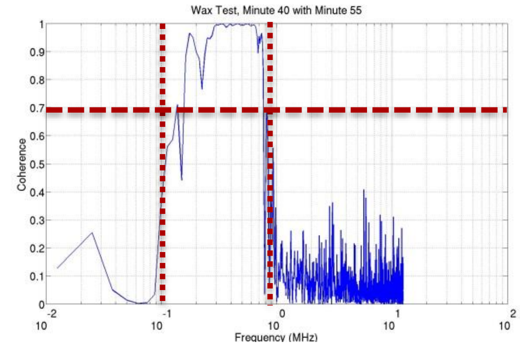
$$\gamma_{xy}^2(f) = \frac{\left|G_{xy}\right|^2}{G_{xx}G_{yy}}$$

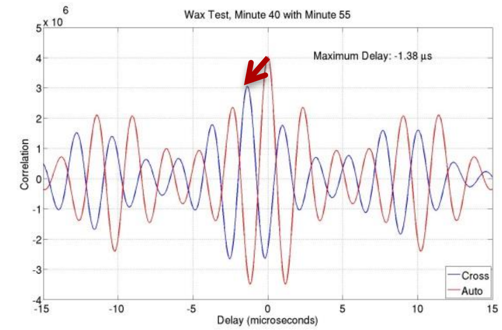
- Correlation: measure of the linear shift between two signals (delay)

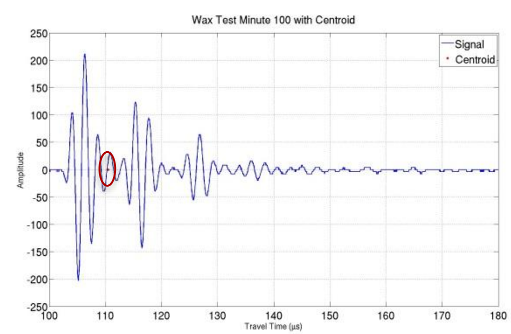
$$R_{x,y}(\delta) = \frac{1}{T}\sum_{i=1}^{T} x_i\, y_{i+\delta}\Delta t$$

