SANDIA REPORT
SAND2017-7282
Unlimited Release
July 2017

HedgeHOGS

A Rapid Nuclear Hedge Sizing and Analysis Tool

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Abstract

The U.S. nuclear stockpile hedge is an inventory of non-deployed nuclear warheads and a force structure capable of deploying those warheads. Current guidance is to retain this hedge to mitigate the risk associated with the technical failure of any single warhead type or adverse geopolitical developments that could require augmentation of the force. The necessary size of the hedge depends on the composition of the nuclear stockpile and assumed constraints. Knowing the theoretical minimum hedge given certain constraints is useful when considering future weapons policy. HedgeHOGS, an Excel-based tool, was developed to enable rapid calculation of the minimum hedge size associated with varying active stockpile composition and hedging strategies.

ACKNOWLEDGMENTS

The authors would like to thank Paul Nielan and Sheryl Hingorani for their support and guidance in this project. Thanks is also due to Staci Dorsey as well as the faculty and staff of the U.S. Military Academy at West Point for their coordination and support of the Military Academic Collaboration program that enabled this work.

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FIGURES

1. INTRODUCTION

The U.S. nuclear stockpile hedge is an inventory of non-deployed nuclear warheads and a force structure capable of deploying those warheads. As specified in the *Nuclear Matters Handbook*, the hedge is to be used to mitigate risks associated with unforeseen catastrophic failure of a class of delivery vehicles or weapon types or family of weapons or a reversal of geopolitical situations that would require more weapons available for use than currently deployed.[1] The June 2013 *Report on Nuclear Employment Strategy of the United States Specified in Section 491 of 10 U.S.C.* specifies the United States' nuclear hedge policy as achieving "a credible deterrent, with the lowest possible number of nuclear weapons…" while "maintain[ing] a sufficient number of non-deployed weapons to hedge against the technical failure of any single weapon type or delivery system at a time." [2] It is important to know what size hedge is required when making policy decisions. A hedge that is too small risks national security, while keeping an excessive hedge does not align with the "obligation to pursue in good faith" a route towards disarmament, as mentioned in the Nuclear Non-proliferation Treaty (NPT).[3] President Barack Obama, on April 5, 2009, in a speech delivered in Prague reiterates this obligation to pursue disarmament, "First, the United States will take concrete steps towards a world without nuclear weapons. To put an end to Cold War thinking, we will reduce the role of nuclear weapons in our national security strategy, and urge others to do the same. Make no mistake: As long as these weapons exist, the United States will maintain a safe, secure and effective arsenal to deter any adversary, and guarantee that defense to our allies…."[4]

Presented with this problem, Sandia National Laboratories has conducted analyses and has now developed a tool to allow technical and policy analysts to quickly and easily determine a minimum hedge size given certain policy-based and practical constraints.[5] This tool could be used by policy-makers to determine the theoretical minimum acceptable hedge size. This could be taken into consideration in future policy decisions and negotiations. Thus, the Hedge Highly Optimized Guidance-based Sizing (HedgeHOGS) tool was created. HedgeHOGS is a Microsoft Excel-based tool that takes inputs describing an active stockpile and outputs a minimum solution for hedge size to satisfy given constraints. HedgeHOGS is user-friendly, implementing GUIs and other features to make the program simple for a technical or policy analyst to use.[6]

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[1] U.S. Department of Defense, Office of the Deputy Assistant Secretary of Defense for Nuclear Matters, *Nuclear Matters Handbook 2016*, Washington, DC, 2016.
[2] 10 U.S. Code § 491 (2012).
[3] Treaty on the Non-Proliferation of Nuclear Weapons (NPT)." *UN Department for Disarmament Affairs*. United Nations, n.d. Web. 17 Oct. 2016.
[4] "Remarks By President Barack Obama In Prague As Delivered." *The White House*. White House, Web. 17 Oct. 2016.
[5] Lafleur, J. and Roesler, A., "Hedge Math: Theoretical Limits on Minimum Stockpile Size Across Nuclear Hedging Strategies," to be published in *Project on Nuclear Issues: A Collection of Papers from the 2015 Conference Series*.
[6] It is important to note that while the total number of weapons in the hedge given by HedgeHOGS is a minimum (and constant), the composition of the hedge could be just one of a family of solutions.

The general problem that is solved by the HedgeHOGS tool is shown to be a linear programming problem of the form shown below in Eq. (1).[7]

$$\begin{aligned} \text{Minimize:} \quad & N = \sum_{i=1}^{n} d_i \\ \text{Subject To:} \quad & g(x,p) = \begin{bmatrix} g_1(x,p) \\ \vdots \\ g_m(x,p) \end{bmatrix} \leq 0 \\ \text{By Varying:} \quad & x = \{h_j\}_{j=1}^{n} \end{aligned} \qquad \text{Eq. (1)}$$

where

d_i	=	quantity of weapon i in the deployed stockpile
h_j	=	quantity of weapon j in the hedge
$g_i(\boldsymbol{x}, \boldsymbol{p})$	=	algebraic relation describing the inequality constraint
n	=	total number of weapon types in the deployed stockpile
$\boldsymbol{p}$	=	parameters (fixed variables such as the number of deployed weapons) of the optimization

Throughout this report, the weapon numbers and yields are completely notional and provided only to illustrate the design and operation of the HedgeHOGS tool. However, the HedgeHOGS tool can be used with past, current, and projected future stockpile data (e.g., data provided in the Production and Planning Directive or the Nuclear Weapons Requirements Planning Document) in the appropriate computing environment.

[7] Lafleur, J. and Roesler, A., "Hedge Math: Theoretical Limits on Minimum Stockpile Size Across Nuclear Hedging Strategies," to be published in *Project on Nuclear Issues: A Collection of Papers from the 2015 Conference Series*.

2. HEDGEHOGS FEATURES

2.1. Basic Inputs

When the HedgeHOGS Excel file is initially opened and the *Main GUI* tab is selected, it will display a stockpile input sheet similar to that shown in Figure 1.[8] The Stockpile Table allows the user to specify the weapon types to be considered for both deployment and hedging. A *Start* button is located on the right hand side of the Stockpile Table along with two embedded files: a user Help file and this SAND report. Upon clicking the *Start* button, the Hedge Analysis dialog box appears which assists in populating and clearing the Stockpile Table, specifying additional options for the hedge strategy, and running the analysis.[9] Clicking the Help button pops up a document describing how to use the tool in more detail, while clicking on the SAND Report button pops up this SAND report.

