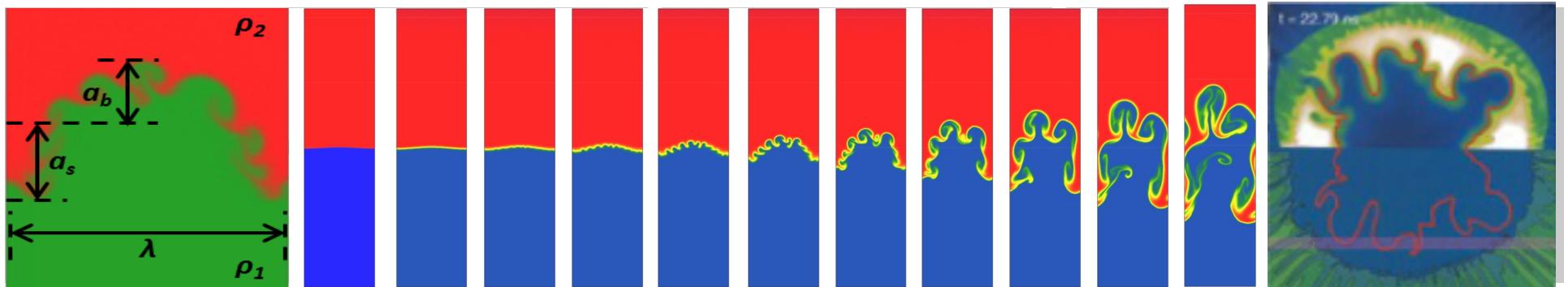


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## DSMC Investigation of Hydrodynamic Instabilities in Gases

**M. A. Gallis, T. P. Koehler, J. R. Torczynski, S. J. Plimpton**

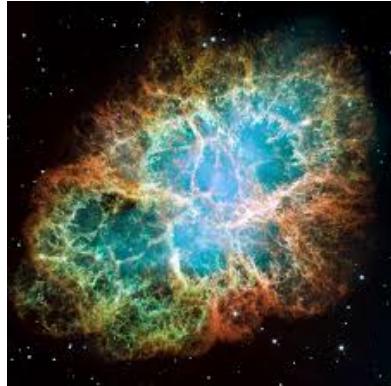
Engineering Sciences Center & Computing Research Center  
Sandia National Laboratories; Albuquerque, New Mexico, USA

30<sup>th</sup> Rarefied Gas Dynamics Symposium  
University of Victoria, Victoria B.C., Canada  
July 10-15, 2016



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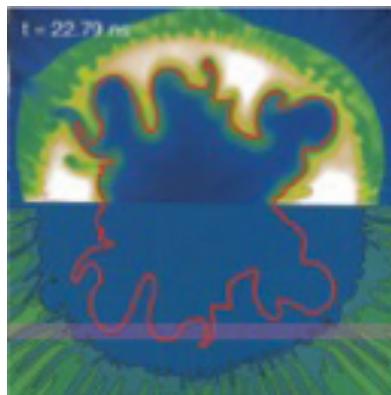
# Instabilities: When do they occur?



Infinitesimal disturbances amplify spontaneously and ultimately dominate the flows.

The growth of the instability is influenced by viscosity, compressibility, three-dimensionality, density ratio.

It is postulated that the failure to achieve ignition at NIF can be attributed to the **Rayleigh-Taylor instability (RTI)**.



Clark et al. 2013

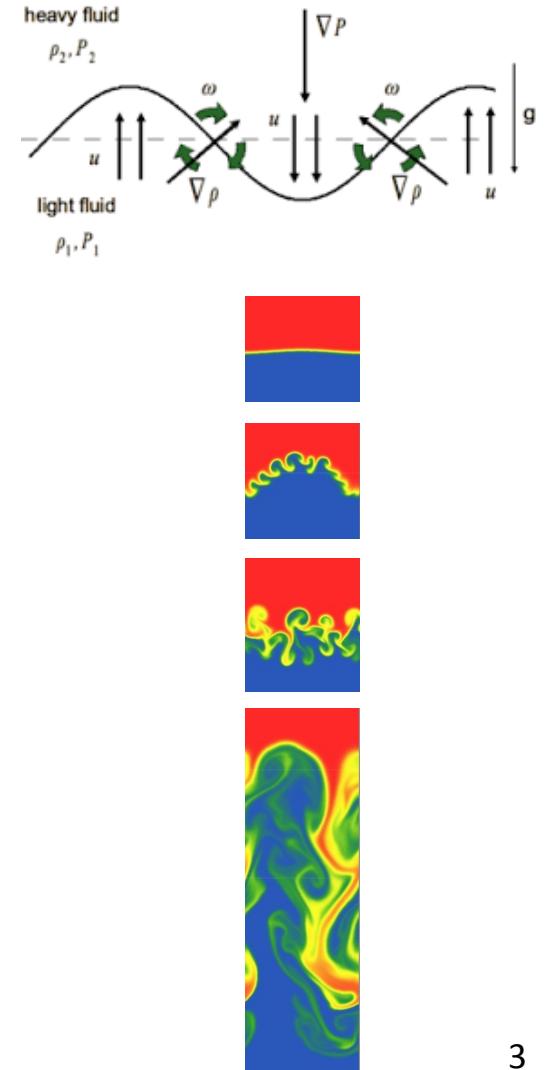
RTI can produce the Kelvin-Helmholtz instability (KHI) and has the Richtmyer-Meshkov instability (RMI) as a limiting case.

Applications range from Inertial Confinement Fusion (ICF, mm) to the formation of supernova remnants (light-years).

Molecular methods like DSMC are becoming increasingly popular for investigating the effects of viscosity and diffusivity in ICF applications, which are known as “kinetic” or “ion-kinetic” effects. (Larroche et al. 2016)

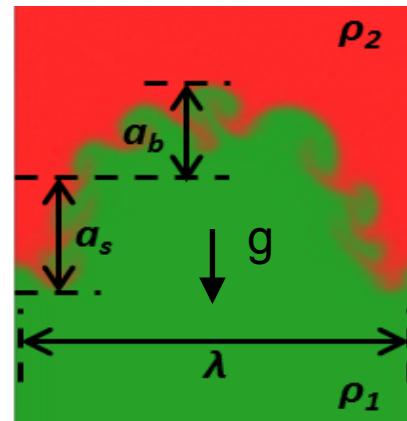
# Rayleigh-Taylor Instability (RTI)

- RTI is an instability of an interface between two fluids of different densities which occurs when the lighter fluid is pushing the heavier fluid.
- Baroclinic torque at the interface creates vorticity and induces a velocity field that increases the amplitude, which in turn increases the baroclinic torque.
- RTI has four main stages:
  - Linear (linear growth of initial perturbations)
  - Nonlinear (mushroom-like structures appear)
  - Structures interact and compete (as in RMI, KHI)
  - Turbulent mixing



# Characteristic & Nondimensional Quantities

Initial Interface



Bubble

Spike

$$a = a_b + a_s$$

$$A = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

$$Re_p = \frac{\lambda \sqrt{\frac{A}{1+A}} g \lambda}{\nu}$$

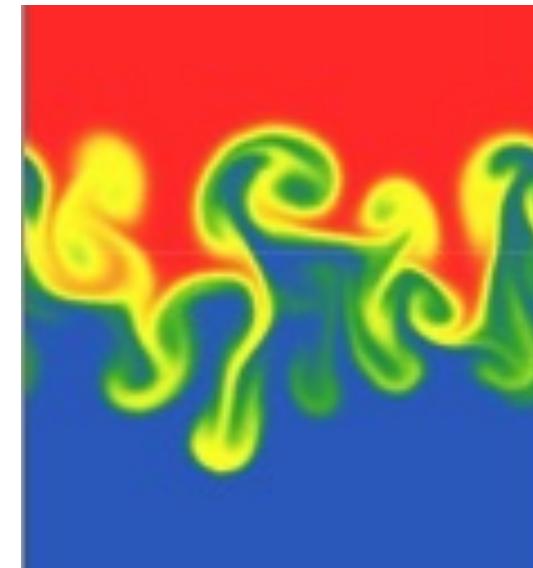
Amplitude

Atwood Number

Reynolds Number  
(Wei & Livescu 2012)

