SAND2016-5291PE

Sandia National Laboratories

Entanglement and the Jaynes-Cummings model with Rydberg-Dressed atoms

Grant Biedermann

Albuquerque, New Mexico

UNM CQuIC

U.S. DEPARTMENT OF ENERGY

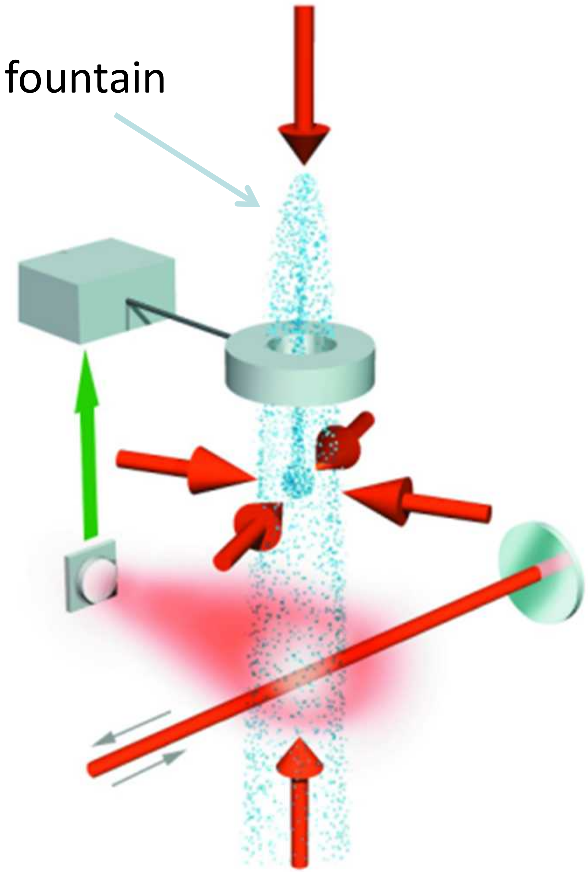
NNSA National Nuclear Security Administration

LDRD

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Quantum-Coherence

Qubit fountain



Atomic fountain principle

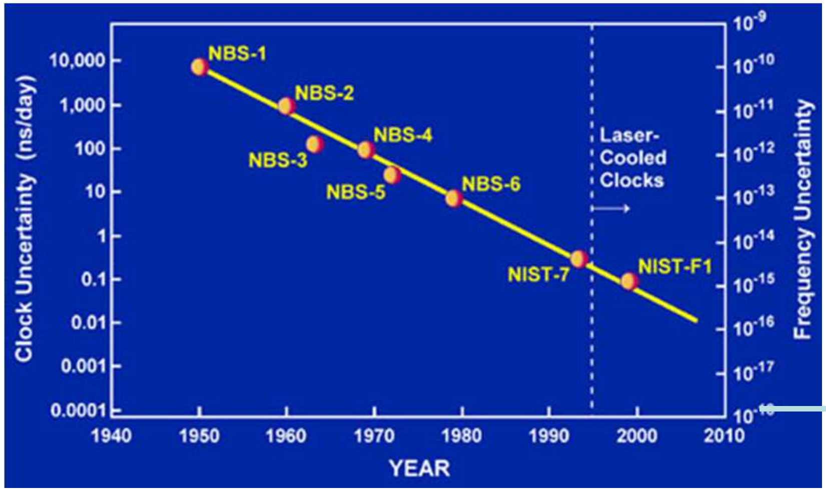
http://smsc.cnes.fr/PHARAO/GP_instrument.htm

Outstanding quantum coherence in neutral atoms enables precision metrology and quantum information

- Example: atomic clocks

$$\left|6^2S_{1/2};F=3,M_F=0\right\rangle \leftrightarrow \left|6^2S_{1/2};F=4,M_F=0\right\rangle$$

$$\left|0\right\rangle \leftrightarrow \left|1\right\rangle$$



Optical clocks

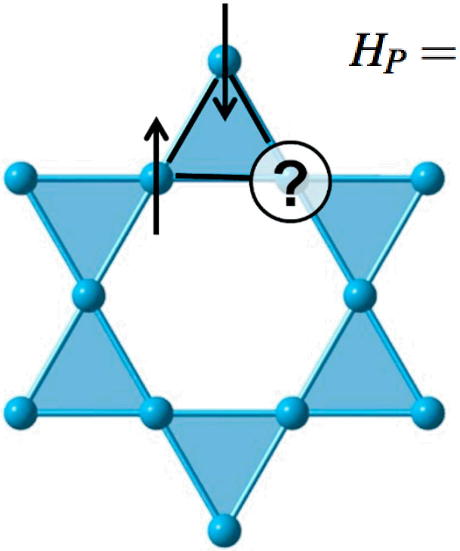
http://www.nist.gov

Applications

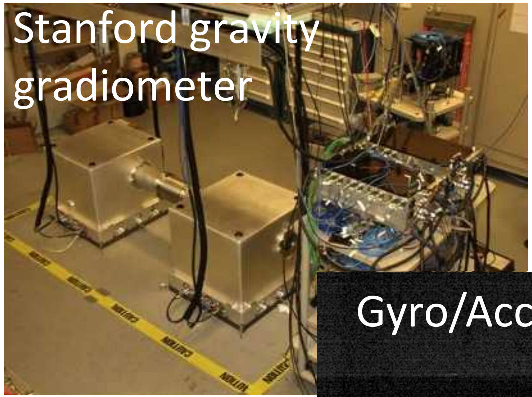
Sandia National Laboratories

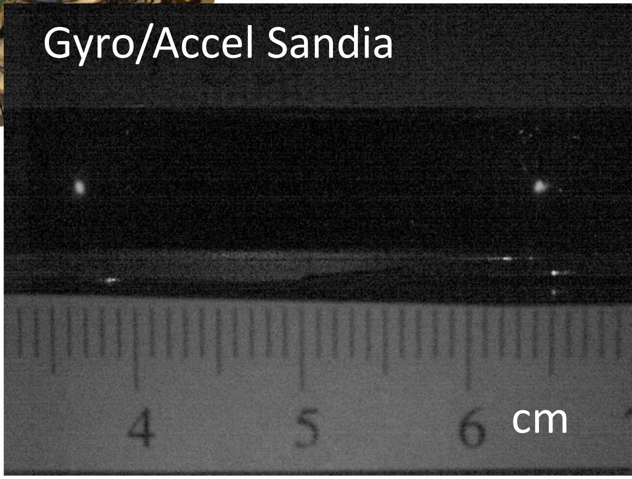
- Atom interferometer inertial sensors
- Clocks
- Magnetometers
- Quantum information
  - improved sensors
  - quantum simulation
  - quantum computation

$$H_P = \sum_{i=1}^{N} \tilde{h}_i \sigma_z^{(i)} + \sum_{i,j=1}^{N} \tilde{J}_{ij} \sigma_z^{(i)} \otimes \sigma_z^{(j)}$$



Frustrated magnetism





2-bit QUBO problem

The two-bit QUBO problem can be stated as

$$\begin{aligned} \text{minimize} \quad & a\,x_1 + b\,x_2 + c\,x_1 x_2 \\ \text{subject to} \quad & x_i \in \{0,1\} \end{aligned}$$

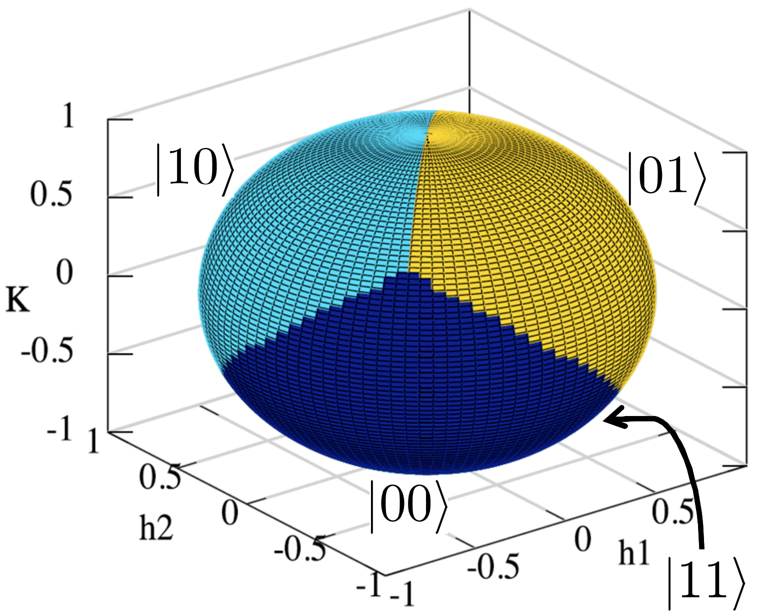
This problem can be mapped to a Hamiltonian (QUBO),

$$\begin{aligned} H &= \frac{a}{2}\left(1+\sigma_z^{(1)}\right) + \frac{b}{2}\left(1+\sigma_z^{(2)}\right) + \frac{c}{4}\left(1+\sigma_z^{(1)}\right)\left(1+\sigma_z^{(2)}\right) \\ &= \left(\frac{a}{2}+\frac{c}{4}\right)\sigma_z^{(1)} + \left(\frac{b}{2}+\frac{c}{4}\right)\sigma_z^{(2)} + \left(\frac{c}{4}\right)\sigma_z^{(1)}\sigma_z^{(2)} + \alpha\, I^{(1)} I^{(2)} \end{aligned}$$

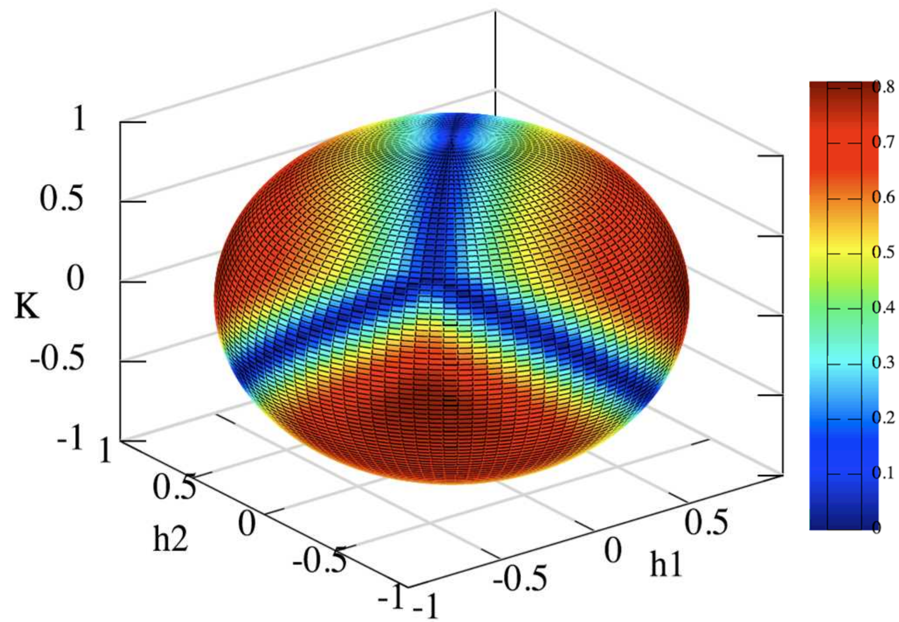
The ground state of this Hamiltonian solves the QUBO problem.