Stockpile Table

Weapon ID	Weapon Name	Leg	Number of Warheads	Yield (kT)
1	Weapon1	Land	4500	1000
2	Weapon2	Land	200	500
3	Weapon3	Land	620	700
4	Weapon4	Air	2000	750
5	Weapon5	Air	1500	850

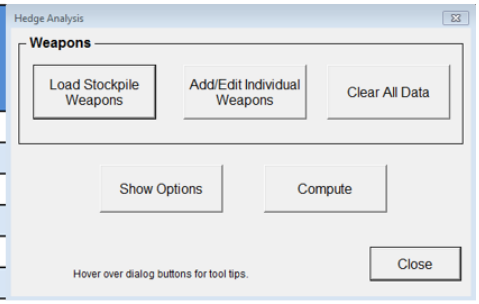

Figure 1. Weapon Input Sheet, populated with fictitious sample data

Weapon information can be entered into the Stockpile either manually or by using the *Add/Edit Individual Weapons* dialog found in the Weapons section *after* clicking the Start button. In either case, there are several pieces of required information in order to conduct an analysis:

Weapon ID: An ordinal identifier used for referencing the weapon
Weapon Name: A string input to identify the weapon
Leg: The leg in which the weapon is deployed. Allowable input for the Leg are: {Air, Land,
Sea, Other}
Number of Warheads: The number of deployed weapons in the stockpile
Yield: The yield in kilotons of the weapon[10]

In addition, three additional pieces of information can be enabled in order to perform advanced hedging analysis of the stockpile. These are:

[8] If the HedgeHOGS Excel file is saved and then reopened all information, including selected options, will remain in the Stockpile Table.
[9] Hovering over any button in the dialog boxes will provide additional guidance regarding the intent of the button.
[10] This is only required when performing a Weighted Hedging Strategy when trying to maintain a constant aggregate yield.

Substitutable Weapon IDs: The allowable Weapon IDs for weapons that can substitute for the given weapon[11]
Multiple Failure Weapon IDs: An ordinal identifier used for referencing the weapon that fails with a failure of the current weapon
Weapon Name: A string input to identify the weapon
Weight: A quantity used to weight weapon usage in a Weighted Hedging Strategy[12]

These are further discussed in Section 2.2.

2.1.1. Manually Entering Stockpile Information

One method of entering stockpile information is by manually typing the information into the Stockpile Table. At a minimum, this requires the user to manually type the Weapon ID, the Weapon Name, the Leg, the Number of Warheads and the Yield for each weapon in the stockpile being analyzed into the Stockpile Table. The default setting of the tool is to assume all weapons within the same selected leg can substitute for each other when hedging (*i.e.*, complete intra-leg hedging). Additional options such as allowing alternative hedging strategies (*i.e.*, not complete intra-leg hedging, inter-leg hedging, etc.), correlated weapon failures, and a maximum quantity of weapons allowable are also available to be manually entered, but require enablement through selecting the Start button, followed by selecting the Show Options button. These options, along with alternative hedging strategies, are further discussed in Section 2.2, Advanced Inputs.

2.1.2. Modify Stockpile Table Dialog

Another method of adding or changing weapon information is through the Modify Stockpile Table dialog that can be accessed by clicking the *Add/Edit Individual Weapons* button. This dialog, shown in Figure 2, allows the user to easily input or modify the required data for basic analysis.

To edit an existing weapon in the Stockpile Table, navigate to the Edit Existing Weapon portion of the Modify Stockpile Table dialog. The Weapon Name drop-down list contains all weapons currently defined in the Stockpile Table. Alternatively, the user could utilize the *Next Weapon* and *Previous Weapon* buttons to scroll up and down through the weapon types in the Stockpile Table. The remaining fields (# of Warheads, Leg, and Yield) will be populated with the information from the Stockpile Table. These can then be edited as desired.

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[11] For example, assuming Weapon1, Weapon2, and Weapon3 are all in the same leg with Weapon IDs 1, 2, 3, respectively, and can substitute for each other for Weapon 1, "2, 3" would be input into this column as these are the ordinal Weapon IDs for Weapon2 and Weapon 3. Similarly, for Weapon 2, "1, 3" would be input, and for Weapon 3, "1, 2".

[12] When selected, the weights are pre-populated to be the yield to allow for a hedging strategy of maintain the aggregate yield of the stockpile. However, arbitrary numbers can be input into this field.

To add a new weapon type not currently in the Stockpile Table, navigate to the Add New Weapons portion of the Modify Stockpile Table dialog. The user can then enter the new weapon type information. The Weapons Table is auto-populated with that weapon type after the *Add Weapon* button is pressed.

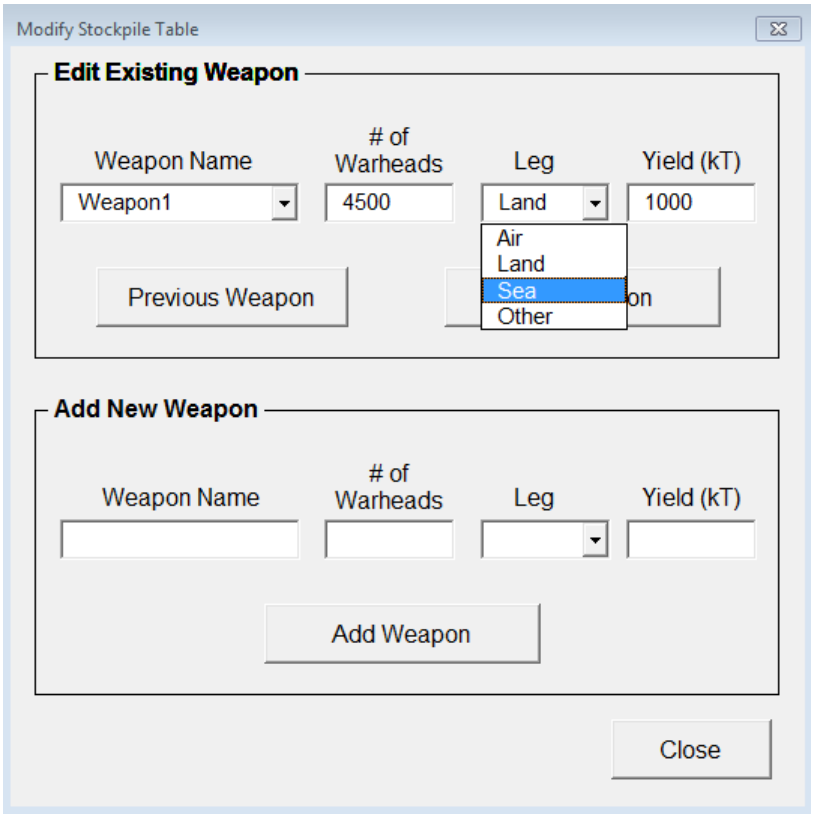

Figure 2. Modify Stockpile Table Dialog, populated with fictitious sample data

2.2. Advanced Options

To access more advanced options for the minimum hedge analysis, including alternative hedging strategies, multiple failures, ensuring a population of weapons of a given yield, or ensuring that a maximum number of weapons is not exceeded, select the *Show Options* button in the Hedge Analysis dialog (obtained after clicking the *Start* button from the *Main GUI* tab). Selecting the *Show Options* button will bring up the Options Dialog, shown in Figure 3.

