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Combining Full-Field Measurements and Inverse Techniques for Smart Material Testing

Identification of Viscoplastic Material Properties using the Virtual Fields Method

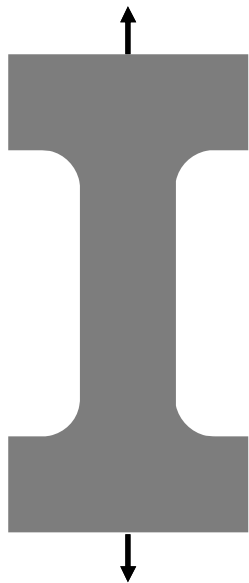
EMC Jones, JD Carroll, KN Karlson, SLB Kramer, PL Reu, DZ Turner



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Traditional Material Identification

- Accurate simulations of material deformation rely on well characterized material properties.
- Traditional material identification consists of a series of uniaxial dog bones
- May not adequately describe material behavior under complex stress states

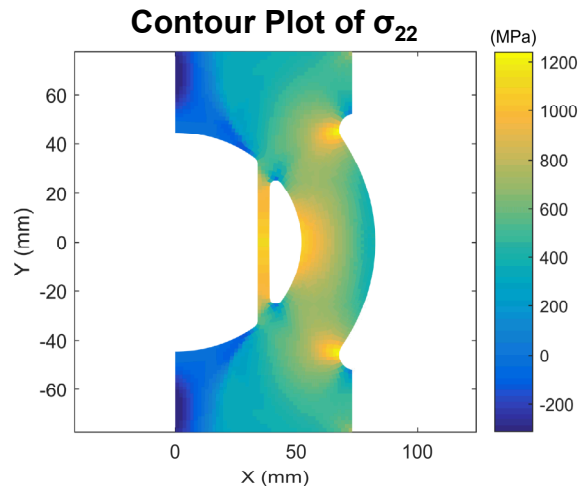


Complex material
Complex loading conditions



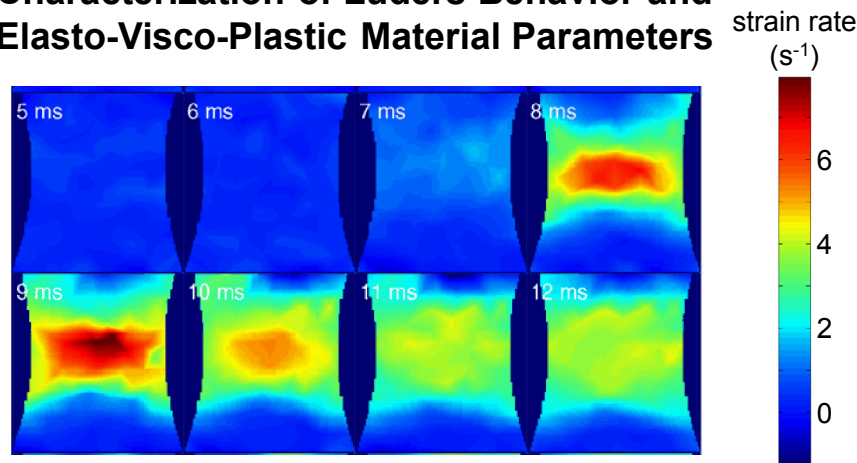
Test matrix		
Rolling Direction	Strain Rate	Temperature
Transverse	0.1	100
Rolling	0.1	100
Transverse	10	100
Rolling	10	100
Transverse	0.1	500
Rolling	0.1	500
Transverse	10	500
Rolling	10	500

High-Throughput, High-Quality Material Identification



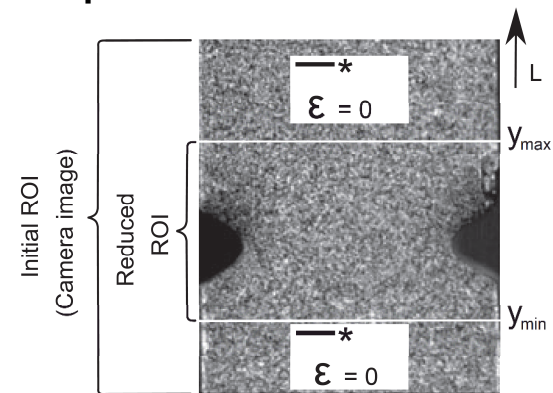
- Complex specimen geometry
 - Range of strain rates in one test
 - Heterogeneous stress state
- Capitalize on rich data from full-field measurements such as Digital Image Correlation (DIC)
- Employ the Virtual Fields Method (VFM) to identify viscoplastic material properties from a single experiment

Characterization of Lüders Behavior and Elasto-Visco-Plastic Material Parameters



Avril et al. (2008) *Mech Mater*

Determination of Johnson-Cook Viscoplastic Material Parameters



Notta-Cuvier et al. (2013) *Strain*

Outline

- Virtual Fields Method (VFM)
- BCJ-MEM Material Model
- Sample Geometry and VFM Cost Function Sensitivity
- Results
 - Identification of viscoplastic material parameters
 - Uniqueness of parameter set
- Summary and Next Steps

Principle of Virtual Power

Current Configuration

$$\underbrace{\int_{V_t} \sigma \cdot \delta D \, dV}_{P_{internal}} = \underbrace{f \cdot \overline{\delta v}}_{P_{external}}$$

$$\partial D = \frac{1}{2} (\nabla \delta v + \nabla^T \delta v)$$

Reference Configuration

$$\underbrace{\int_{V_o} ((\det F) \sigma \cdot F^{-T}) : \delta \dot{F} \, dV}_{P_{internal}} = \underbrace{f \cdot \overline{\delta v}}_{P_{external}}$$

$$\delta \dot{F} = \nabla \delta v$$

Key Features:

- Large deformation formulation
- No plane stress assumption

Assumptions:

- Neglecting acceleration and body forces
- Constant virtual fields on boundary where load is applied

σ	Cauchy Stress
F	Deformation Gradient
f	Resultant Load
V	Sample Volume
∂v	Virtual Velocity
∂D	Virtual Rate of Deformation
$\partial \dot{F}$	Virtual Velocity Gradient

Virtual Fields Method for Viscoplasticity

Set-Up

1. Choose a material model
2. Choose a sample geometry
3. Acquire full-field displacements through the volume
4. Select initial guesses for material parameters
5. Select virtual velocity field(s)

VFM Algorithm

1. Calculate stress
2. Compute cost function
 - $\Psi = \sum_{\text{time}} [P_{\text{internal}} - P_{\text{external}}]^2$
3. Iterate on material parameters until cost function is minimized

■ Von Mises Flow Criterion

- Equivalent Stress:

$$\bar{\sigma} = \sqrt{\frac{3}{2} s:s} \quad \left(s = \sigma - \frac{1}{3} \text{tr}(\sigma) I \right)$$