- Centroid: measure of where the energy of one signal is centered

$$\tau = \frac{\int t|x(t)|^2 dt}{\int |x(t)|^2 dt}$$

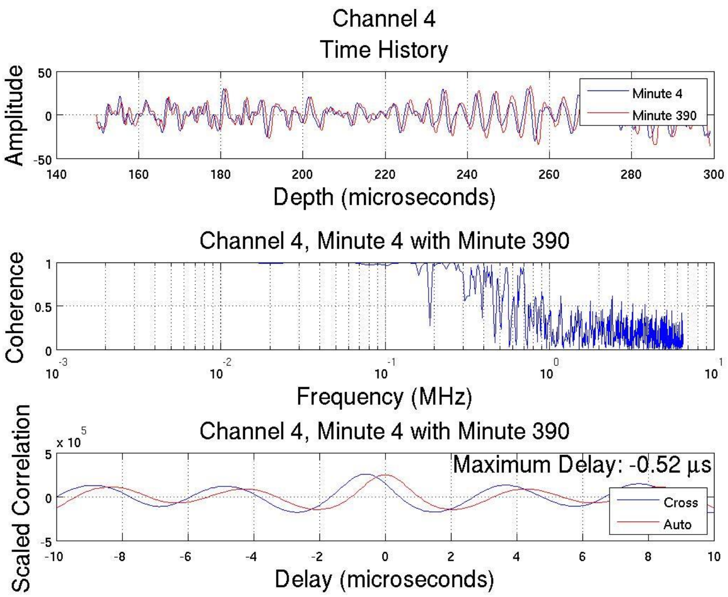




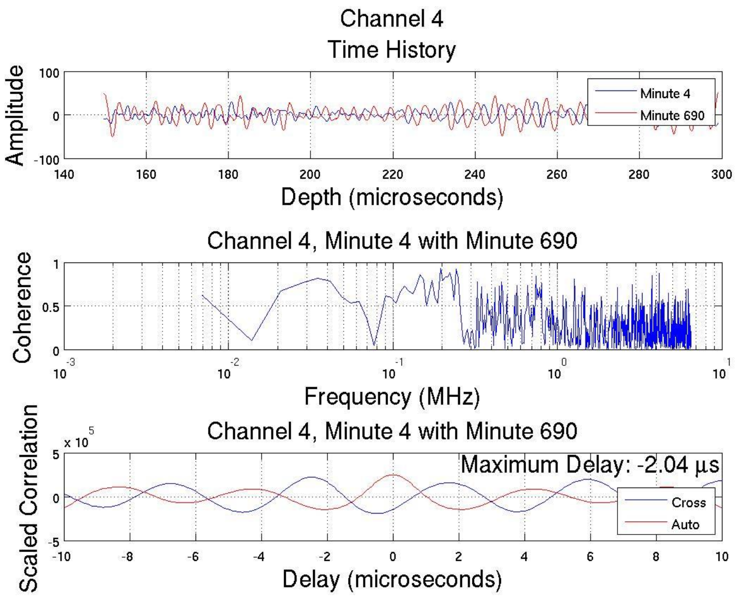


Coherence/Correlation Examples

Good: Coherence is equal to 1 at low frequencies. Early dips in the coherence may indicate natural frequencies of the system. There is a clear 'cliff' in the coherence, after which the data is noise. The cross correlation is clearly a linear shift of the auto correlation.



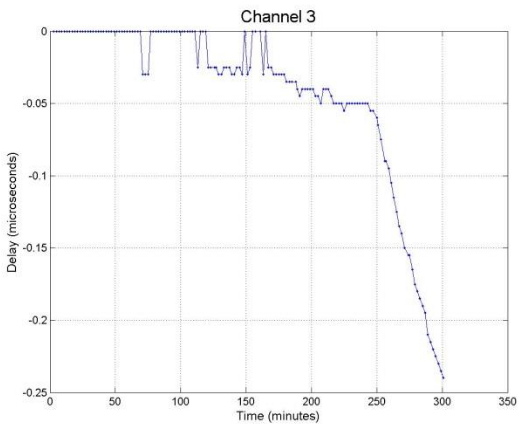
Poor: Coherence is frequently less than 0.5. It is difficult to see if there is a relationship between the cross correlation ad the auto correlation. In the time histories it is apparent that the signals are entirely different, not just shifted.



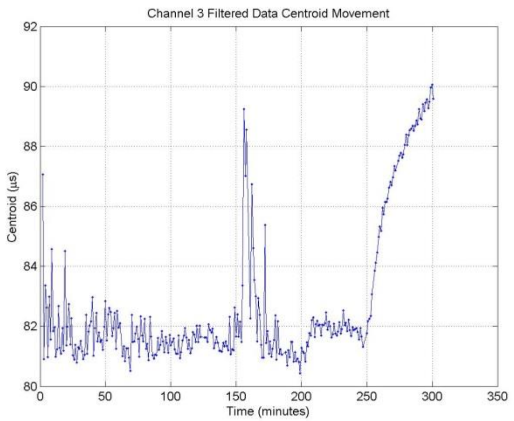
Note: When checking coherence between signals, it is also good to examine the PSDs of the signals to establish which frequencies are actually present in the data sets (and therefore should have high coherence). Low coherence at some frequency range may be due to the absence of that frequency range in one of the data sets.

Algorithm Options - Results

- Time Delay
  - Total time shift in signal from beginning
  - Only applicable if coherence is high

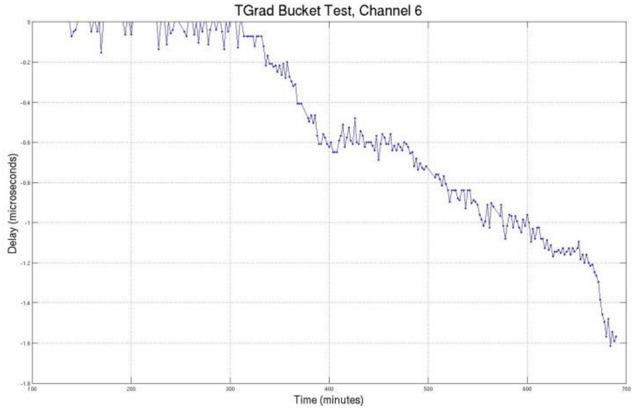


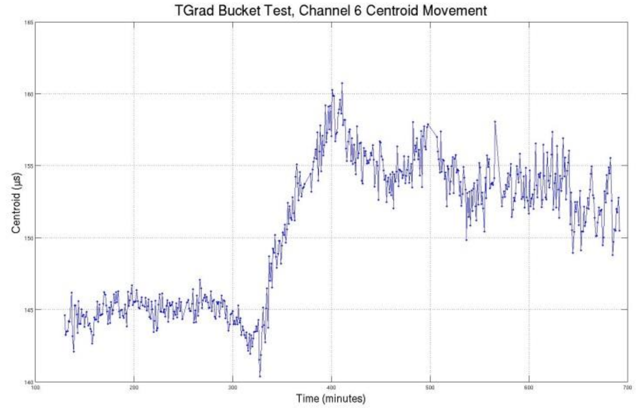
- Centroid Tracking
  - Indicates horizontal spread of signal
  - Only catches ‘big’ changes
  - Gate width must be constant
  - (Spike in middle is known noise)