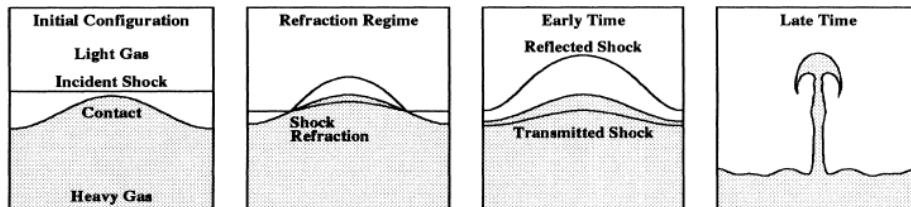
# Most Unstable Wavelength

- The initial growth rates of small-amplitude perturbations are influenced by the fluid properties.
- Viscosity and diffusivity inhibit small-wavelength perturbations from growing, allowing a particular wavelength, the **most unstable wavelength**, to emerge, **outpacing** the growth of all other wavelengths:



$$\lambda_m \approx 4\pi(v^2 / Ag)^{1/3}$$

# RTI Is Related to RMI



Grove et al., Phys. Rev. Lett., 71 (21), 3473 (1993).

## Shock propagation

- Incident shock travels down in upper gas
- Transmitted shock travels down in lower gas
- Reflected shock travels upward in upper gas

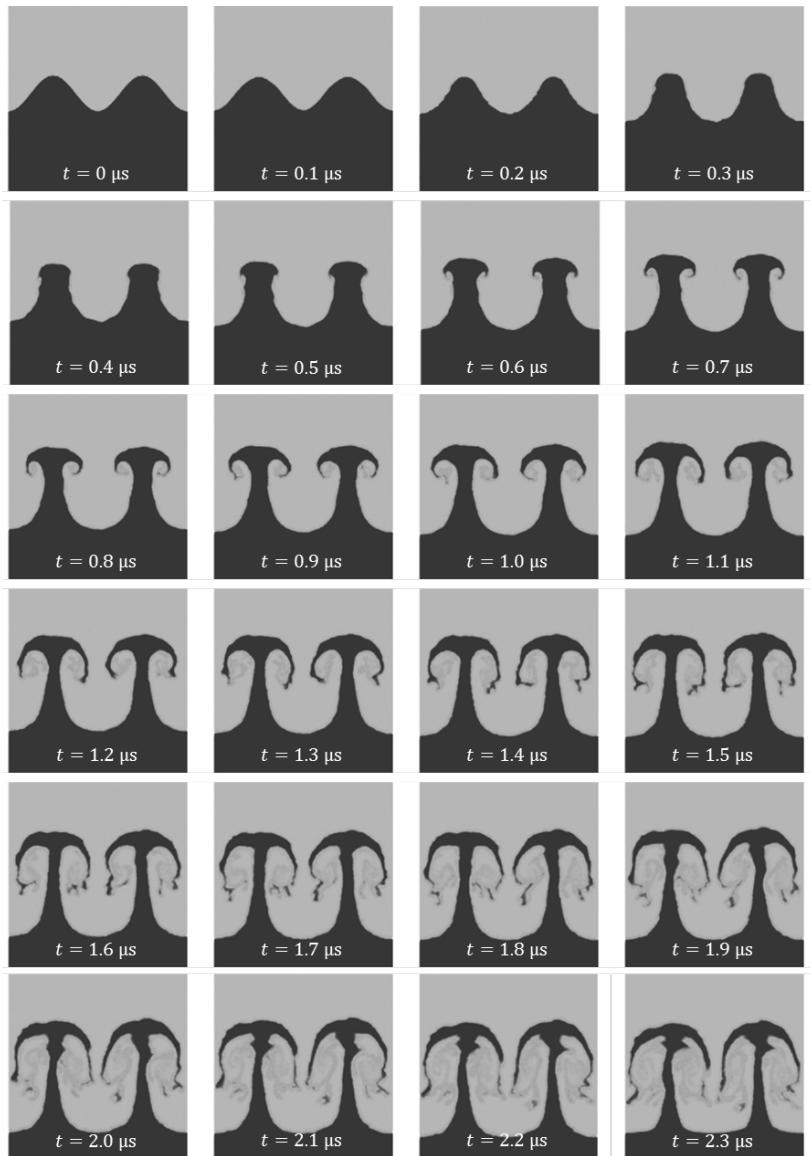
## Interface motion

- Interface is accelerated to constant velocity
  - Travels in same direction as shock
- Vorticity generated baroclinically at interface
  - Density & pressure gradients misaligned

## Perturbation growth

- Initially, amplitude growth is linear with time
- Later, amplitude growth becomes nonlinear
  - When amplitude is similar to wavelength
- Bubbles, spikes, roll-up, more instabilities

Gallis et al. Physics of Fluids (2015)

