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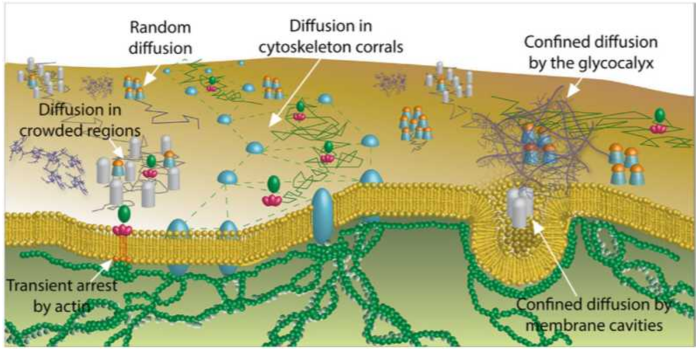


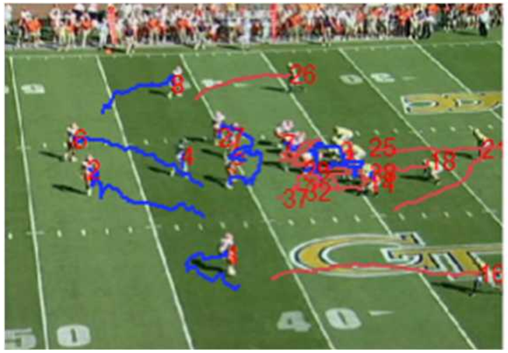

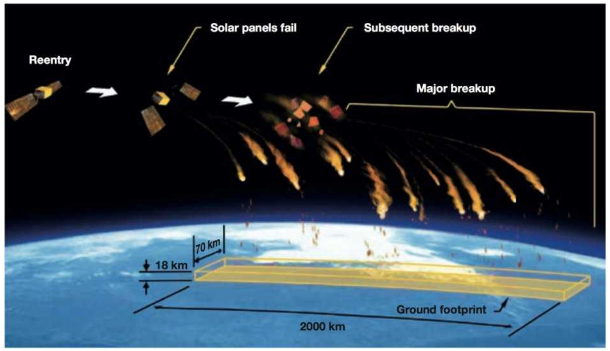


# Single Particle Tracking of Blinking Particles

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NNSA National Nuclear Security Administration

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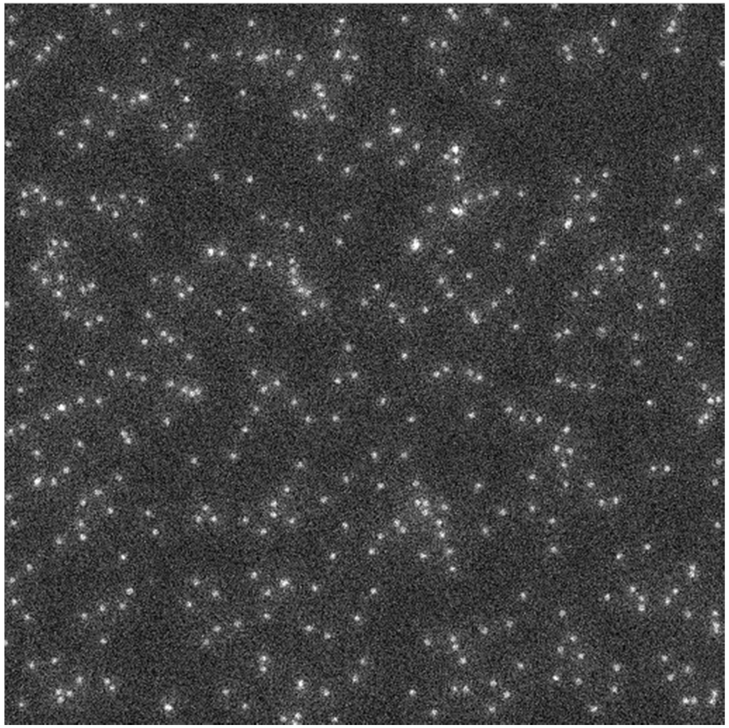
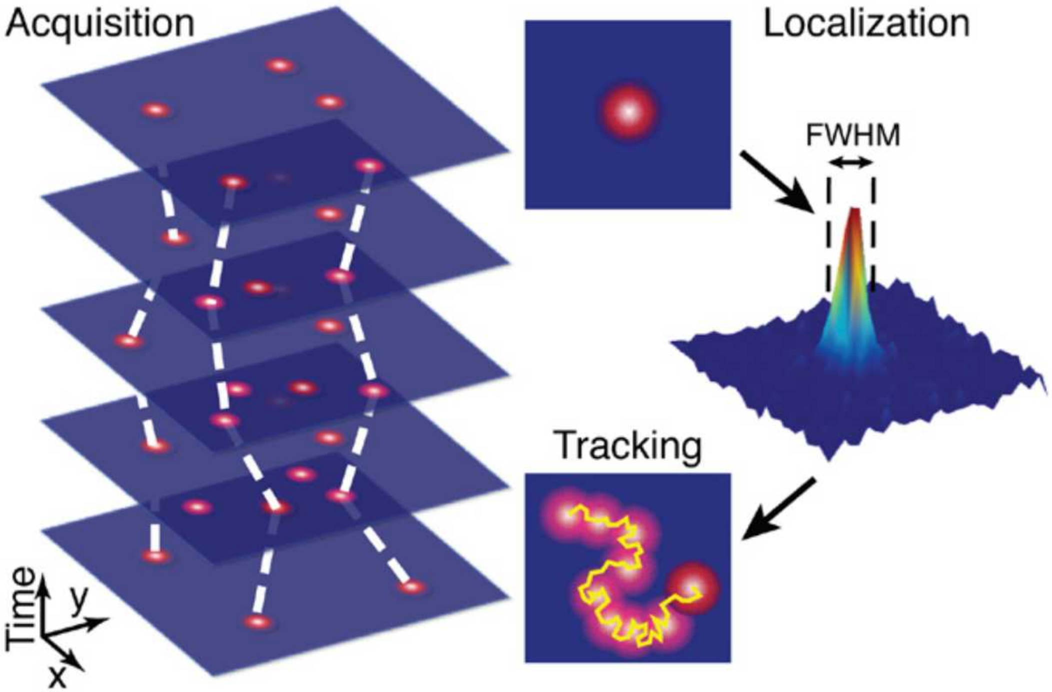
# Outline