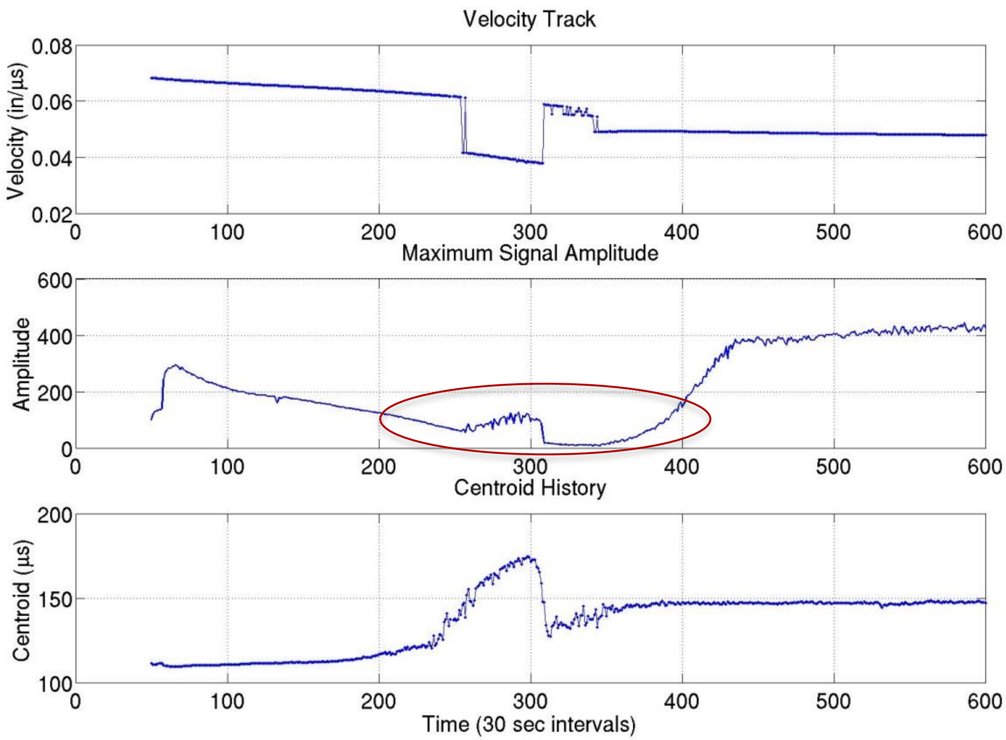
Algorithm Options - Results

- Time Delay
  - Total time shift in signal from beginning
  - Only applicable if coherence is high
  - Requires user oversight
- Centroid Tracking
  - Indicates horizontal spread of signal
  - Only catches 'big' changes
  - Gate width must be constant
  - Almost automated; must 'zoom in' on desired section of signal





