# A spacetime finite element method for coupled acoustic fluid-structure interactions

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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# Background: sonic crystals

- Sonic crystals are made from periodic arrays of scatterers inside a matrix
- Acoustic band gaps form for certain arrangements, frequencies, etc.
  - Acoustic waves attenuate quickly in these gaps!
- Application areas: frequency filters, sound diffusers, etc.
- Determination of the band gaps
  - Need to compute the center frequency and the width
  - (Extended) plane wave expansion method, finite difference time domain (FDTD)
  - Either approximate or expensive!

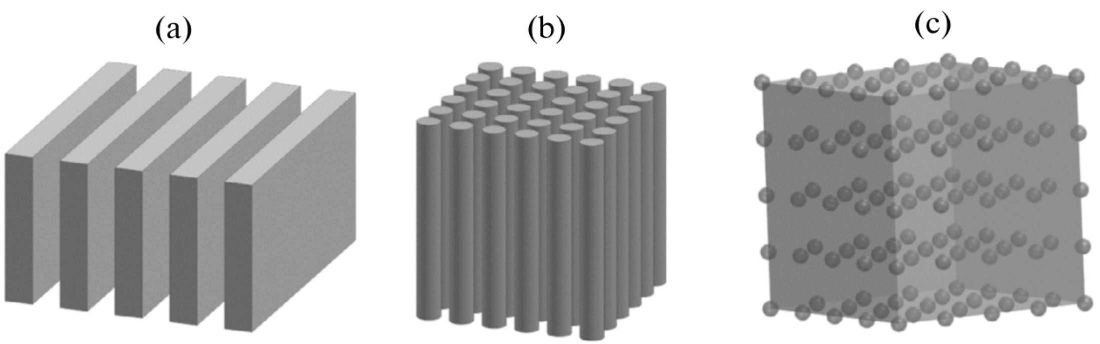
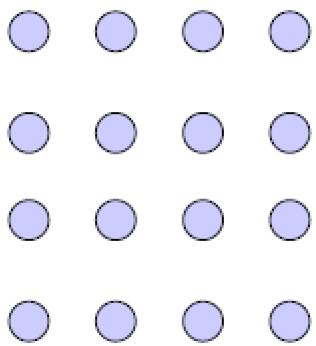
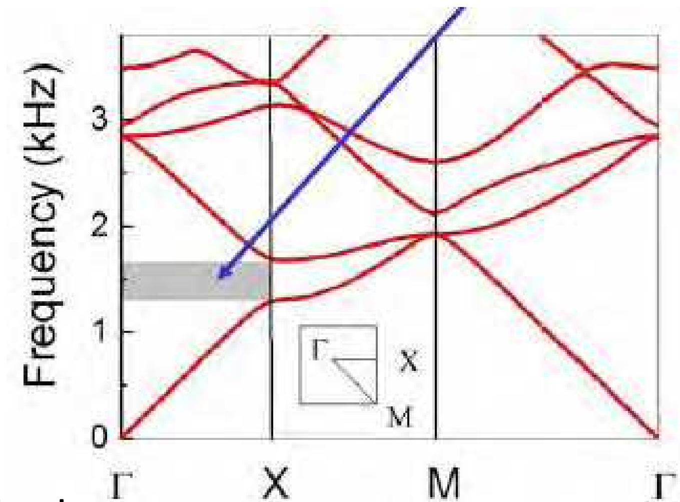



**Fig. 1.** Different types of sonic crystals. (a) 1-D sonic crystal consisting of plates arranged periodically; (b) 2-D sonic crystal with cylinders arranged on a square lattice; (c) 3-D sonic crystal consisting of periodic arrangement of sphere in simple cubic arrangement. Gupta, *Acoustical Physics,* 2014

# Background: sonic crystals

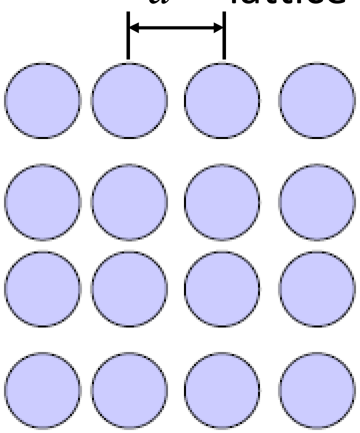
- Filling fraction plays an important parameter for determining band gap