Applications:
- Machine learning
- Image classification
- Neural networks

QUBO solution space



Phase Diagram

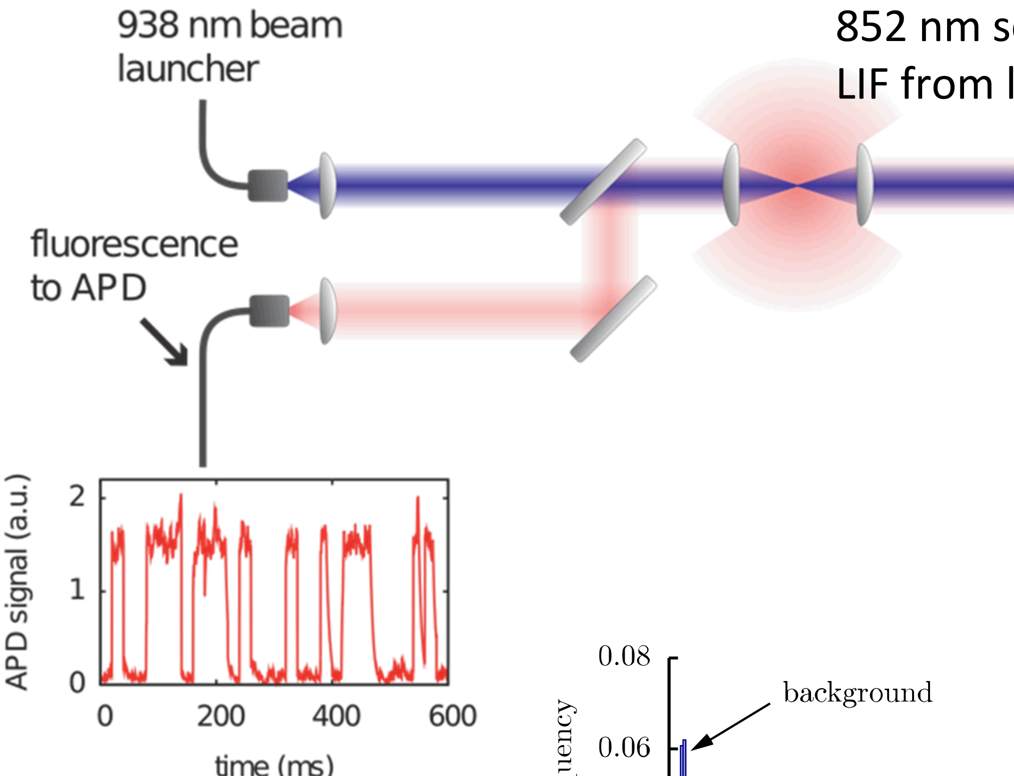


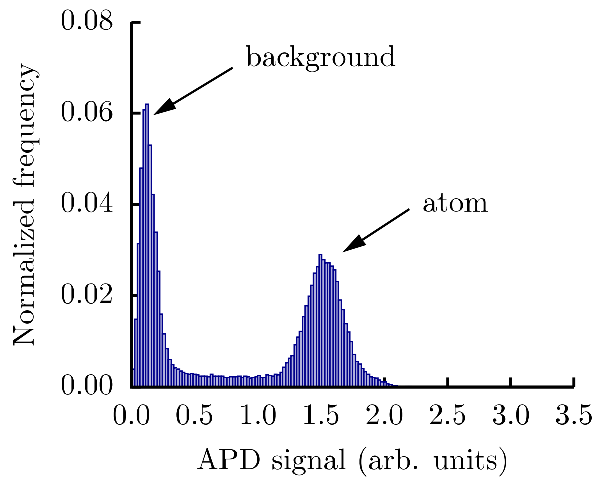
Minimum Gap

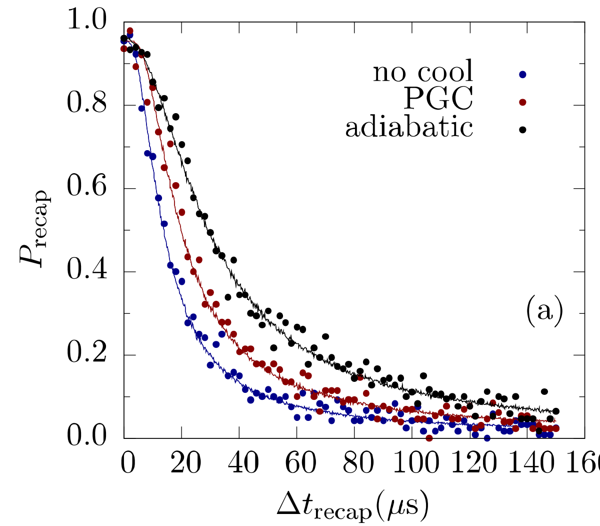
$$H = h_1\sigma_z^{(1)} + h_2\sigma_z^{(2)} + K\sigma_z^{(1)}\sigma_z^{(2)}$$

Single atom control

852 nm scatter:
LIF from laser cooling light

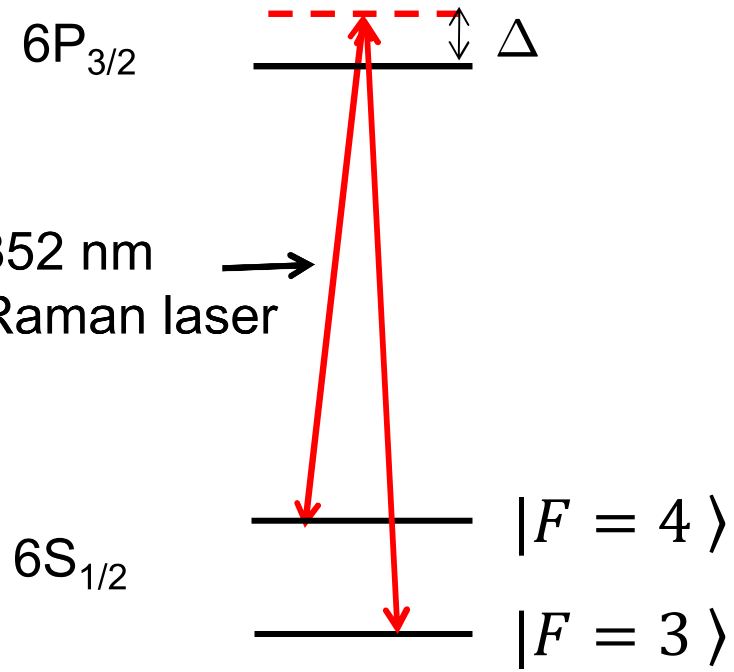






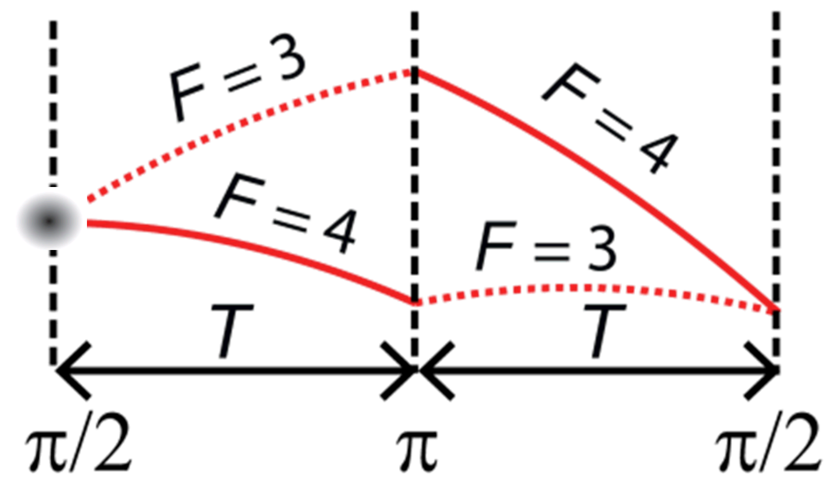
Drop and recapture

Light-pulse atom interferometry



stimulated Raman transition

wavepacket trajectory

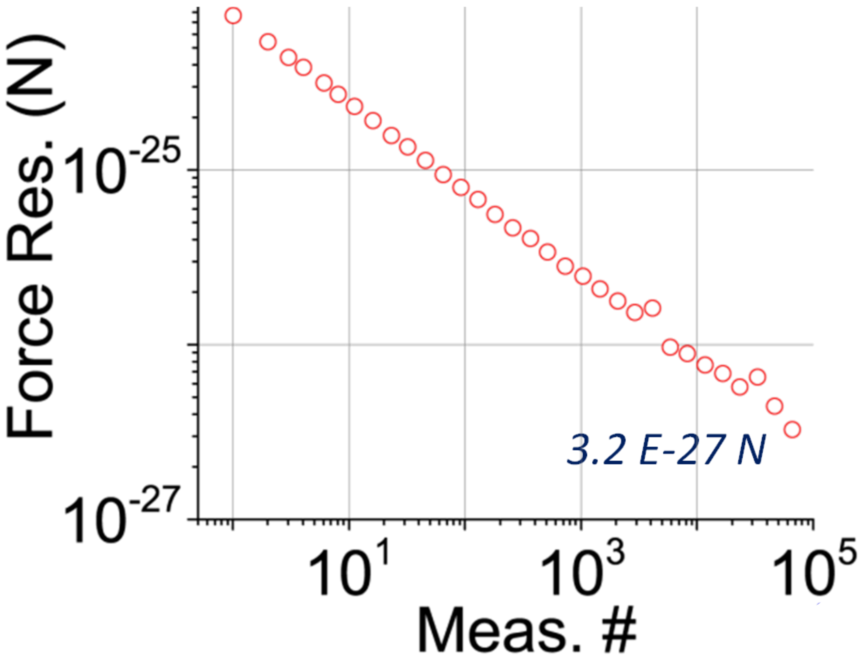


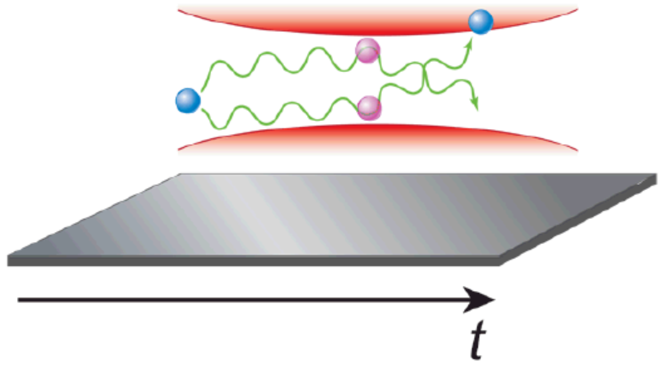
- Exceptional accelerometers and gyroscopes nrad/√Hz, n*g*/√Hz to p*g*/√Hz
- Large commercial and govt. interest in fielding this technology

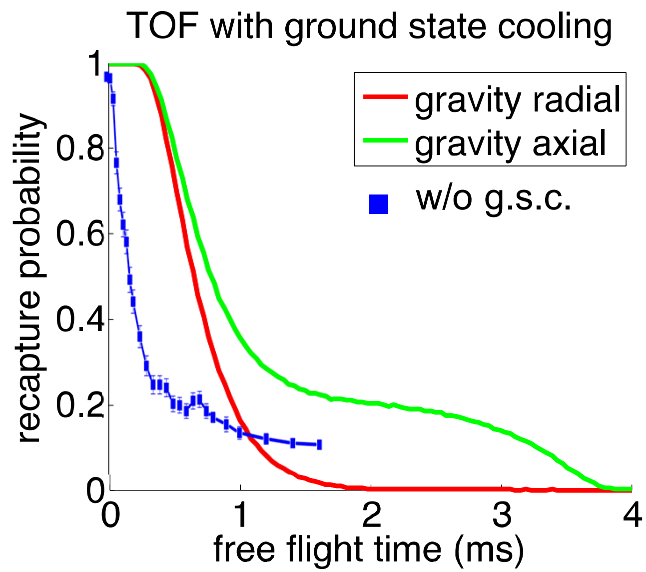
Kasevich, and Chu, Phys. Rev. Lett. 67, 181–184 (1991)

Gustavson, Landragin, and Kasevich, Class. Quantum. Grav 17, 2385–2398 (2000).