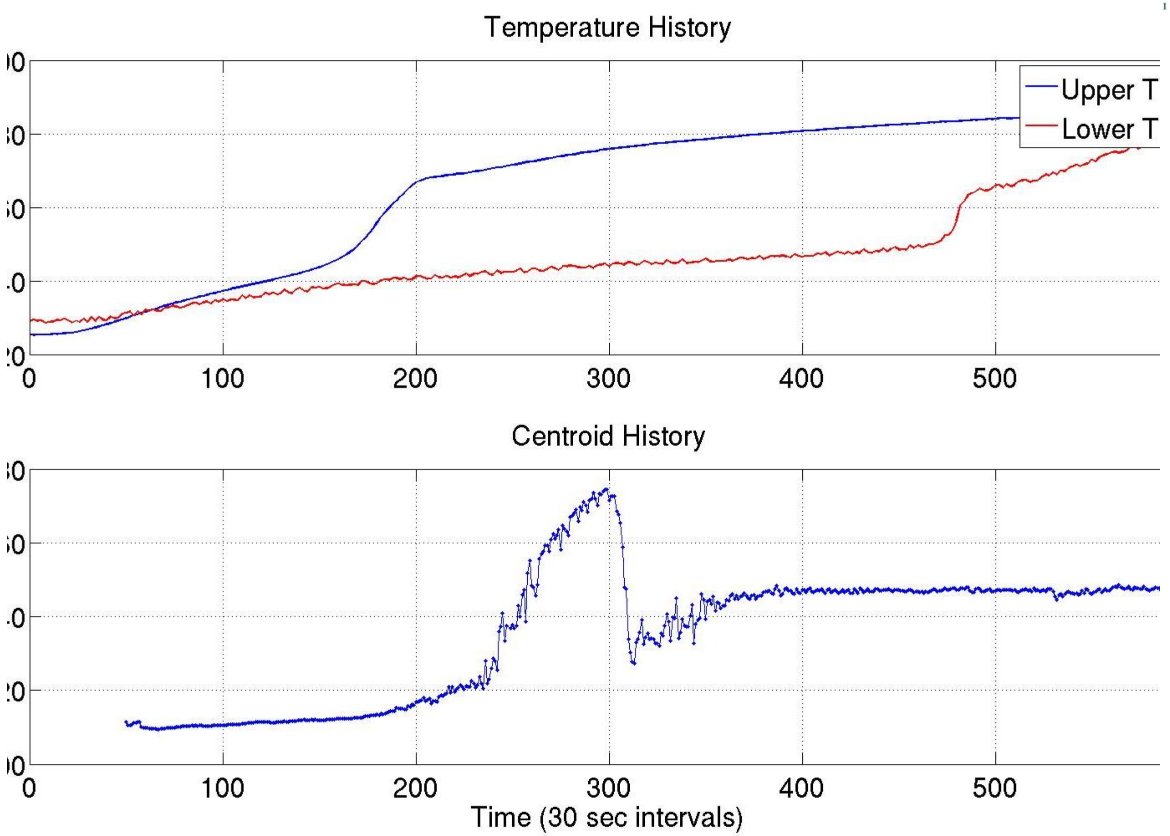
Wave Tracking Method



Signal slows down and spreads out as wax melts

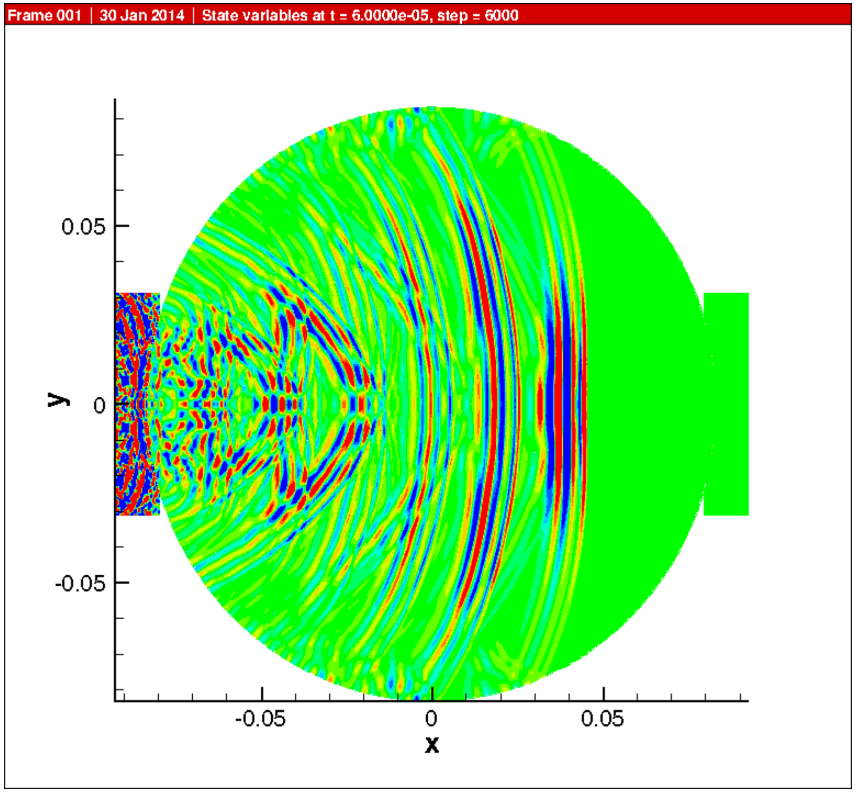
Signal lost in middle of test; low amplitude and noise in centroid catch this