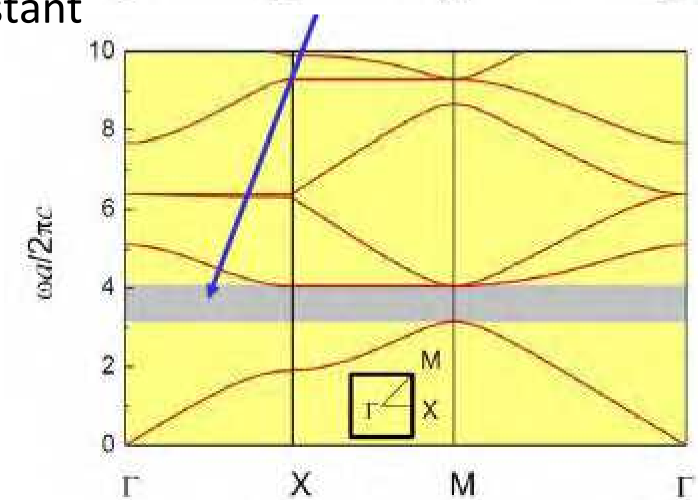





Partial band gap for waves in some directions only

$a$ = lattice constant






Complete band gap for waves in all directions

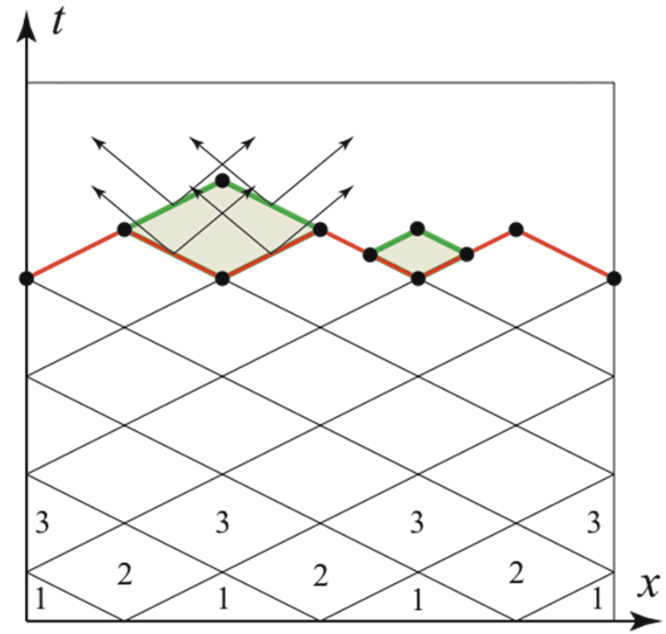
Andrew Norris, Rayleigh Lecture, ASME NCAD IMECE 2011

# Direct simulation of sonic crystals

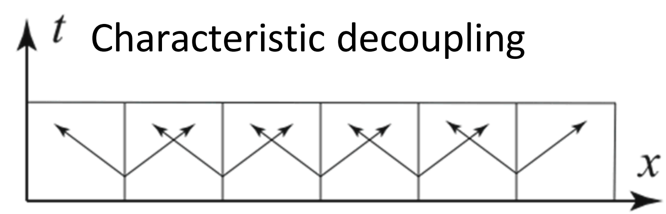
- Spatial and temporal fidelity must resolve
  - Scatterer size and entire domain size (orders of magnitude difference)
  - Frequencies of interest: must fill out the band gap diagram
  - Acoustic-structure interactions when structures are excited
- Standard computational challenges must be overcome
  - Time step limitations (Courant) for explicit integration
  - Algebraic linear solver expense for implicit
  - Spatial locations where high resolution necessary change with time
- Spacetime discontinuous Galerkin finite element method
  - Satisfy the resolution requirements and overcome the computational challenges!

# Discontinuous Galerkin Finite Element

- **Applicable to a wide range** of physical processes
  - Nonlinear hyperbolic systems of PDEs
  - Uses spacetime Discontinuous Galerkin (SDG) finite element method
- **Superior performance:**
  - Multi-scale simulations with discontinuities solved faster
  - Linear cost instead of exponential cost as complexity grows
  - Highly parallelizable, asynchronous patch-by-patch finite element solver
- **Local decoupled problems:**
  - Small patches of finite elements decouple from the global solution domain and are solved individually
  - Local solution structure enables a highly efficient *h-* or *hp-adaptive* solution strategy while maintaining linear scaling
  - Local and global conservation/balance is ensured
  - Linear *O(N)* complexity in the number of patches



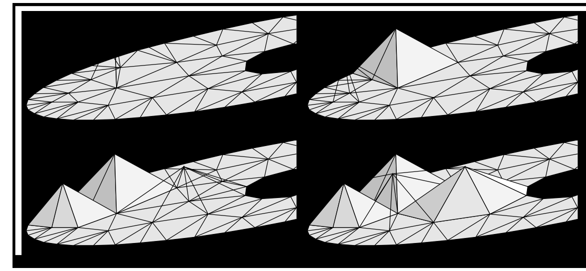

Characteristic decoupling

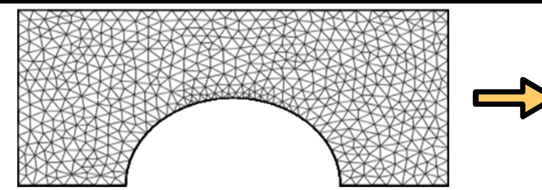


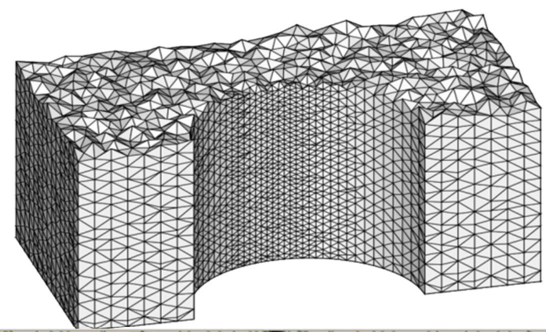

Traditional: no element decoupling

Miller, et. al., *Comp. Meth. In App. Mech. Eng.* **198** (2008).