- What is single particle tracking (SPT)
- Why does SPT matter?
- Expressing SPT as a graph theory problem
- Finding an approximate solution to the NP-hard SPT problem in O(N log N)

# What is Single Particle Tracking?



Chenouard, N., et al. (2014). Nat Meth **11(3): 281-289.**




Manzo, C. and M. F. Garcia-Parajo (2015). Reports on Progress in Physics **78(12).**

Single particle tracking: The process of connecting multiple discreet observations of many particles throughout time and linking together all observations of a single particle into a trajectory.

# Who Uses Single Particle Tracking?

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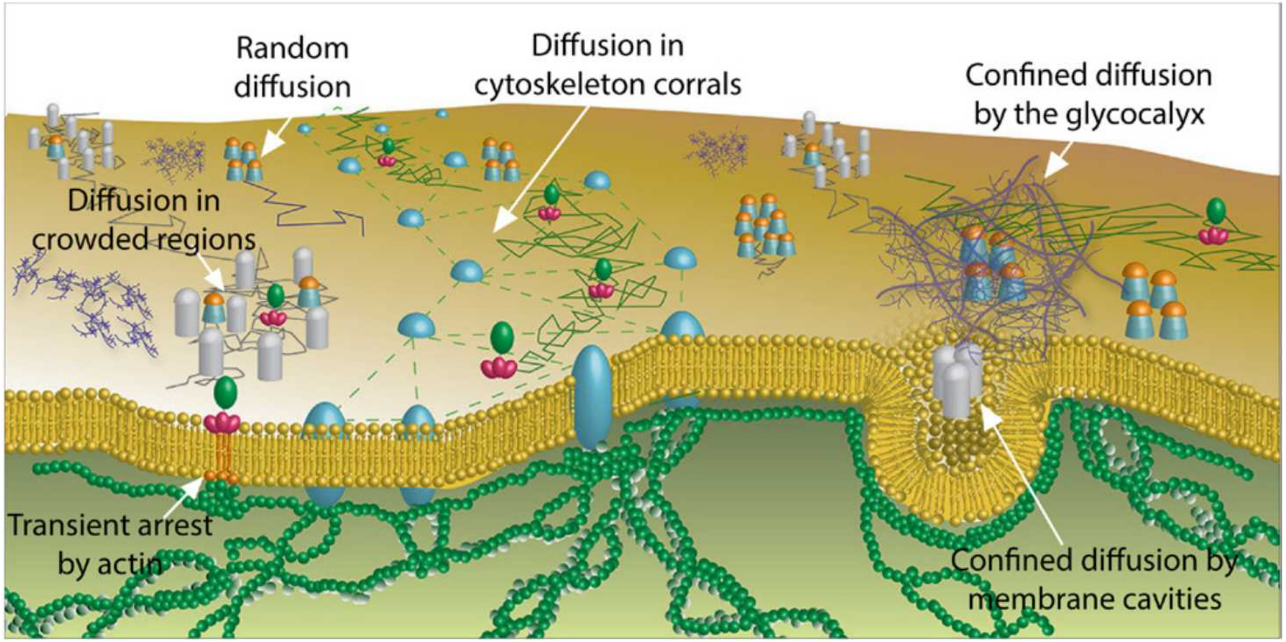