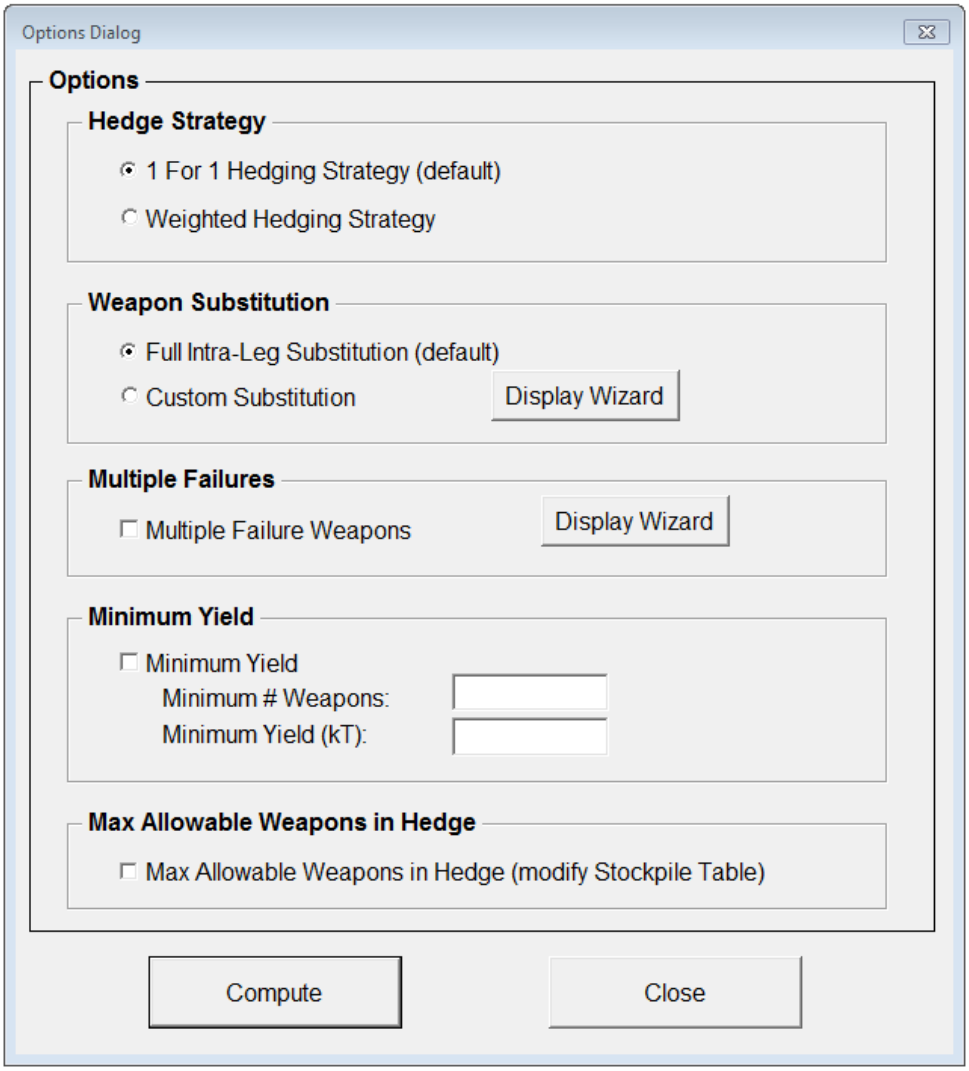

Figure 3. Options Dialog

2.2.1. Hedge Strategy Option

By default, HedgeHOGS assumes a one for one hedging strategy. That is to say, should a weapon fail in the deployed leg, a weapon from the hedge can be uploaded to substitute for that weapon (according to the appropriate substitution rules). This option is indicated by the radio button next to *1 For 1 Hedging Strategy (default)* being selected. In this case, the constraint equations in Eq. (1) relating to the one for one hedging strategy are of the form

$$g_k(x,p) = \sum_i d_i - \sum_j h_j,\ k = \{1,\ldots,m\} \qquad \text{Eq. (2)}$$

where

d_i = quantity of weapon i in the deployed stockpile

h_j = quantity of weapon j in the hedge

and the index i indicates the failed weapon, the index j indicates the allowable substitutable weapons for that failed weapon, and the index k indicates the k^{th} constraint. Note that this is just the general form and the summations are over different indices to allow for alternate weapon substitution rules.

The user also has the option to implement a weighted hedge strategy by selecting the radio button next to *Weighted Hedging Strategy*. This alters the form of the constraint equations in Eq. (1), to

$$g_k(x,p) = \sum_i \alpha_i d_i - \sum_j \alpha_j h_j,\ k = \{1,\ldots,m\} \qquad \text{Eq. (3)}$$

where

d_i = quantity of weapon i in the deployed stockpile
h_j = quantity of weapon j in the hedge
α_i or α_j = weight associated with weapon i or j in the hedge

In the case where the weights, that is the α_i and α_j, in Eq. (3) are chosen to be the weapon yields (the default for weighted hedging), the constraints in Eq. (3) defines a relationship such that the total aggregate yield of the stockpile is maintained. That is to say that the summation of the yield of the weapons that can be uploaded from the hedge is equal to the summation of the yield of the failed weapons in the deployed stockpile (e.g., ten 5 kT weapons are required to replace one 50 kT weapon). However, this also gives a mechanism to preferentially bias the selection of substitutable weapons through the choice of appropriate weights.

2.2.2. Weapon Substitution Option

By default, HedgeHOGS assumes only weapon types within the same leg can substitute for each other (complete intra-leg hedging). In this case, the domain of j in Eqs. (2) and (3) consists of the other weapons in that leg. This may not be the case if (1) inter-leg hedging is permitted, or (2) an alternative, defined weapon substitution strategy is desired. As mentioned previously, changing the allowable weapon substitution strategy alters Eqs. (1) and (2) by changing the domain j to be the set of allowable substitutions. Alternate weapon substitution strategies can be enabled by selecting the radio button next to *Custom Substitution*. After selecting the *Custom Substitution* radio button, the user can either manually enter the allowable substitutions or follow a dialog by selecting the *Display Wizard* button located next to the *Custom Substitution* radio button.

After selecting the *Custom Substitution* radio button, a column entitled Substitutable Weapon IDs will appear. By default, the allowable substitutable weapons will be the remaining weapons on the same leg. However, the desired substitution strategy can be manually modified by typing the Weapon IDs of all of the desired allowable substitutable weapon types in the Stockpile Table, separated by commas. The interpretation of this column is that if the weapon in the Weapon

ID/Weapon Name column fails then the weapons identified in the Substitutable Weapon IDs column can substitute for it.

The Substitutable Weapons Wizard, shown in Figure 4, is accessed by clicking the *Display Wizard* button located next to the *Custom Substitution* radio button field. The dialog allows for the user to select the appropriate substitutable weapons by first selecting the failed weapon from the drop-down list, then moving the desired weapons from the left list to the right list using the *Add ->* button. If there are weapons that are not desired to be substituted, select the *<-Remove* button to remove them from the list. As an example, in Figure 4, if Weapon 1 fails then Weapon 2, Weapon 3, or Weapon 4 can substitute for it.