# Spacetime DG FEM exploits hyperbolic structure







- **Accuracy**: the SDG FEM is arbitrarily high order accurate in space *and* time
  - User defined basis function accuracy
  - Resolves sharp solution features in both space and time
- **Adaptive** space-time mesh generation:
  - Information propagation speeds are used to construct *causal* space-time meshes
  - Unstructured and asynchronous mesh generation for time-dependent problems
  - Unstructured meshes in space-time can handle moving boundaries easily
- **Multi-scale:**
  - element size ratios for a simulation can be $O(10^8)$ or larger
  - Shocks/discontinuities resolved and propagated naturally due to discontinuous basis functions

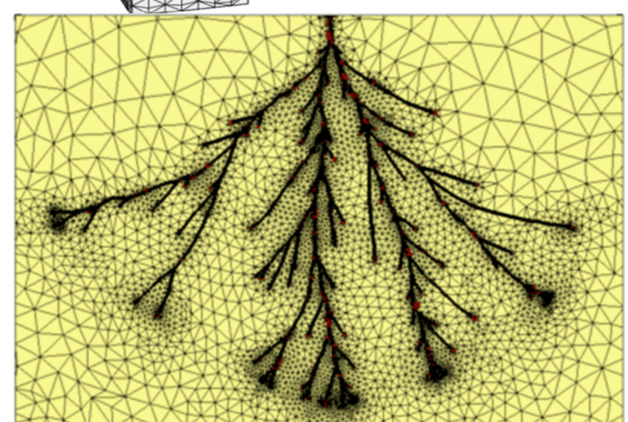


Figure: Adaptive mesh following dynamic crack propagation, branching, and arrest. Abedi et al., CMAME, 2014.

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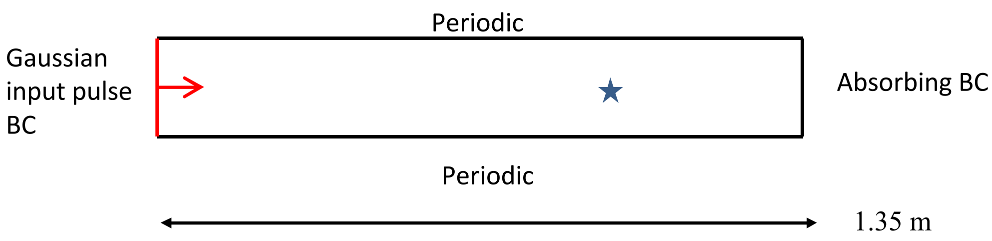
# SDG application areas

- Elastodynamics, thermal conduction, fracture, inviscid fluid flow, atomistic-to-continuum coupling
- Development and collaboration credits: Reza Abedi (UTSI), Robert Haber (UIUC), Jeff Erickson (UIUC), Shripad Thite (google), Shuo-Heng Chung (microsoft), Aaron Becker (google), Laxmikant Kale (UIUC), Duane Johnson (Ames Lab, Iowa State), Brent Kraczek (ARL)