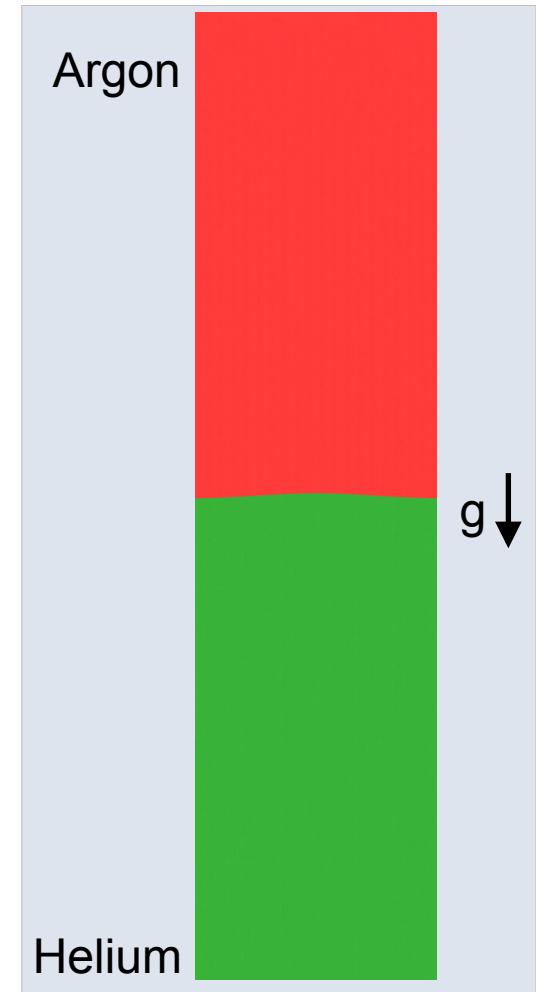


# Why DSMC for Rayleigh-Taylor Instability?

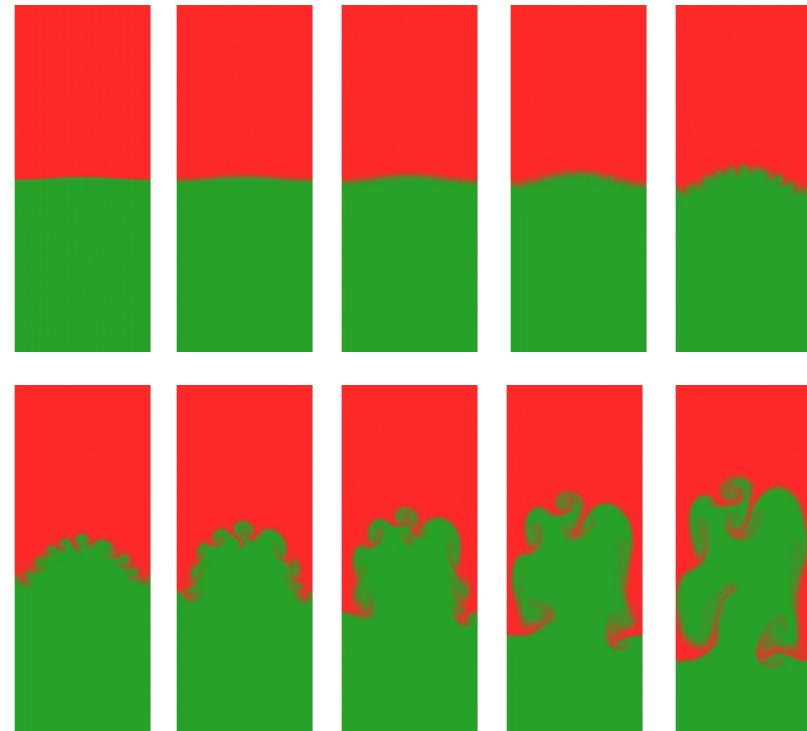
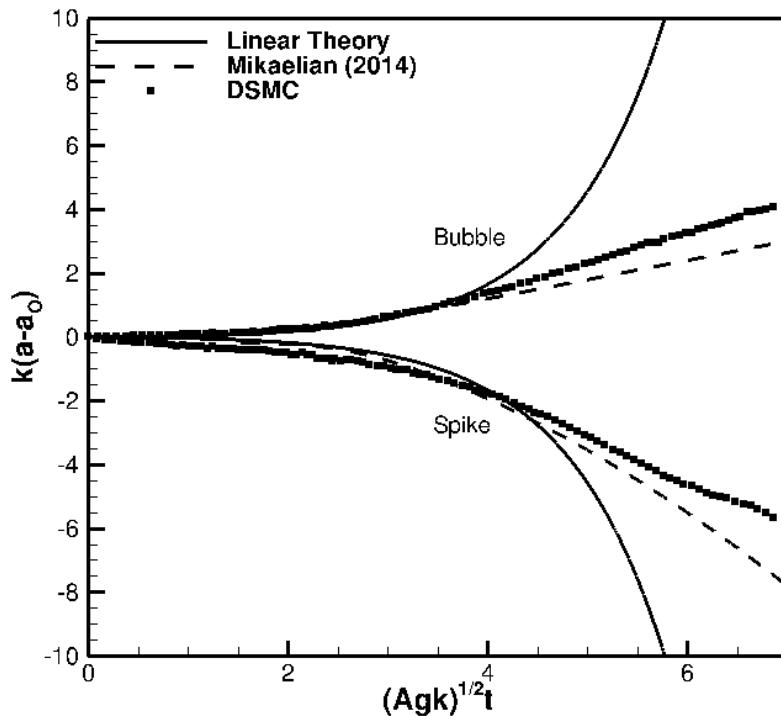
- DSMC provides a molecular-level description of the hydrodynamic processes that may be physically more realistic for **large accelerations and chemically reacting flows and the representation of naturally occurring thermal fluctuations**
- DSMC inherently accounts for transport properties
- The DSMC method offers the potential to identify the impact of molecular level effects (e.g., rotational and vibrational energy exchange, gas-phase chemical reactions, and gas-surface interactions) on hydrodynamic instabilities.
- Typical 2D DSMC simulation characteristics:
  - Physical Domain: 1 mm x 4 mm (ICF-pellet-size domain)
  - # Cells: 4 billion
  - # Particles: 400 billion
  - # Cores:  $\frac{1}{4}$ - $\frac{1}{2}$  million
  - Run time: 30 hrs (= 900, 1800 CPU years)
  - Time steps:  $200,000 \times 0.1 \text{ ns} = 20 \mu\text{s}$

# DSMC Simulations of the Rayleigh-Taylor Instability in Gases

- The interface between argon (red) and helium (green) gases is slightly perturbed:  
 $\lambda = 0.001\text{m}$ ,  $a = 10\mu\text{m}$ ,  $A = 0.81$
- Initial state hydrostatic equilibrium
- Acceleration of the system excites the RTI
  - Initially, **thermal fluctuations and diffusion** perturb the interface
  - The initial perturbation amplitude grows exponentially
  - A second growth stage occurs at the most unstable wavelength, which forms **“bubbles”** and **“spikes”**
  - Additional instabilities break up the larger structures, resulting **in turbulent mixing of the gases**

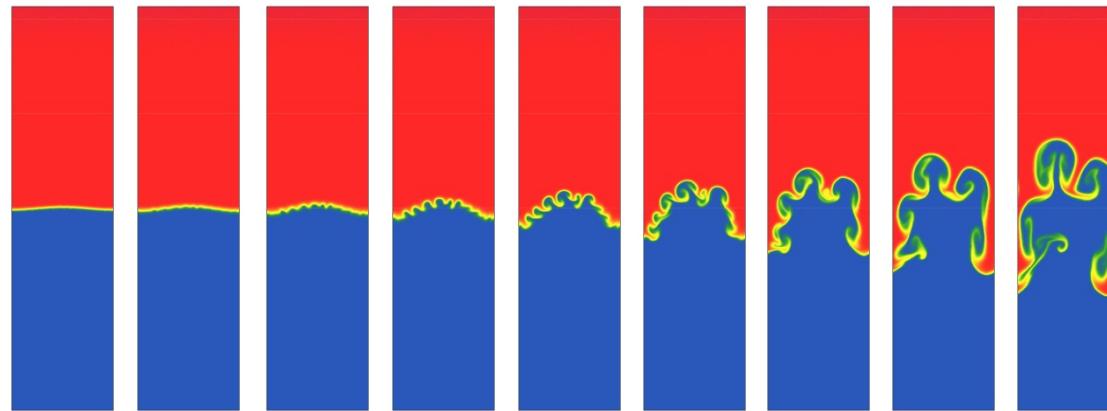


# Development of 1-mm Wavelength Perturbation for Gravity $10^8 \text{ m/s}^2$

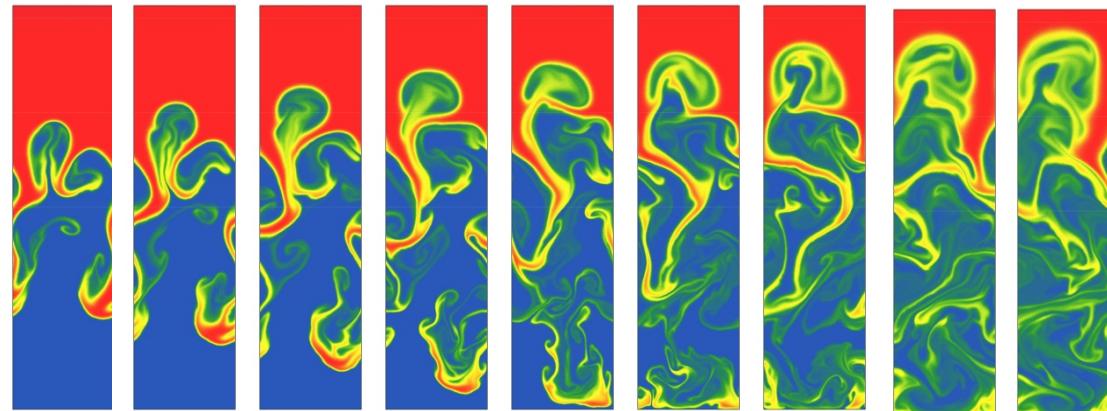


- Initial perturbations of a small wavelength develop and grow exponentially.
- Smaller structures, corresponding to the most unstable wavelength, appear.
  - Their number is independent of the molecular simulation ratio and the random number seed.
- Larger structures emerge as the smaller disturbances interact and combine.