Single atom interferometry



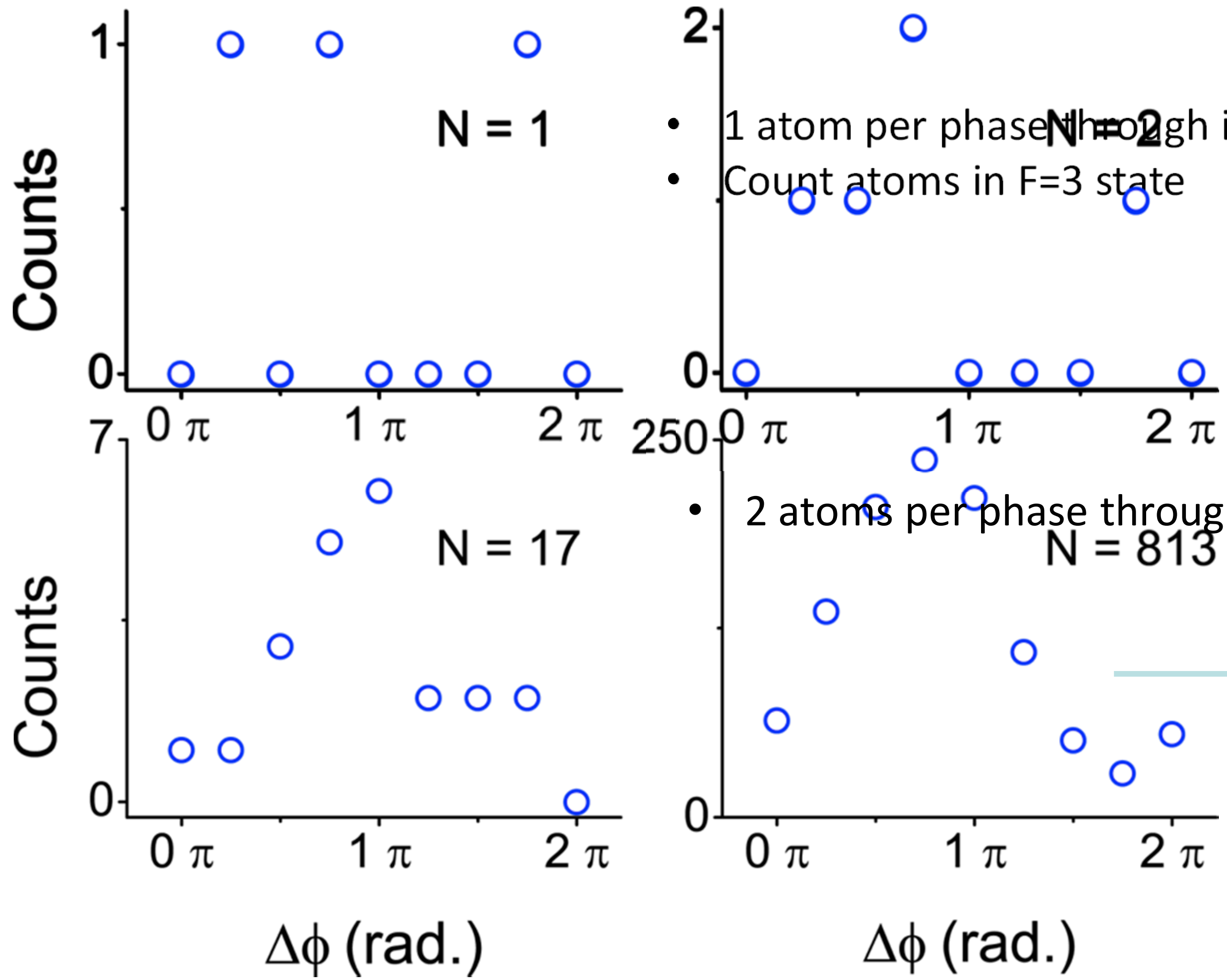




- We showed one can use single atoms
- Single atom control: gateway to harnessing quantum control in sensing
- 10^{-27} N ≈ m*g* for a cesium atom

Parazzoli, et al., Phys. Rev. Lett. **109**, 230401 (2012)

Building a fringe, one atom at a time

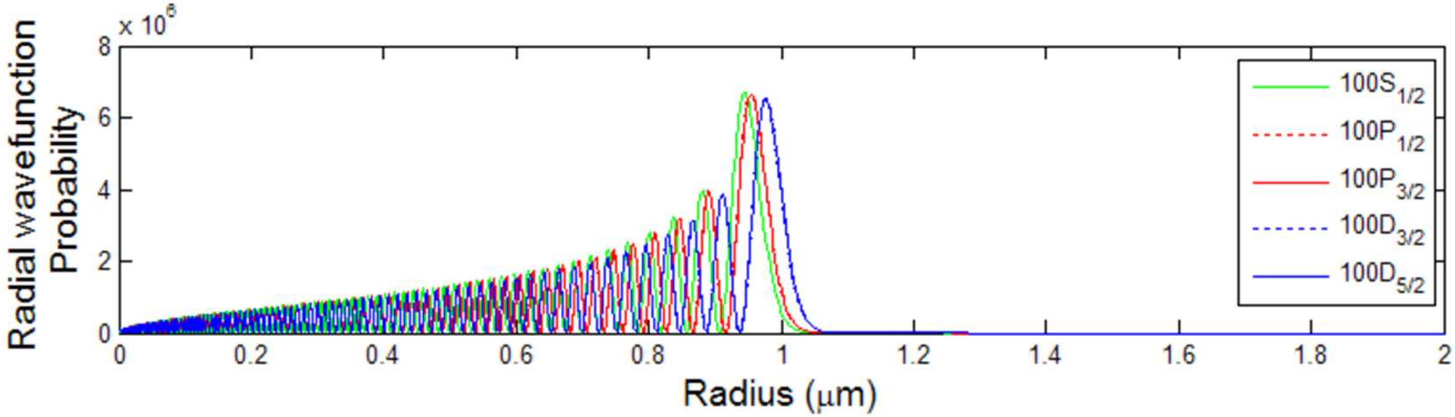


- 1 atom per phase through interferometer.
- Count atoms in F=3 state
- 2 atoms per phase through interferometer.

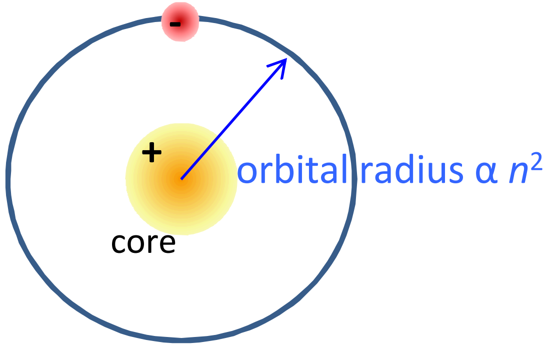
→ 7,317 atoms

Rydberg state mediated interaction

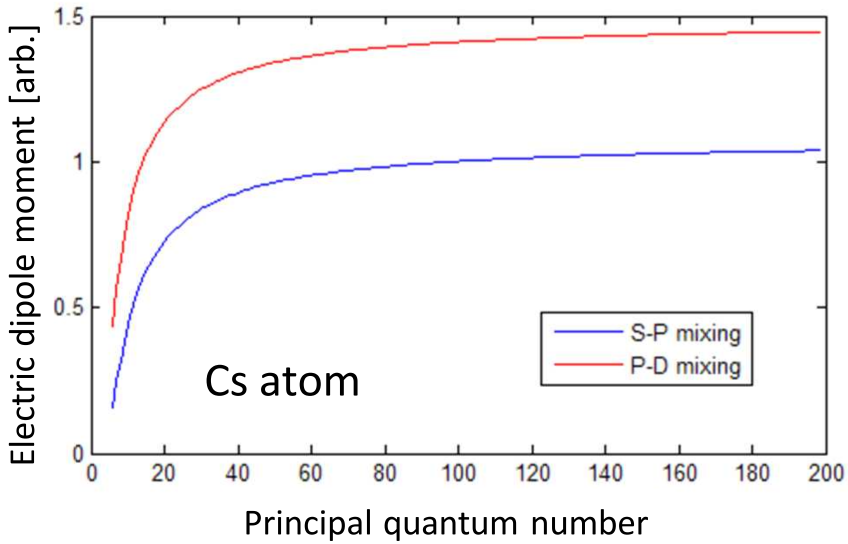
An example of the radial wavefunctions of a Cs atom at n = 100:



A Rydberg atom can have a strong electric dipole moment.

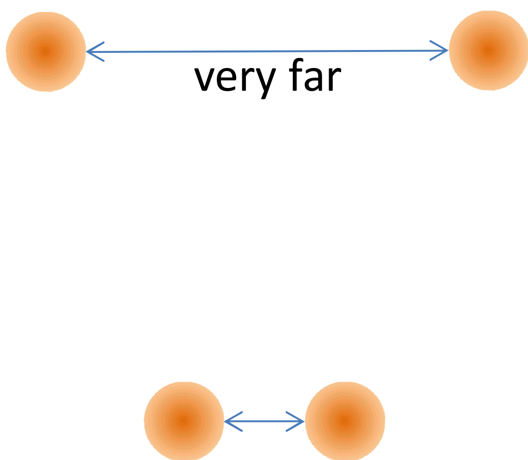


A classical picture of an atom



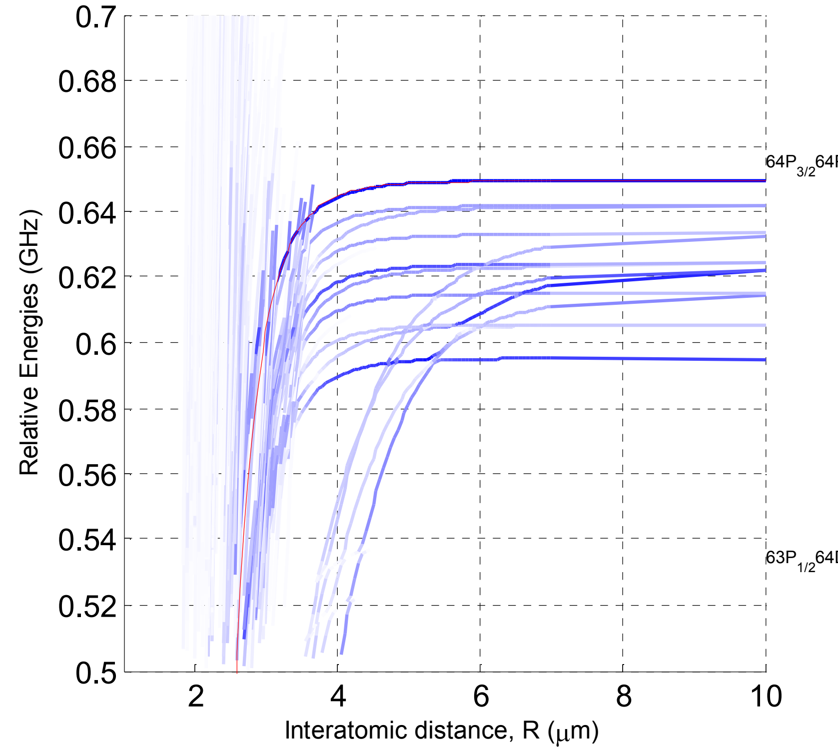
Electric dipole-dipole interaction

$$H_{\text{atoms}} = \sum_i H_0^{(i)} + \frac{1}{4\pi\varepsilon_0 r^3} \sum_{i \neq j} \left(\mathbf{D}^{(i)} \cdot \mathbf{D}^{(j)} - 3\mathbf{D}^{(i)} \cdot \hat{\mathbf{r}}\hat{\mathbf{r}} \cdot \mathbf{D}^{(j)} \right)$$

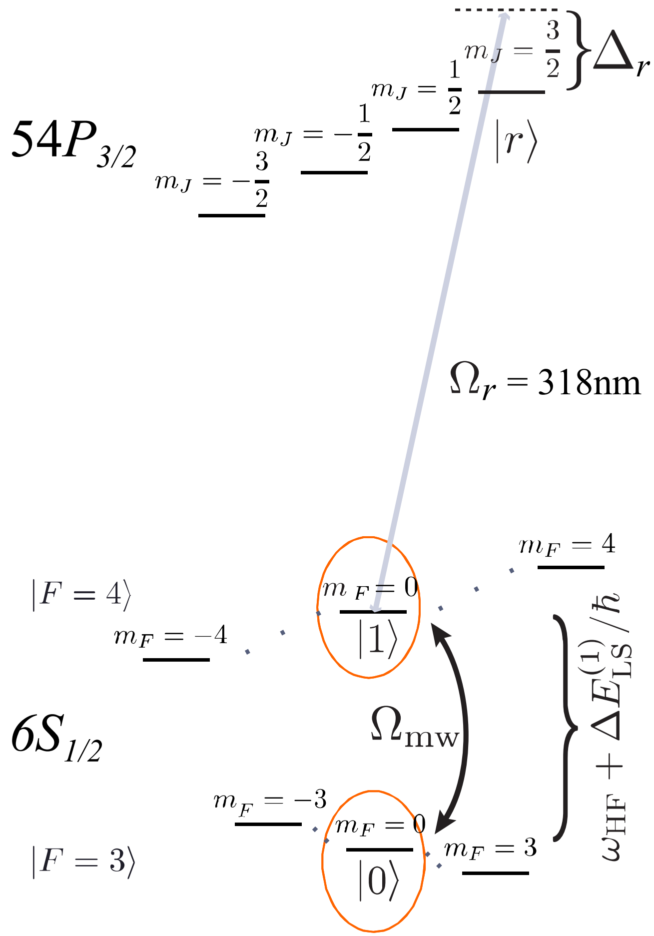


close enough (< 6 μm)