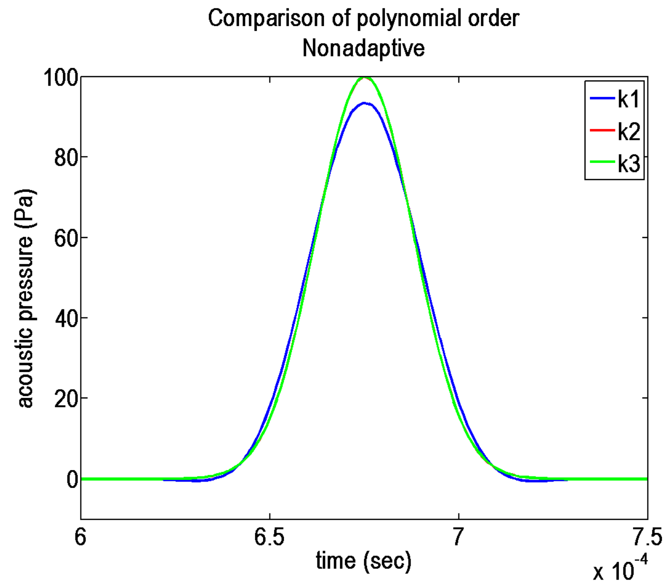
Another cool movie here

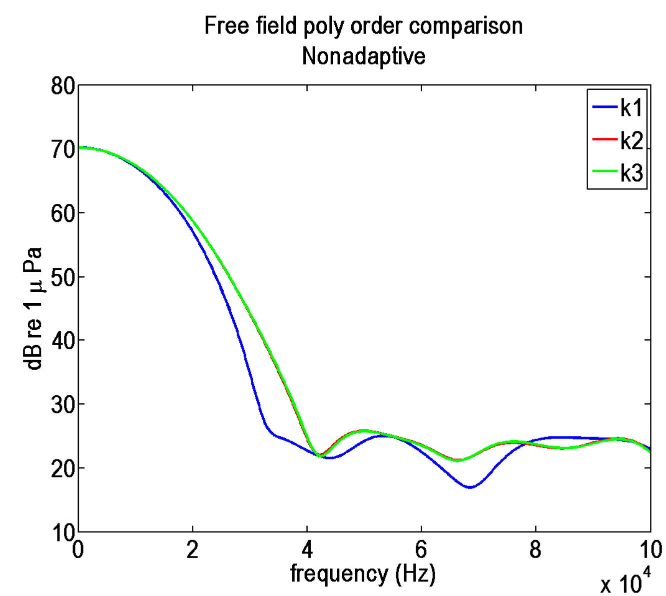
# Demonstration problem

- 2D Acoustic propagation in free space



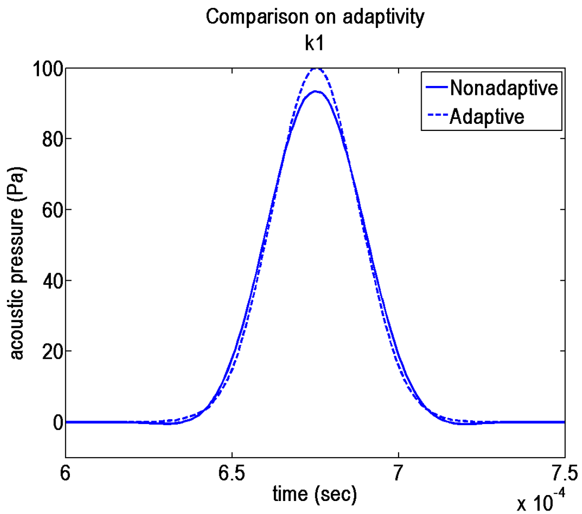

- Evaluation of
  - Element polynomial order
  - Adaptive vs non-adaptive meshes
  - Numerical losses
  - Post processing

