

Linear Models for Treaty Verification Tasks

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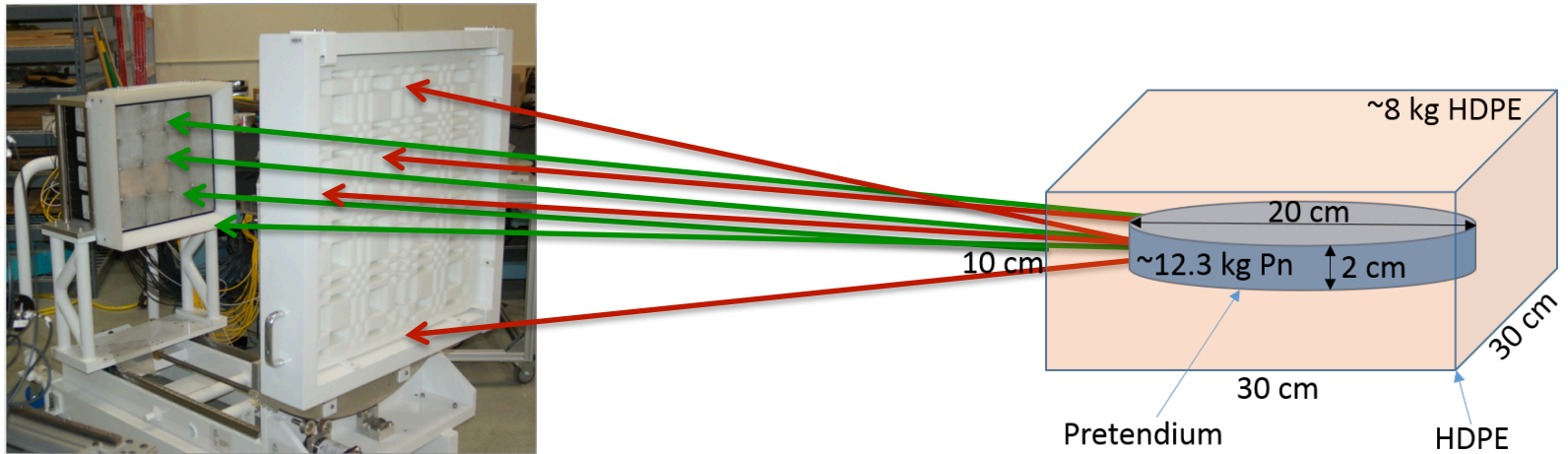
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Outline

- Approach to arms-control-treaty verification
 - Task-based imaging
 - List-mode processing
 - Models that store non-sensitive information
- Linear models to classify items (using BeRP ball study)
 - Channelized Hotelling observer
 - Reducing each channel's discrimination ability
 - Penalizing discrimination on nonsensitive information
- Pretendium item discrimination

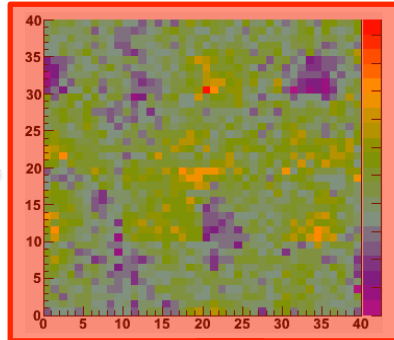
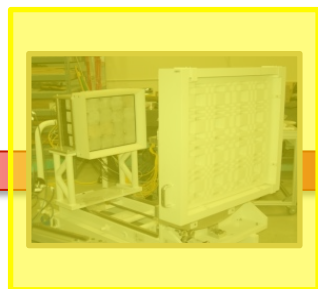
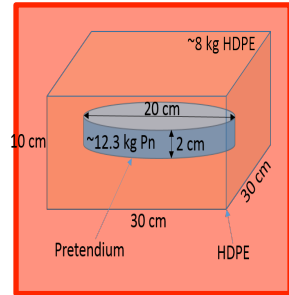
Verification task

Is it really a warhead?



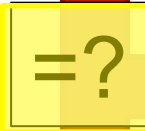
“Traditional” Template Matching

Trusted object

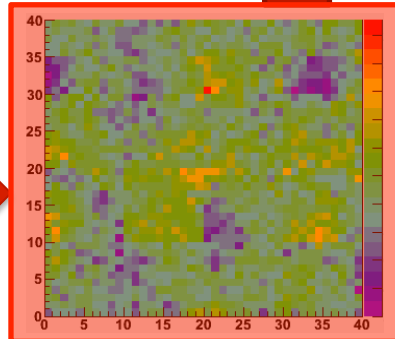
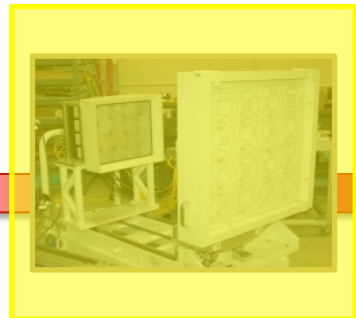
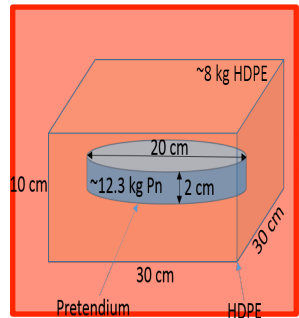


Calibration data is sensitive
IB required

LEGEND	
Red	No Access
Yellow	Access Before & After
Green	Full Access



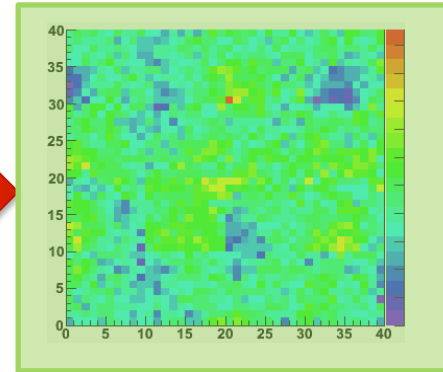
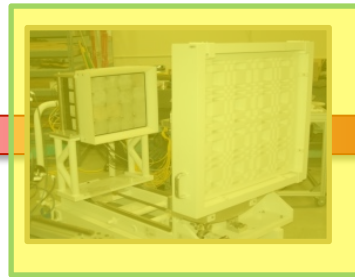
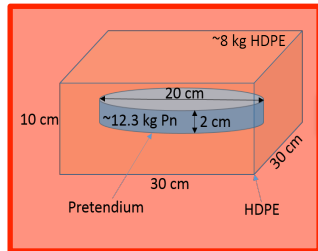
Tested object



Tested detector data is sensitive
IB required

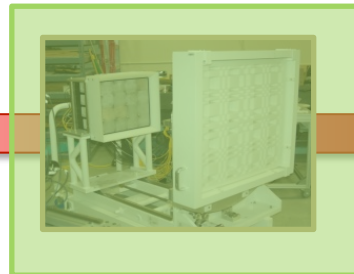
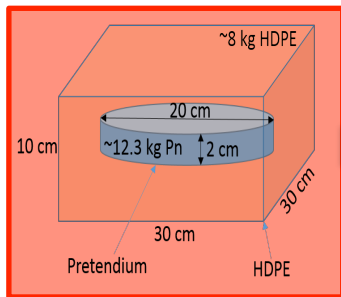
Our proposal

Trusted object

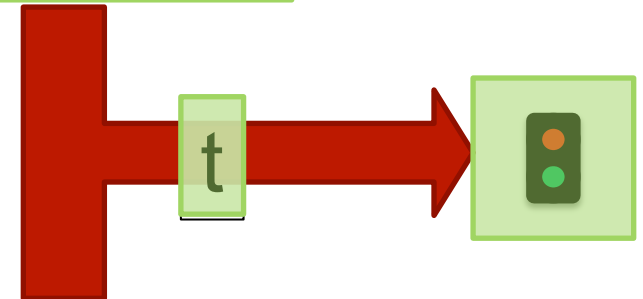


Hypothetical observer stores info sufficient for confirmation but not sensitive

Tested object



LEGEND	
Red	No Access
Yellow	Access Before & After
Green	Full Access



Testing data is processed event by event, only updating test statistic.

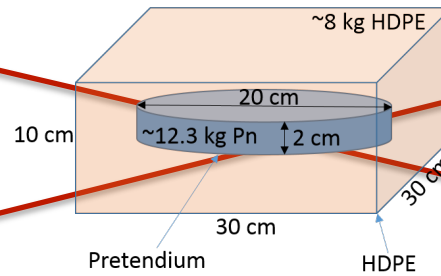
Data not aggregated

Think snapchat!

Verification Task

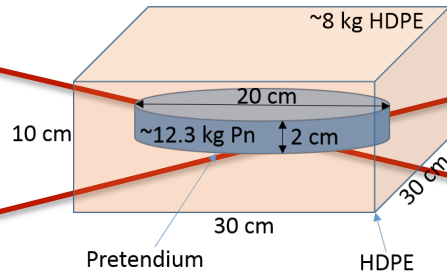
Hypothesis tests can't be used with list-mode data.

Is it really a warhead?