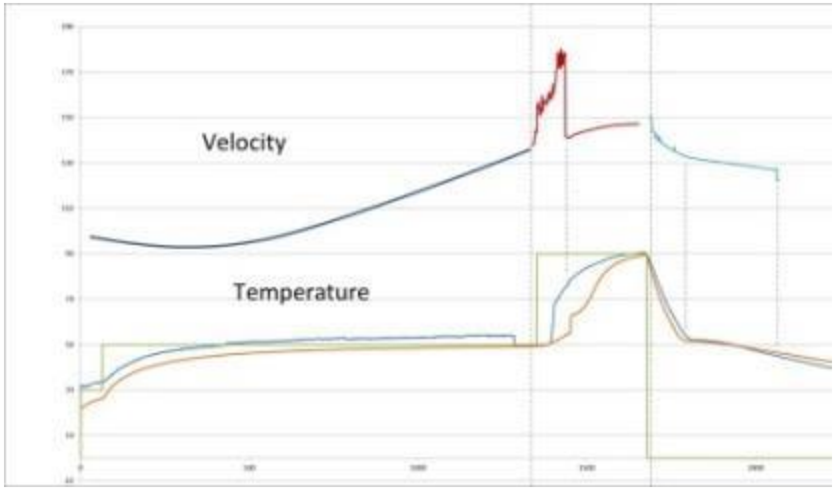
Conclusion: centroid comparable to velocity track



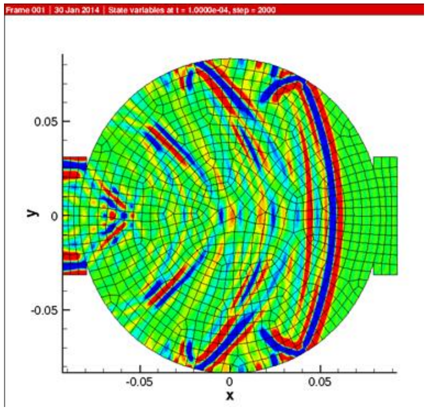
Event Check: Wax Data

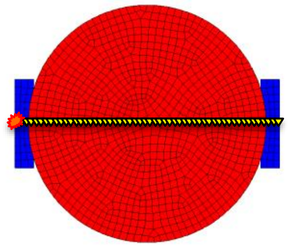
- Upper TC in wax that melts early; Lower TC in wax that melts later
- Conclusion: not a very clear connection between these two data types

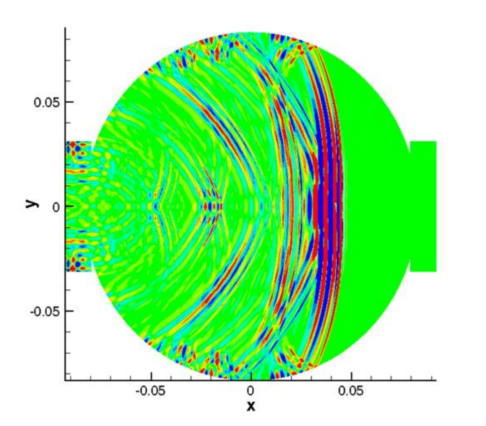

Frame 001 | 30 Jan 2014 | State variables at t = 6.0000e-05, step = 6000
0.05
y
0
-0.05
-0.05
0
0.05
x