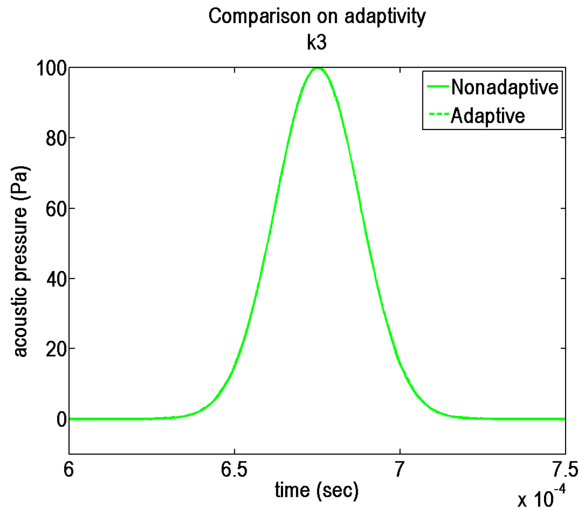


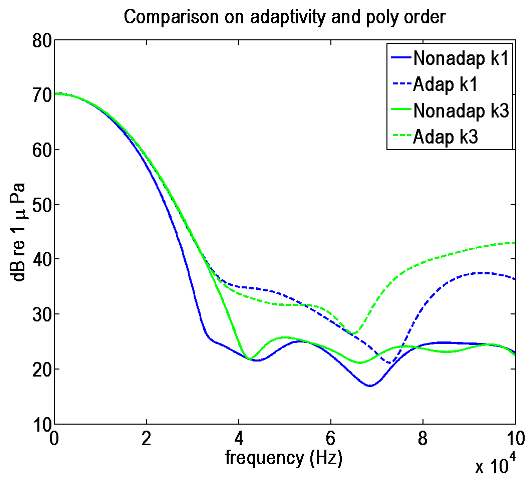



# Demonstration problem

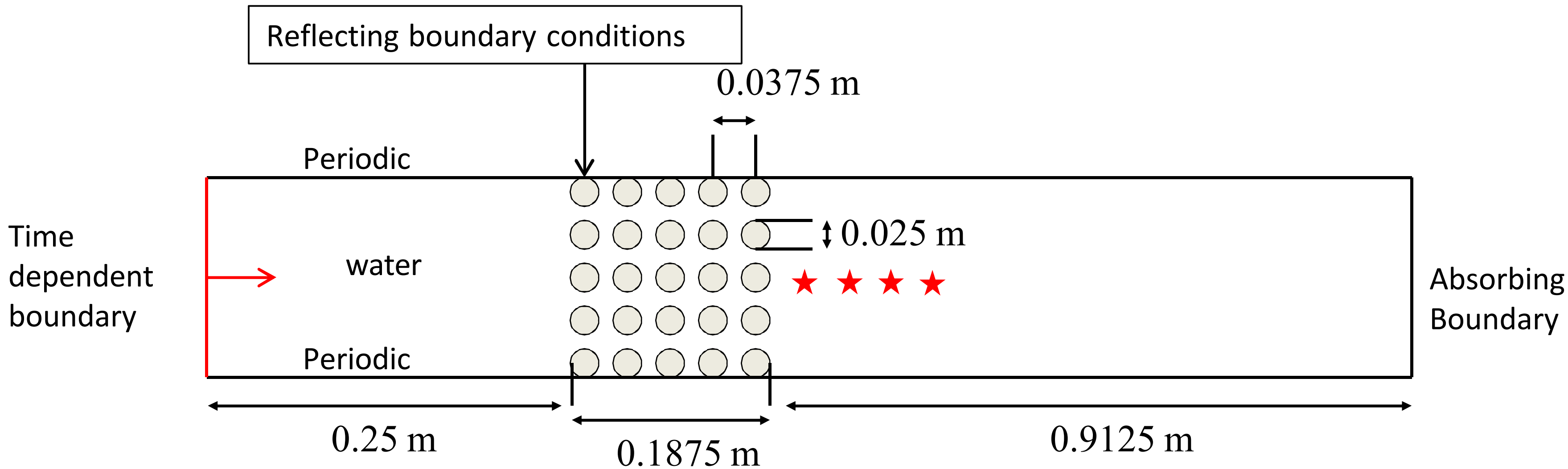
- Differences between adaptive and non-adaptive are more pronounced for lower polynomial order
- Adaptive computations are in good comparison with non-adaptive with polynomials of order k > 1
  - But, computational savings with adaptive computations
- Numerical losses are most severe for low polynomial order