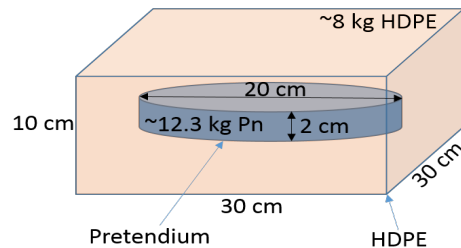


Verification Task

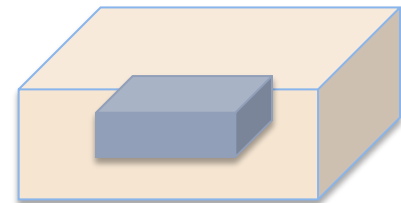
Is it really a warhead?



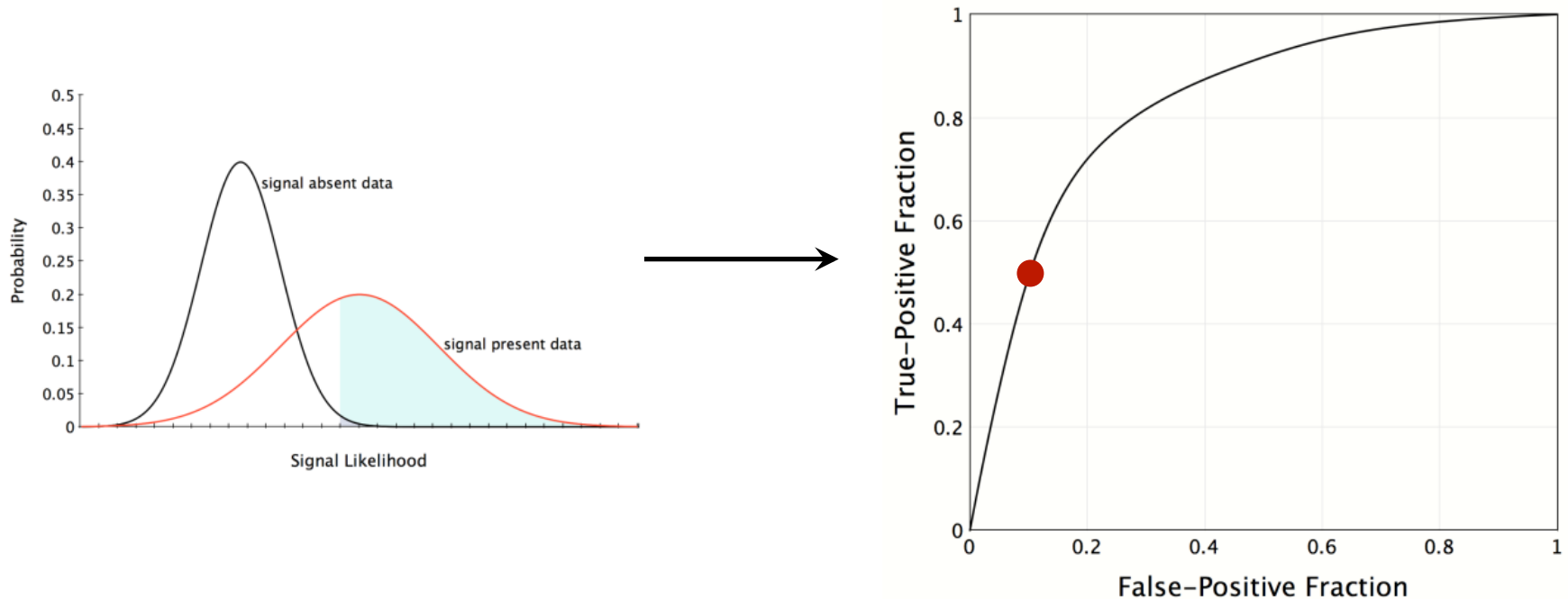
Is it warhead A or warhead B?



OR



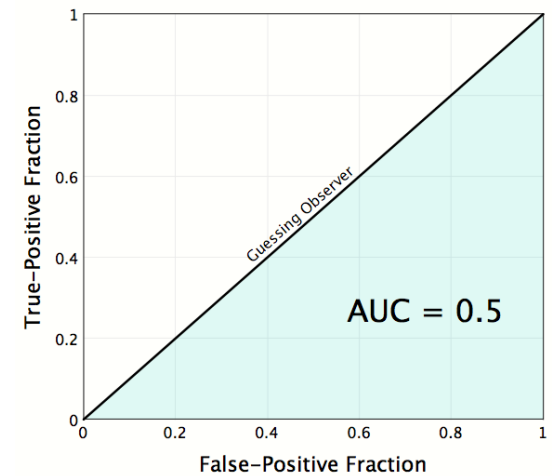
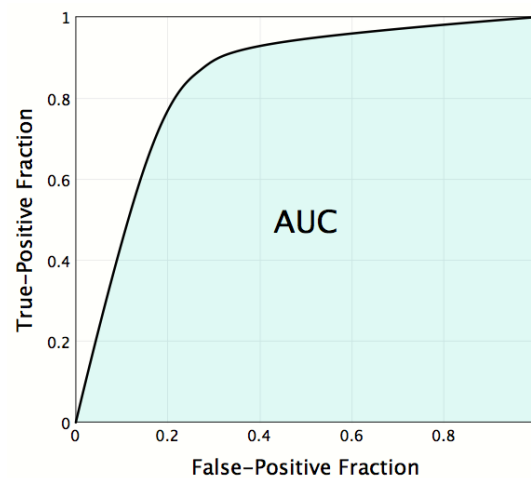
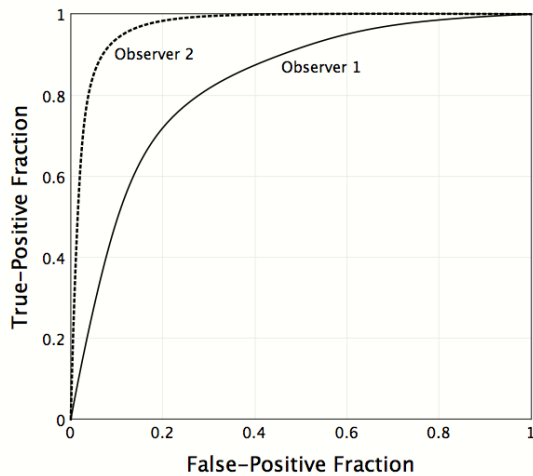
Task performance of observer models are assessed and compared



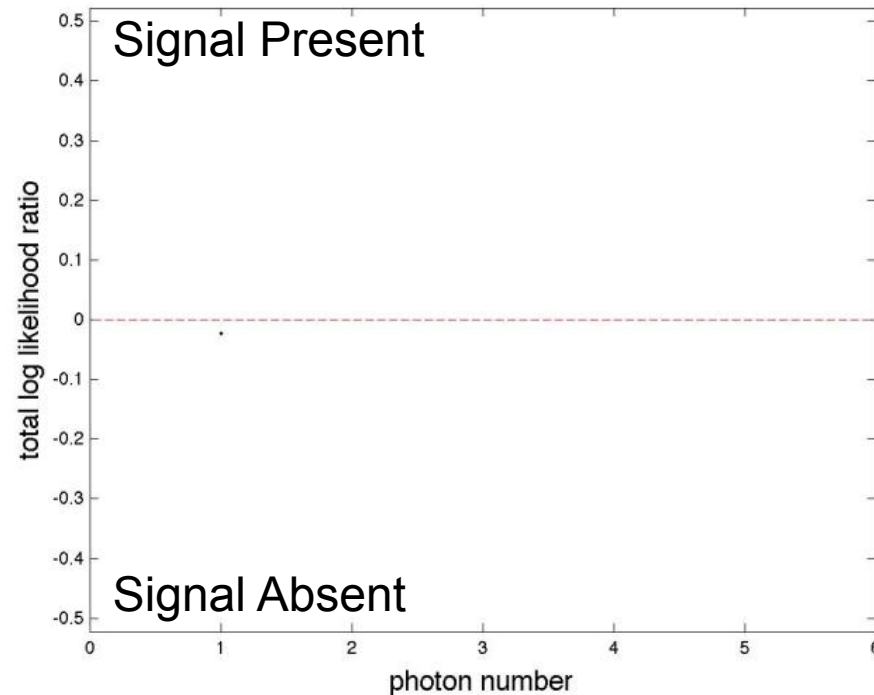
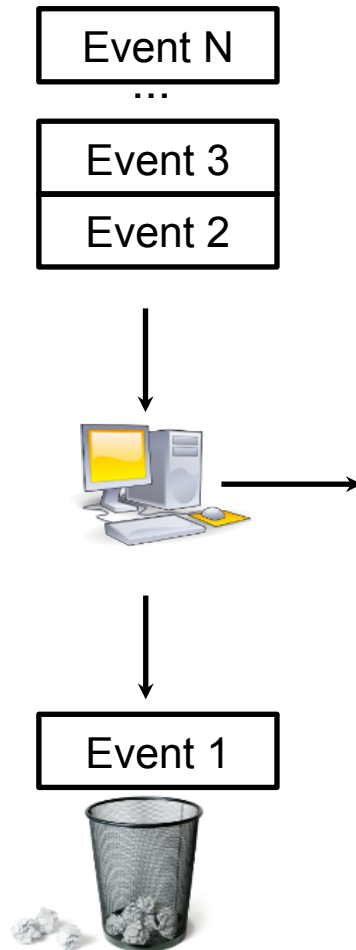
ROC curve plots the sensitivity vs. the false-positive fraction for all possible thresholds.