- Equivalent Plastic Strain Rate:

$$\dot{p} = \sqrt{\frac{2}{3} \dot{\varepsilon}^p : \dot{\varepsilon}^p}$$

- Flow Criterion:

$$f = \bar{\sigma} - \sigma_f = 0$$

■ Hardening Law

- $\sigma_f(p, \dot{p}, \xi) = \sigma_o \left\{ 1 + \text{asinh} \left[\left(\frac{\dot{p}}{b_\sigma} \right)^{1/m_\sigma} \right] \right\} + \kappa \left\{ 1 + \text{asinh} \left[\left(\frac{\dot{p}}{b_\kappa} \right)^{1/m_\kappa} \right] \right\}$

- $\dot{\kappa} = (H - R_d \kappa) \dot{p}$

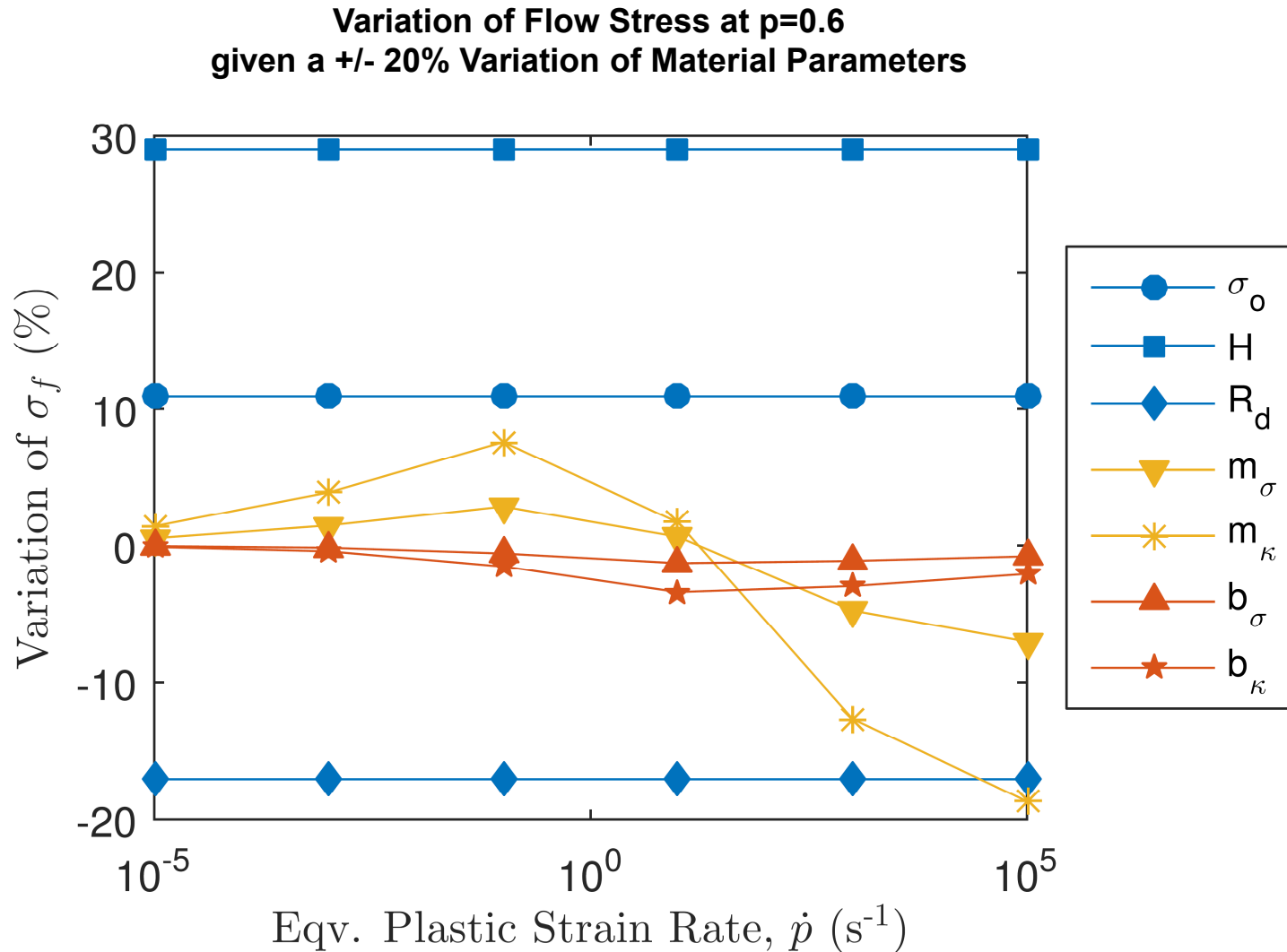
- Constant strain rate: $\kappa = \frac{H}{R_d} [1 - \exp(-R_d p)]$

Viscoplastic Material Parameters for 304L Stainless Steel

Parameter	Symbol	Value	Units
Quasi-static Yield Stress	σ_o	307.9	MPa
Hardening Variable	H	2.692	GPa
Dynamic Recovery	R_d	2.601	--
Rate-Dependent Exponent (Yield)	m_σ	3.169	--
Rate-Dependent Exponent (Hardening)	m_κ	3.169	--
Rate-Dependent Coefficient (Yield)	b_σ	16.25	s ⁻¹
Rate-Dependent Coefficient (Hardening)	b_κ	16.25	s ⁻¹

$$\sigma_f(p, \dot{p}, \xi) = \sigma_o \left\{ 1 + \operatorname{asinh} \left[\left(\frac{\dot{p}}{b_\sigma} \right)^{1/m_\sigma} \right] \right\} + \frac{H}{R_d} [1 - \exp(-R_d p)] \left\{ 1 + \operatorname{asinh} \left[\left(\frac{\dot{p}}{b_\kappa} \right)^{1/m_\kappa} \right] \right\}$$