# Density Profiles for 1-mm Wavelength Perturbation for Gravity $10^8 \text{ m/s}^2$

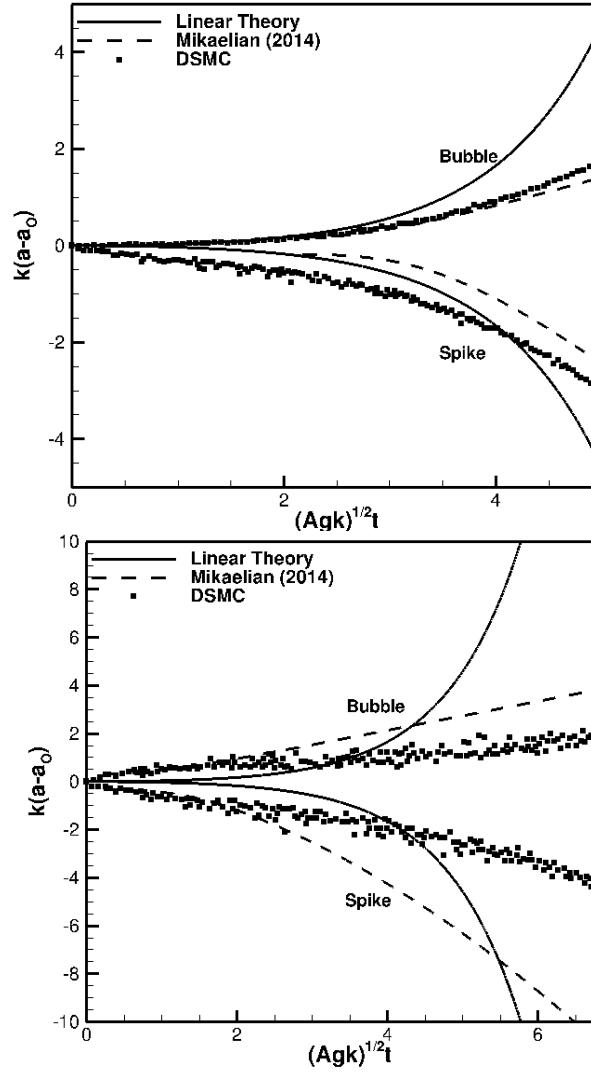
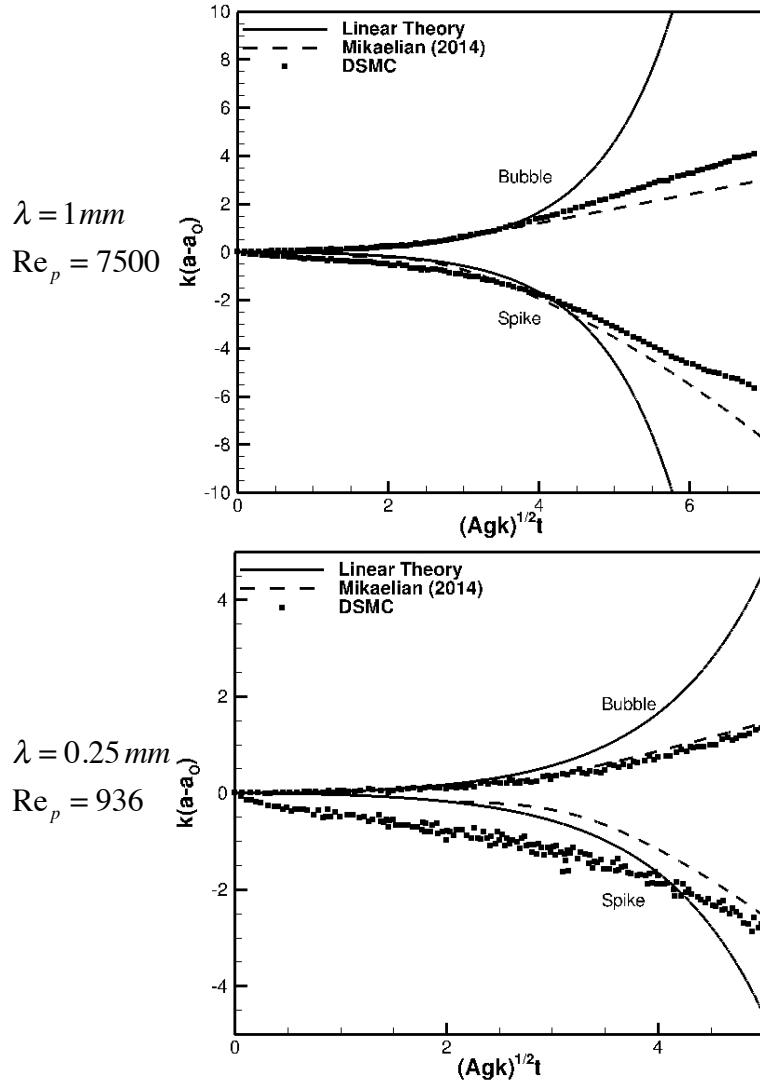


Images progress at 10,000 time step increments



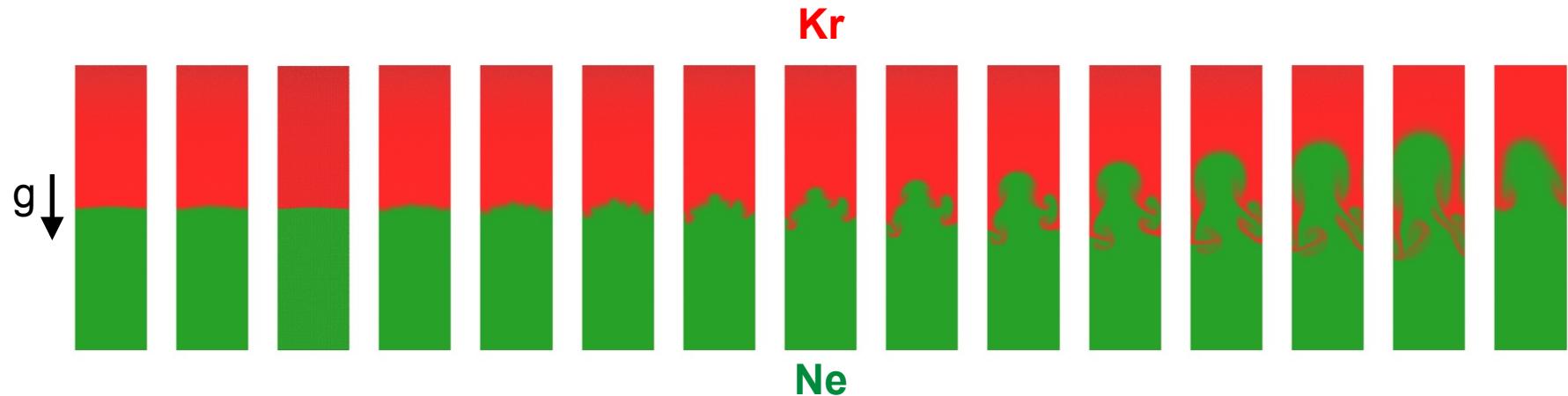
Initially, diffusion thickens the interface. Subsequently, bubbles and spikes appear.