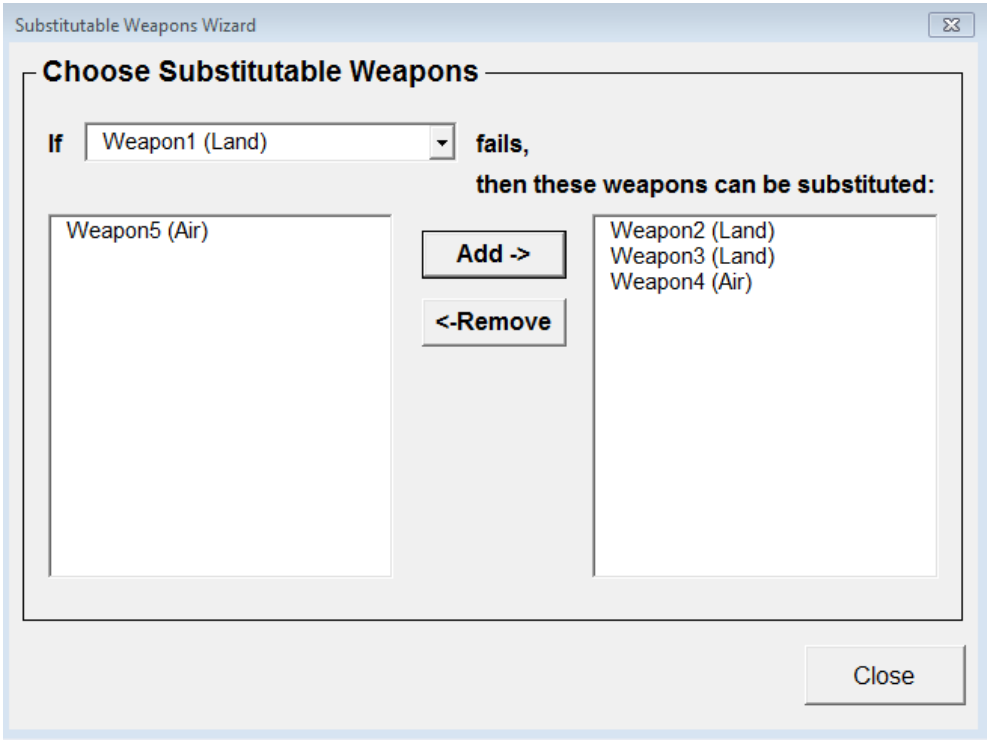

Figure 4. Substitutable Weapons Dialog, populated with fictitious sample data

Mathematically, the constraints are identical as previously described. However, the domain of *j* in either Eq. (2) or Eq. (3) may have been restricted (if some weapons from within the same leg are not allowed) or expanded (if inter-leg hedging is allowed).

As an example of the Stockpile Table for substitutable weapons, see Figure 5.

Stockpile Table					
Weapon ID	Weapon Name	Leg	Number of Warheads	Yield (kT)	Substitutable Weapon IDs
1	Weapon A	Other	50	100	2
2	Weapon B	Other	50	100	1
3	Weapon C	Other	50	100	1
4	Weapon D	Other	50	100	1
5	Weapon E	Other	50	100	1
6	Weapon F	Other	50	100	

Figure 5. Weapons Table Showing Substitutable Weapons, populated with fictitious sample data

In Figure 5, Weapon B (Weapon ID 2) is substitutable for Weapon A, and Weapon A (Weapon ID 1) is substitutable for Weapons B, C, D, and E. No weapons can substitute for Weapon F.

2.2.3. Multiple Failures Option

HedgeHOGS allows multiple failure hedging, which allows the user to model situations where, if one weapon type experiences a technical failure, others may as well due to common components or failure modes. For the case of multiple failures, the constraint equations defined by Eqs. (2) and (3) are modified such that domain of i, the number of deployed weapons, is increased to include the weapons that fail simultaneously. Similarly, the domain of j, the number of weapons in the hedge, is reduced by the weapons that fail simultaneously.

As an illustration of this idea, consider the case where there are three weapons, Weapon 1, Weapon 2, and Weapon 3, and all can substitute for each other. The deployed quantities of Weapon 1, Weapon 2, and Weapon 3, are given by d_1, d_2, d_3, respectively. Similarly, the hedge quantities for these three weapons are h_1, h_2, and h_3. Under the one for one hedging strategy assumption, the optimization problem solved by HedgeHOGS is given by

$$\begin{aligned} \text{Minimize:} \quad & N = d_1 + d_2 + d_3 \\ \text{Subject To:} \quad & g_1(x,p) = d_1 - h_2 - h_3 \le 0 \\ & g_2(x,p) = d_2 - h_1 - h_3 \le 0 \\ & g_3(x,p) = d_3 - h_1 - h_2 \le 0 \\ \text{By Varying:} \quad & x = \{h_1, h_2, h_3\} \end{aligned} \qquad \text{Eq. (4)}$$

Assume that if Weapon 2 fails at the same time as Weapon 1 (but that Weapon 1 does not fail at the same time as Weapon 2), the first constraint equation in the optimization problem defined by Eq. (4) becomes

$$g_1(x,p) = d_1 + d_2 - h_3 \leq 0 \qquad \text{Eq. (5)}$$

Note that the other constraints are not altered by enabling this multiple failure option.

To enable this capability, the user needs to navigate to the Multiple Failures portion of the Options Dialog. The user then selects the checkbox next to *Multiple Failure Weapons*. After selecting this, the user can either manually update the Stockpile Table in the Multiple Failure Weapon IDs column or select the *Display Wizard* button to cause the Multiple Failures wizard to appear. If manually updating the Stockpile Table, the Weapon IDs for the weapon that fails simultaneously with the weapon selected should be entered, separated by commas. When using the Multiple Failures Wizard, as shown in Figure 6, the user first selects a weapon that fails from the drop-down list. This will populate a list on the left side of the dialog with the possible weapons that could fail simultaneously with the weapon selected from the drop-down list. Move the desired weapons to fail simultaneously from the left list to the right list using the *Add ->* button. If there are weapons that are selected to be failed simultaneously that are not desired to do so, select the *<-Remove* button to remove them from the list on the right.