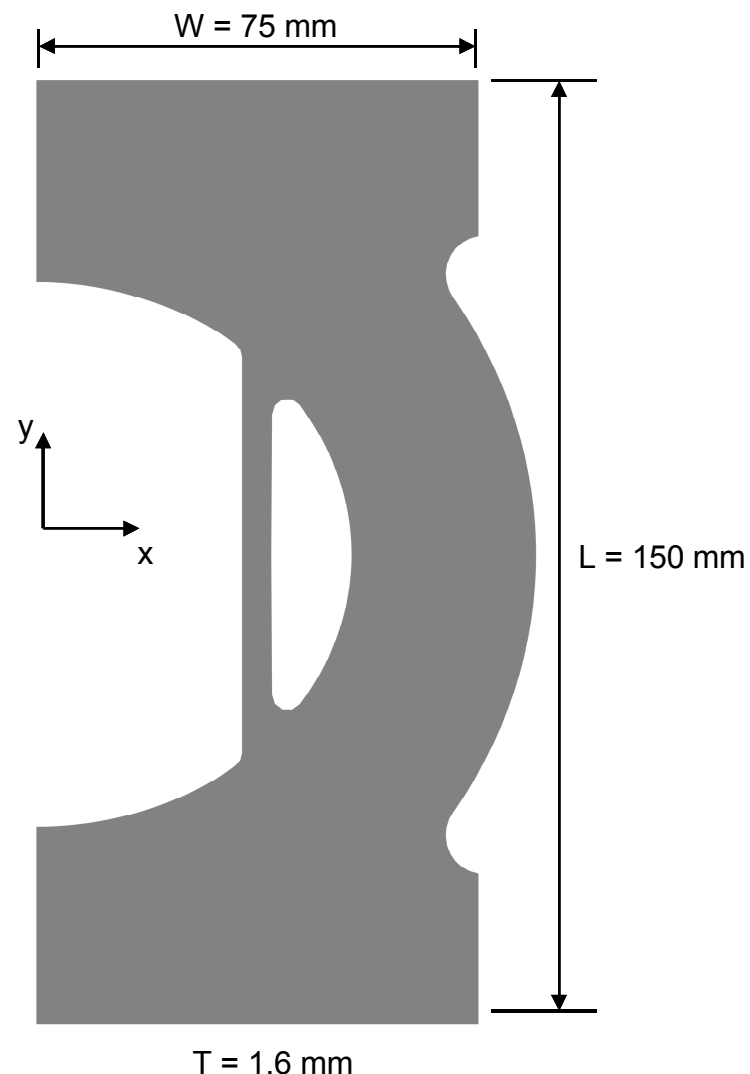
Flow Stress Sensitivity to Material Parameters



Novel Sample Geometry

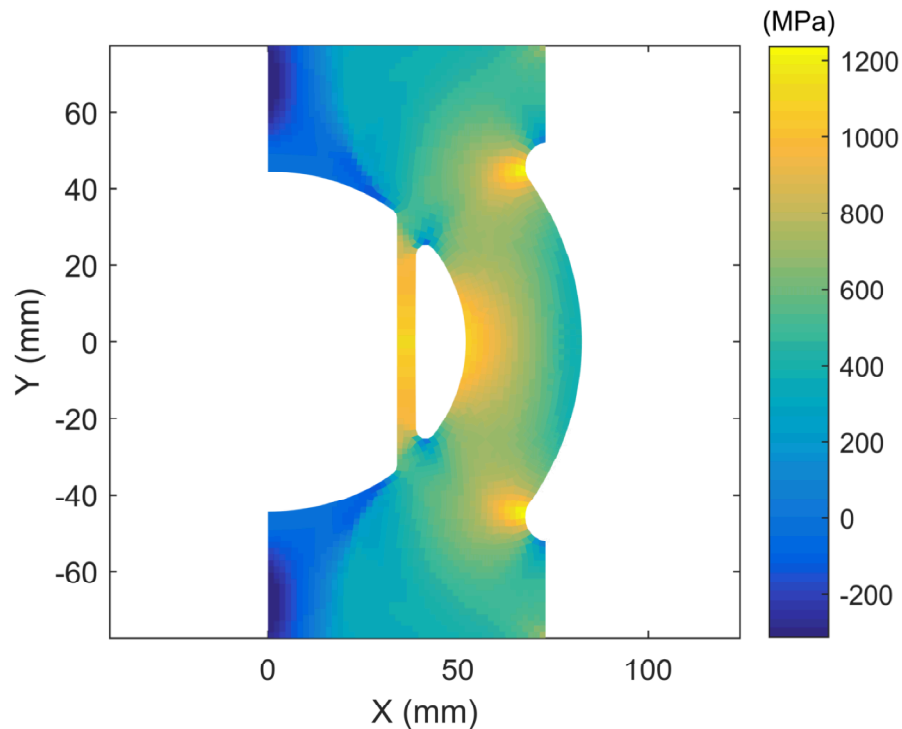
Design Criteria:

- Maximize strain/stress heterogeneity
- Maximize range of strain rates
- Minimize amount of key data near sample edges
- Planar sample with complex 2D geometry

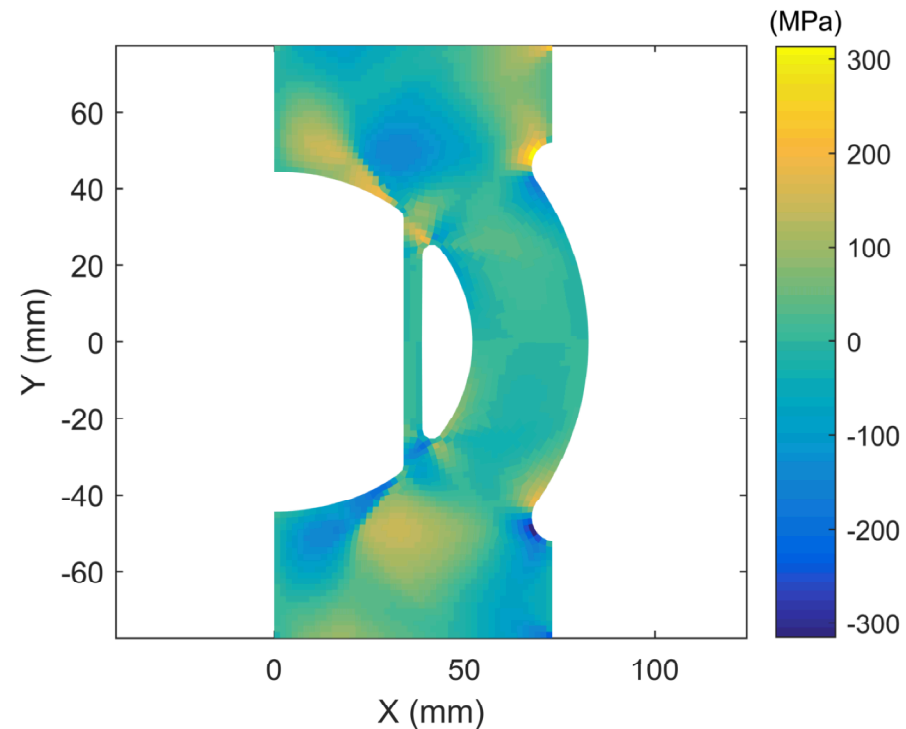


Stress Heterogeneity

Contour Plot of σ_{22}
(Time Step 175)