# RTI for Ar/He with Different Wavelengths



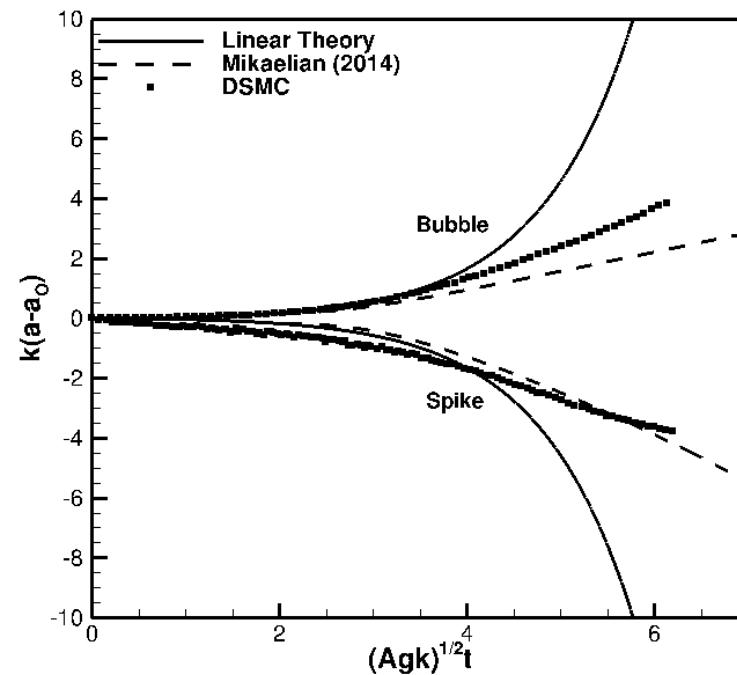
For each simulation, the domain width is one wavelength, and the amplitude is 1% of the wavelength.