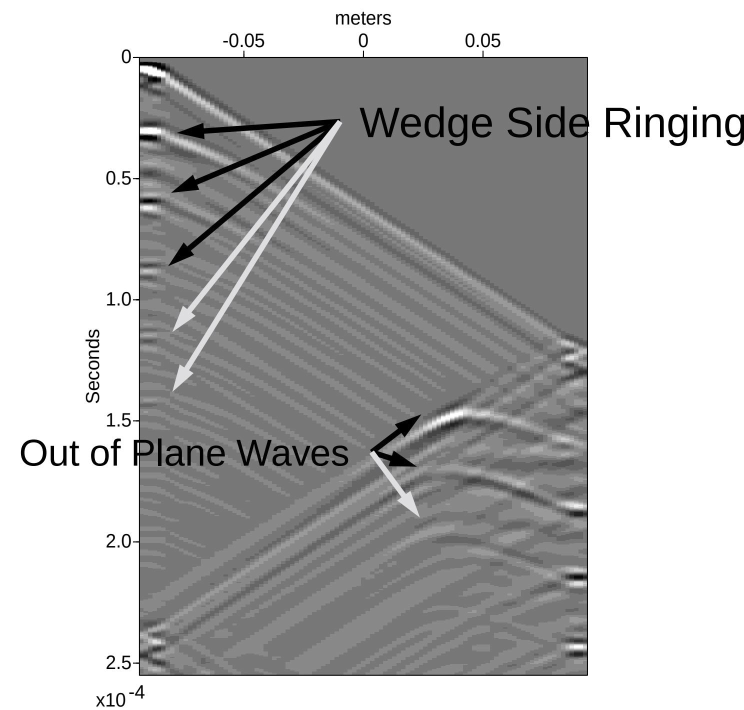

Velocity
Temperature

Point Source

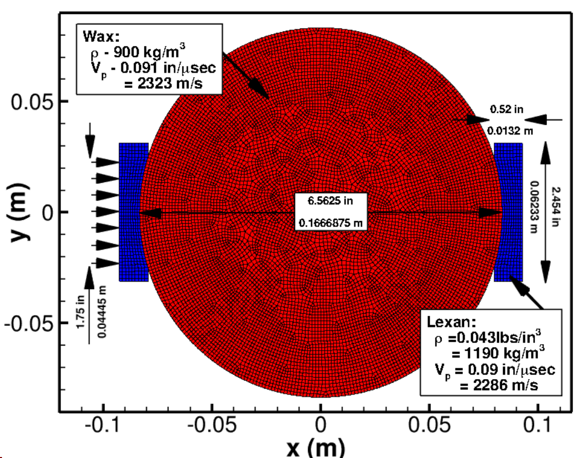


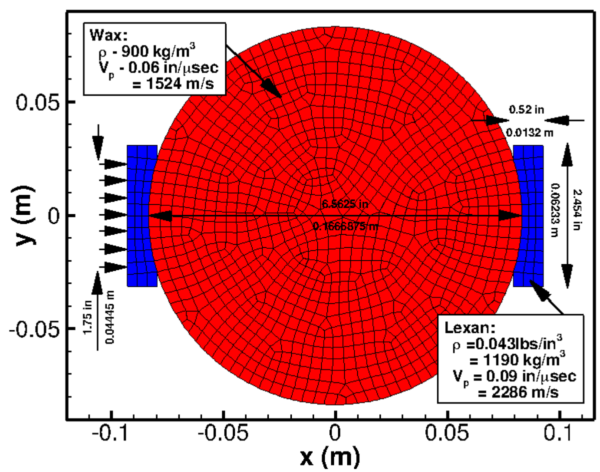






Wax01 Point Source



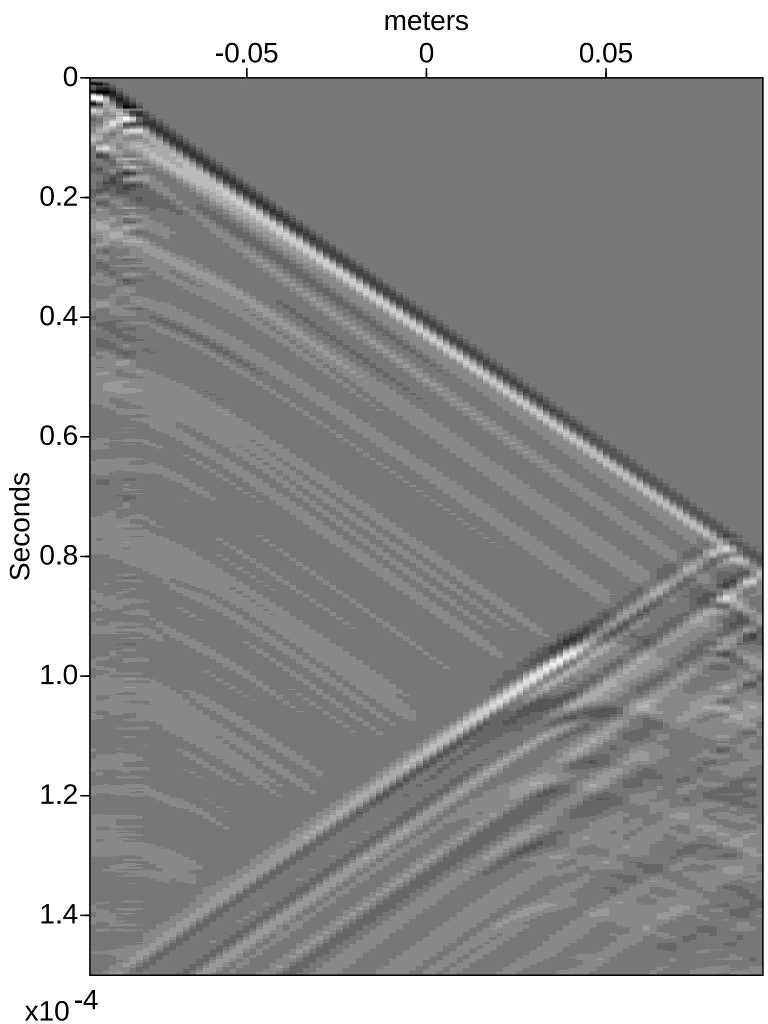


$$\left[(\lambda+\mu)\nu_x\nu_x + \left(\mu-\rho\nu^2\right)\right]A_x + (\lambda+\mu)\,\nu_x\nu_y A_y + (\lambda+\mu)\,\nu_x\nu_z A_z = 0$$