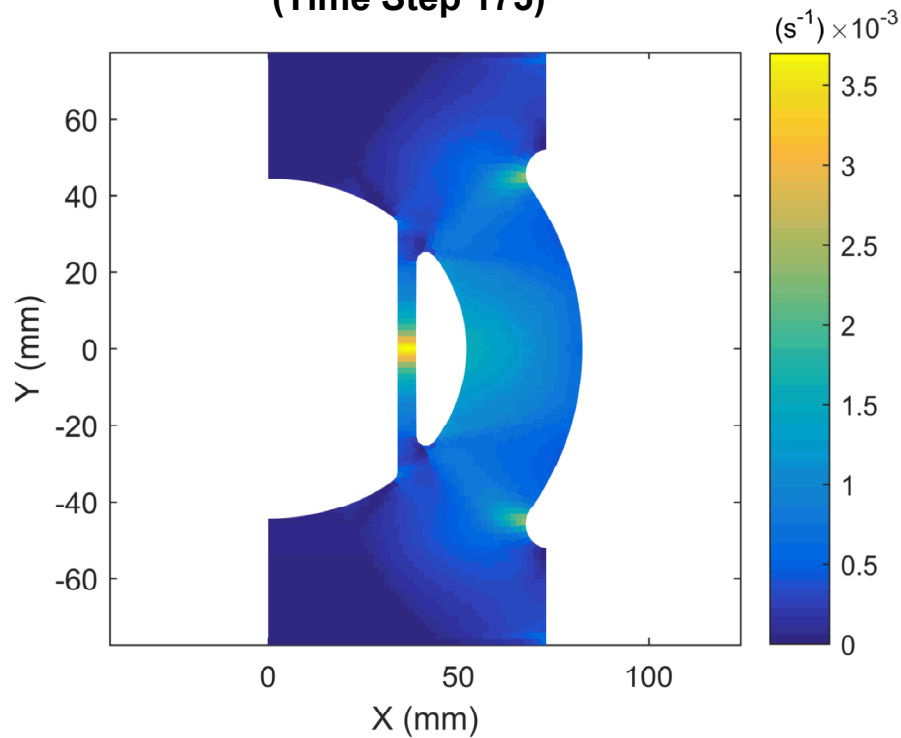
Contour Plot of σ_{12}
(Time Step 175)



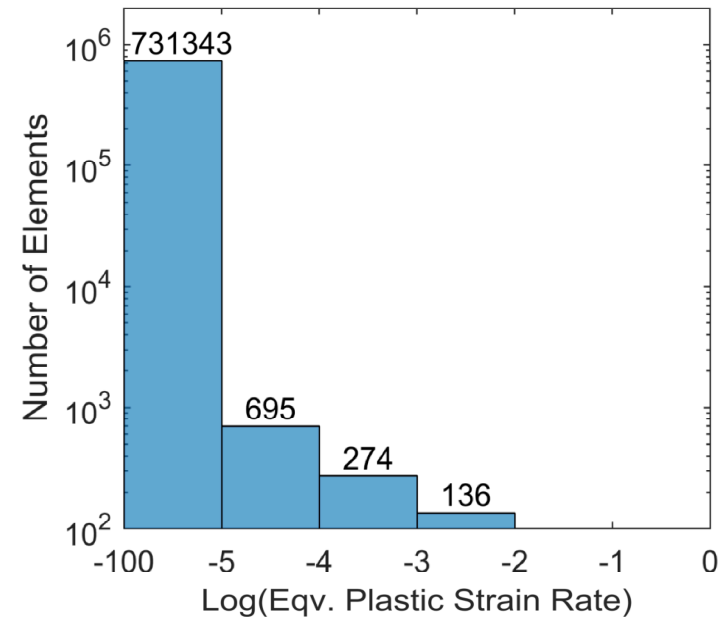
Finite Element Method simulation

Strain Rate Heterogeneity

**Contour Plot of Equivalent Plastic Strain Rate
(Time Step 175)**



**Histogram of Equivalent Plastic Strain Rates
(All Elements, All Time Steps)**



Finite Element Method simulation

Cost Function Sensitivity

Virtual Velocity Field

$$\delta v_x = \cos\left(\frac{y\pi}{L}\right)$$

$$\delta v_y = \frac{2y + L}{2L}$$

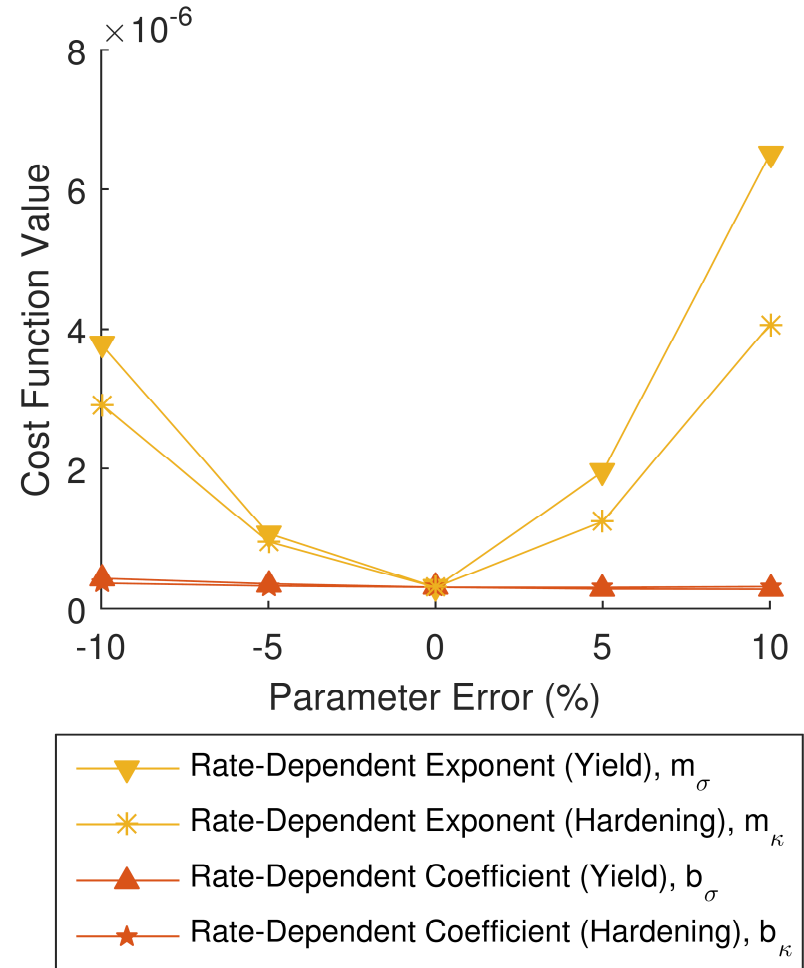
$$\delta v_z = 0$$

Cost Function

$$\Psi = \sum_{\text{time}} [P_{\text{internal}} - P_{\text{external}}]^2$$

Displacement Field

“Perfect” displacements from
FEM simulation



Simulated Experiments

Three displacement fields:

1. “Perfect” displacements from FEM simulation
2. 0.1 μm random noise added to FEM displacements
 - $\sim 1/500$ pixel DIC noise floor
3. 1.0 μm random noise added to FEM displacements
 - $\sim 1/50$ pixel DIC noise floor

VFM Algorithm Inputs:

- Initial Guess: +20 % from exact parameter value
- Bounds: +/- 90 % from exact parameter value

Material Identification Results

			Exact Displacements	0.1 μm noise	1.0 μm noise
Parameter	Exact Value	Units	% Error		
σ_o	307.9	MPa	-1.5		
H	2.692	GPa	2.3		
R_d	2.601	--	-0.5		
m_σ	3.169	--	17.9		
m_k	3.169	--	-17.4		
b_σ	16.25	s^{-1}	10.8		
b_k	16.25	s^{-1}	10.9		
Cost Function Residual			2.1e-7		