# DSMC Simulations of the RTI for Kr/Ne

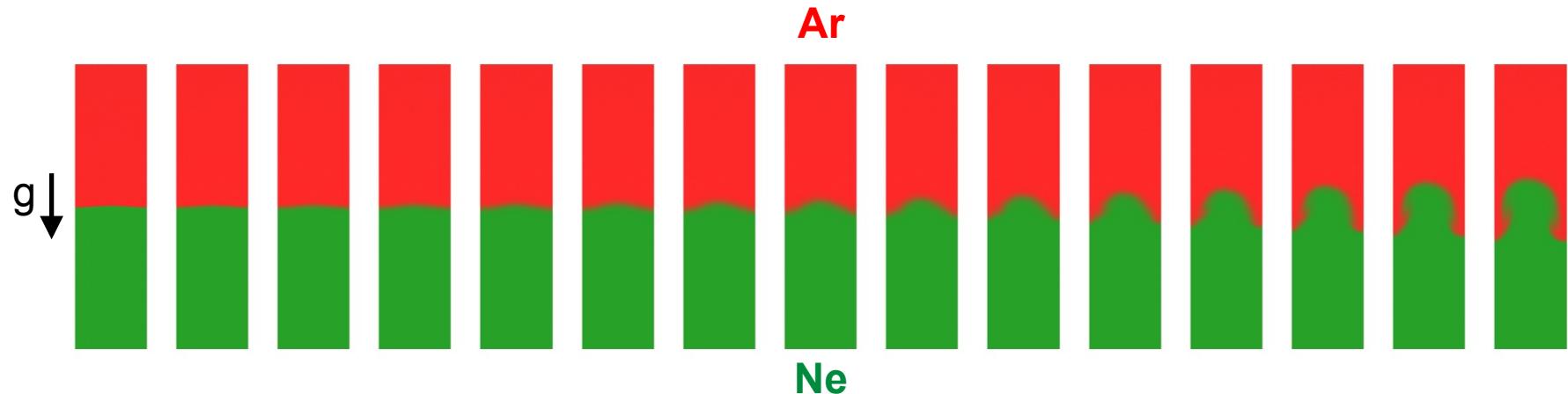


$A = 0.61$   
Gravity =  $10^8 \text{ m/s}^2$

$\text{Re}_p = 12800$

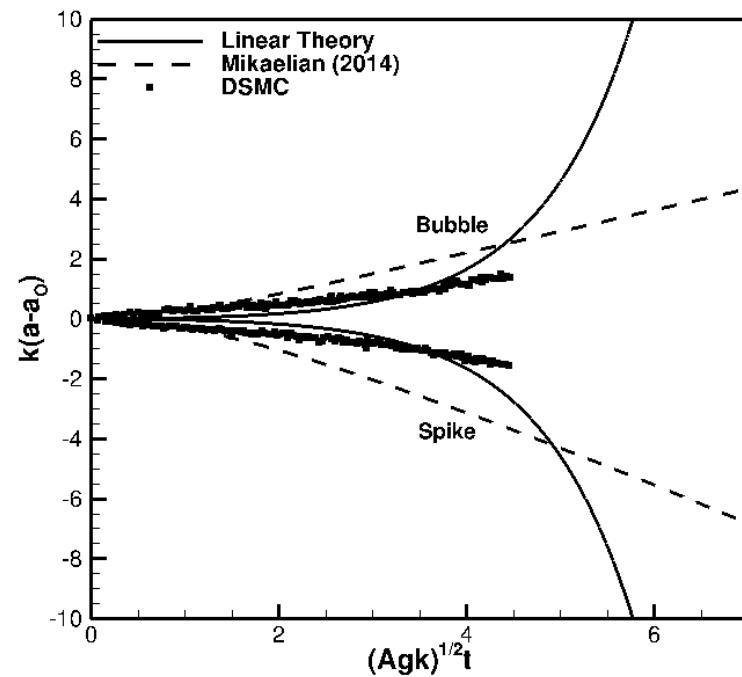


# DSMC Simulations of the RTI for Ar/Ne



$A = 0.32$   
Gravity =  $10^8 \text{ m/s}^2$

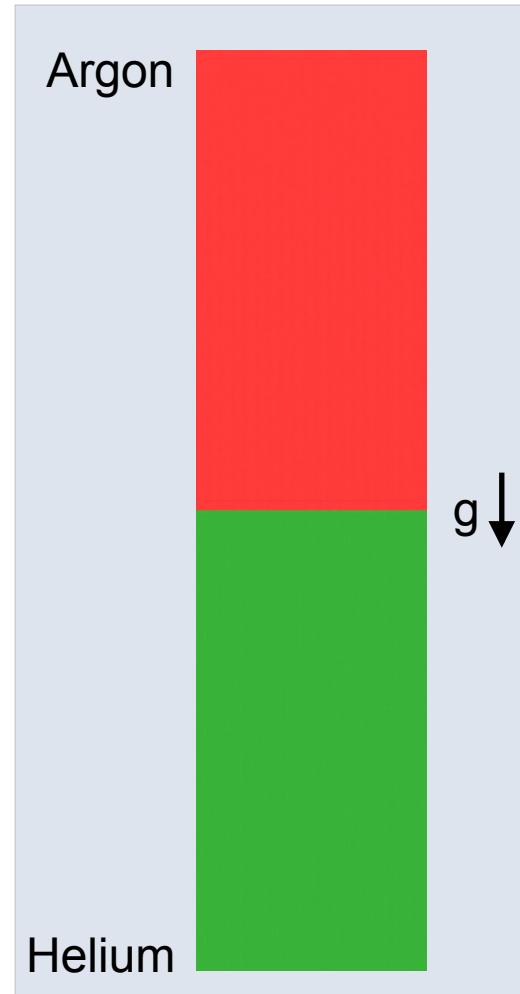
$\text{Re}_p = 6000$



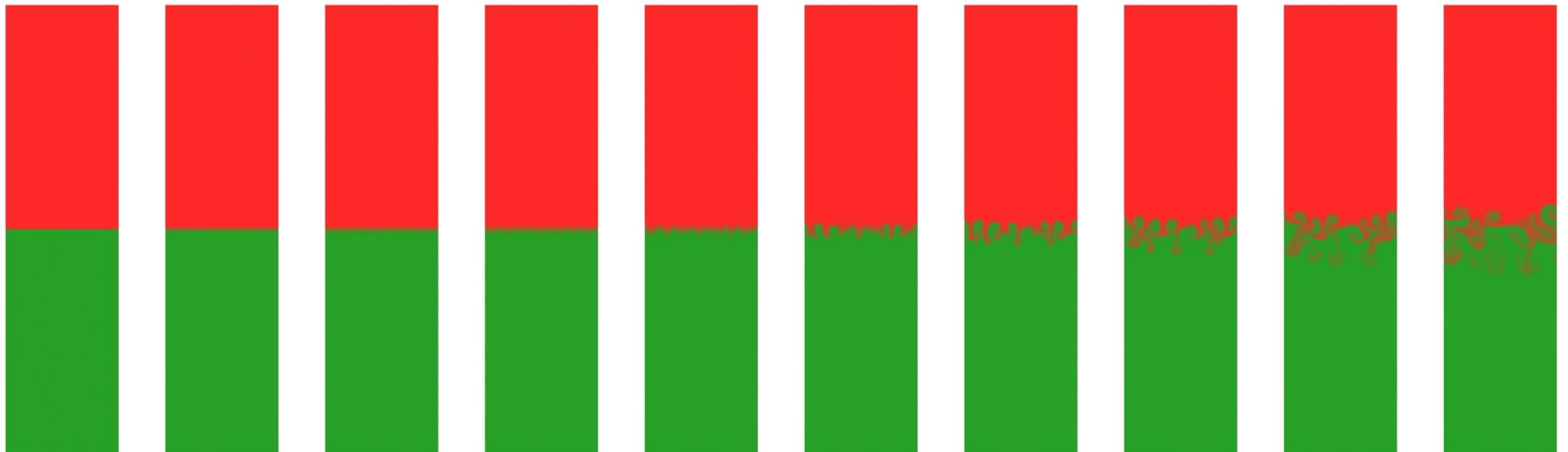
# RTI from an Initially Molecularly Flat Interface



- The interface between argon (red) and helium (green) gases is **initially flat**
- Acceleration of the system excites the RTI
  - Initially, thermal fluctuations and diffusion perturb the interface
  - The amplitude of thermal fluctuations grows exponentially
  - Gases penetrate each other differently, forming **“bubbles”** and **“spikes”**
  - Finally, additional instabilities break up the larger structures resulting in turbulent and chaotic mixing of the gases



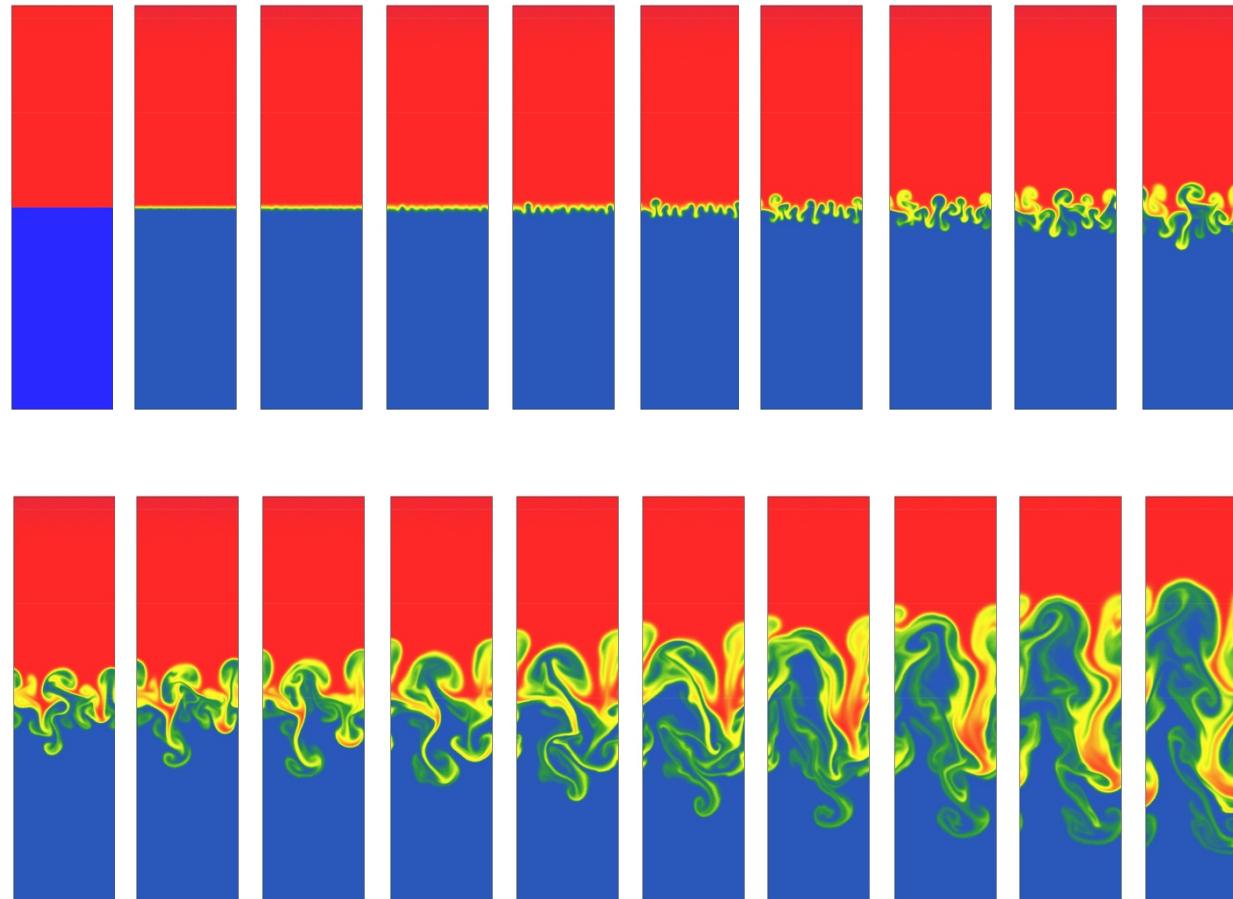
# RTI from an Initially Flat Interface



Images progress at 10,000 time step increments

The numbers of bubbles and spikes correspond to the most unstable wavelength.