Weighted Rydberg Energy levels: Excitation from ground-state to 64P3/2
x-polarized light; B = 4.8 G; E = 6.4 V/m;



Rydberg-Dressed states



Single atom picture

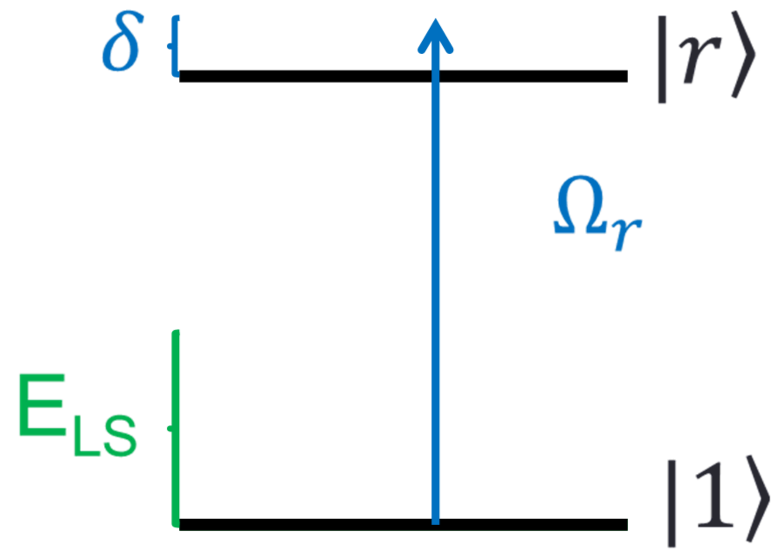
$$\mathbf{H} = \frac{\hbar}{2}\begin{pmatrix} -2\,\Delta_L & \Omega_L \\ \Omega_L & 0 \end{pmatrix}$$

$$\Delta\mathbf{E} = \frac{\hbar\,\Omega_L{}^2}{4\,\Delta_L}$$

- As atoms approach one another, Rydberg state energy shifts which changes the detuning—atom-atom interaction
- Controllable with laser intensity and detuning—on demand

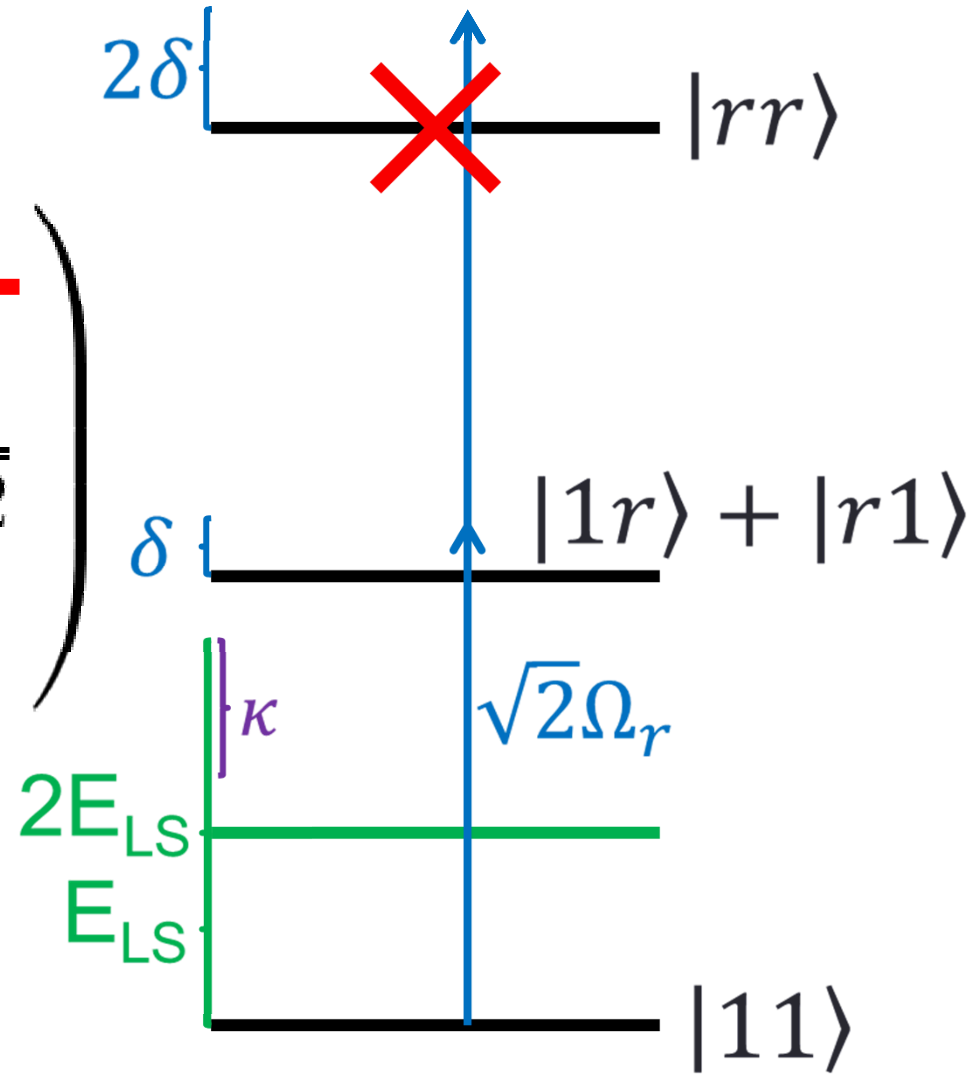
Rydberg-Dressed states

$$H^{(1)} = \begin{pmatrix} -\delta_r & \frac{\Omega_r}{2} \\ \frac{\Omega_r}{2} & 0 \end{pmatrix}$$



Rydberg-Dressed states

$$H^{(2)} = \begin{pmatrix} V_{dd} & \frac{\Omega_r}{\sqrt{2}} & 0 \\ \frac{\Omega_r}{\sqrt{2}} & -\delta_r & \frac{\Omega_r}{\sqrt{2}} \\ 0 & \frac{\Omega_r}{\sqrt{2}} & 0 \end{pmatrix}$$



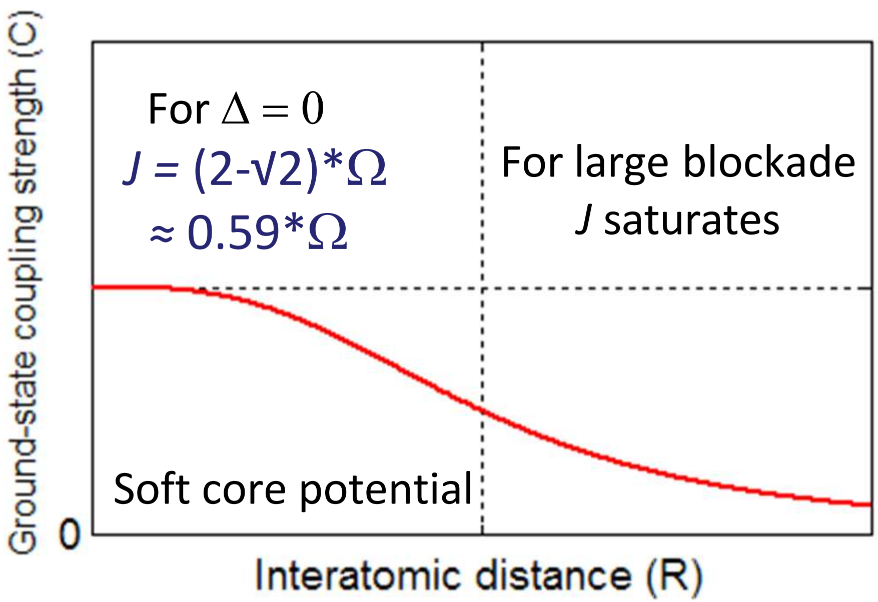
$$\kappa = E_{LS}^{(2)} - 2E_{LS}^{(1)}$$

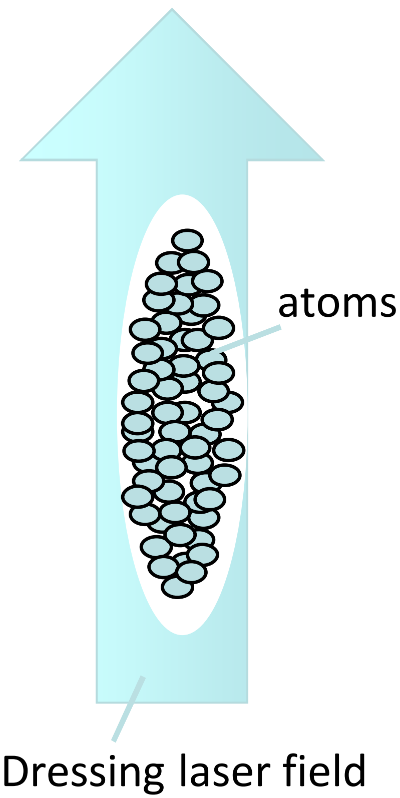
Rydberg-dressed interactions

Tunable interaction strength (J), low sensitivity to atom motion, and effectively strong ground-state interactions.

For $\Omega << \Delta$

$$J = \Delta\nu_{BLS} - \Delta\nu_{LS} = \frac{\Omega^4}{8\Delta^3} - \frac{3\Omega^6}{16\Delta^5} + \frac{35\Omega^8}{128\Delta^7} - \cdots$$

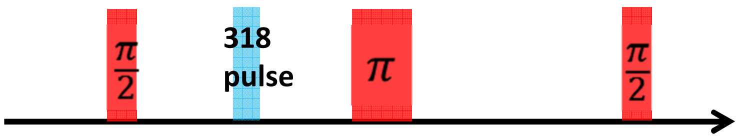




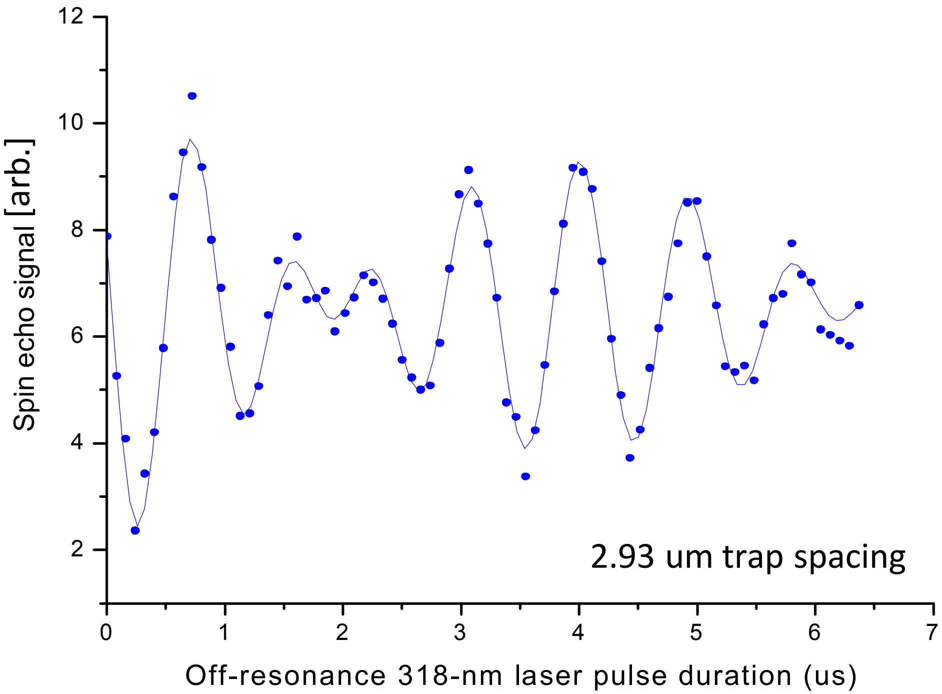
I. Bouchoule, K. Mølmer, Phys. Rev. A 65, 041803 (2002).
J. Johnson, S. Rolston, Phys. Rev. A 82, 033412 (2010).