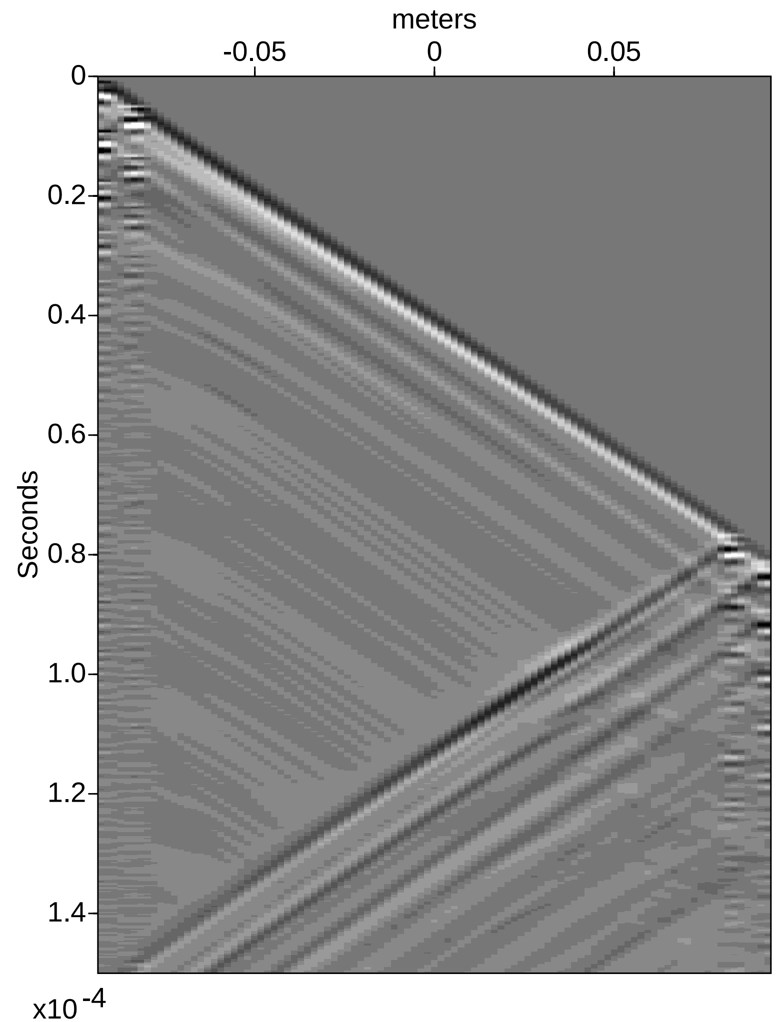
$$(\lambda+\mu)\nu_y\nu_x A_x + \left[(\lambda+\mu)\nu_y\nu_y + \left(\mu-\rho\nu^2\right)A_y\right] + (\lambda+\mu)\nu_y\nu_z A_z = 0$$

$$(\lambda+\mu)\nu_z\nu_x A_x + (\lambda+\mu)\nu_z\nu_y A_y + \left[(\lambda+\mu)\nu_z\nu_z + \left(\mu-\rho\nu^2\right)A_z\right] = 0$$