Material Identification Results

			Exact Displacements	0.1 μm noise	1.0 μm noise
Parameter	Exact Value	Units	% Error		
σ_o	307.9	MPa	-1.5	-3.5	
H	2.692	GPa	2.3	0.6	
R_d	2.601	--	-0.5	5.4	
m_σ	3.169	--	17.9	40.8	
m_k	3.169	--	-17.4	9.9	
b_σ	16.25	s^{-1}	10.8	10.3	
b_k	16.25	s^{-1}	10.9	10.4	
Cost Function Residual			2.1e-7	3.2e-7	

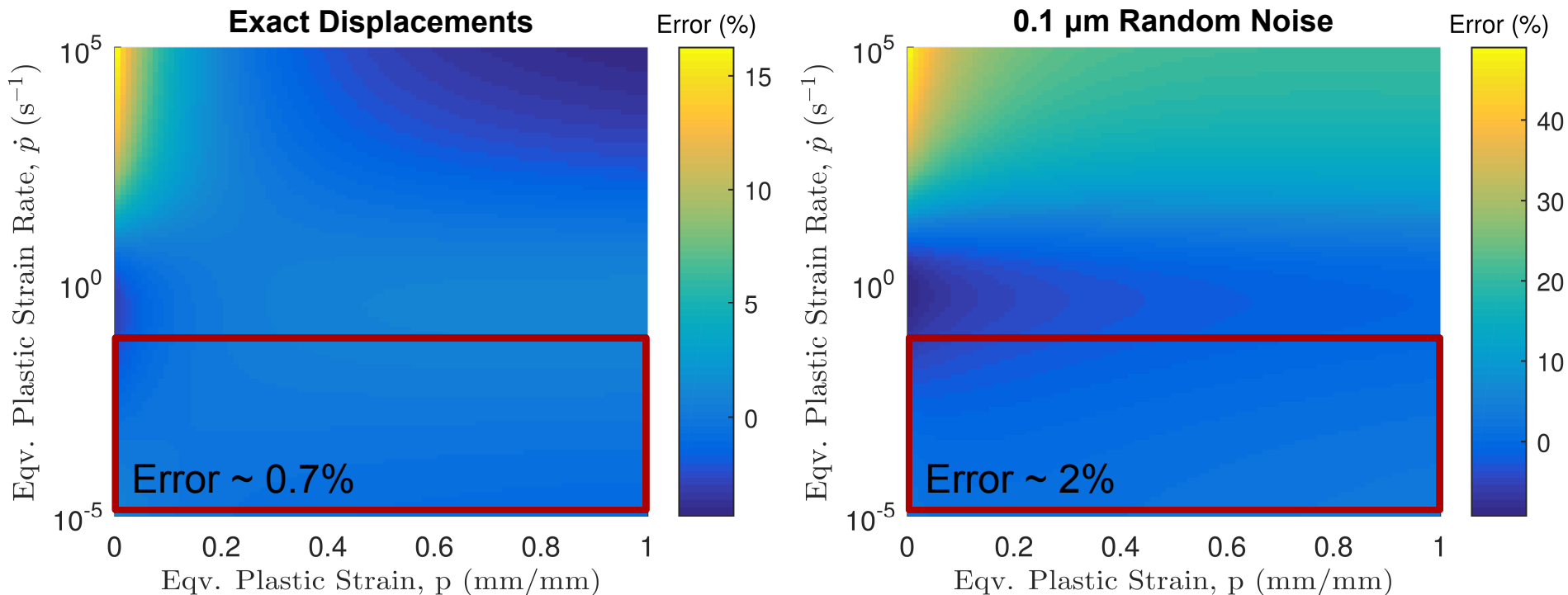
Material Identification Results

			Exact Displacements	0.1 μm noise	1.0 μm noise
Parameter	Exact Value	Units	% Error		
σ_o	307.9	MPa	-1.5	-3.5	-6.3
H	2.692	GPa	2.3	0.6	20.4
R_d	2.601	--	-0.5	5.4	90.0
m_σ	3.169	--	17.9	40.8	9.0
m_k	3.169	--	-17.4	9.9	11.8
b_σ	16.25	s^{-1}	10.8	10.3	90.0
b_k	16.25	s^{-1}	10.9	10.4	90.0
Cost Function Residual			2.1e-7	3.2e-7	1.1e-4

Uniqueness of Parameter Set

$$\sigma_f(p, \dot{p}, \xi) = \sigma_o \left\{ 1 + \operatorname{asinh} \left[\left(\frac{\dot{p}}{b_\sigma} \right)^{1/m_\sigma} \right] \right\} + \frac{H}{R_d} [1 - \exp(-R_d p)] \left\{ 1 + \operatorname{asinh} \left[\left(\frac{\dot{p}}{b_\kappa} \right)^{1/m_\kappa} \right] \right\}$$