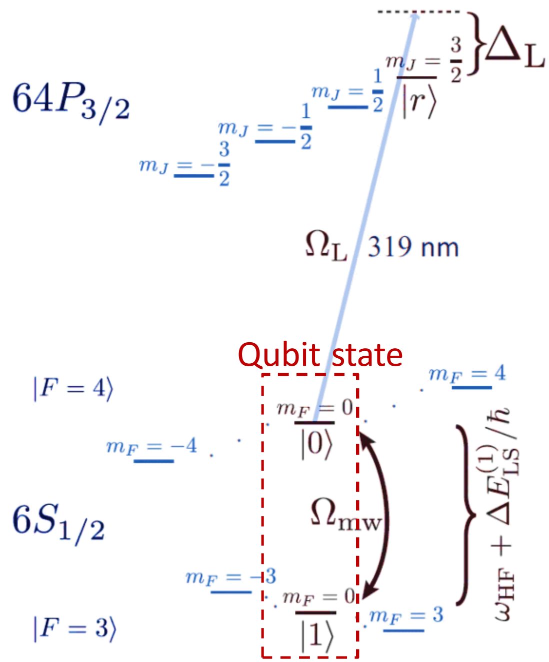
Rydberg-dressed Ramsey dynamics

Microwave transition is via Raman laser

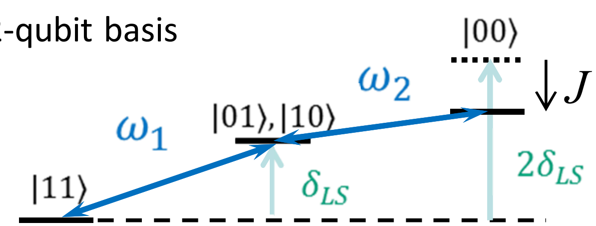


318-nm laser dressed spin echo sequence





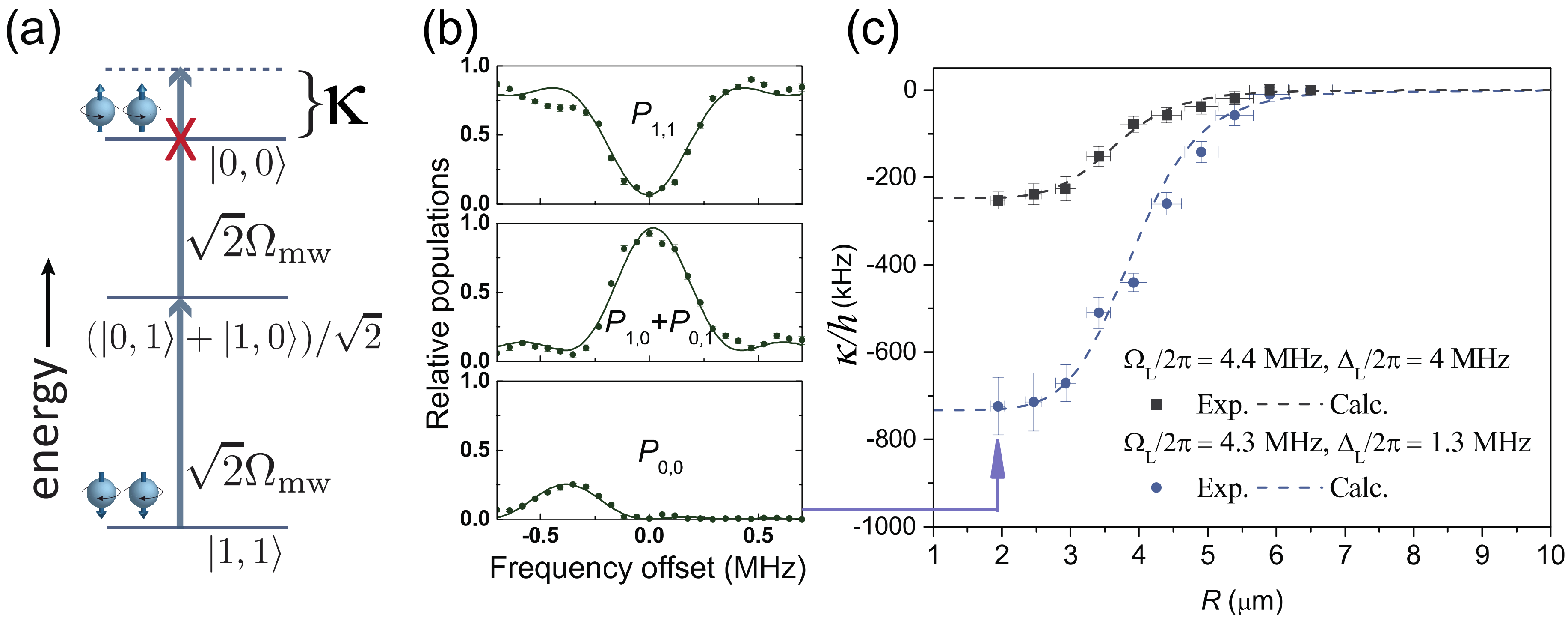
2-qubit basis



A frequency beat note is generated if $\omega_1 \neq \omega_2$

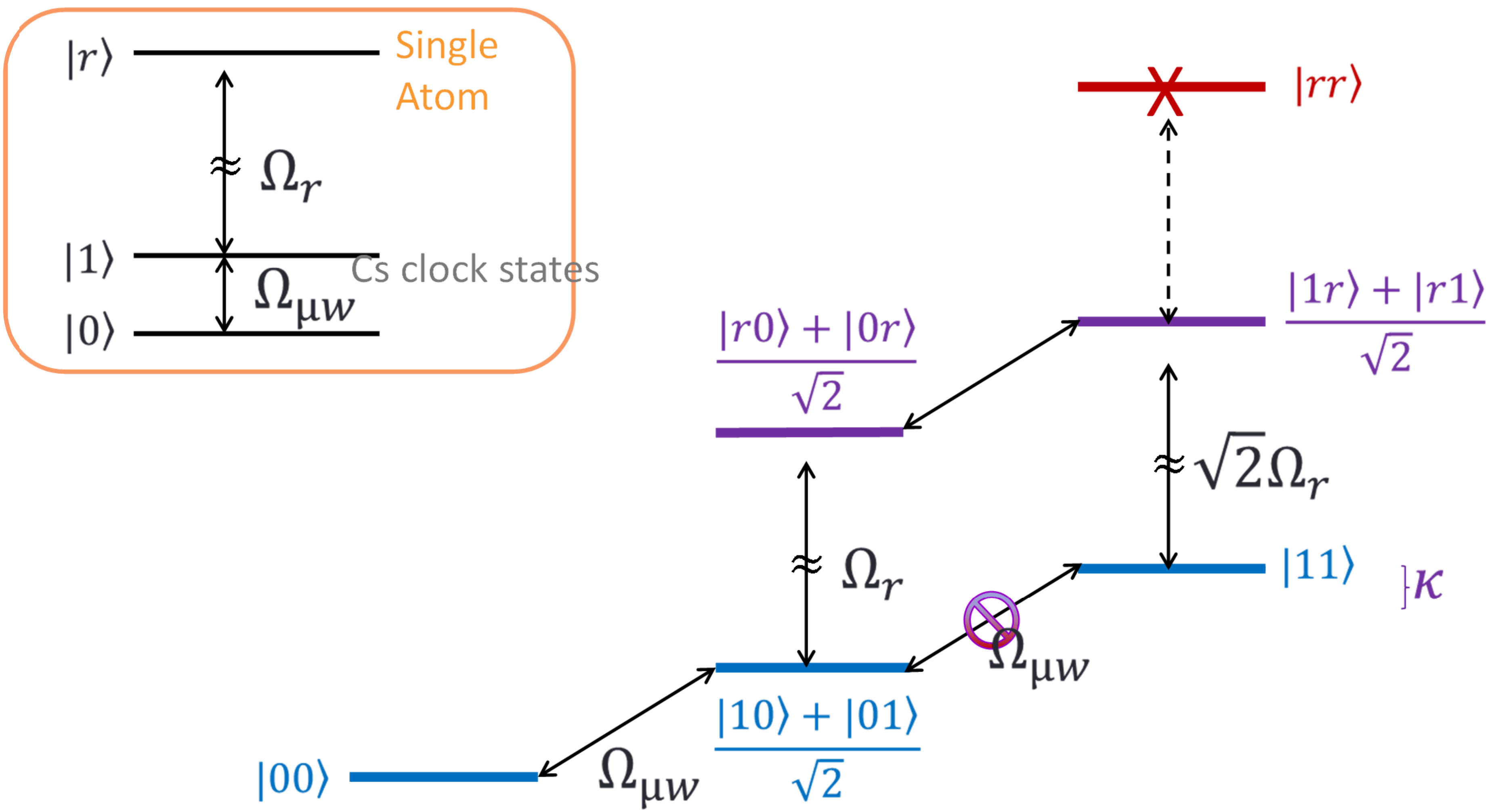
κ vs. R

Direct measurement of two-qubit interaction strength κ as a function of two-atom separation with two conditions.



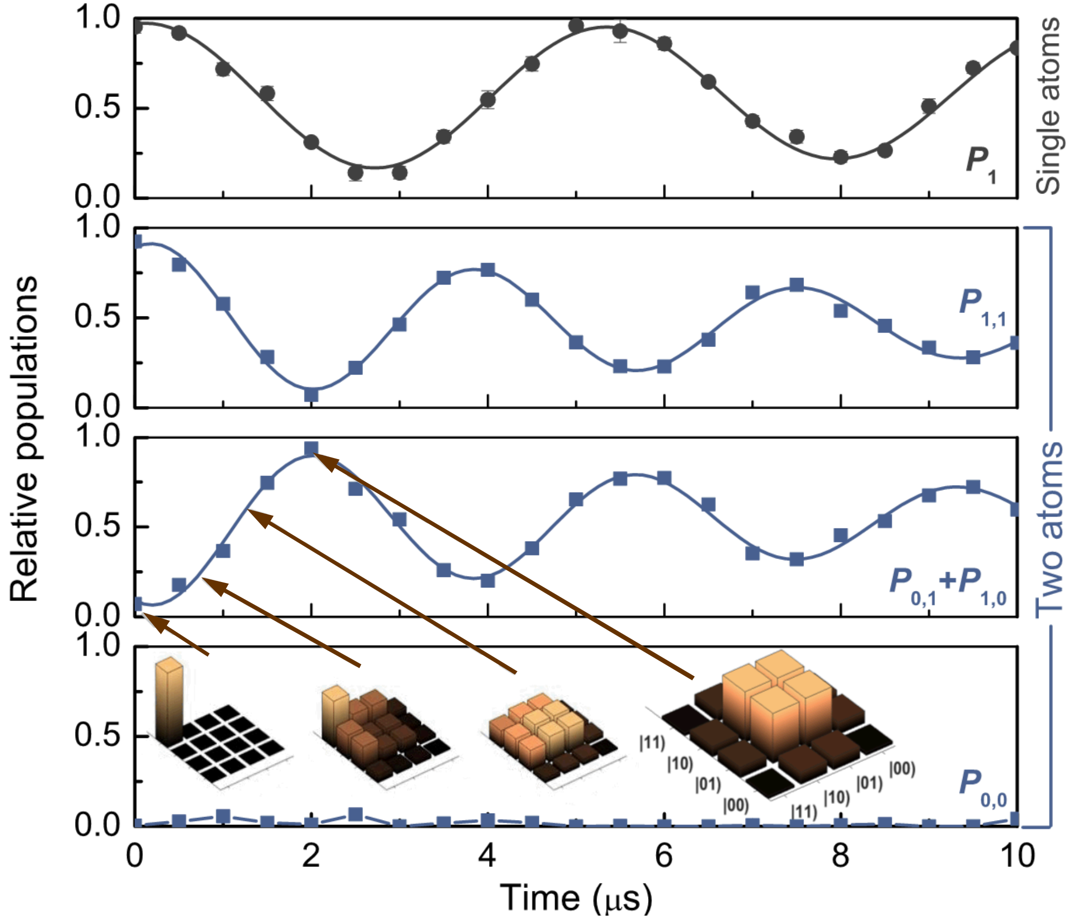
Jau, et al., Nature Phys. (2016)