Calculated Values - Pressure and Velocity

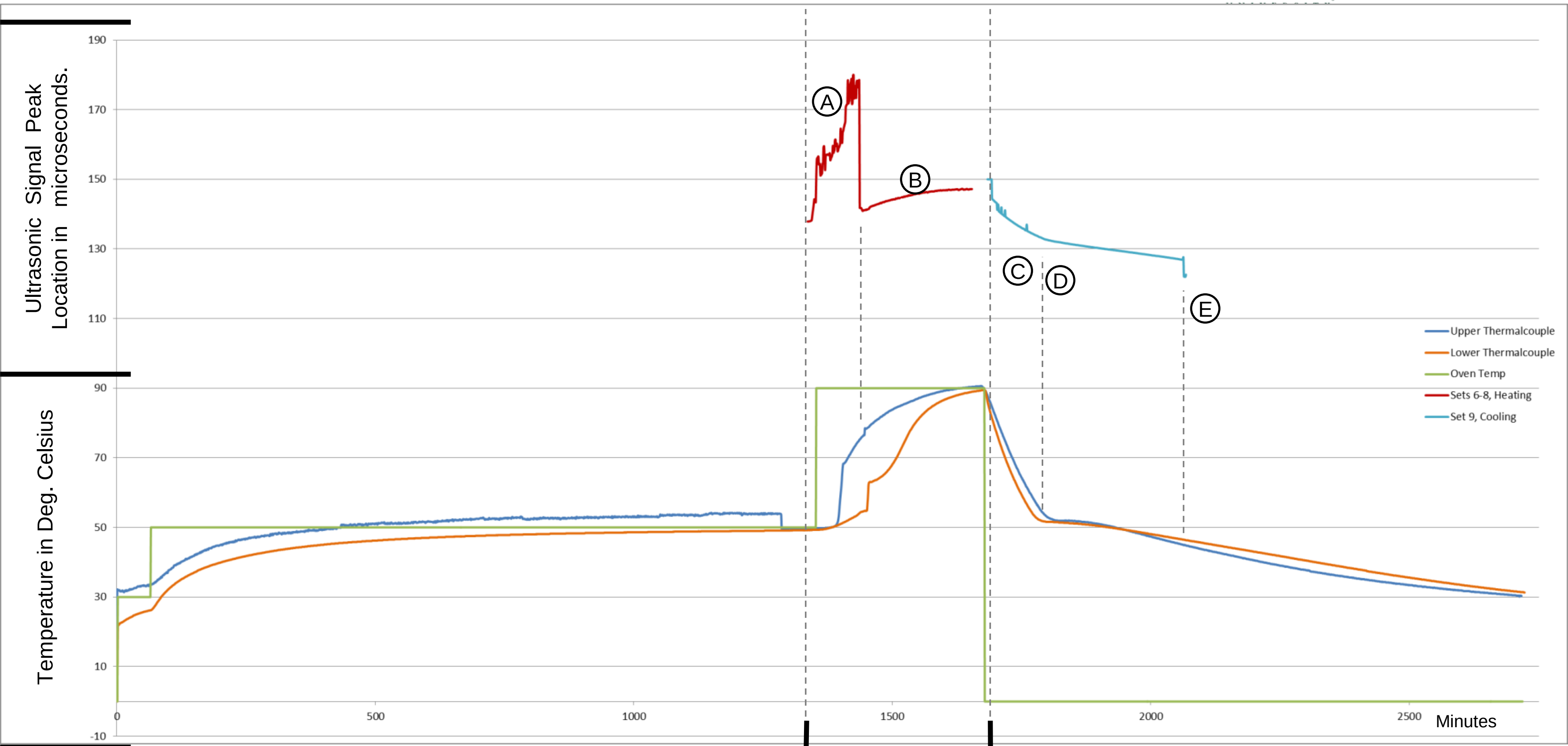


Wax01 Piezoelectric BC - Pressure



Wax01 Piezoelectric BC - U-velocity

Heating and Cooling of Wax



Heating the Wax to thermal equilibrium at 50˚C.

Once Equalized, Oven temp was raised to 90˚C, and held until the melted wax was stable at 90˚C. As the wax melted from the outside in, the un-melted center mass continually shifted and caused unstable signals, Ⓐ, until the wax was completely melted, Ⓑ, and equal temp.

Once liquified, the oven was turned off to allow the wax to cool. The wax cooled quickly as a liquid, Ⓒ, and slower when it started to Ⓓ solidify, Ⓔ Data recording capability was lost when the wax shrunk away from the sides of the bucket and contact was no longer made,

.

Wax Melting/Cooling Experiment #1