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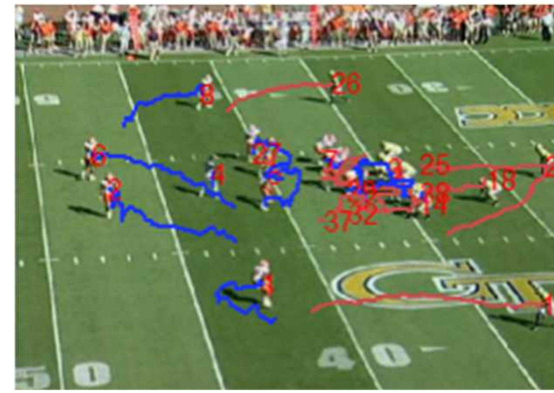
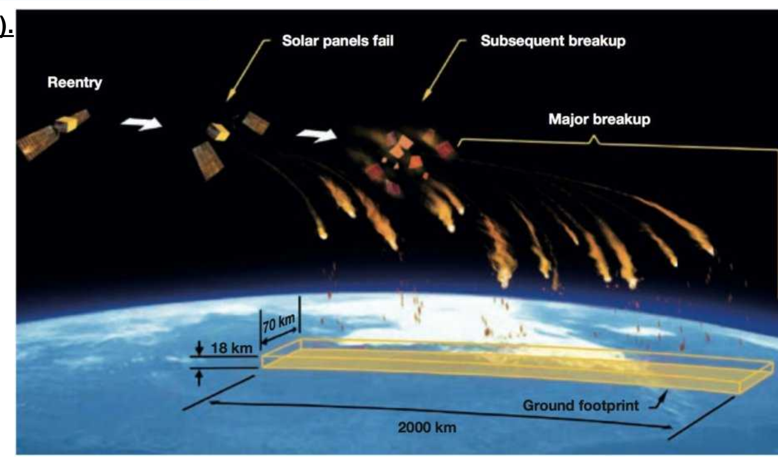


http://www.umiacs.umd.edu/~taheri/myHomepage/Research.html

SPT is useful across many fields, where our primary interest is biology.

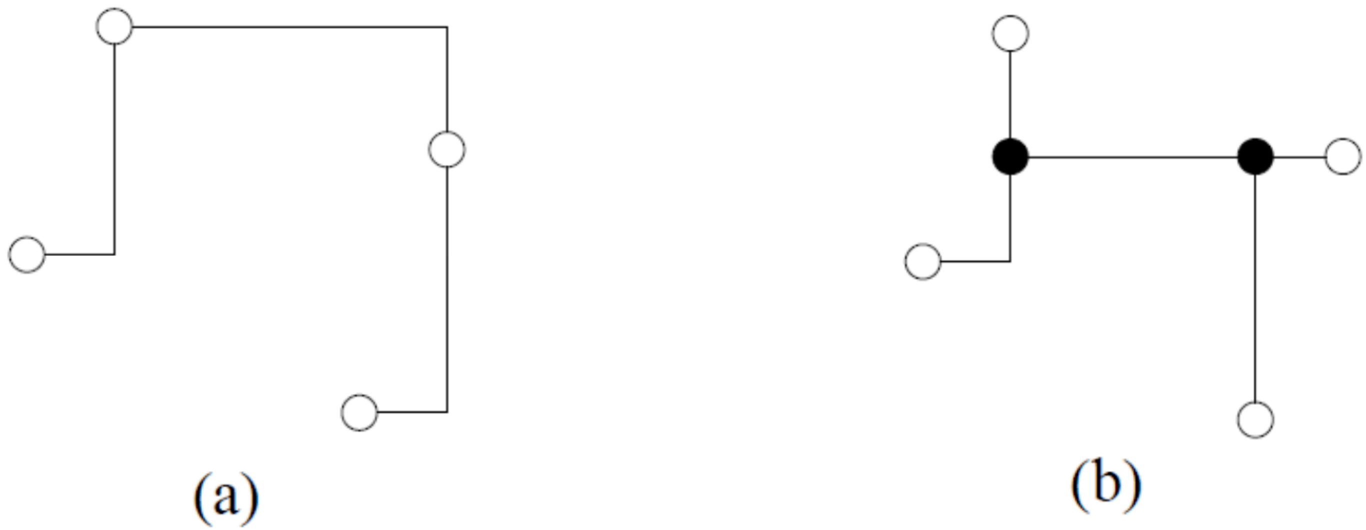



http://www.spacesafetymagazine.com/space-on-earth/malaysia-flight-370/space-debris-meteorite-forecast-safer-aviation/