Generating Entanglement



Producing Bell-state entanglement

Initial state is $|1\rangle$ or $|11\rangle$, then apply 318-nm and Raman lasers
Experimental data with $J/h \approx 750$ kHz



Single-atom Rabi oscillation: $|1\rangle \leftrightarrow |0\rangle$

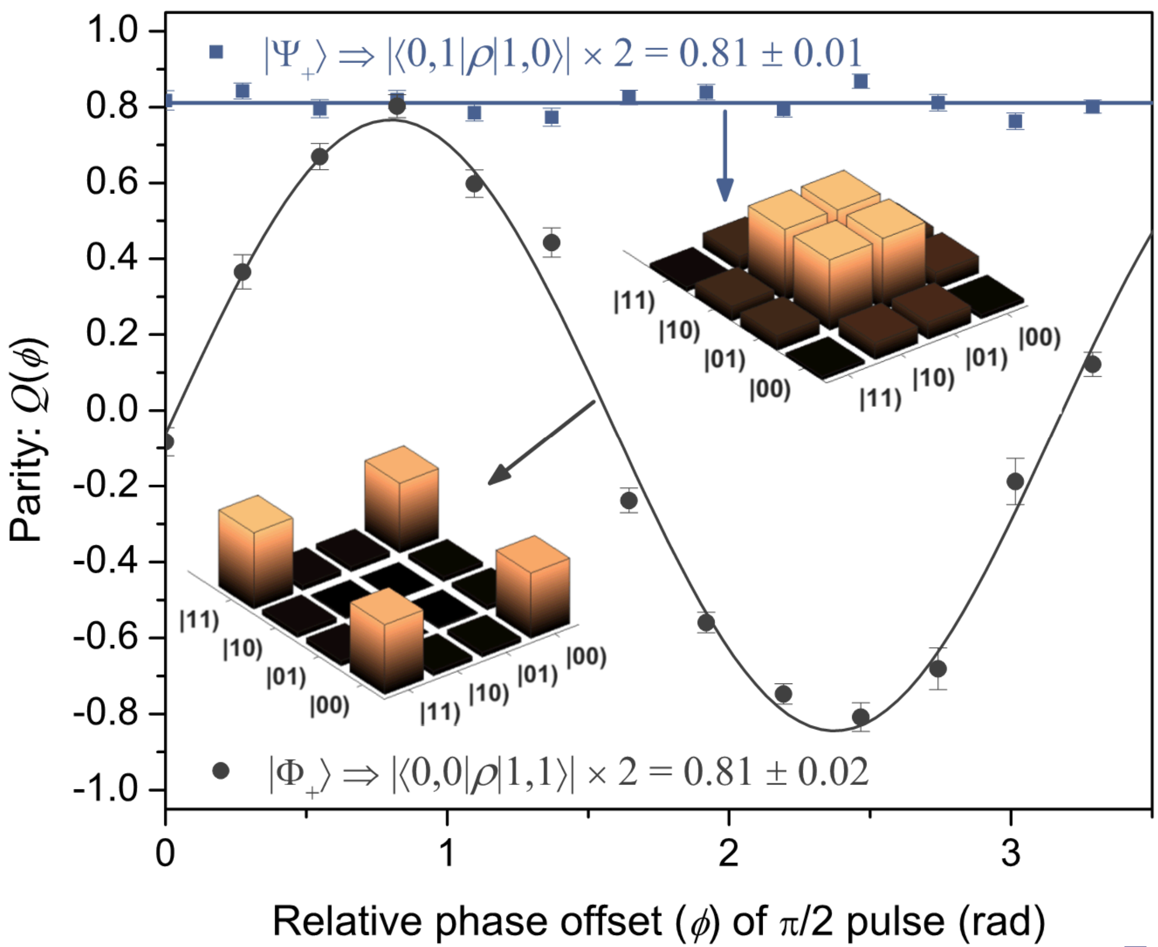
Two-atom Rabi oscillation: $|11\rangle \leftrightarrow (|10\rangle + |01\rangle)/\sqrt{2}$

- $\sqrt{2}$ times faster
- No significant population being transferred to $|00\rangle$
- Bell state $|\Psi_+\rangle$ is produced at $t = \pi/\sqrt{2}\Omega_{mw}$

Process occurs entirely and directly in the ground state

Entanglement Fidelity ≥ 81%

Verify the entanglement via parity measurements



Prepare two Cs atoms in Bell state $|\Psi_+\rangle$ or $|\Phi_+\rangle$

Apply a global $\pi/2$ rotation with a given phase

Perform parity measurement $Q = P_{11} + P_{00} - (P_{01} + P_{10})$

Obtain the two-qubit entanglement fidelity F, where $Q \leq F \leq 1$.

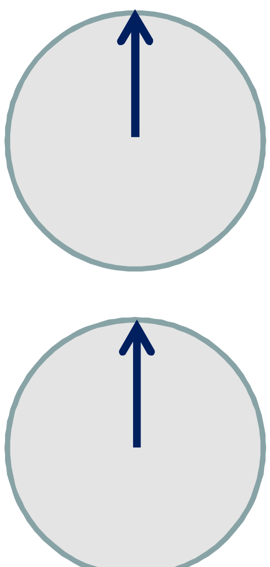
Two atom coherence = 0.405 ± .01

Generating Entanglement

- Schrödinger picture: $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$
- Interaction picture: $|\psi(t)\rangle = e^{-i\kappa t}|\psi(0)\rangle$

Phase:

Laser ON OFF



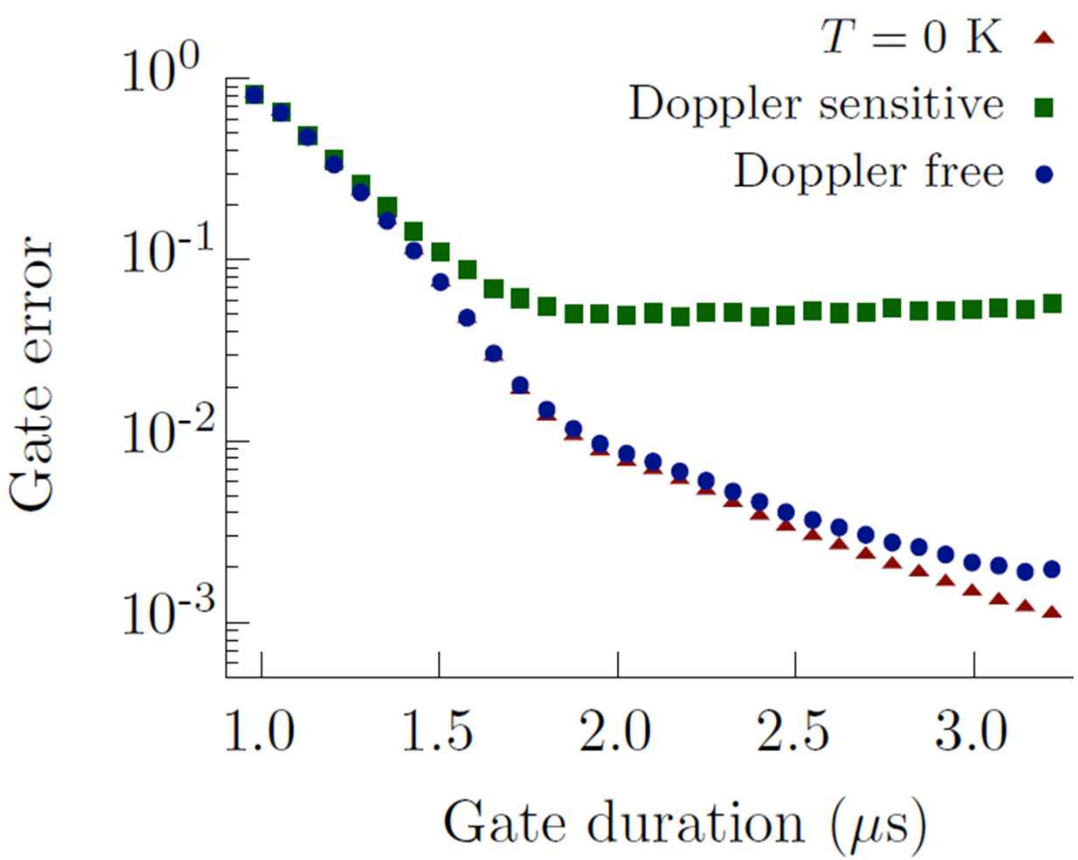
$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \mathrm{CPHASE}$$

$|11\rangle$

$|01\rangle$ $|10\rangle$

and $|00\rangle$

Simulated CPHASE gate fidelities

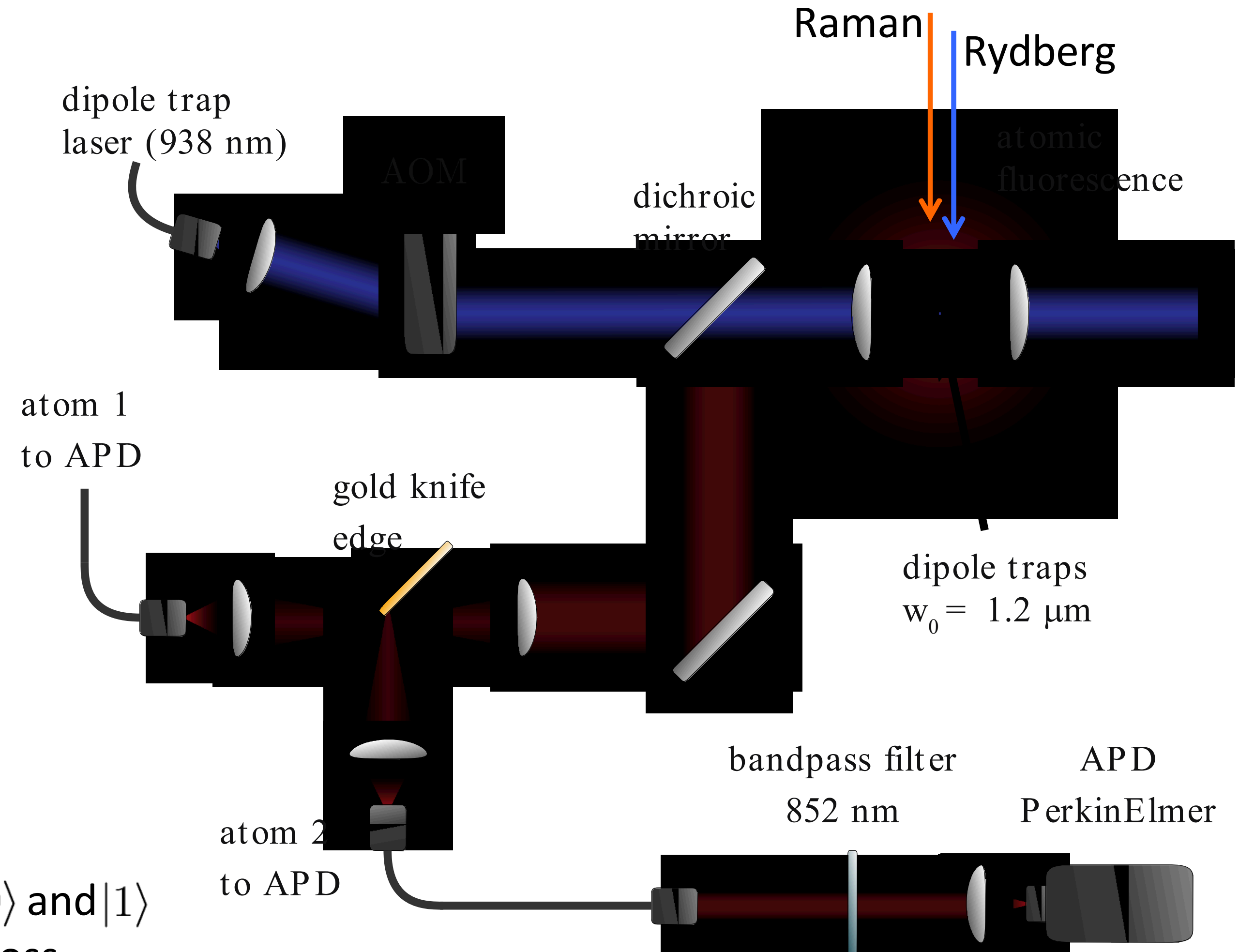


$\Omega = 0 \rightarrow 3$ MHz
$\Delta/2\pi = 6 \rightarrow 0$ MHz
$\Gamma = 3.7$ kHz
$T = 16\ \mu$K

- Motional errors set a high floor on error for the single-beam scheme.
- The Doppler-free scheme is limited by the much smaller photon scattering rate.