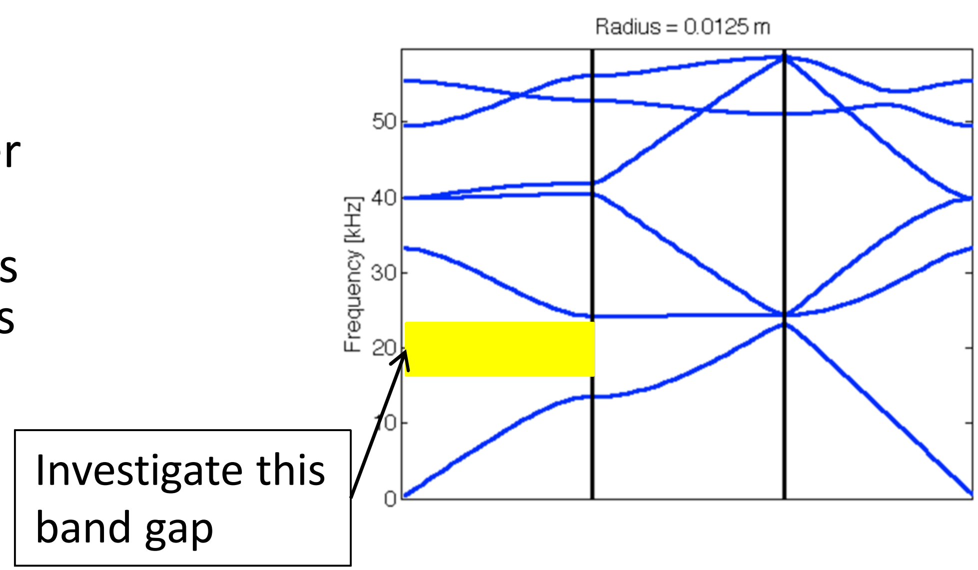



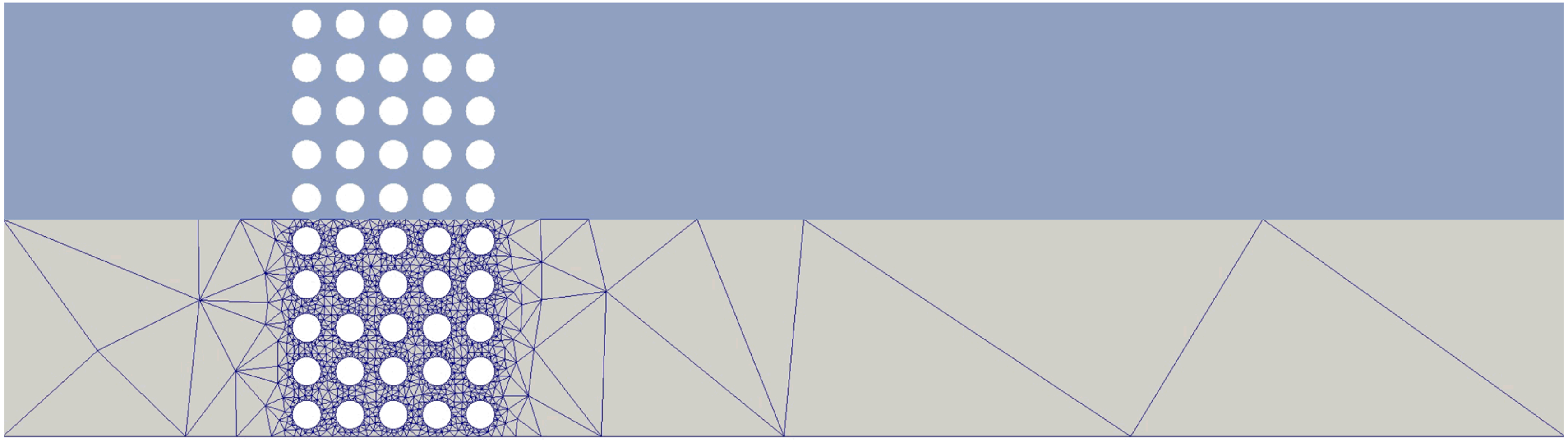
# Multiscale acoustic problems

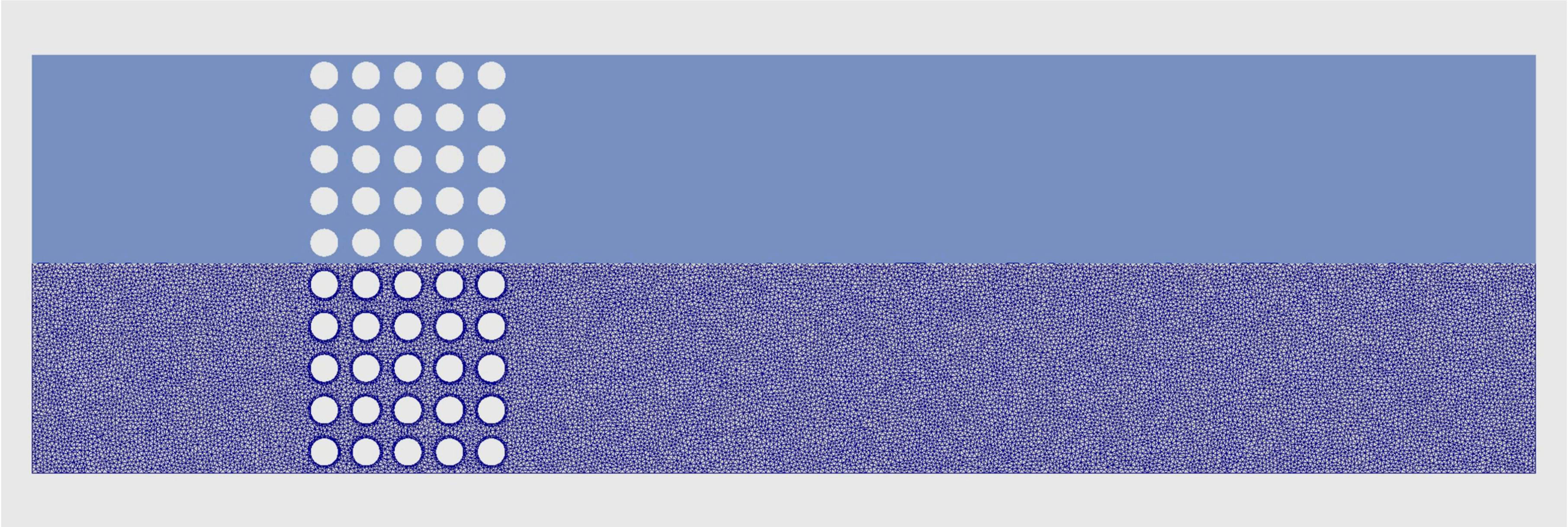



- Geometry designed for band gap center frequency of 20 kHz
- Computed using Linearized Euler Equations
- Cubic polynomial basis functions show best accuracy for acoustics propagation
  - Increased computational cost over linear basis functions