# Why is this a hard problem?

- Connecting observations between adjacent frames is an assignment problem – maximum bipartite matching.
  - A naïve algorithm would perform an O(V!) exhaustive search – impractical except for the smallest problems
  - Much more efficient algorithms, $O(V^3)$ or better, allowing large problems to be solved exactly in reasonable time
- Single particle tracking (SPT) is more complicated – not just a set of assignment problems
  - Assignment is not always 1 to 1 – can be one to many or many to one
  - Not every particle is seen at all times – **assignment must account for missing observations**
  - As a result, SPT is an NP-hard combinatorial optimization problem

# Steiner Problem Analogy

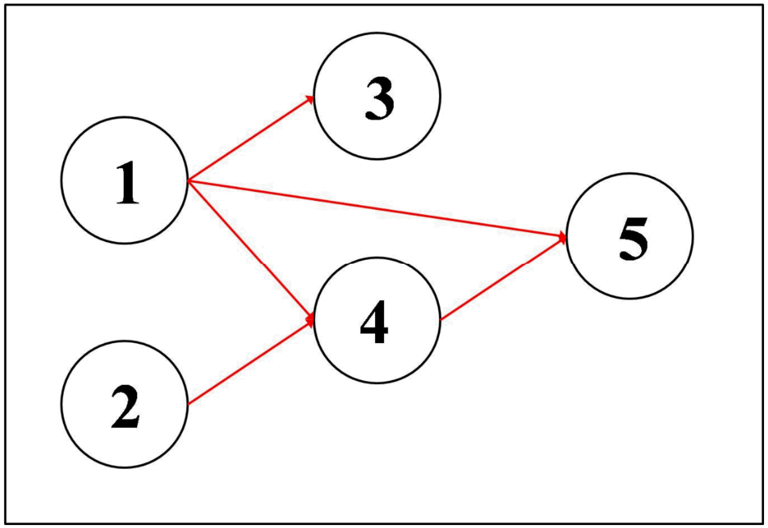



Gabriel Robins http://www.cs.virginia.edu/~robins/papers/Steiner_chapter.pdf

In the rectilinear plane, a) The minimum spanning tree (MST) and (b) the Steiner minimal tree (SMT). Hollow dots correspond to the points to be connected while solid dots represent Steiner points.

**O(E log V)** **NP-complete**

# Framing SPT as an Adjacency Matrix




**Sample Set of Observations**

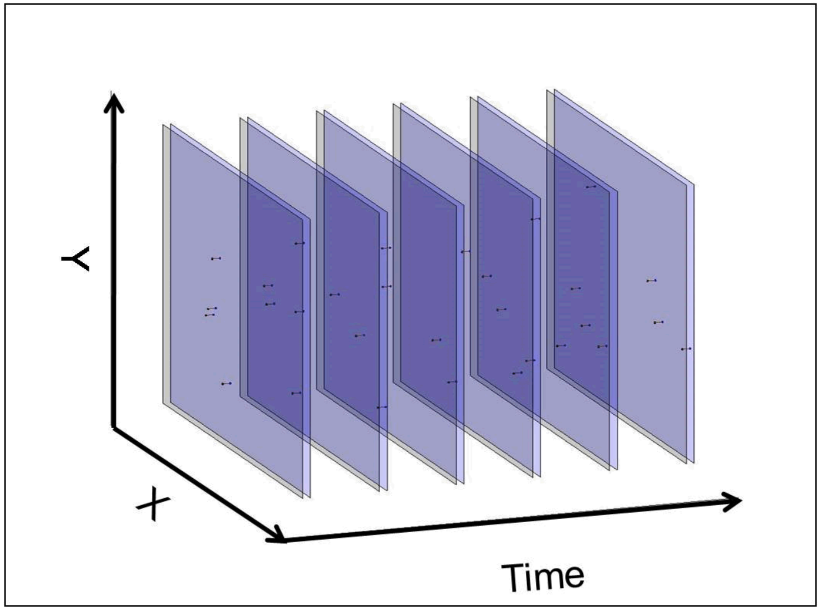
| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | | | 1 | 1 | 1 |
| 2 | | | | 1 | |
| 3 | | | | | |
| 4 | | | | | 1 |
| 5 | | | | | |

**Corresponding Sparse Directed Adjacency Matrix**

Simplified example – actual adjacency matrix would have weights to represent the relative probability of the edges.