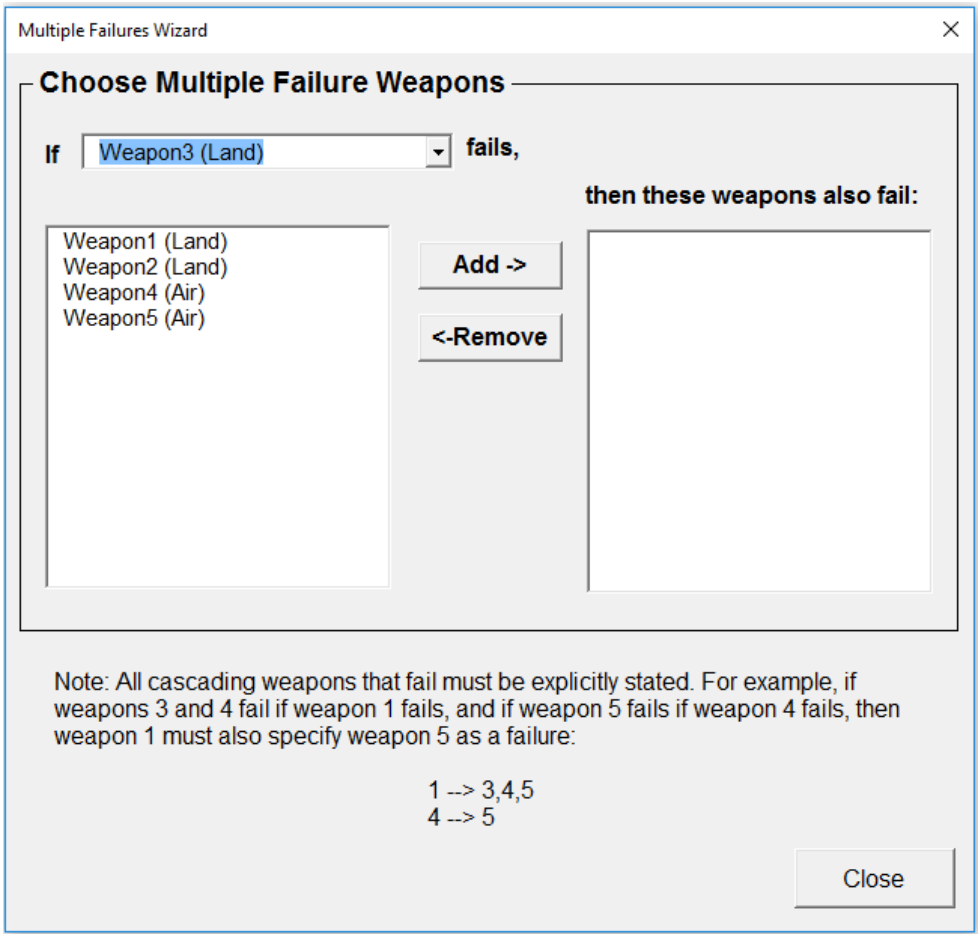

Figure 6. Multiple Failures Dialog, populated with fictitious sample data

Figure 7 shows an example of a modified Stockpile Table with multiple failures. This table says that if Weapon A fails, Weapon B (Weapon ID 2) and Weapon C (Weapon ID 3) fail. Also, if Weapon B fails, Weapon A (Weapon ID 1) fails.

Stockpile Table

Weapon ID	Weapon Name	Leg	Number of Warheads	Yield (kT)	Multiple Failure Weapon IDs
1	Weapon A	Other	50	100	2,3
2	WeaponB	Other	50	100	1
3	Weapon C	Other	50	100	
4	Weapon D	Other	50	100	
5	Weapon E	Other	50	100	
6	Weapon F	Other	50	100	

Figure 7. Weapons Table Showing Correlated Hedging, populated with fictitious sample data

As an important caveat, HedgeHOGS does not allow for cascading failures implicitly. That is to say that if Weapon 2 fails with Weapon 1 and Weapon 3 fails with Weapon 2, Weapon 3 will not fail if Weapon 1 fails. In order to handle this case, the failure of Weapon 2 with Weapon 1 and the failure of Weapon 3 with Weapon 2 need to be explicitly defined.

2.2.4. Minimum Yield Option

HedgeHOGS also allows the user to specify a minimum number of weapons with a yield greater than a specified amount that can be used in the hedge. By requiring this, an additional constraint will be added that is of the form

$$g(x,p) = n_y - n_r \qquad \text{Eq. (6)}$$

where

$n_y = \sum (h_i | y_{h_i} \geq y)$ = number of weapons in the hedge with yield greater than y

n_r = number of weapons required with yield greater than y

To enable this option, navigate to the Options Dialog (shown in Figure 3) and then select the checkbox next to *Minimum Yield*. Enter the minimum number of required weapons of the minimum required yield in the *Minimum # Weapons* field. Then enter the minimum yield in kilotons required by the weapons in the *Minimum Yield (kT)* field. For example, notionally assume that it is required to have 5,000 weapons that have at least 1 MT of yield. The user would type 5000 in the *Minimum # Weapons* field and 1000 in the *Minimum Yield (kT)* field.

2.2.5. Maximum Allowable Weapons in Hedge Option

HedgeHOGS also allows the user to specify a maximum number of hedge weapons. Practically, this can be interpreted as a maximum number of produced weapons since the total number of weapons produced equals the sum of the deployed weapons and the weapons in the hedge. When implementing this option, additional constraints of the form

$$g_{k+m}(x,p) = h_{max,k} - h_k,\ k = \{1,\ldots,m\} \qquad \text{Eq. (7)}$$

where

h_k	=	quantity of weapon k in the hedge
$h_{max,k}$	=	maximum allowed of weapon k in the hedge

are added to the optimization problem given in Eq. (1).

In HedgeHOGS this option is implemented by navigating within the Options Dialog (shown in Figure 3) to the Max Allowable Weapons in Hedge portion of the interface. The checkbox next to *Max Allowable Weapons in Hedge (modify Stockpile Table)* is then selected. The Max Hedge Weapons column then appears in the Stockpile Table, allowing the user to directly input the maximum allowable weapons in the hedge for the desired weapons (note: only weapons with a maximum allowable number in the hedge need to be specified).

Stockpile Table

Weapon ID	Weapon Name	Leg	Number of Warheads	Yield (kT)	Max Hedge Weapons
1	Weapon1	Land	4500	1000	200
2	Weapon2	Land	200	500	100
3	Weapon3	Land	620	700	
4	Weapon4	Air	2000	750	
5	Weapon5	Air	1500	850	

Figure 8. Weapons Table Showing Max Weapon Option, populated with fictitious sample data

2.3. Practical Interpretation of Hedging Options

Table 1 provides a reference between the options that were discussed in Section 2.2, Advanced Options, and some practical interpretations of their implementation should these options be selected.

Table 1. Practical Interpretation of HedgeHOGS Options

HedgeHOGS Implementation	Practical Interpretation
No Options Selected	Complete Intra-Leg 1 For 1 Hedging
Weighted Hedging Strategy	Bias Weapon Selection (e.g., Maintain Total Aggregate Yield, Reliability Based Hedging)
Substitutable Weapons	Arbitrary Substitution Strategies (e.g., Inter-leg Hedging, Allowable Substitution Policies)
Multiple Failures	Account for Common Failure Modes Across Systems
Minimum Yield	For Targeting, Require a Minimum Number of Weapons of a Given Yield
Max Allowable Weapons in Hedge	Limit the Number of Weapons to be Considered to the Number Produced

2.4. Output

HedgeHOGS takes the user's deployed stockpile composition inputs and runs Excel's Simplex LP Linear Solver algorithm to find the minimum acceptable hedge size and a representative hedge composition. Note that, as in many optimization problems, there may exist an entire family of solutions (i.e., hedge compositions) that can achieve the same optimum (i.e., minimum hedge size solution). Only one of these hedge compositions is shown in the Results Report.

To execute the solver, the *Compute* button (either in the Hedge Analysis or Options Dialogs) must be selected. After selecting the Compute button, a dialog will appear noting the exit condition from the Solver algorithm. After closing this result (and assuming a solution was found) the user will be taken to the Results Report tab. The Results Report gives a summary of the relevant inputs, the options selected, the minimum hedge size both in quantity and as a fraction of the stockpile, and a representative composition. An example Results Report is shown in Figure 9 and the fields are described further below.