# Results of sonic crystal





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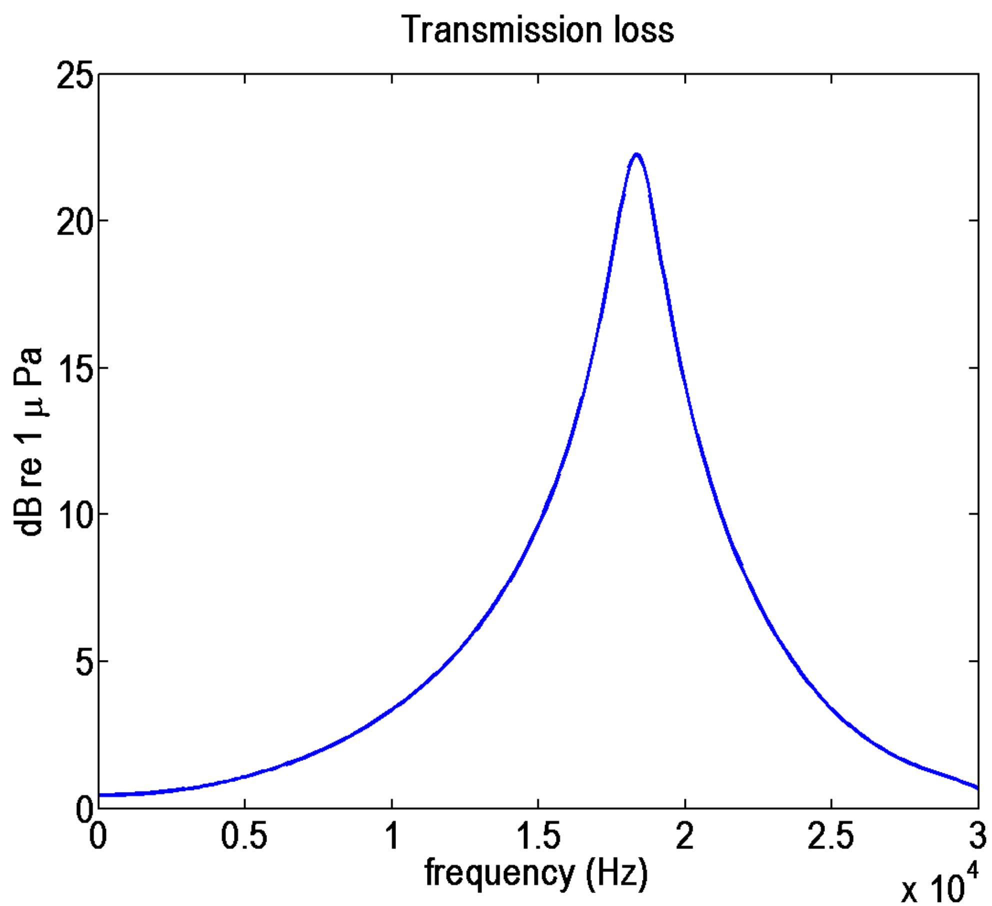
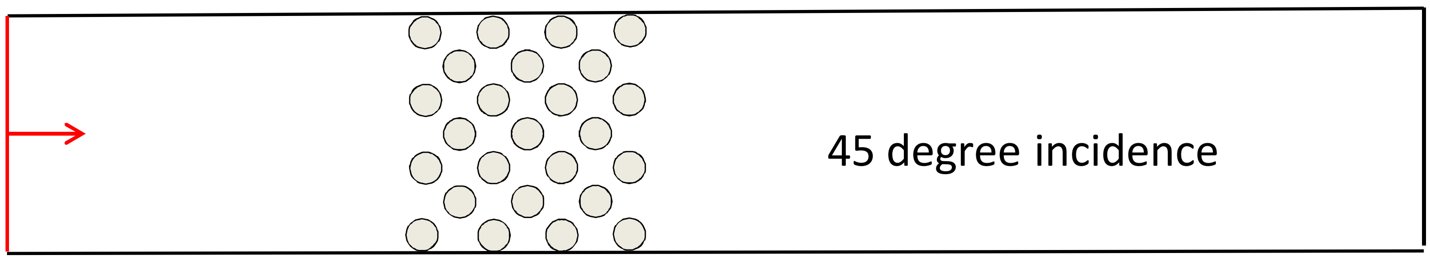
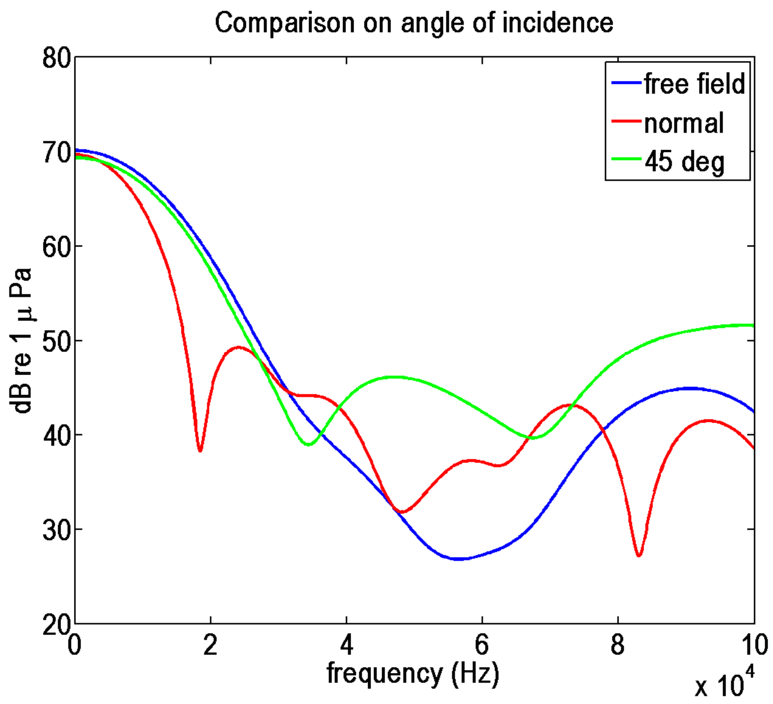
# Band gap analysis