# Accounting for Noise, Missing Particles

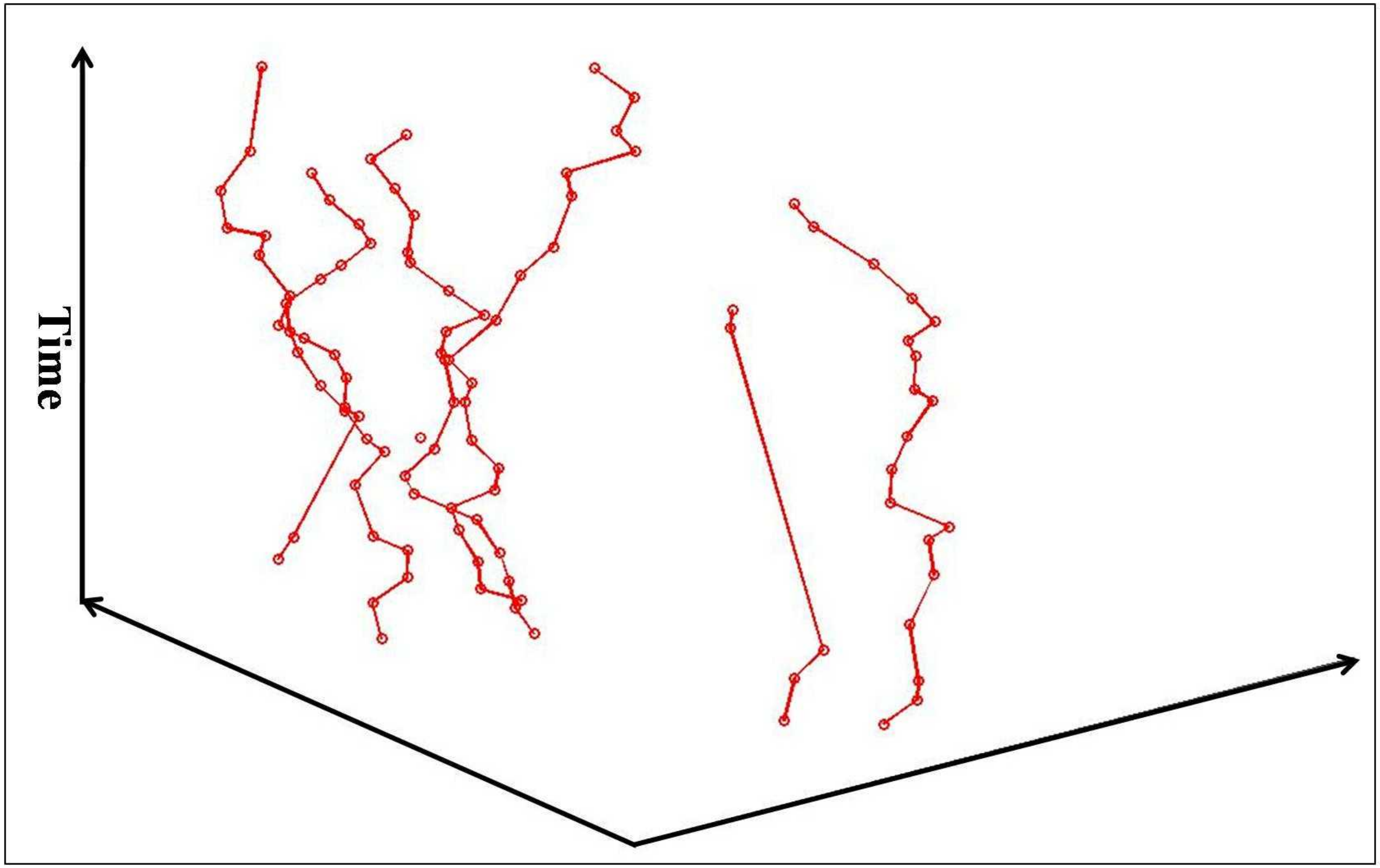
Prior example allowed weights to be assigned to each edge.
The probability a particle is noise is associated with that vertex.




| | $1_-$ | $1_+$ | $2_-$ | $2_+$ | $3_-$ | $3_+$ | $4_-$ | $4_+$ | $5_-$ | $5_+$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $1_-$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $1_+$ | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| $2_-$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| $2_+$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| $3_-$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| $3_+$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $4_-$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $4_+$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| $5_-$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| $5_+$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**One solution:** Split each frame into two, splitting particle X into particles $X_-$ and $X_+$, adding an edge between them. Particle probabilities can be assigned to that edge.