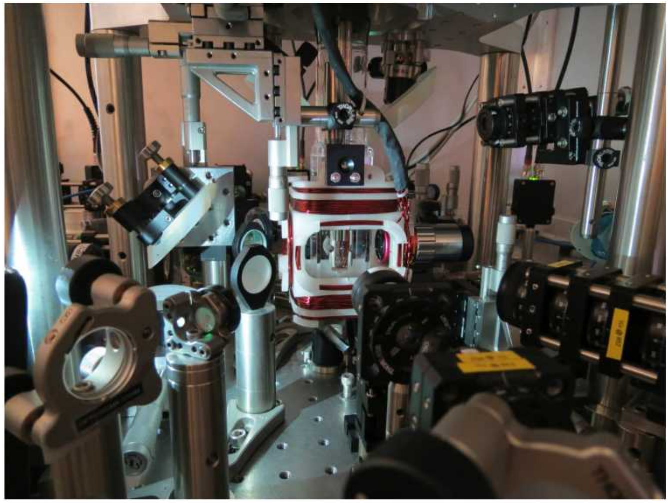
Phys. Rev. A 91, 012337 (2015)

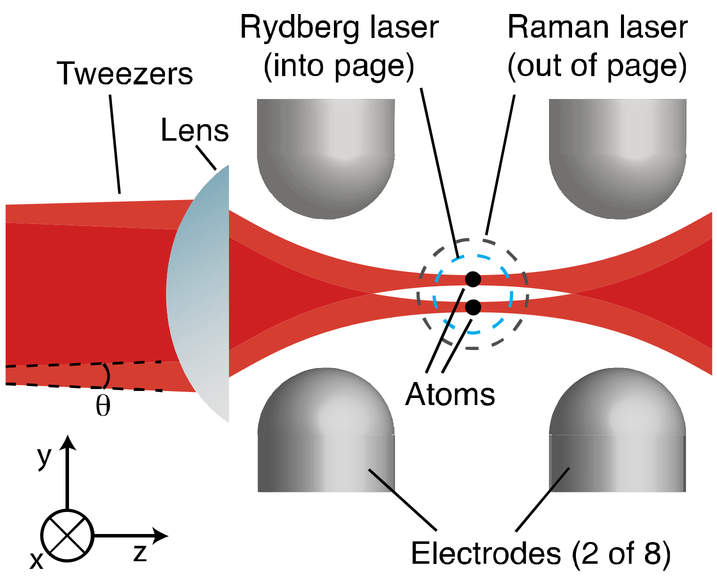
Experiment schematic—2 atoms

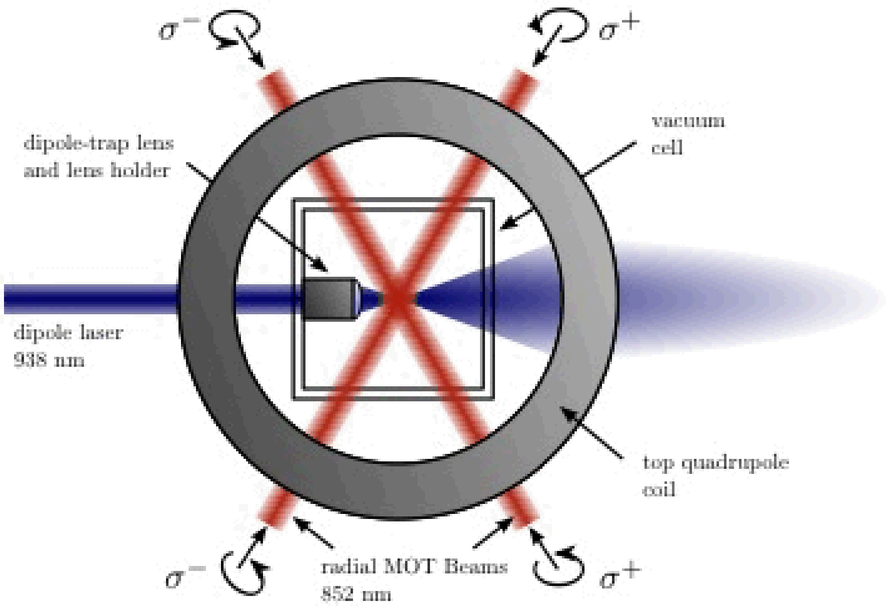


- Directly detect both $|0\rangle$ and $|1\rangle$
- Distinguishable from loss
- FPGA control: Reuse atoms
- Data rate ≈ 10 s^{-1} for two atoms

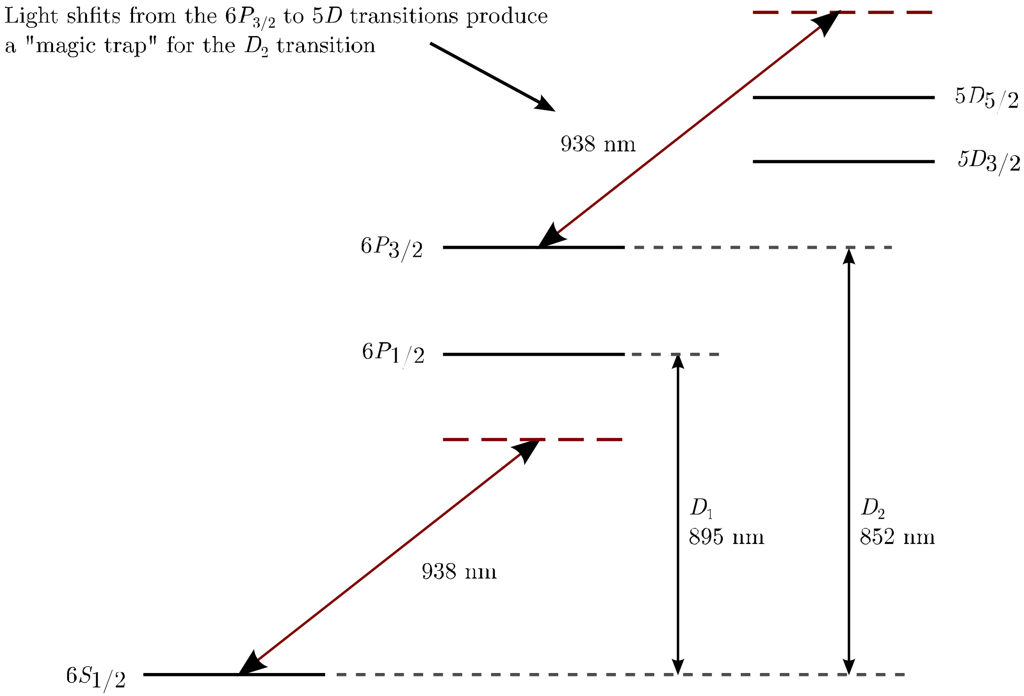
Apparatus



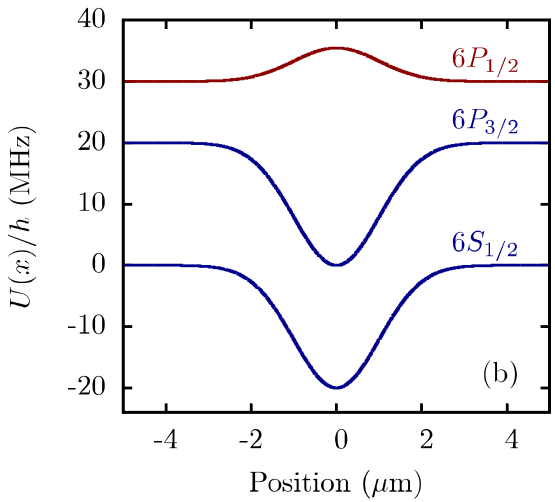

Rydberg laser
(into page)
Raman laser
(out of page)
Tweezers
Lens
θ
Atoms
y
x
z
Electrodes (2 of 8)


σ−
σ+
dipole-trap lens
and lens holder
vacuum
cell
dipole laser
938 nm
top quadrupole
coil
σ−
radial MOT Beams
852 nm
σ+

Single atom control



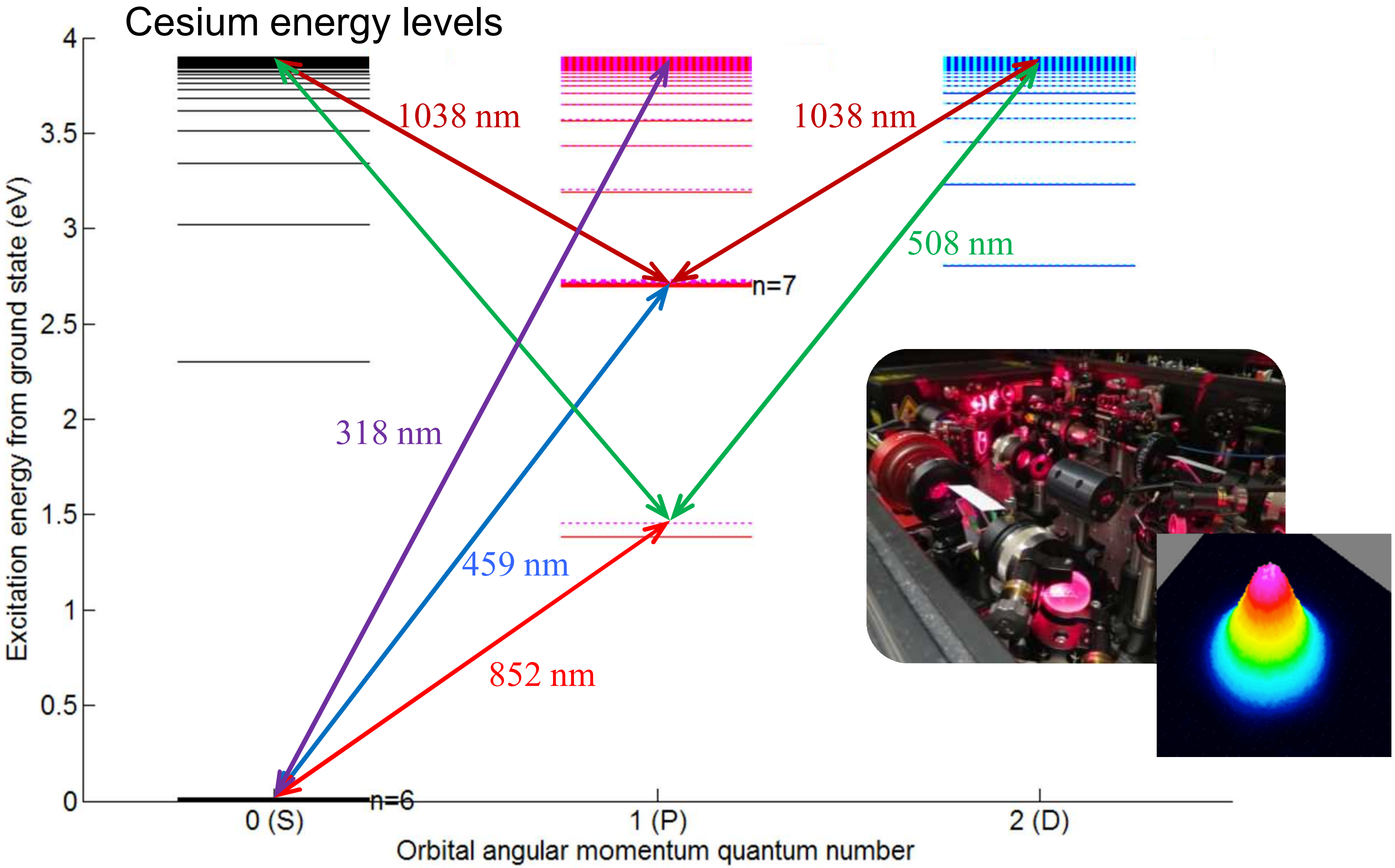
Why 938 nm? It's magic for the cooling transition.