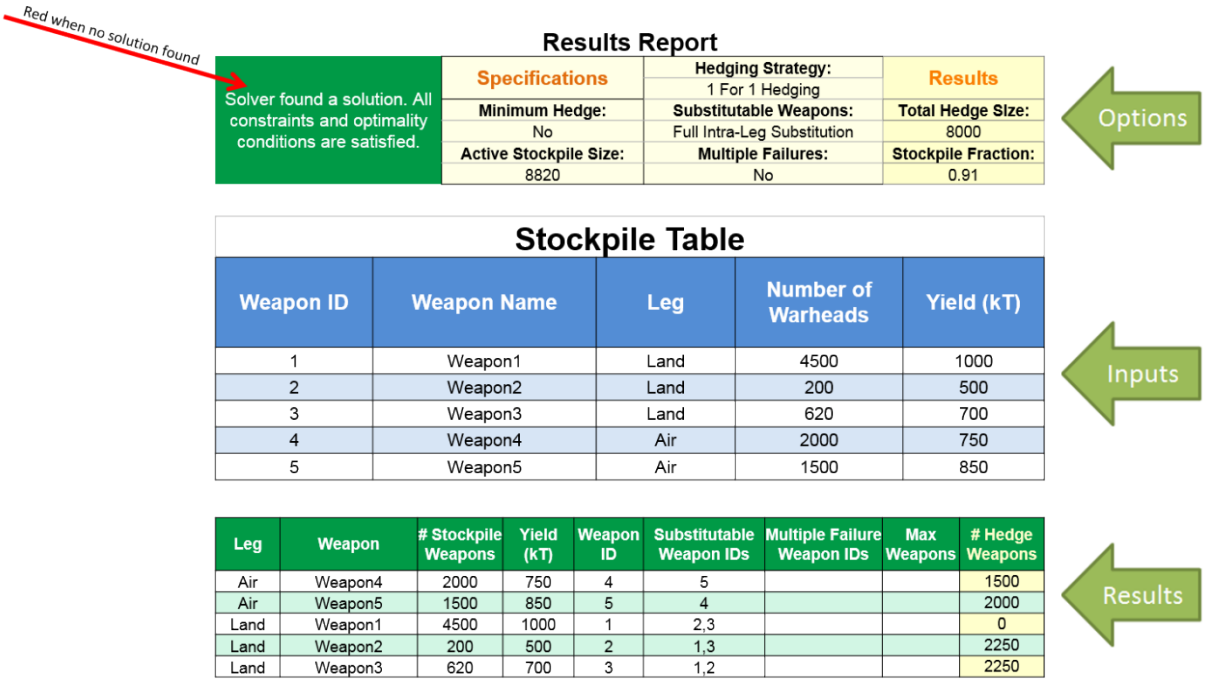

Figure 9. Example Report Sheet, populated with fictitious sample data

Upper Left Field: This field is green with the text "Solver found a solution. All constraints and optimality conditions are satisfied." if a minimum hedge size solution is found. If not, this field will be red with the exit condition from Solver.

Minimum Hedge: This field reports whether or not the option to include a minimum number of weapons of a given yield is chosen. If not chosen, this field will report "No" otherwise it will report the quantity of weapons and their yield.

Active Stockpile Size: This field reports the number of weapons in the deployed stockpile.

Hedging Strategy: This field indicates whether a 1 for 1 hedging strategy or a weighted hedging strategy is used. A 1 for 1 hedging strategy is indicated by "1 For 1 Hedging" and a weighted hedging strategy is indicated by "Weighted Hedging."

Substitutable Weapons: This field indicates the substitution strategy utilized. By default, this will be Full Intra-Leg Substitution (i.e., weapons within the same leg can substitute for each other). If an alternative substitution strategy is defined, the field will be populated with "Custom Substitution." The substitution strategy can be found in the table below where the allowable substitutions are defined under the Substitutable Weapon IDs column.

Multiple Failures: This field indicates whether multiple failures are enabled. If multiple failures are not enabled, this field will be populated with "No." Should multiple failures

be enabled, this field will be identified with a "Yes" and the weapons that fail simultaneously will be identified in the Multiple Failure Weapon IDs column in the table.

Total Hedge Size: This field indicates the minimum size of the hedge given the hedging options specified.

Stockpile Fraction: This field indicates the ratio of the minimum hedge size to the active deployed stockpile.

2.5. Assumptions and Limitations

The weapon substitution constraints driven by today's hedge sizing guidance are typically linear in form and can be solved with well-established optimization algorithms. Consequently, the objective functions considered and the possible constraints that HedgeHOGS can process are linear functions (i.e., $f(\boldsymbol{x}) = \Sigma\alpha_i x_i$, where α_i are weights on the vector-valued $\boldsymbol{x}$). Thus, HedgeHOGS can utilize the Simplex LP Linear Solver in Excel. For HedgeHOGS to run, the Solver Add-In and Macros must be enabled for Excel. Enabling the Solver Add-In is an automated process the first time the user runs HedgeHOGS. Alternatively, on the PC using Excel 2010 or later, the Solver Add-In can be added by:

1. Go to *File* > *Options*
2. Click *Add-Ins*, and then in the *Manage* box, select *Excel Add-ins*
3. Click *Go*
4. In the *Add-Ins available* box, select the *Solver Add-in* check box, and then click *OK*
    a. If the *Solver Add-in* is not listed in the *Add-Ins available* box, click *Browse* to locate the add-in
    b. If you get prompted that the Solver Add-in is not currently installed on your computer, click *Yes* to install it

In Excel 2016 for Mac, the Solver Add-In can be added by:

1. On the *Tools* menu, select *Add-Ins*
2. In the *Add-Ins available* box, select the *Solver Add-In* check box, and then click *OK*
    a. If the *Solver Add-in* is not listed in the *Add-Ins available* box, click *Browse* to locate the add-in
    b. If you get prompted that the Solver Add-in is not currently installed on your computer, click *Yes* to install it

As noted in Section 2.2, HedgeHOGS will arrive at a stockpile composition solution that achieves the minimum hedge size given the constraints it is supplied; however, this solution is not necessarily the only solution that achieves the minimum hedge size. Deeper analysis is required to determine if alternative hedge compositions exist. A full characterization of all possible minimum-hedge-size compositions is beyond the current capability of HedgeHOGS. It is also important to keep in mind that HedgeHOGS provides the theoretical minimum hedge solutions for the constraints it is supplied. These solutions may not always be practical or achievable if other unmodeled constraints exist. Such constraints will cause the "true" minimum hedge size to either remain the same or increase.

HedgeHOGS is designed to handle up to 98 different deployed weapon types in four different legs (Air, Land, Sea, and Other).

HedgeHOGS has been verified to work on Excel 2013 on the PC (Version 15.0.4911.1002) and Office 2016 on the Mac (Version 15.33).

2.6. Inter-Leg Hedging

To run HedgeHOGS using full inter-leg hedging (in which every weapon type can substitute for each other), the easiest method is to make the leg of every weapon type the same. Figure 10 gives an example of this where every weapon type's leg is listed as "Other."

Stockpile Table

Weapon ID	Weapon Name	Leg	Number of Warheads	Yield (kT)
1	Weapon1	Other	4500	1000
2	Weapon2	Other	200	500
3	Weapon3	Other	620	700
4	Weapon4	Other	2000	750
5	Weapon5	Other	1500	850

Figure 10. Full Inter-Leg Hedging Example, populated with fictitious sample data

To specify inter-leg hedging only between a few different weapon types, the Custom Substitution option can be utilized. This allows the user to either manually enter information in the "Substitutable Weapon ID" column is the Stockpile Table or by using the Weapon Substitutions Wizard. If manually entered, Weapon IDs must be separated by commas. Figure 11 shows a case where Weapons D and F in the hedge can substitute for Weapon A. Weapon A can substitute for Weapon D, but it cannot substitute for Weapon F.