Figure of Merit

Observer models evaluated by comparison of area under ROC curve

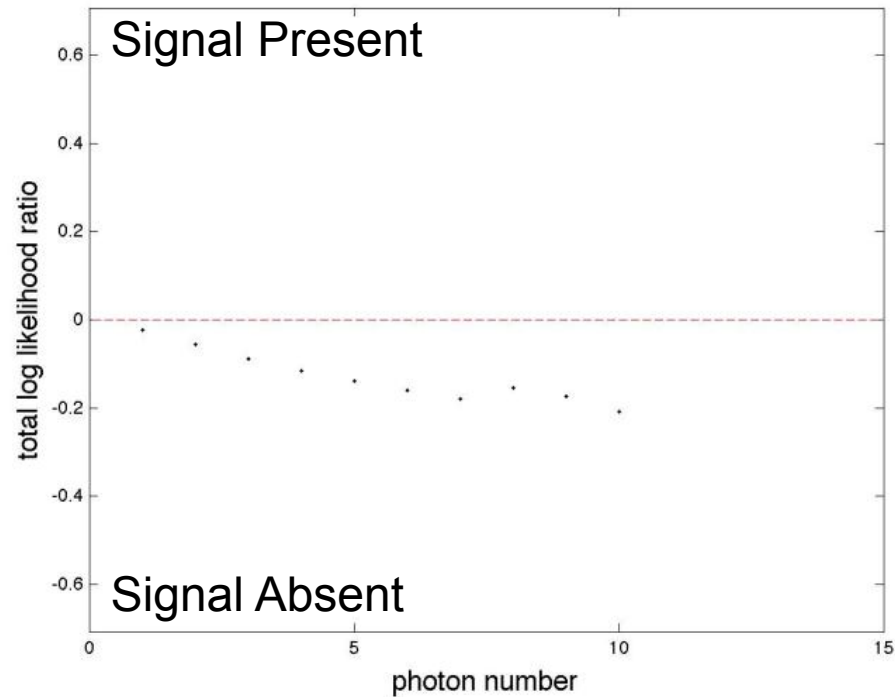
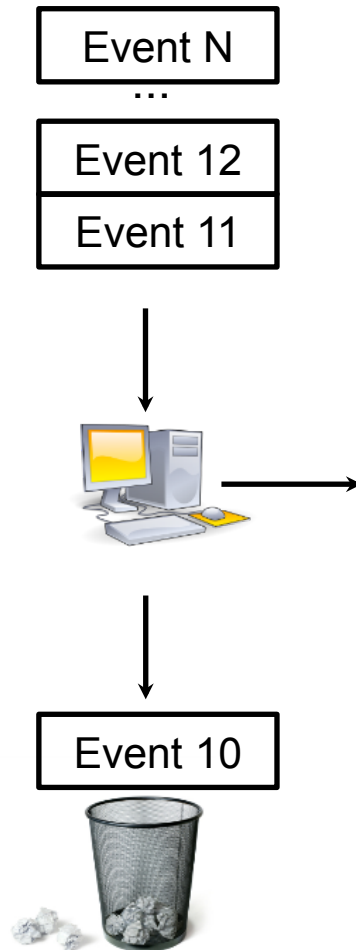


List-mode Processing



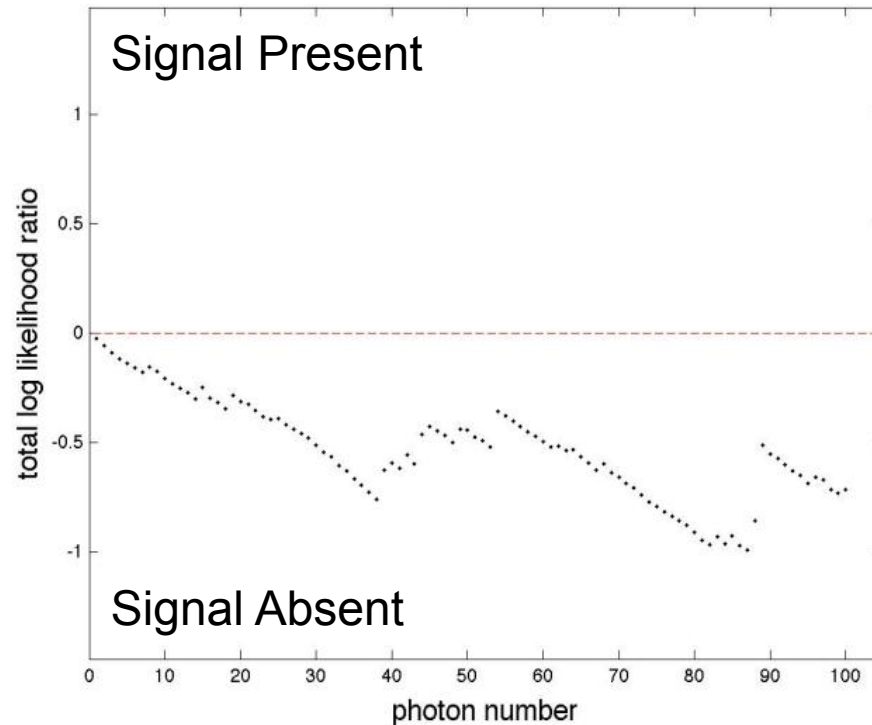
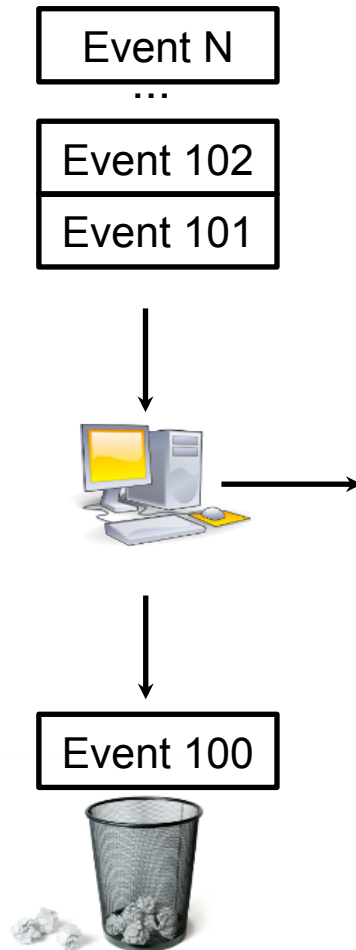
Output running sum is the likelihood of a signal being present, which is thresholded to make a decision.

List-mode Processing



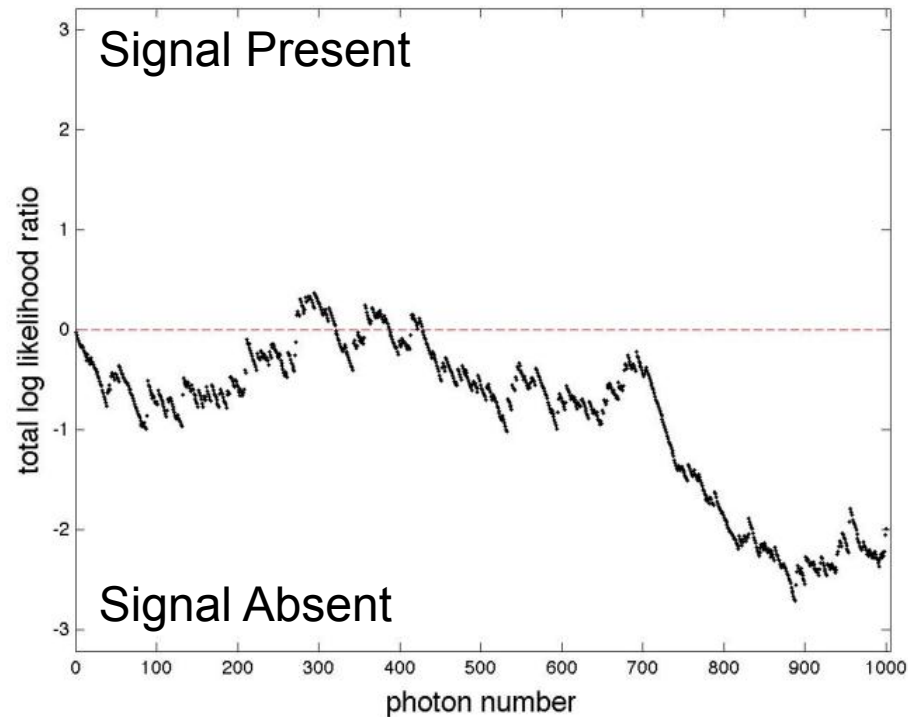
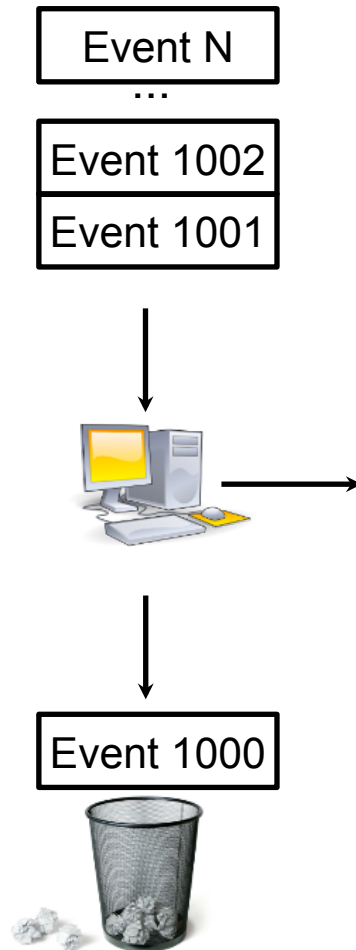
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List-mode Processing