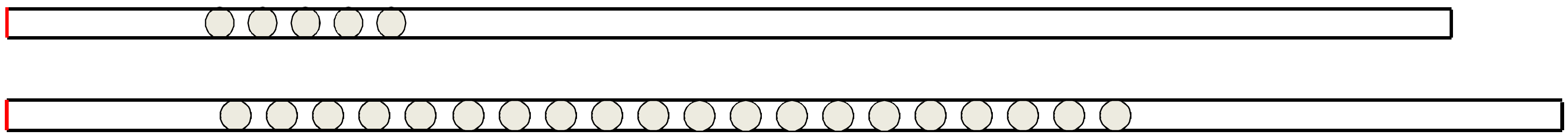
# Different propagation direction



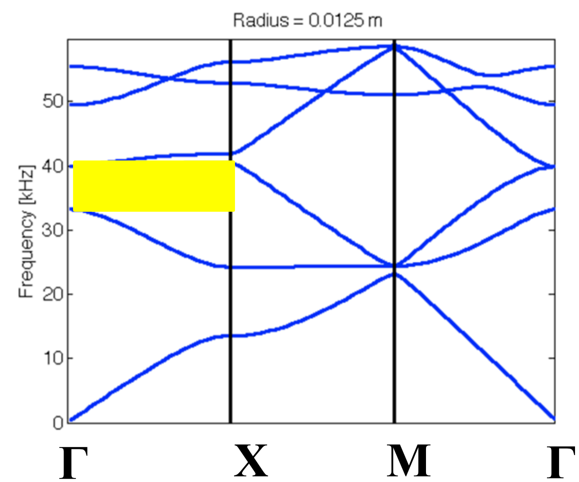




Interference patterns due to scatterer arrangement change band gaps.

# A few other examples



- Different frequency content
- Time and space resolution study
- Method of manufactured solutions
- Results all physically meaningful



# Acoustic fluid-solid coupling

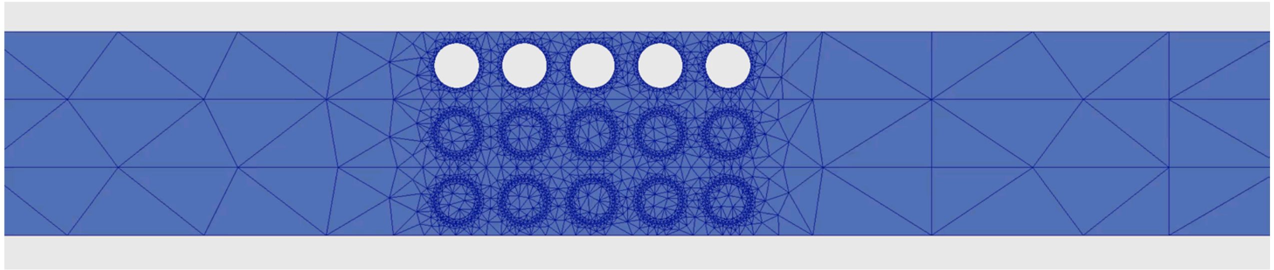
- **Acoustic fluid**: a Newtonian fluid that propagates small amplitude pressure and velocity disturbances; does not support shear waves

- **Elastic solid**: small amplitude deformations; supports both pressure and shear waves

- **Acoustic fluid-solid (AFS) coupling**:
  - Solid displacement does not affect wave propagation
  - Traction forces and velocities are coupled at the AFS interface
  - Accurately predicts acoustic wave propagation in coupled media

- **Spacetime DG method for AFSI:**
  - Permits asynchronous time evolution based on local material properties
  - Weakly enforces continuity between elements at material interfaces
  - Utilizes highly adaptive meshes to resolve physics at all length/time scales

# Acoustic fluid-solid coupling



# Conclusions & future work

- Spacetime discontinuous Galerkin finite element method is a viable tool for computational acoustics and acoustic-FSI
  - Capabilities lacking for 3 spatial dimensions, future work!
- Adaptive spatial *and* temporal resolution greatly improves efficiency
  - High order (k=3) interpolation can also lead to reduced simulation time
- Direct simulation of acoustic fluid-structure interaction could be used to verify other analysis methods, e.g., plane wave expansion or homogenization approahes