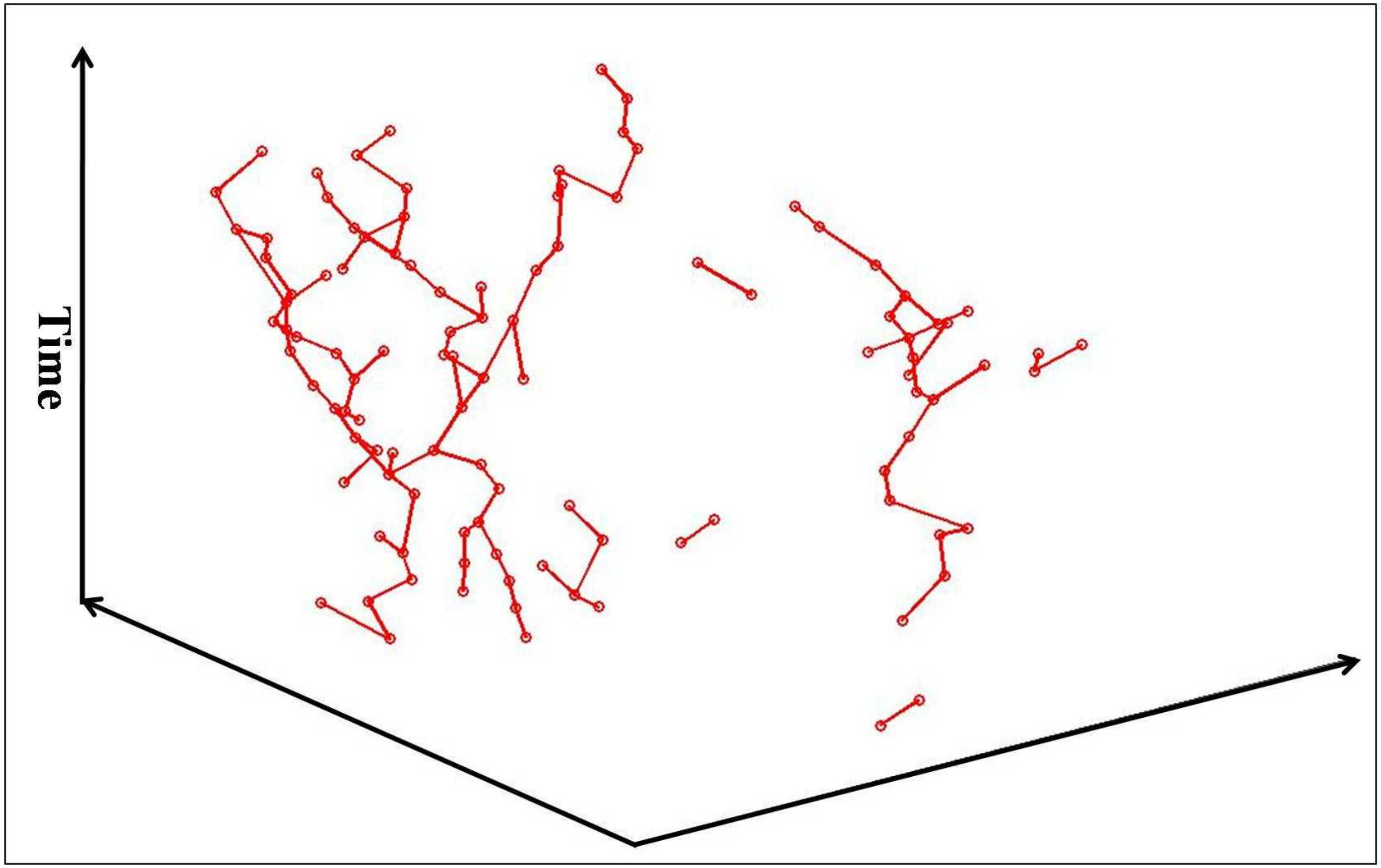
# Objective – Forest of Shortest Paths




Simulated data – optimal solution is a forest of shortest paths

- Many such forests exist. How do we find the one we want?