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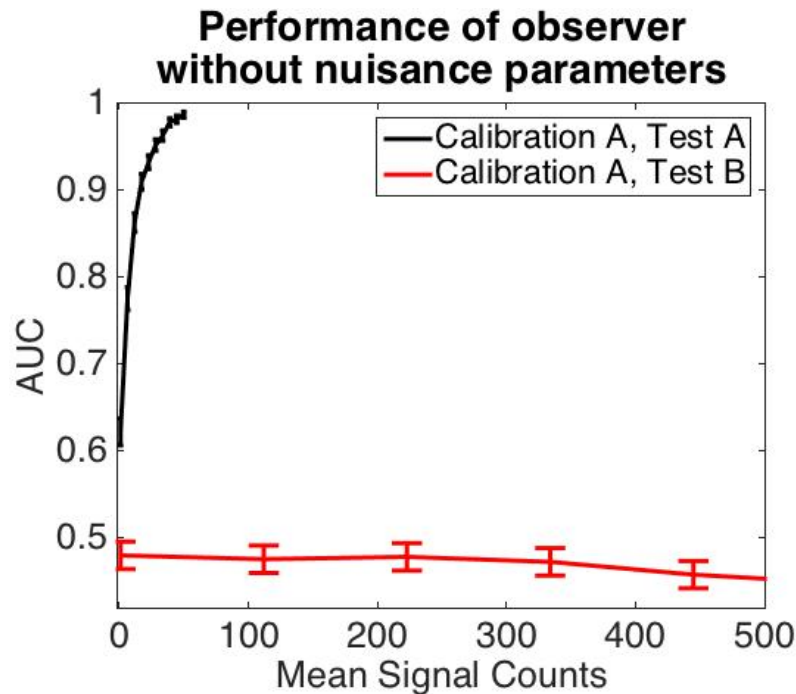
Nuisance parameters

Nuisance parameter: Any variable that affects the data but is not of interest to task.

- Relevant examples:
 - Source material age. Affects detected count rate and gamma energy spectra.
 - Disk orientation
 - Disk location in polyethylene

Importance of Nuisance Parameters

- Nuisance parameters must be taken into account for results to generalize
- Orientation as a nuisance parameter



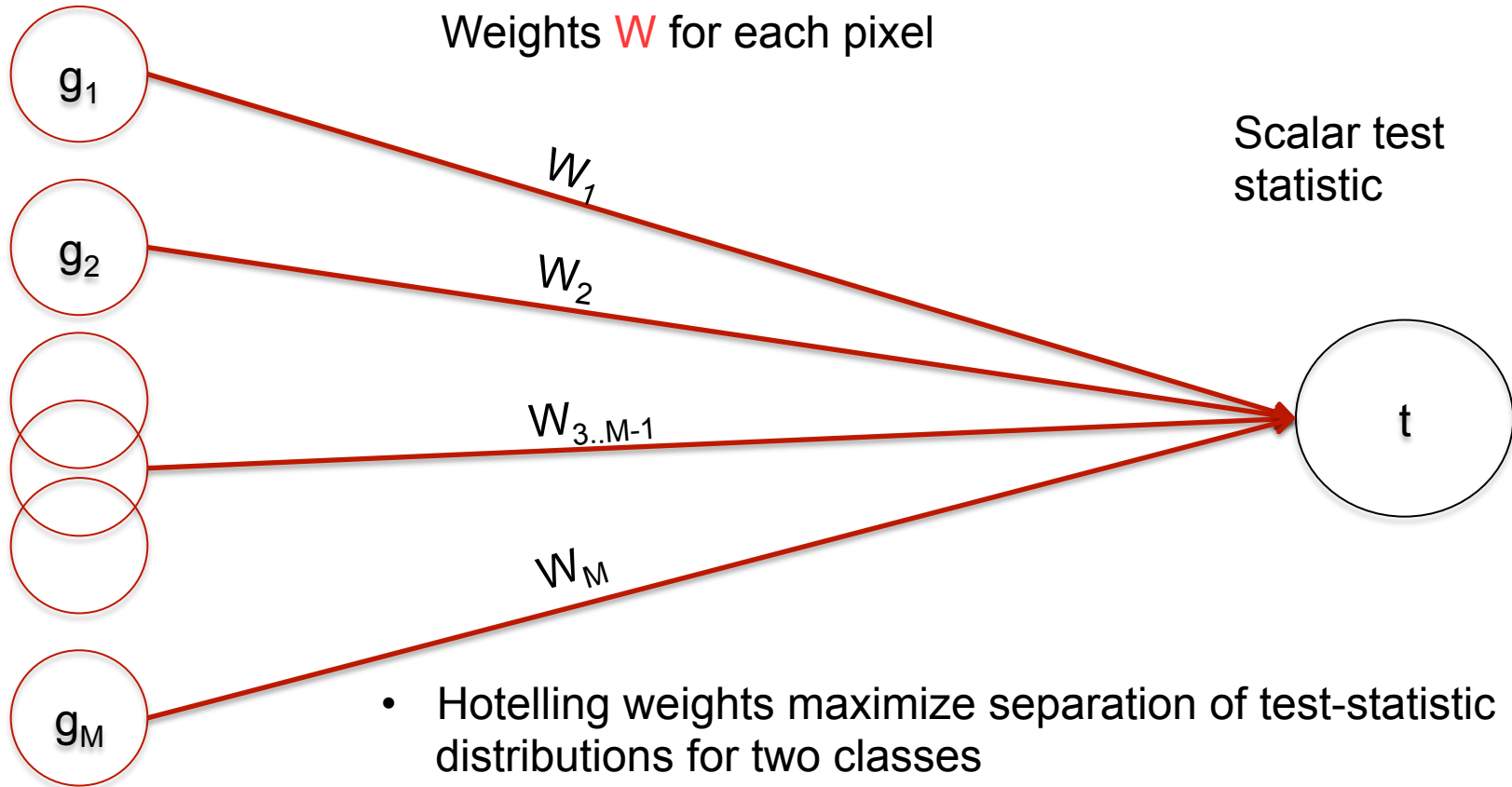
MacGahan, C. J., M. A. Kupinski, N. R. Hilton, E. M. Brubaker, and W. C. Johnson (2016). Development of an Ideal Observer that Incorporates Nuisance Parameters and Processes List Mode Data. *JOSA A*, 33(4), pp. 689–697.

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Linear Template Observers

Projection data **g**
(sensitive count map)



- Hotelling weights maximize separation of test-statistic distributions for two classes
- Take into account randomness in objects.
- Generally sensitive

Linear template observers

- Series of weights \mathbf{W} act on binned testing data \mathbf{g}_{test} , result is scalar that is thresholded to make a decision.

$$t_{test} = \mathbf{W}^\dagger \mathbf{g}_{test}, t_{test} \lesseqgtr t_{thresh}$$

- Hotelling observer is the ideal set of weights \mathbf{W} defined as:

$$\mathbf{W} = \mathbf{K}_g^{-1} \Delta \bar{\mathbf{g}} \quad \mathbf{K}_g = \frac{\mathbf{K}_1 + \mathbf{K}_2}{2}$$
$$\Delta \bar{\mathbf{g}} = \bar{\mathbf{g}}_2 - \bar{\mathbf{g}}_1$$

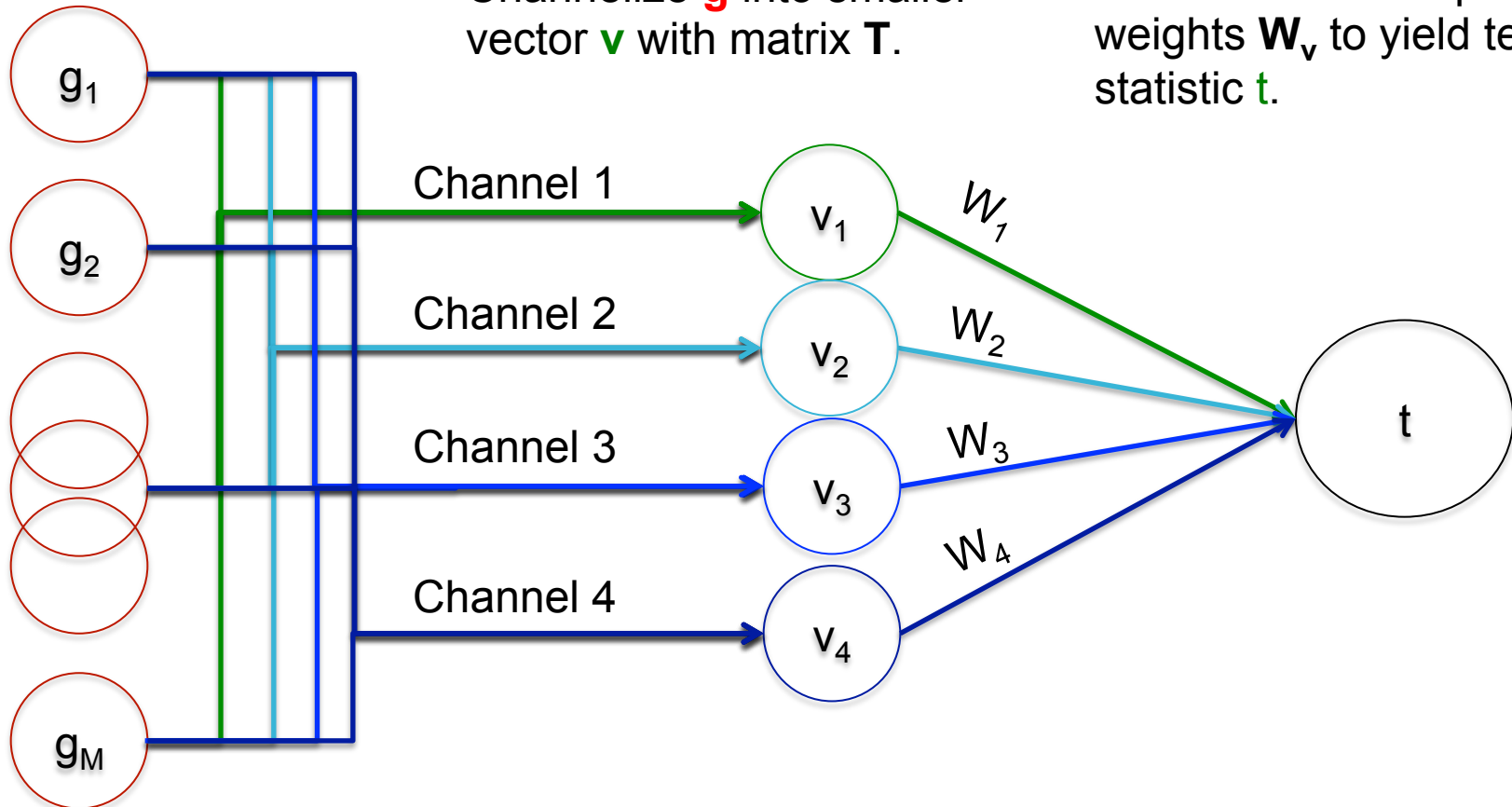
- Averages are over all randomness (Poisson noise, object variability)