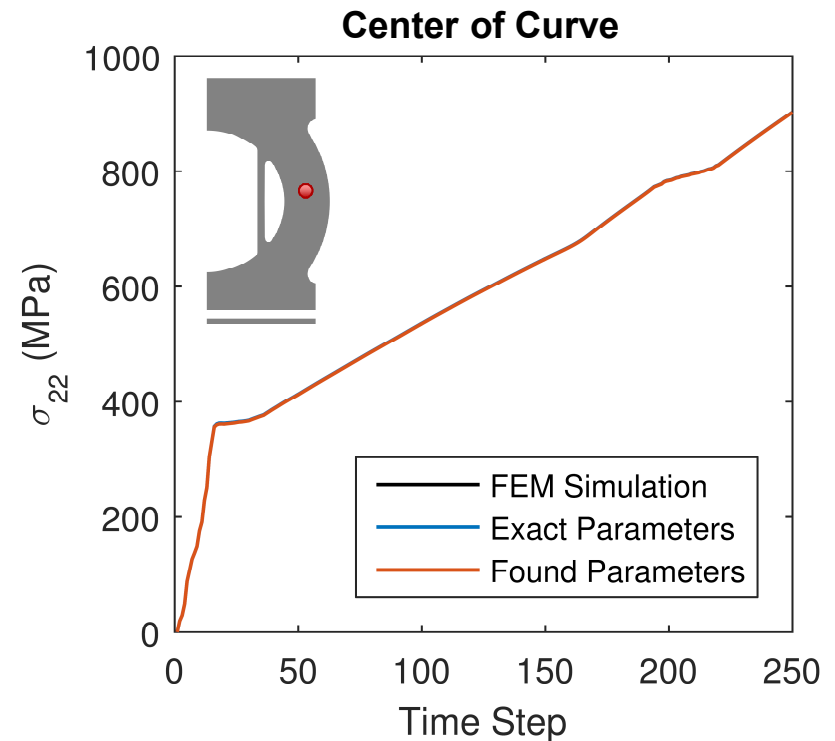
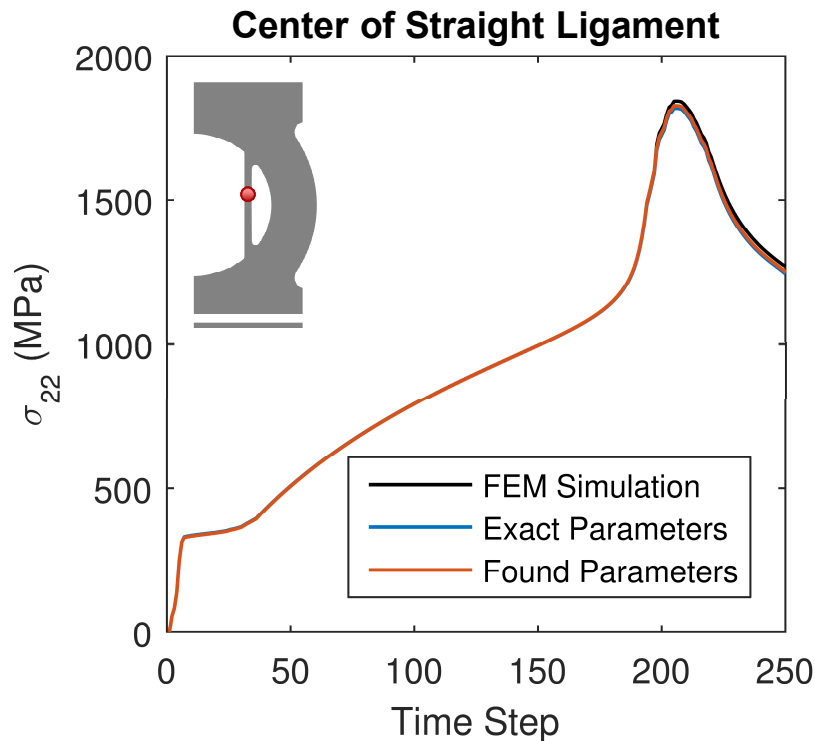
Error of BCJ-MEM Flow Stress Calculated from Different Parameter Sets



Reconstructed Stress Field

Exact Displacements

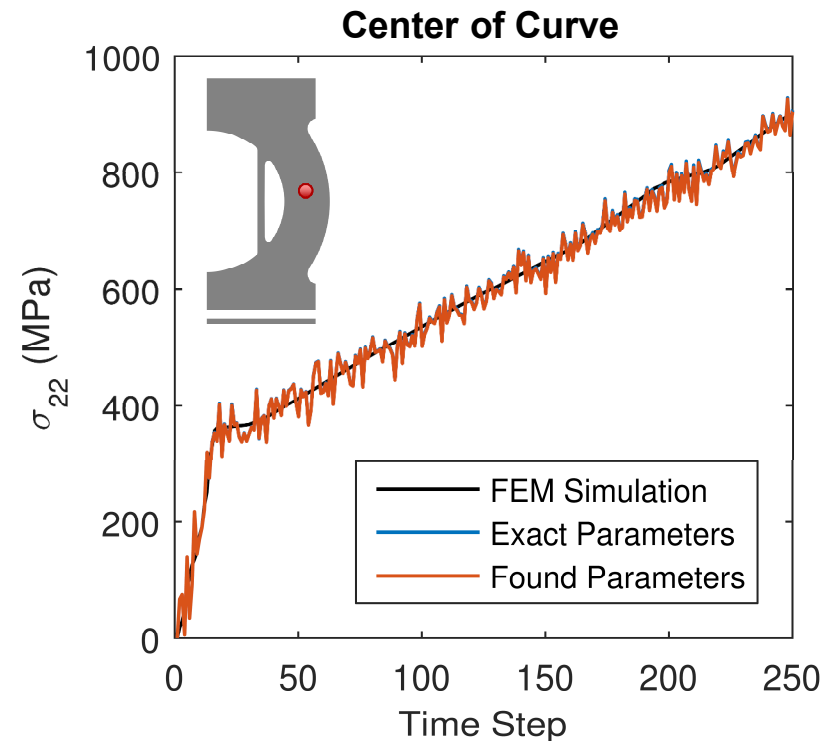
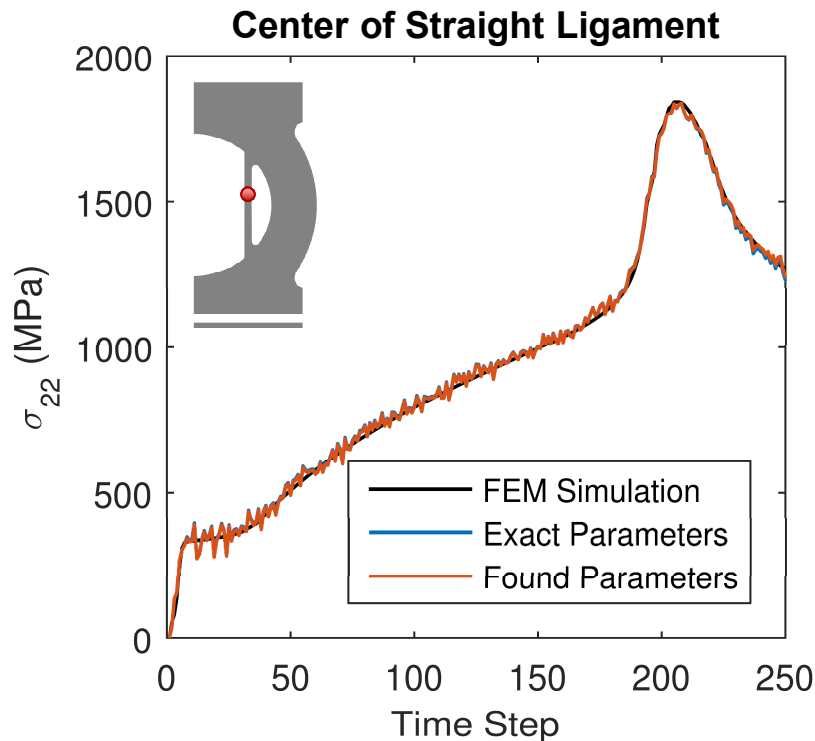
< 2% Error in Von Mises Stress
Calculated with Exact Parameters
versus Found Parameters



Reconstructed Stress Field

0.1 μm Random Noise

< 5% Error in Von Mises Stress
Calculated with Exact Parameters
versus Found Parameters



Summary and Next Steps

Summary:

- Developed a Virtual Fields Method (VFM) framework:
 - Viscoplastic material properties from the BCJ-MEM material model
 - 3-dimensional (no plane stress assumption)
 - Finite deformations (no small strain assumption)
- Validated VFM implementation using simulated displacements from a FEM model

Next Steps:

- Optimize specimen geometry, loading path, cost function to activate all material parameters
- Investigate implications of non-orthogonal material parameters
- Perform a parametric study of noise influence on material ID using a stereo-DIC simulator (collaboration with Ruben Balcaen and Dr. Pascal Lava at KU Leuven)
- Utilize VFM methodology to experimentally determine material parameters of 304L stainless steel