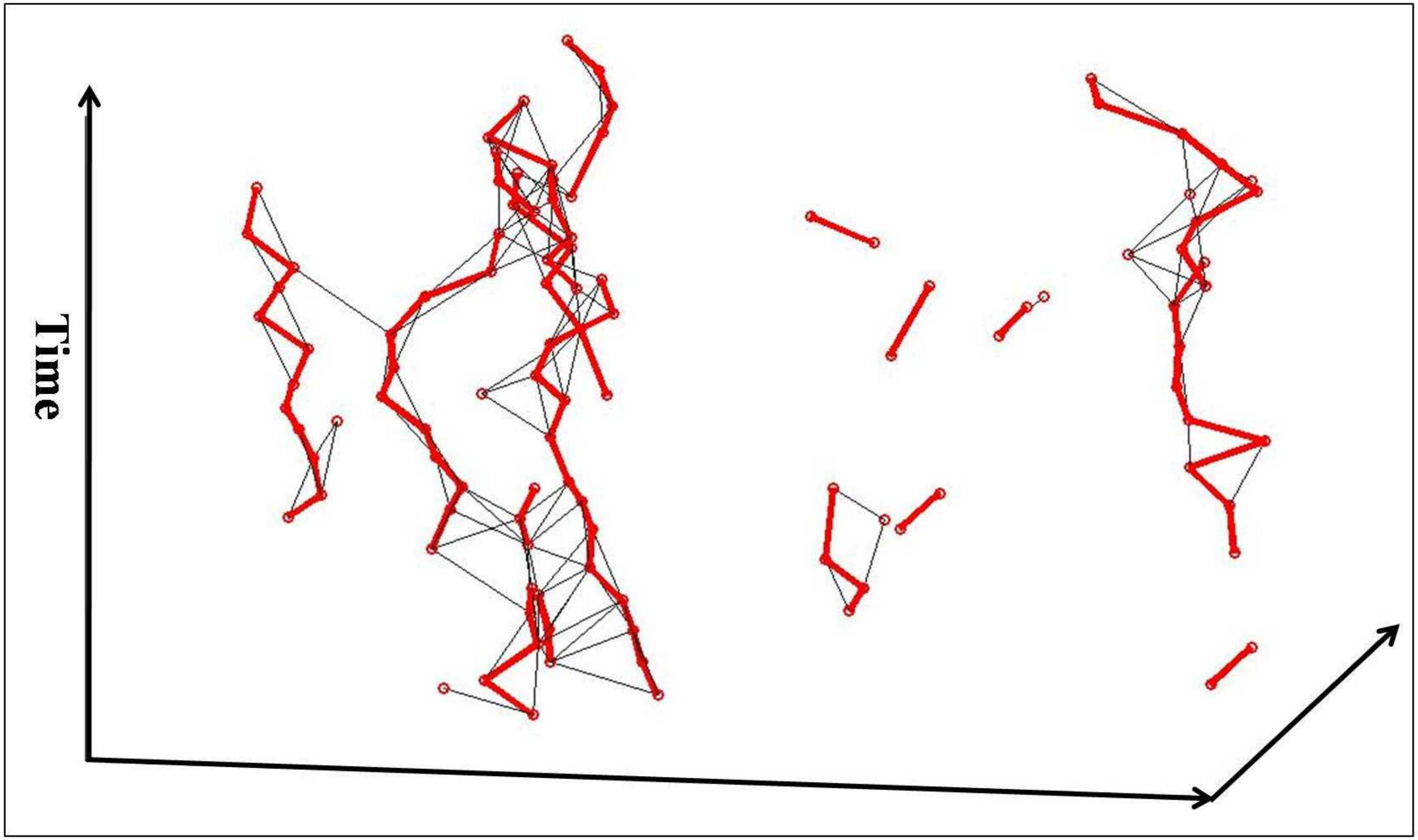
# Approximation – Minimum Spanning Tree




Minimum spanning tree on simulated data (including missing and spurious observations and localization uncertainty)

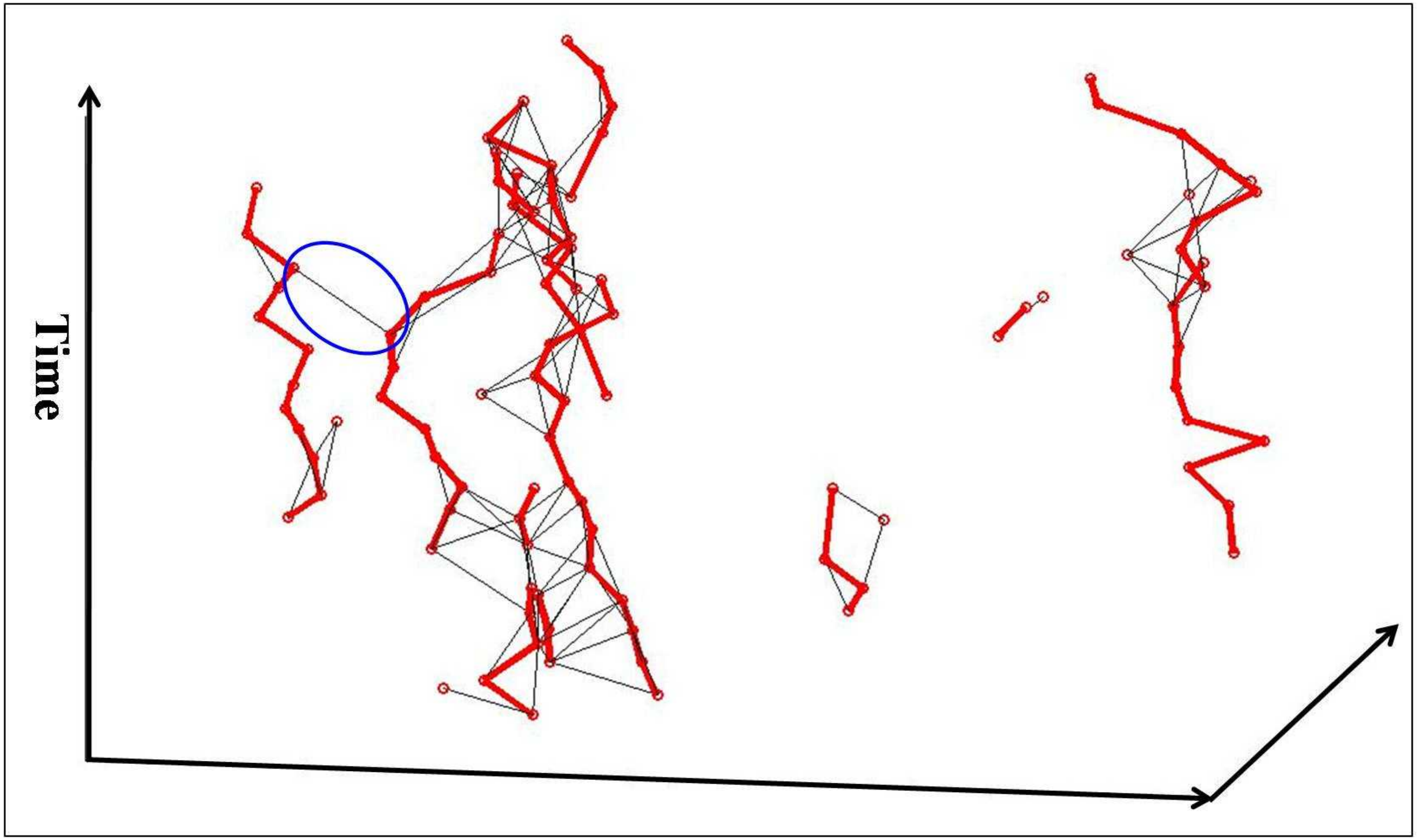
- Looks quite similar to optimum solution – O(E log V)