Channelized Hotelling Observer

Projection data \mathbf{g}
(sensitive count map)

Channelize \mathbf{g} into smaller
vector \mathbf{v} with matrix \mathbf{T} .

\mathbf{v} combined with optimal
weights \mathbf{W}_v to yield test
statistic t .



Channelized Hotelling

- Channelize vector \mathbf{g} ($P \times 1$) with operator \mathbf{T} ($Q \times P$) into much smaller vector \mathbf{v} ($Q \times 1$) with Q values.

$$\mathbf{v} = \mathbf{T}\mathbf{g}$$

- Optimal set of weights for these channelized values are a function of \mathbf{T} :

$$\mathbf{W}_{\mathbf{v}} = \mathbf{K}_{\mathbf{v}}^{-1} \Delta \bar{\mathbf{v}}$$

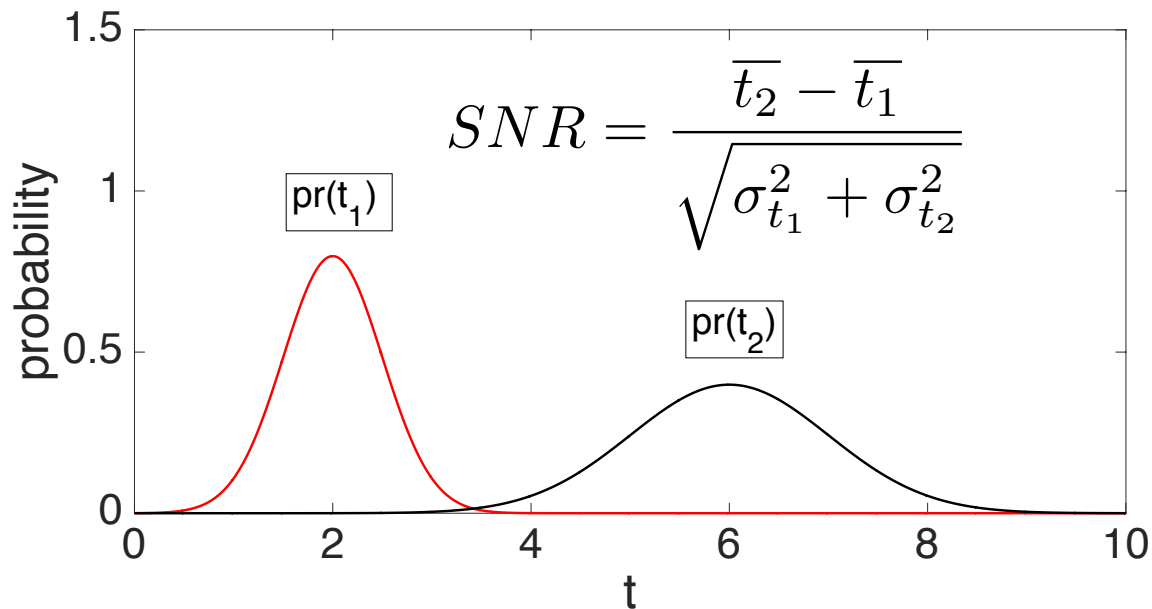
- Inner product of weights and channelized vector

$$\mathbf{W}_{\mathbf{v}} \mathbf{v}_{\text{test}} \lesseqgtr t_{\text{thresh}}$$

Optimizing T

- T can be optimized to maximize SNR^2 of test statistic distributions for best performance.
 - Gradient descent with backtrack

$$f_{obj}(\mathbf{T}) = SNR^2(\mathbf{T})$$

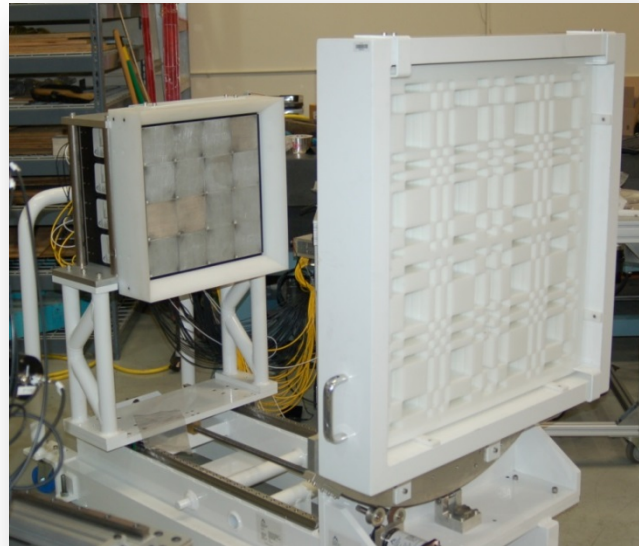
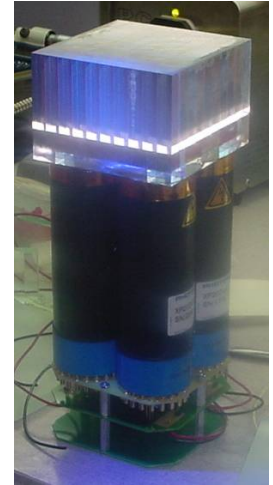


BeRP ball location discrimination example

- The goal in this toy problem is to classify an item at one of two locations
 - T (TAI) is a BeRP ball at (0,0)
 - F (spoof) is BeRP ball at (2,2)

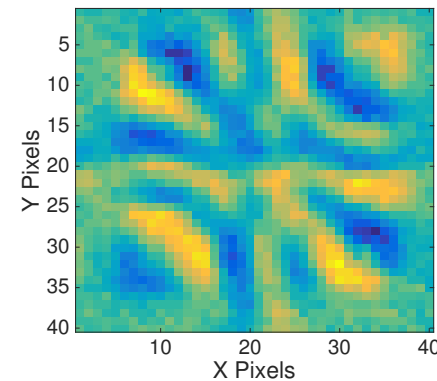
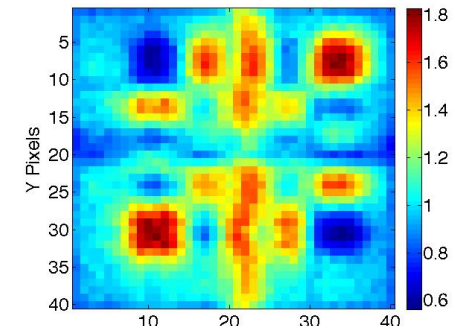
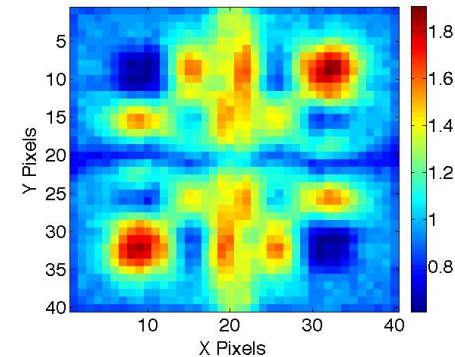
Imaging System

- ORNL/SNL fast neutron coded-aperture imager developed for arms control treaty verification.
- Image plane consists of 16 organic scintillator pixelated block detectors
 - Each block consists of a 10x10 array of 1 cm. pixels.
 - PSD and pixel id accomplished by 4 photomultiplier tubes.