Food for Thought



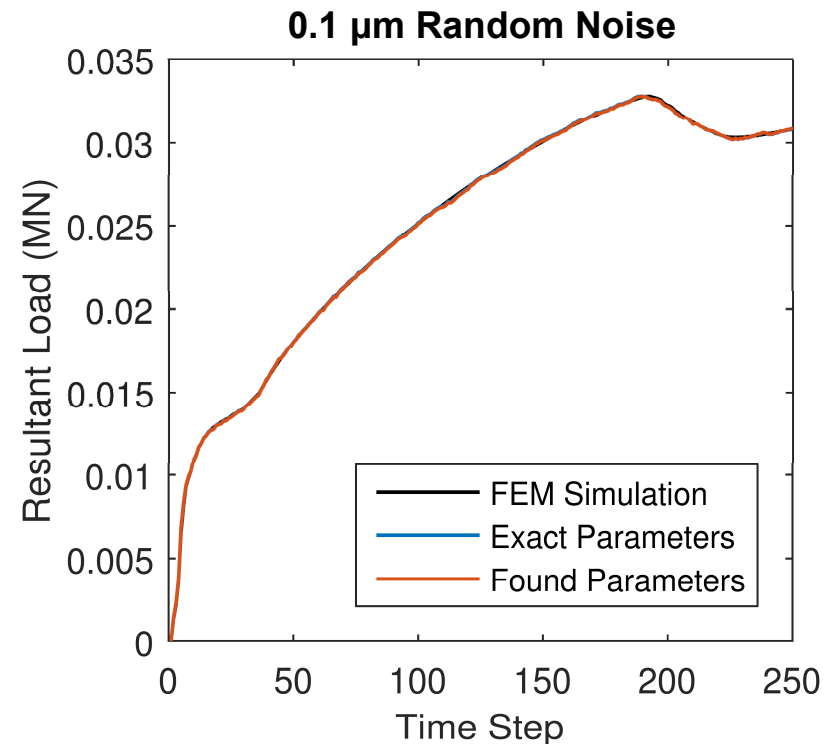
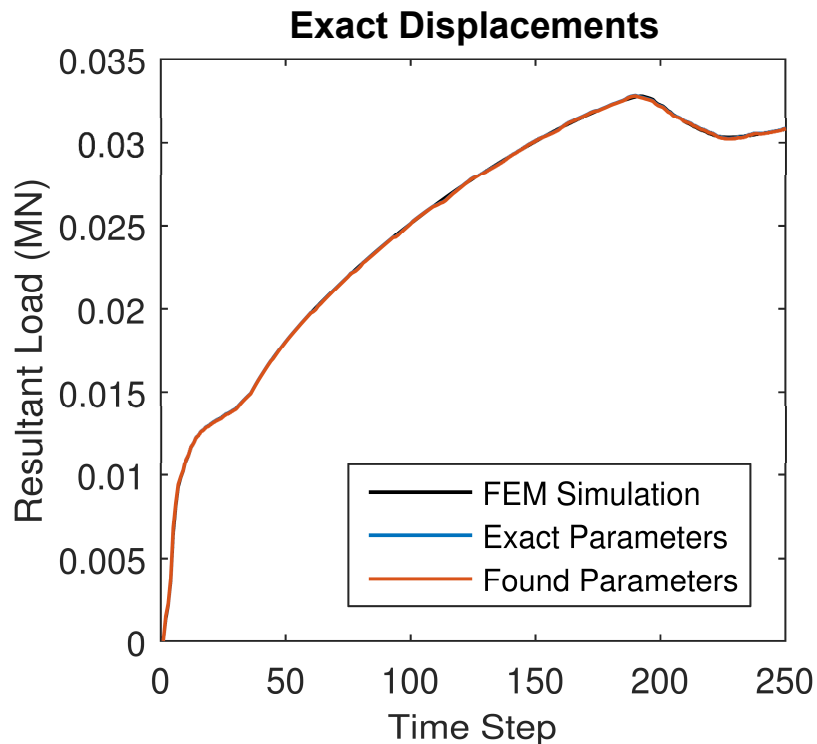
Material: Acrylonitrile butadiene styrene (ABS)

Open Questions:

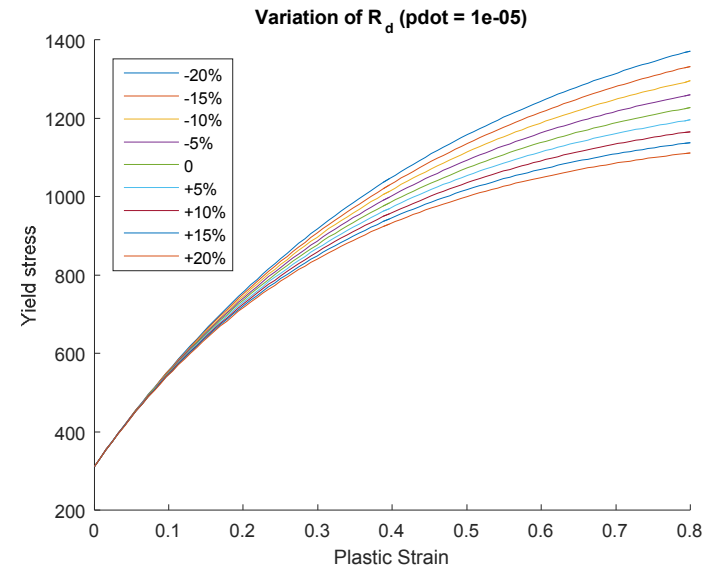
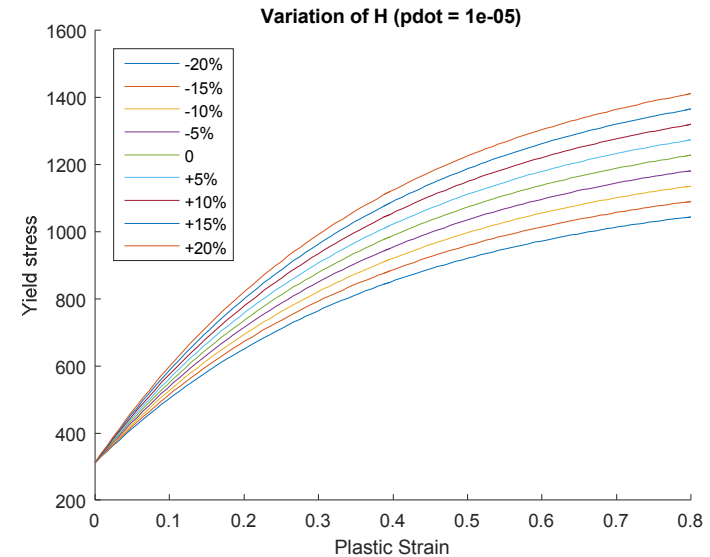
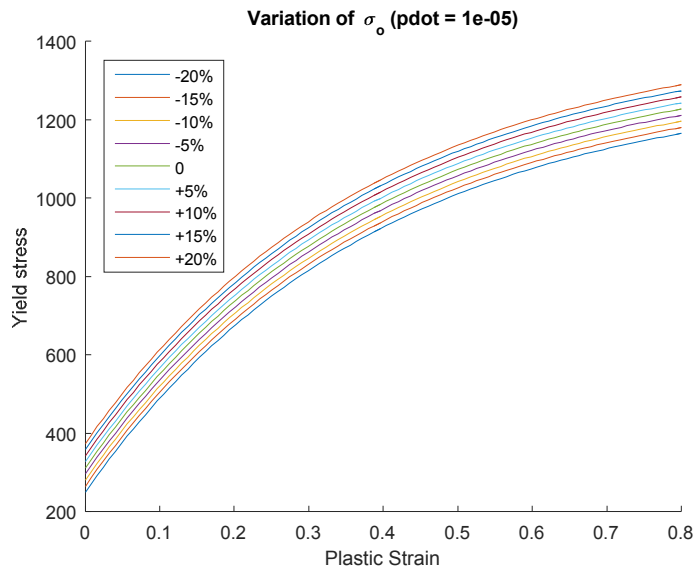
- Do material parameters identified from a uniaxial stress state adequately describe material behavior subjected to complex loading?
- How does orthogonality of a parameter set affect predictions of material behavior?
- Which is better: a material model with physics-based (though covariant) parameters, or a model with completely independent parameters?