# Density Profiles for an Initially Flat Interface

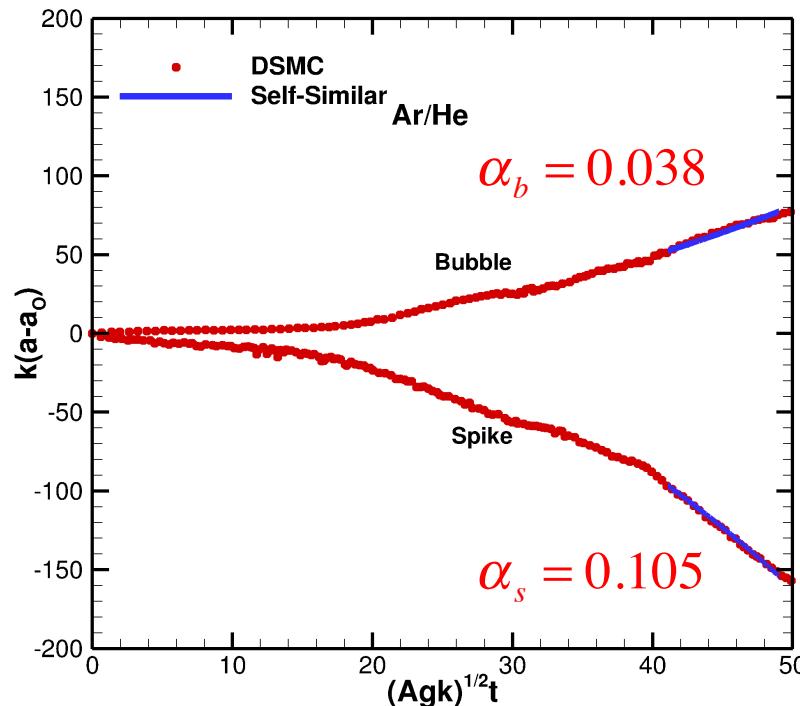


Images progress at 10,000 time step increments

# Late-Time Behavior: Self-Similarity

At late times, under certain idealized conditions, the flow can forget its initial conditions and enter a self-similar growth phase (Fermi and von Neumann 1953) described by the following equation:

$$a_{b,s} = \alpha_{b,s} A g t^2$$



Self-similar behavior is observed for long times.  
Waviness is due to competition between bubbles and spikes.

# RTI in 3D: Density Profile

Typical 3D DSMC simulation characteristics:

Physical Domain: 1 mm x 1 mm x 4 mm

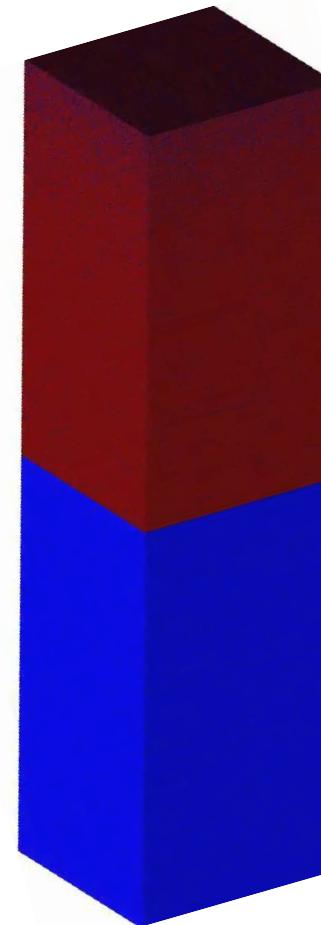
# Cells: 62.5 billion

# Particles: 1.2 trillion

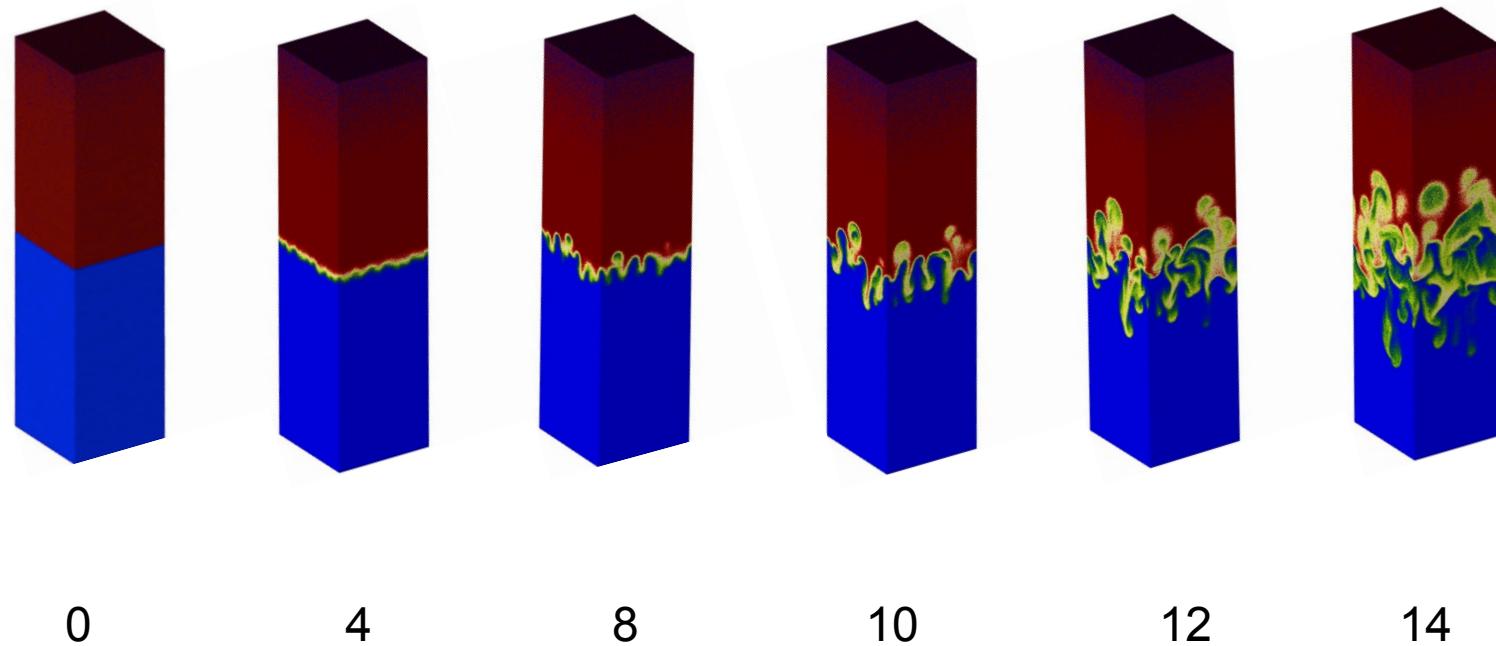
# Cores: ½ million

Run time: 90 hrs (5400 CPU years)