Hotelling observer – location discrimination

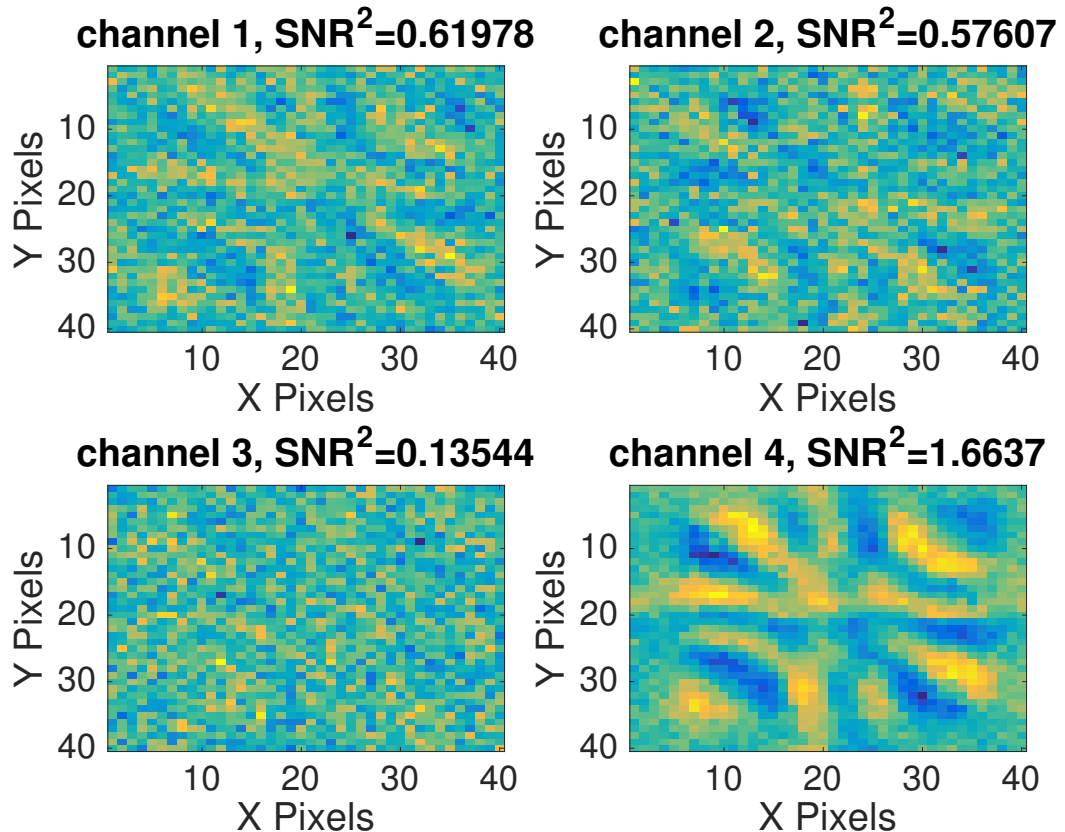
- T - BeRP ball image at (0,0)
- 5M counts
- F - BeRP ball image at (20mm, 20mm)
- 5M counts
- Hotelling weights.



CHO example optimization

Each channel corresponds to a template.

The optimally weighted sum of channels corresponds to the Hotelling weights (SENSITIVE)



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Nonsensitive channels

- Can we develop a channelizing matrix where each channel has poor performance in the discrimination task, but optimally combined the model performs well?

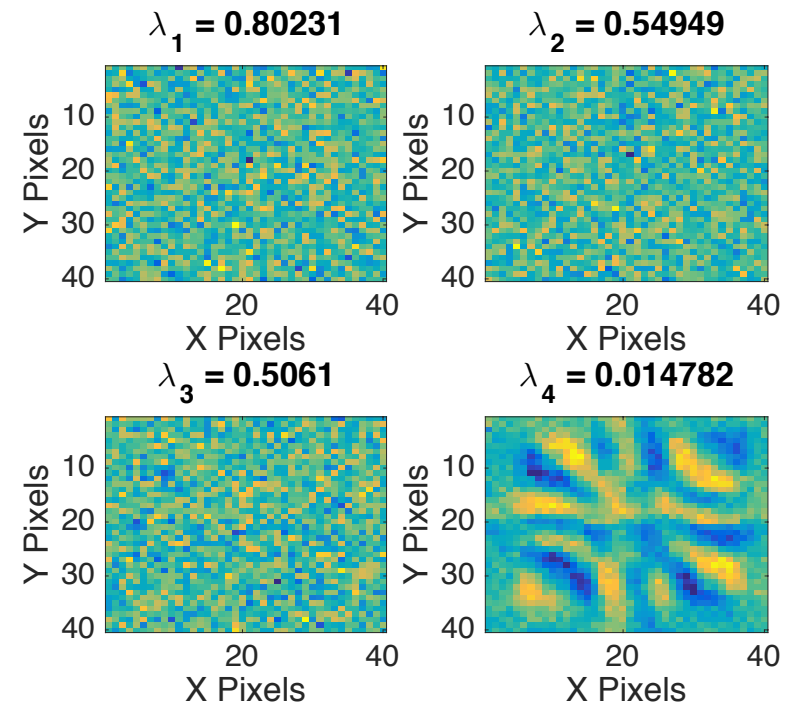
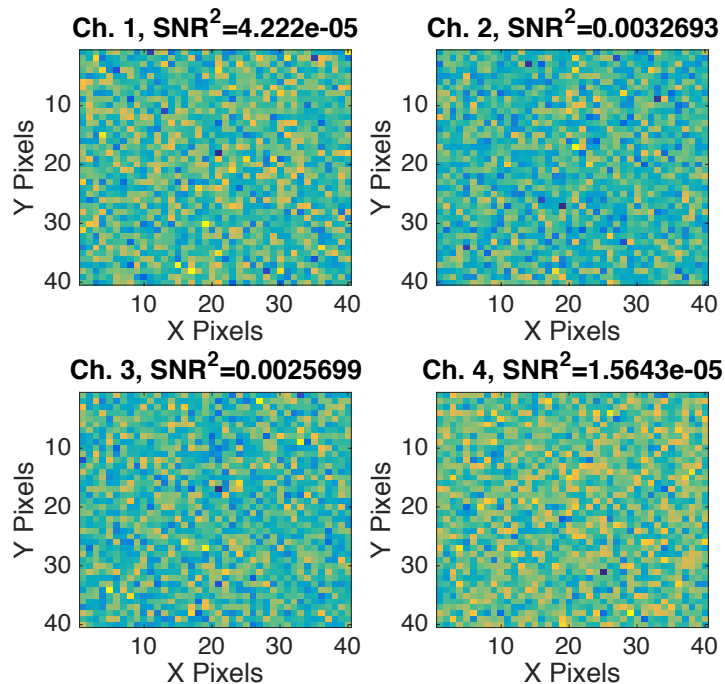
$$f_{obj}(\mathbf{T}) = SNR_{[T,F]}^2(\mathbf{T}) - \eta \sum_{l=1}^L SNR_{[T,F]}^2(T_l)$$

- Above objective function accomplishes this.

Results for nonsensitive channel penalty

Optimally weighted sum of these channels is equivalent to the Hotelling weights.....

But an SVD of the channelizing matrix reveals the Hotelling weights.



Results for nonsensitive channel penalty

Because this routine emphasizes the relationship between channels, removing just a small percentage drastically reduces performance.

L	L_{mon}	SNR^2 for L_{mon} channels	% Performing Poorly (total $SNR^2 < 0.1$)
4	3	0.269	88
10	9	0.7105	50
10	7	0.0195	98
25	24	0.975	18
25	22	0.291	46
25	18	0.0314	98

Host could give monitor roughly 70% of the channels (to help in verification and identifying spoofs)

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Non-sensitive model

- Optimal \mathbf{T} inherently contains information on geometry/isotopic composition of items.
- Ideally, the host could be given the channelizing matrix, channelized values and test statistics for its measured TAIs to the monitor.
- Since the monitor has the model, it could measure its own items, trying to replicate the test-statistic distribution of the TAIs.
- The ideal model would return the same test statistic for any TAIs that have sensitive construction parameters (mass, size) within some tolerance.

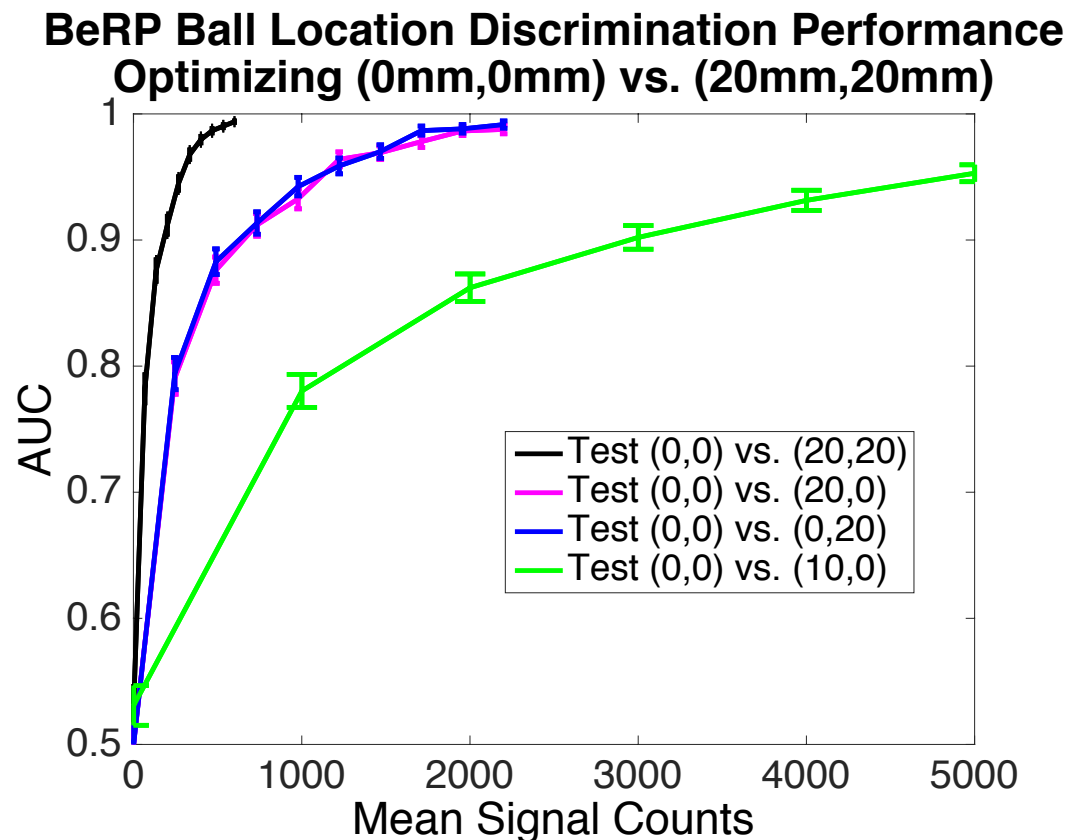
BeRP ball location discrimination example