Resultant Load

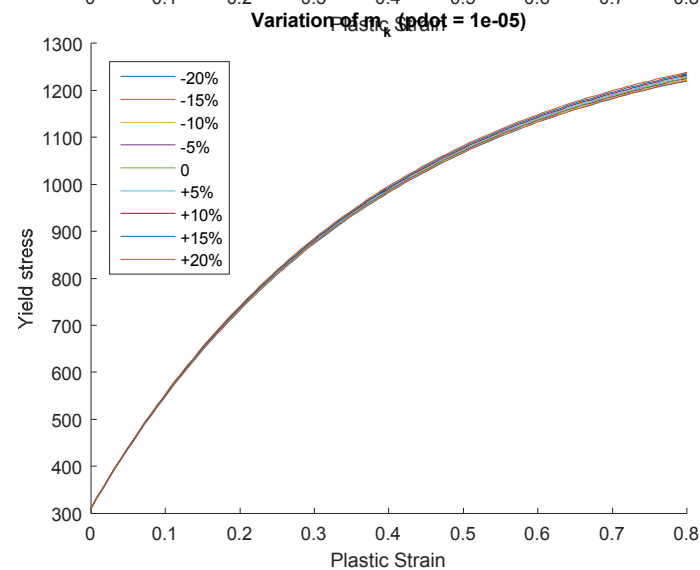
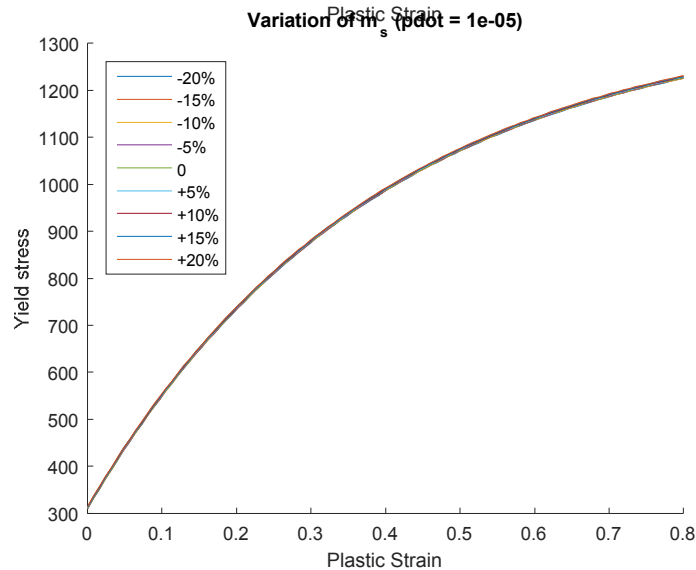
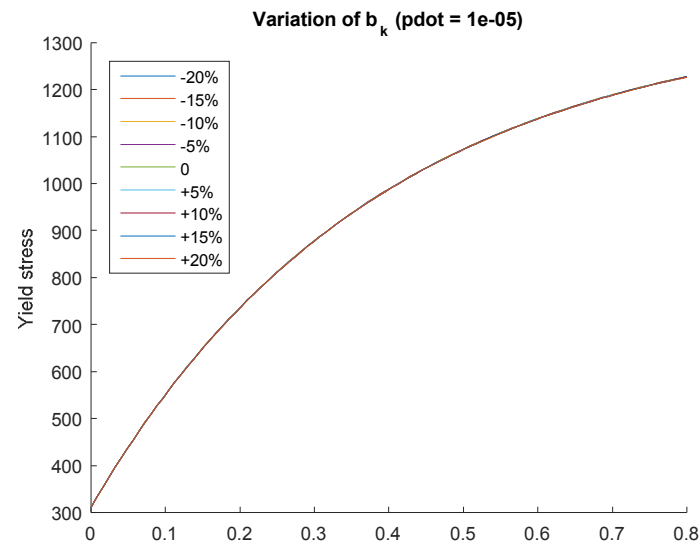
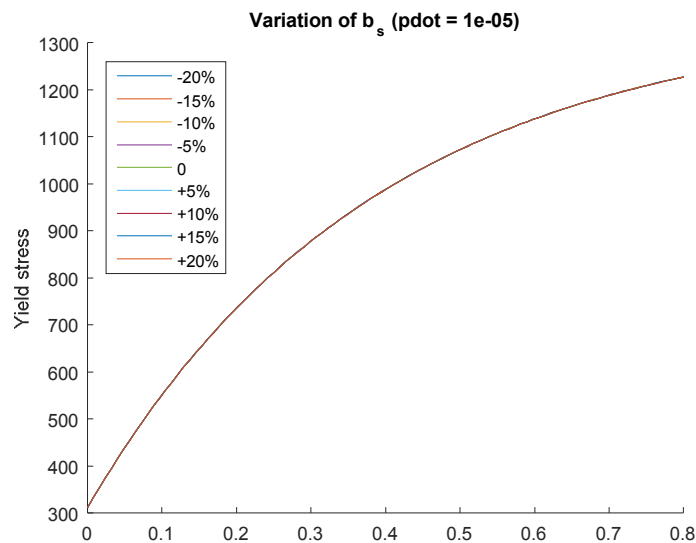
Resultant load averaged from 10 individual horizontal slices distributed evenly along height of sample.



Model Sensitivity to Quasi-Static Material Parameters



Model Sensitivity to Strain-Rate Dependent Material Parameters



Model Sensitivity to Strain-Rate Dependent Material Parameters