Time steps:  $200,000 \times 0.1 \text{ ns} = 20 \text{ } \mu\text{s}$

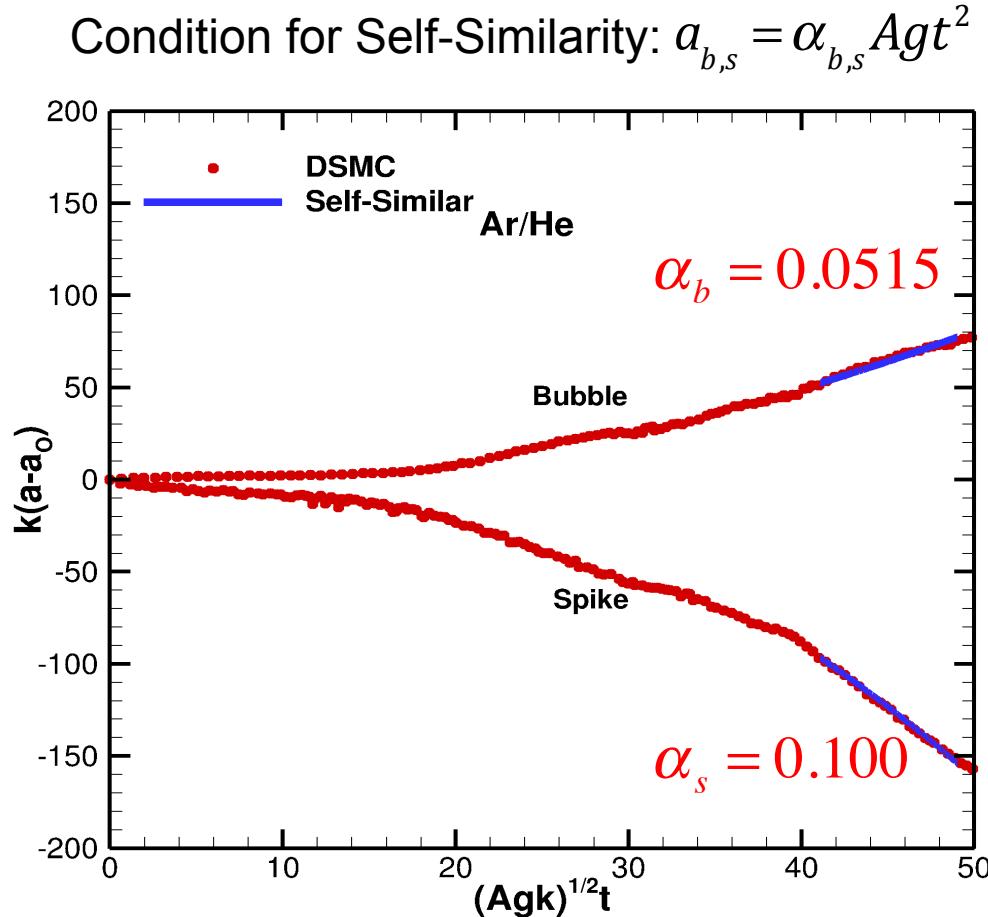


# RTI from a Flat Interface in 3D



Images progress at **multiples** of 10,000 time step increments

# 3D Late-Time Behavior: Self-Similarity



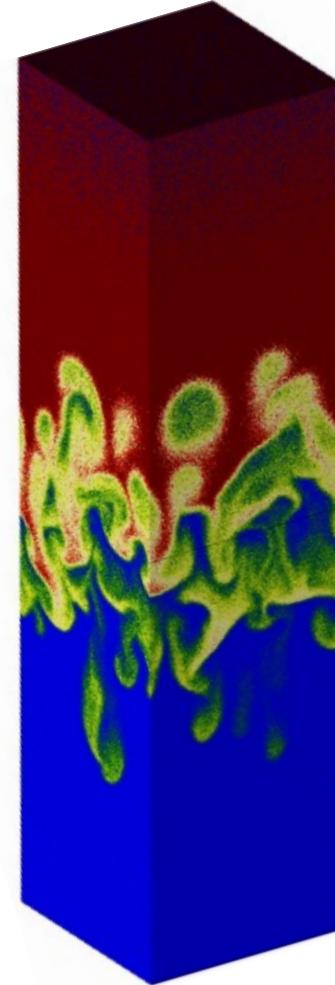
Self-similar behavior is observed at late times.  
Waviness is due to competition between bubbles and spikes

# Conclusions

The Direct Simulation Monte Carlo (DSMC) method can simulate the Rayleigh-Taylor and Richtmyer-Meshkov instabilities.

- Structures are like those in other approaches
- Amplitude growth rates are also similar
- The growth rate in the self-similar regime is within experimental observations

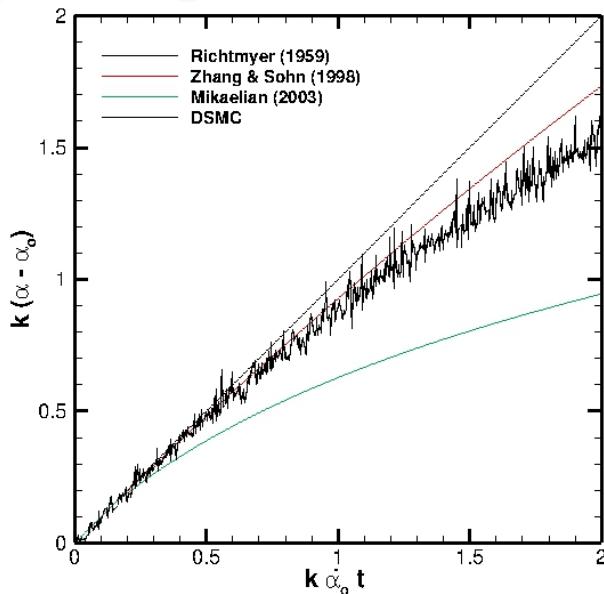
SPARTA is open source: <http://sparta.sandia.gov>



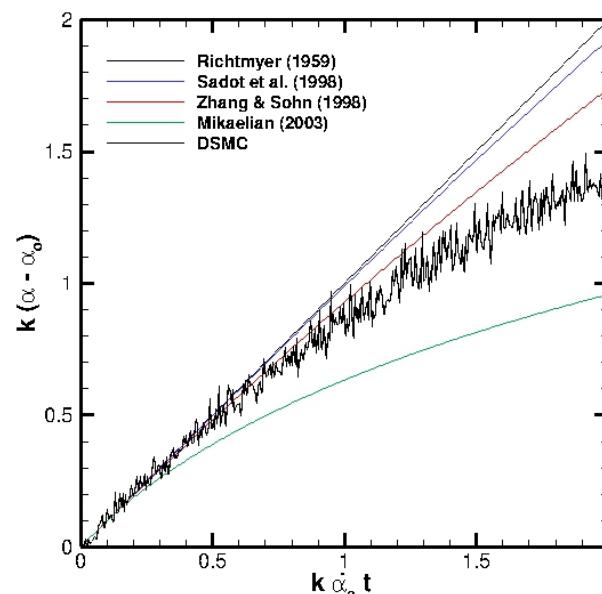
# Comparing DSMC to Theoretical Models



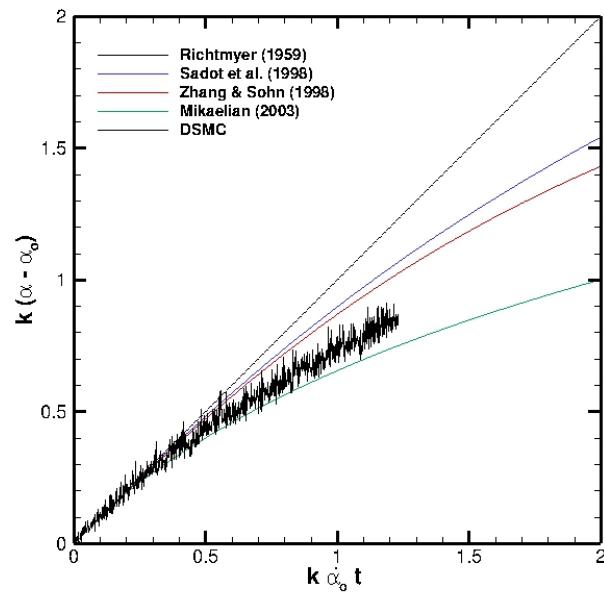
Xe/He  
A=0.94



Ar/He  
A=0.82



Xe/Ar  
A=0.53



Ar/Ne  
A=0.33