- The goal in this toy problem is to classify an item at one of two locations
 - T (TAI) is a BeRP ball at (0,0)
 - F (spoof) is BeRP ball at (20,20)
- Treat x location of TAI as sensitive. Want to penalize ability to discriminate:
 - T (TAI) is a BeRP ball at (0,0)
 - T_1 (TAI) is a BeRP ball at (20,0) (SIMULATED)

$$f_{obj}(\mathbf{T}) = SNR_{[T,F]}^2(\mathbf{T}) - \eta SNR_{[T,T_1]}^2(\mathbf{T})$$

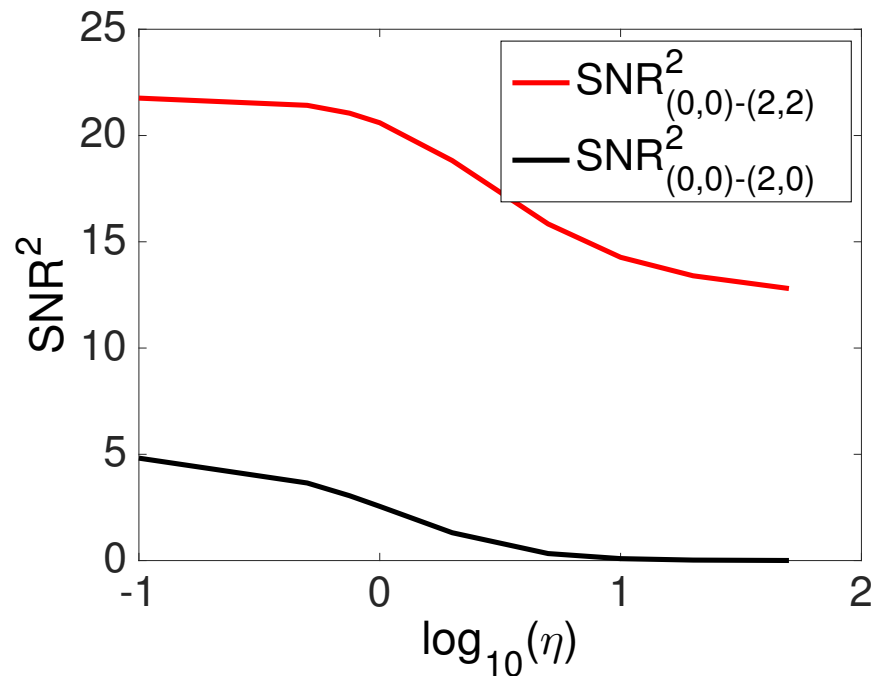
Standard Optimization

Area under ROC curve (AUC) is a measure of test-statistic distribution separation



Effect of penalty term

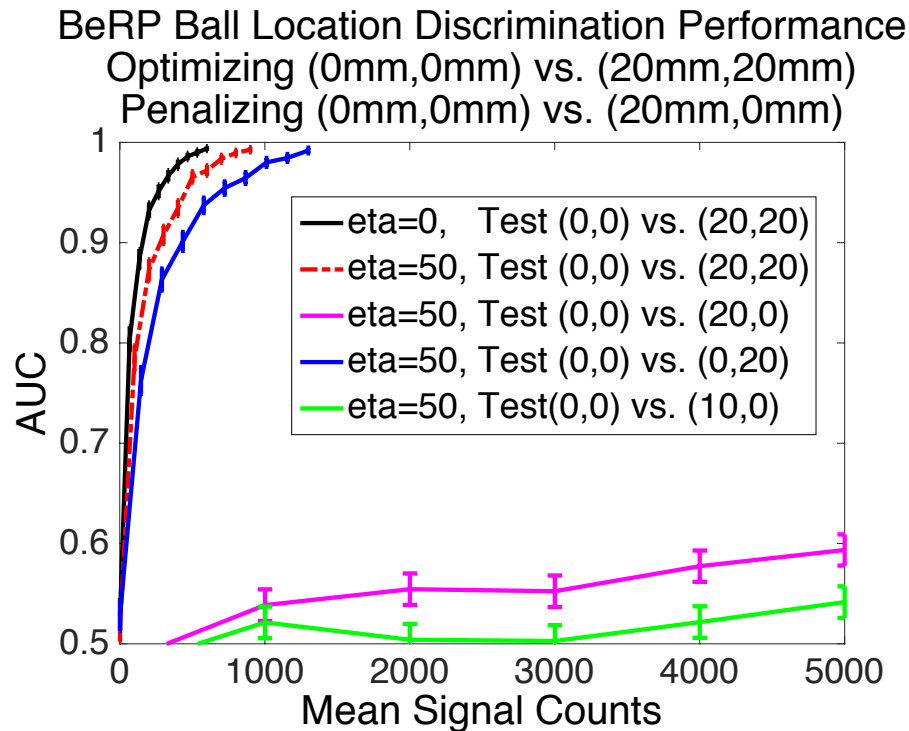
$$f_{obj}(\mathbf{T}) = \text{SNR}_{[T,F]}^2(\mathbf{T}) - \eta \text{SNR}_{[T,T_1]}^2(\mathbf{T})$$



As η is increased, the resulting channelizing Hotelling observer (while no longer optimal) can't distinguish between the source at (0mm,0mm) and source at (20mm,0mm)

Performance Change

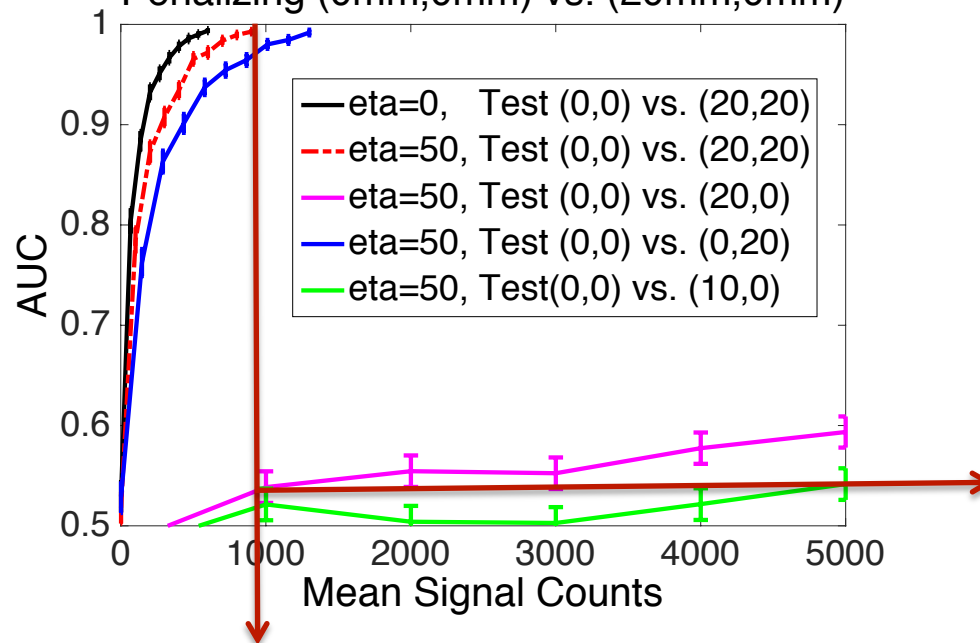
- Ability to differentiate objects based on change in x decreases



Performance Change

- Ability to differentiate objects based on change in x decreases

BeRP Ball Location Discrimination Performance
Optimizing (0mm,0mm) vs. (20mm,20mm)
Penalizing (0mm,0mm) vs. (20mm,0mm)



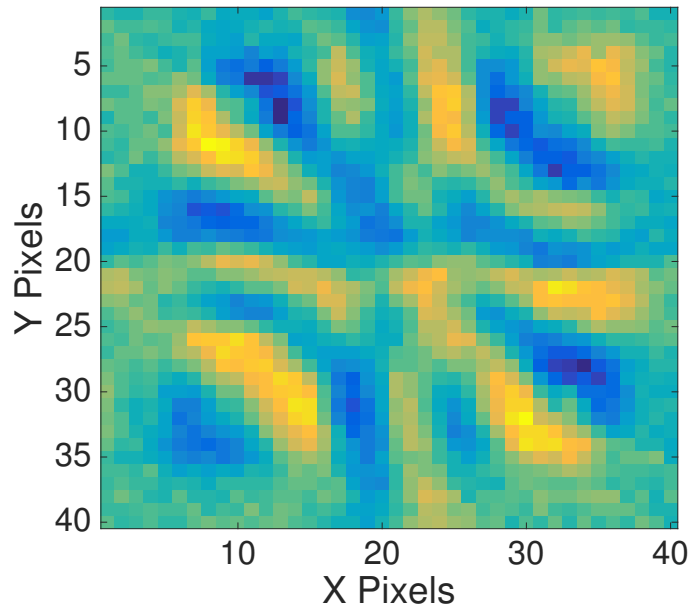
Desired Outcome

Mean and variance of test statistic distributions when imaging BeRP ball at (0,0) and (20,0) can't be discriminated after 1,000 measurements at 900 signal counts each

Operating acquisition time chosen to correspond to 900 signal counts.