Stockpile Table

Weapon ID	Weapon Name	Leg	Number of Warheads	Yield (kT)	Substitutable Weapon IDs
1	WeaponA	Land	50		4,6
2	WeaponB	Land	50		
3	WeaponC	Land	50		
4	WeaponD	Air	50		1
5	WeaponE	Air	50		
6	WeaponF	Air	50		

Figure 11. Partial Inter-Leg Hedging Example, populated with fictitious sample data

3. EXAMPLE CASES

In this section, example cases are shown and verified with previous work pertaining to hedging. Two cases are considered. The first is a situation with three weapons, where every weapon can hedge for any other weapon except itself. The second case has three weapons; however, only intra-leg hedging is allowed. Two weapons are of the same leg and one is in a different leg. One weapon is made adaptable to produce a possible solution.

3.1. Three Weapon Inter-Leg Hedging Case

Figure 12 describes an example in which three weapon types exist and inter-leg hedging is allowed. This means that each weapon type can hedge for the other two weapon types. The solution graph shows that a minimum hedge size that is 50% the size of the deployed stockpile exists within the green triangle. Minimum hedge solutions greater than 50% exist outside of this triangle.

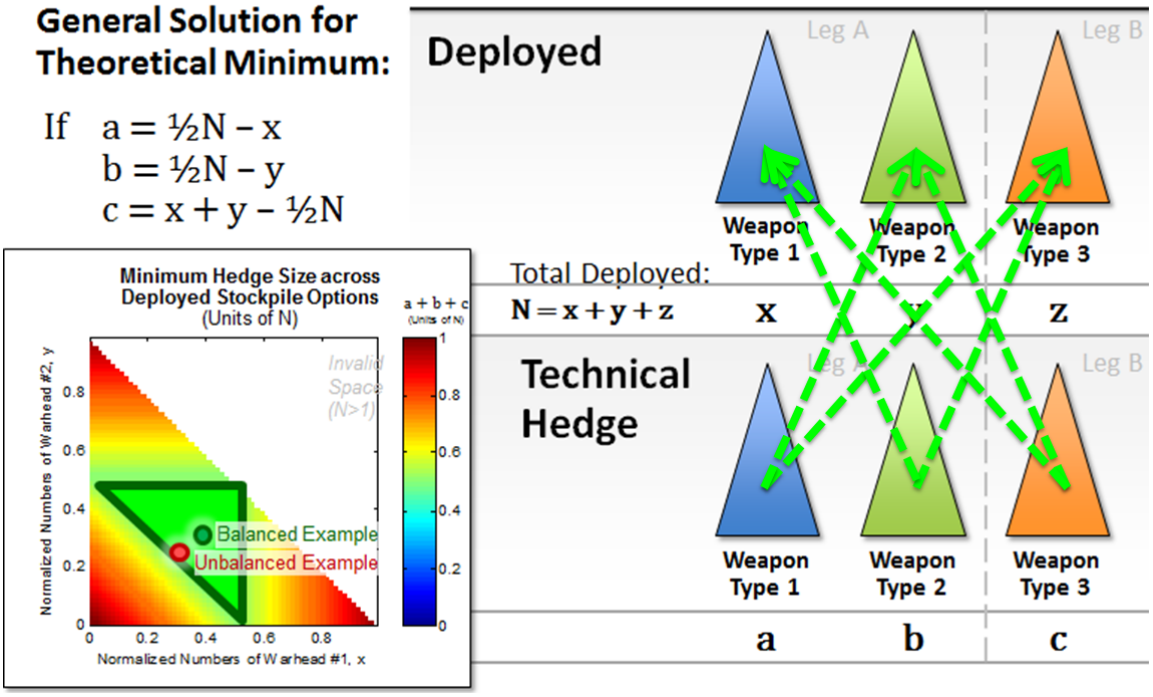

Figure 12. Solution for Three Weapon Inter-Leg Hedging Case[13]

These results are verified by HedgeHOGS in Figure 13, Figure 14, and Figure 15 which contains three separate cases. The first example, Figure 13, shows an optimal solution of a 50% hedge achieved with a balanced example, meaning every weapon in the stockpile has the same number

[13] Lafleur, J. and Roesler, A., "Hedge Math: Theoretical Limits on Minimum Stockpile Size Across Nuclear Hedging Strategies," SAND2016-2995C, April 2016.

of warheads (30). The second example, Figure 14, shows another solution of 50% with an unbalanced example. The third example, Figure 15, shows a result greater than 50% because it is outside of the green triangle shown in Figure 12.

Results Report

Solver found a solution. All constraints and optimality conditions are satisfied.	Specifications	Hedging Strategy:	Results
		1 For 1 Hedging	
	Minimum Hedge:	Substitutable Weapons:	Total Hedge Size:
	No	Full Intra-Leg Substitution	45
	Active Stockpile Size:	Multiple Failures:	Stockpile Fraction:
	90	No	0.50

Leg	Weapon	# Stockpile Weapons	Yield (kT)	Weapon ID	Substitutable Weapon IDs	Multiple Failure Weapon IDs	Max Weapons	# Hedge Weapons
Land	Weapon1	30	1000	1	2,3			15
Land	Weapon2	30	1000	2	1,3			15
Land	Weapon3	30	1000	3	1,2			15

Figure 13. HedgeHOGS Results for Three Weapon Inter-Leg Hedging Case, utilizing fictional weapon data, showing an optimal balanced hedge solution.

Results Report

Solver found a solution. All constraints and optimality conditions are satisfied.	Specifications	Hedging Strategy:	Results
		1 For 1 Hedging	
	Minimum Hedge:	Substitutable Weapons:	Total Hedge Size:
	No	Full Intra-Leg Substitution	45
	Active Stockpile Size:	Multiple Failures:	Stockpile Fraction:
	90	No	0.50

Leg	Weapon	# Stockpile Weapons	Yield (kT)	Weapon ID	Substitutable Weapon IDs	Multiple Failure Weapon IDs	Max Weapons	# Hedge Weapons
Land	Weapon1	28	1000	1	2,3			17
Land	Weapon2	28	1000	2	1,3			17
Land	Weapon3	34	1000	3	1,2			11

Figure 14. HedgeHOGS Results for Three Weapon Inter-Leg Hedging Case, utilizing fictional weapon data, showing an optimal unbalanced hedge solution.

Results Report

Solver found a solution. All constraints and optimality conditions are satisfied.	Specifications	Hedging Strategy:	Results
		1 For 1 Hedging	
	Minimum Hedge:	Substitutable Weapons:	Total Hedge Size:
	No	Full Intra-Leg Substitution	70
	Active Stockpile Size:	Multiple Failures:	Stockpile Fraction:
	90	No	0.78

Leg	Weapon	# Stockpile Weapons	Yield (kT)	Weapon ID	Substitutable Weapon IDs	Multiple Failure Weapon IDs	Max Weapons	# Hedge Weapons
Land	Weapon1	10	1000	1	2,3			35
Land	Weapon2	10	1000	2	1,3			35
Land	Weapon3	70	1000	3	1,2			0

Figure 15. HedgeHOGS Results for Three Weapon Inter-Leg Hedging Case, utilizing fictional weapon data, showing a non-minmal hedge solution.