# Removing Redundant Edges



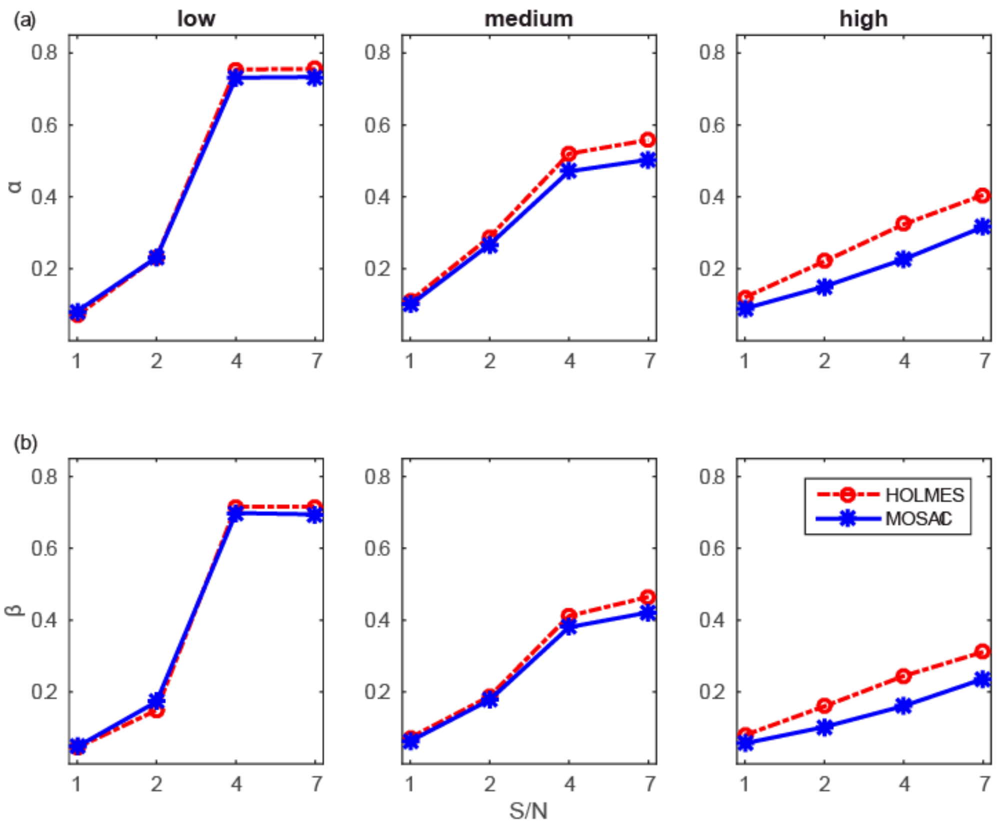

At this point, we have a directed acyclic graph, where any red lines correspond to shortest paths. Some edges are clearly redundant.

# Redundant Edges Removed




The redundant edges could not be part of the optimal solution. Removing them can greatly simplify the graph.

# Performance




Comparison of our algorithm (HOLMES) to one of the best performing algorithms in the ISBI 2012 Particle Tracking Challenge on datasets from the challenge. Higher is better.

# Conclusion

- Single particle tracking (SPT) is very important to many fields including biology
- SPT is a very hard problem (NP-hard)
- Graph theory provides a valuable framework and set of tools for SPT
- Our algorithm is competitive with the best existing algorithms


Image from http://shortlister.com/question-yourself/

**Acknowledgements: Steve Granick and Kejia Chen**

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Currently at Sandia National Laboratories.

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