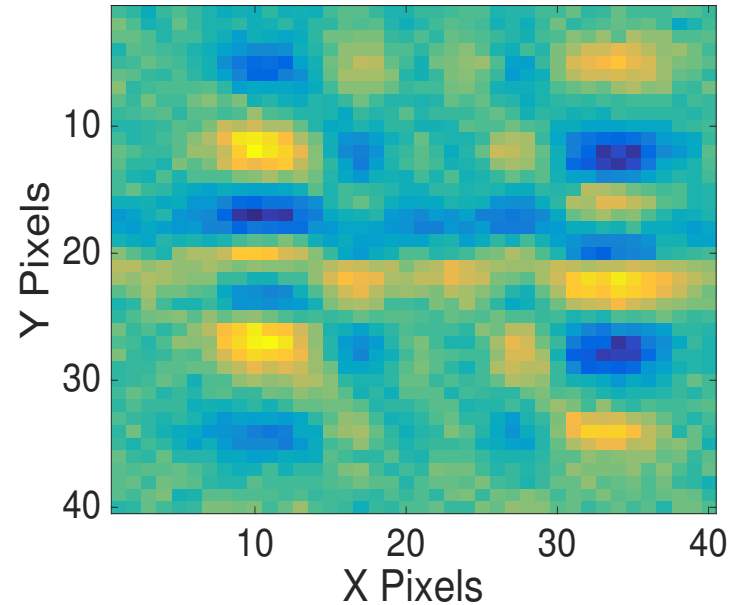
Sensitive Information Removal

- As eta increases, $\mathbf{W}_v^t \mathbf{T}$ is no longer optimal.



$\eta = 0$

$\mathbf{W}_v^t \mathbf{T}$ equal to Hotelling weights



$\eta = 50$

$\mathbf{W}_v^t \mathbf{T}$ corresponds to vertical shift.
No longer x information in template

Outline

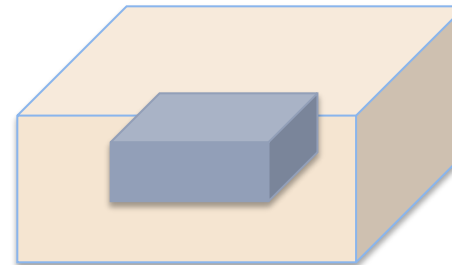
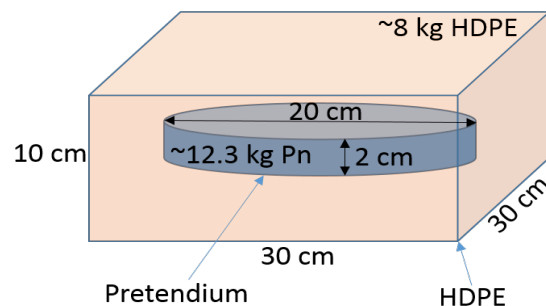
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Task

- Monitor needs to differentiate:
 - T: Cylindrical Pretendium
 - F: Square Pretendium
- Host wants to prevent dissemination of diameter of pretendium ring in object T up to tolerance of 30%. Penalizes ability to distinguish
 - T: Cylindrical Pretendium, 20cm diameter
 - T_1 : Cylindrical Pretendium, 14cm diameter
 - T_2 : Cylindrical Pretendium, 26cm diameter

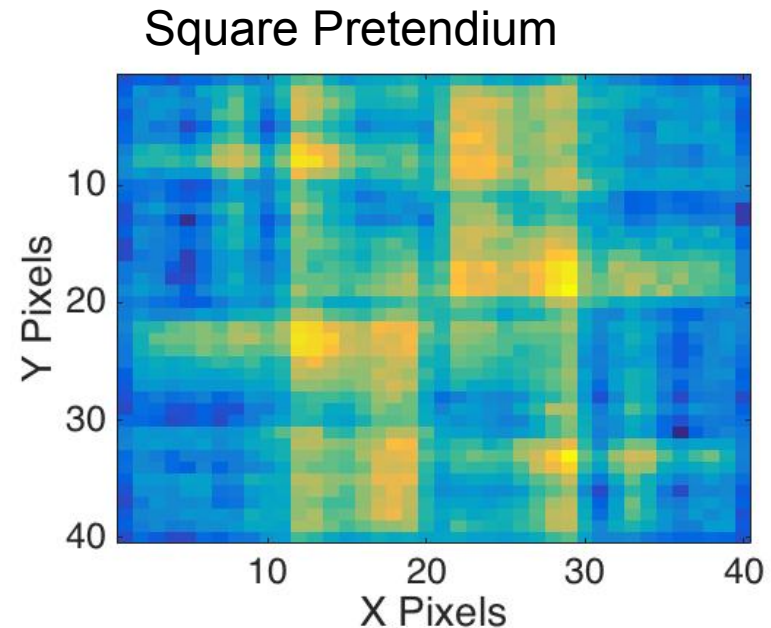
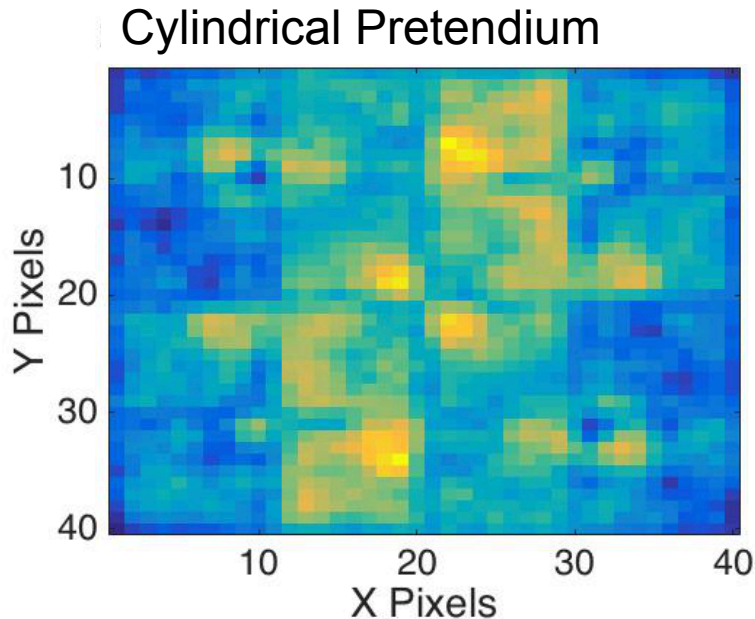
Task – Discriminate pretendium shape

- Binary discrimination differentiating moderated flat cylinder from rectangular prism
 - 15 cm of polyethylene (~6 path lengths for neutrons, ~2 for gammas)
 - Self shielding reduces gamma flux out of objects.
 - Minimal difference in detected energy spectra, use gamma ray or neutron imager



Acquire data

- Host takes calibration data on the TAI and designated spoof.
- Observer model will be built on this data.



New objective function

$$f_{obj}(\mathbf{T}) = SNR_{[T,F]}^2(\mathbf{T}) - \eta \left(SNR_{[T,T_1]}^2(\mathbf{T}) + SNR_{[T,T_2]}^2(\mathbf{T}) \right)$$

This penalizes ability to discriminate test-statistic distributions for items with slightly different diameters.

Procedural outline

- Perform a detector calibration measurement on items T and F.
 - Long acquisition time
 - Try to capture impact of nuisance parameters
- Penalized items T_1 and T_2 would need to be simulated.
 - Simulate detected energy in each pixel
 - Need to simulate with same nuisance parameters as T
 - Convert detected energy for T_1 and T_2 to light output/PMT signal (SNL code)
- Choose optimal acquisition time (point on AUC curve)
 - Take independent measurements on all of the items.
 - Find optimal point where AUC of discriminated pair=1
 - Check that penalization was effective.
 - May need to include more penalized items in optimization routine
- Implement channelized weights through electronic board or some other procedure
 - If all channelized values are between 0 and 1, could use attenuating material

Questions?

Data definition

- List-mode data A_n :
 - Estimated energy, pixel, and particle type (photon or neutron) for event n . Define N to be total number of detected events.
- Data $\{A_n\}$ binned into data vector \mathbf{g} ($P \times 1$).

$$g_p = \sum_{n=1}^N f_p(A_n)$$

- \mathbf{g} is binned detector data – image, spectra or both.