3.2. Three Weapon Intra-Leg Hedging Case with One Adaptable Weapon

Figure 16 describes a case where three weapon types exist and only intra-leg hedging is allowed, with one adaptable weapon type. This means that each weapon type can hedge only for those in the same leg, except for Weapon Type 2, which is the adaptable weapon type. Weapon Type 2 can hedge for Weapon Type 3, which exists on another leg. The solution graph on the left in Figure 16 shows that a minimum 50% hedge can be achieved only if Weapon Type 2 does not exist in the deployed stockpile. This solution is shown in Figure 17. The balanced example is shown in Figure 18.

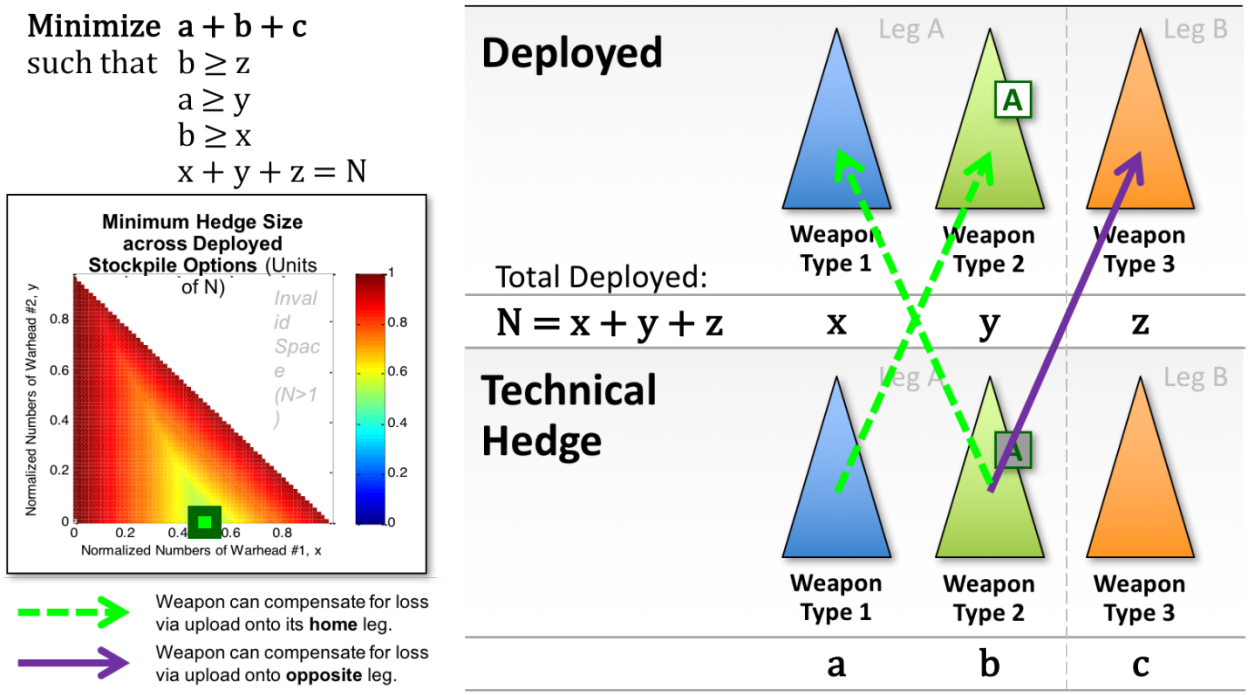

Figure 16. Solutions for Three Weapon Intra-Leg Hedging Case with One Adaptable Weapon[14]

[14] Lafleur, J. and Roesler, A., "Hedge Math: Theoretical Limits on Minimum Stockpile Size Across Nuclear Hedging Strategies," SAND2016-2995C, April 2016.

Results Report

Solver found a solution. All constraints and optimality conditions are satisfied.	Specifications	Hedging Strategy:	Results
		1 For 1 Hedging	
	Minimum Hedge:	Substitutable Weapons:	Total Hedge Size:
	No	Custom Substitution	45
	Active Stockpile Size:	Multiple Failures:	Stockpile Fraction:
	90	No	0.50

Leg	Weapon	# Stockpile Weapons	Yield (kT)	Weapon ID	Substitutable Weapon IDs	Multiple Failure Weapon IDs	Max Weapons	# Hedge Weapons
Other	Weapon 1	45	1000	1	2			0
Other	Weapon 2	0	1000	2	1,3			45
Other	Weapon 3	45	1000	3				0

Figure 17. HedgeHOGS Results for Three Weapon Intra-Leg Hedging Case with One Adaptable Weapon, utilizing fictional weapon data showing (a) that the minimum hedge size of 50% of the active deployed stockpile is only achievable if Weapon Type 2 is not included in the active deployed.

Results Report

Solver found a solution. All constraints and optimality conditions are satisfied.	Specifications	Hedging Strategy:	Results
		1 For 1 Hedging	
	Minimum Hedge:	Substitutable Weapons:	Total Hedge Size:
	No	Custom Substitution	60
	Active Stockpile Size:	Multiple Failures:	Stockpile Fraction:
	90	No	0.67

Leg	Weapon	# Stockpile Weapons	Yield (kT)	Weapon ID	Substitutable Weapon IDs	Multiple Failure Weapon IDs	Max Weapons	# Hedge Weapons
Other	Weapon 1	30	1000	1	2			15
Other	Weapon 2	30	1000	2	1,3			30
Other	Weapon 3	30	1000	3				15

Figure 18. HedgeHOGS Results for Three Weapon Intra-Leg Hedging Case with One Adaptable Weapon, utilizing fictional weapon data showing that including Weapon Type 2 in the stockpile increases the minimum hedge size to 67%.

4. CONCLUSION AND FUTURE IMPROVEMENTS

HedgeHOGS is a tool that uses a linear mathematical optimizer to solve for the theoretical minimum hedge size given a certain stockpile of weapons and user-specified constraints. The tool is designed to model a real world problem, and thus should be changed and improved based on changing circumstances.

Currently HedgeHOGS allows for a multitude of options. Both intra-leg hedging and inter-leg hedging options are incorporated. Additional constraints, such as a minimum number of large yield weapon types, one-for-one hedging, weight-based hedging, maximum hedge size for each weapon type, and multiple failures options are also included. This creates many permutations of options that allow the user to customize the tool.

While the aforementioned options are already implemented, more features could be added to model different scenarios. A combined objective function for multi-objective problems could be included. For example, one could consider having a combined objective function consisting of yield and stockpile size. In addition, more intuitive user inputs and enhancements to the results report could be implemented. A more visual view of the hedging solution obtained, including a graph, could also be added.

HedgeHOGS is portable and easily run on any system that runs Excel. HedgeHOGS utilizes Visual Basic for Applications (VBA) code in addition to Excel tables. Thus, the aforementioned potential improvements could be implemented by any individual with programming experience who is familiar with VBA.

DISTRIBUTION

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