- ≈ 5 mW, 43 nm red
- focused to ≈ 1 μm
- gives ≈ 20 MHz or ≈ 1 mK

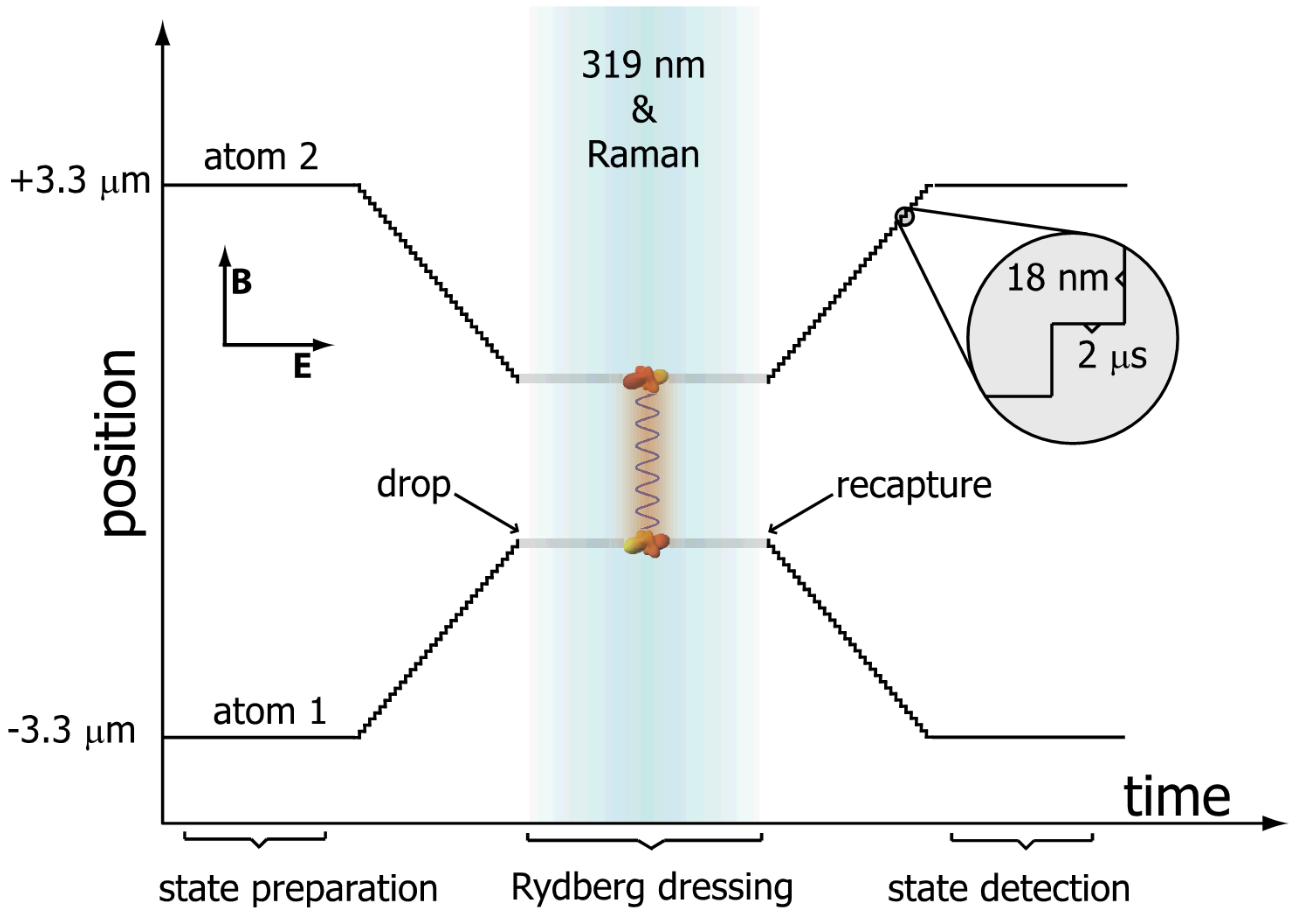
Rydberg excitation laser

Sandia National Laboratories

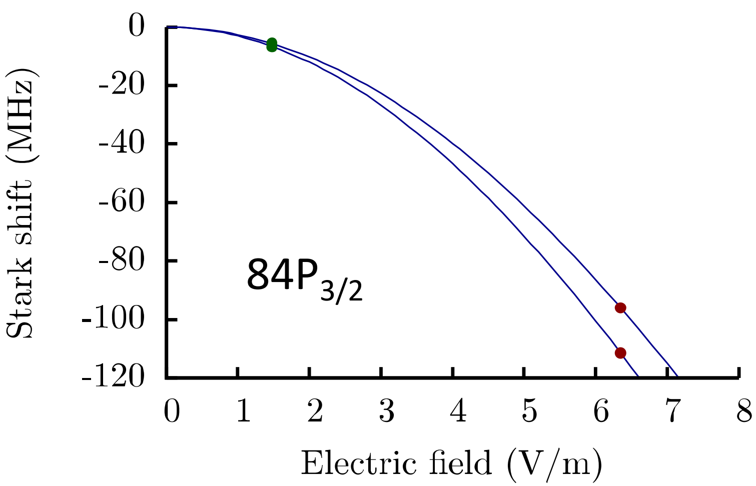


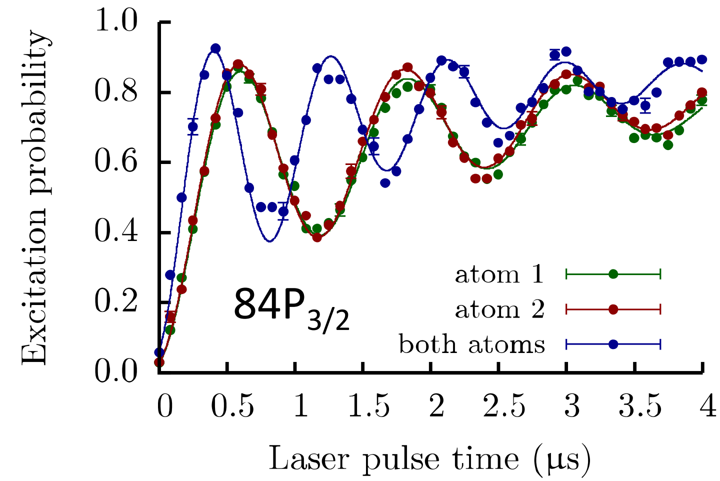
Published: Phys. Rev. A 89, 033416 (2014)

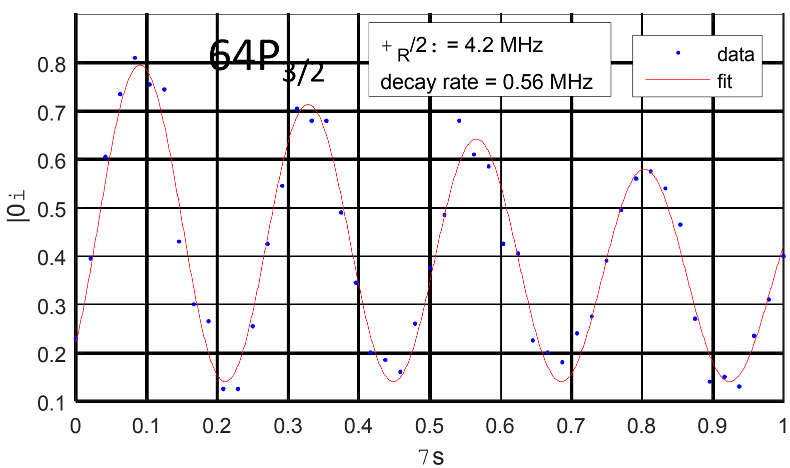
Dynamic atom positioning

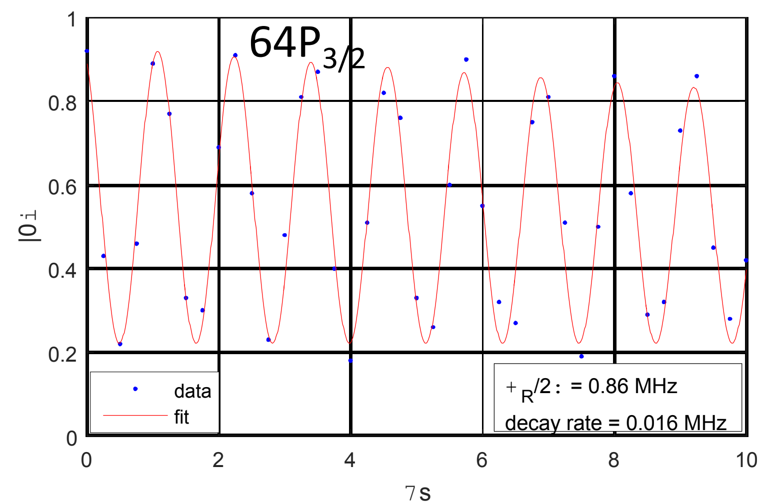


Controlling decay



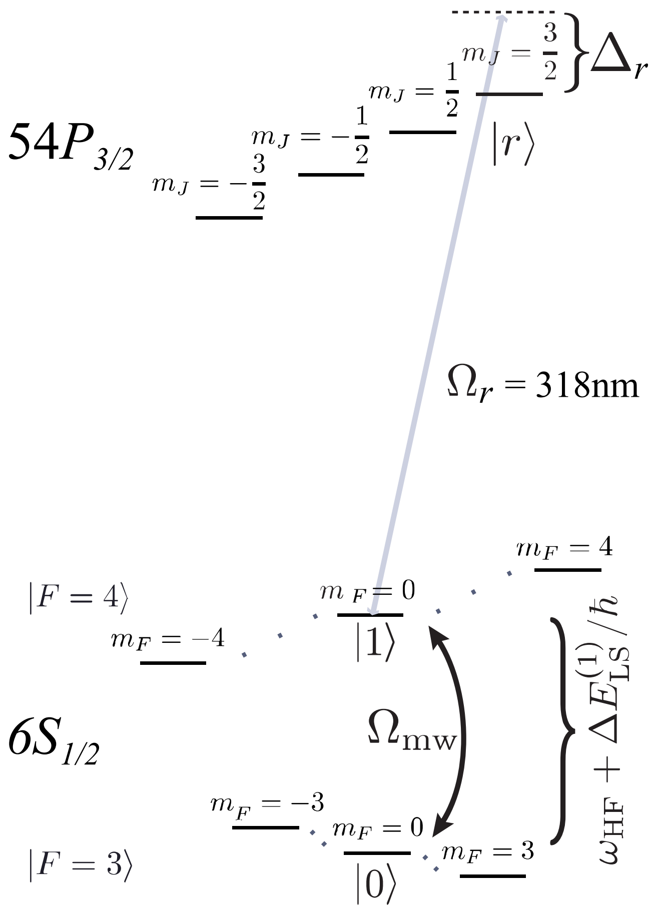


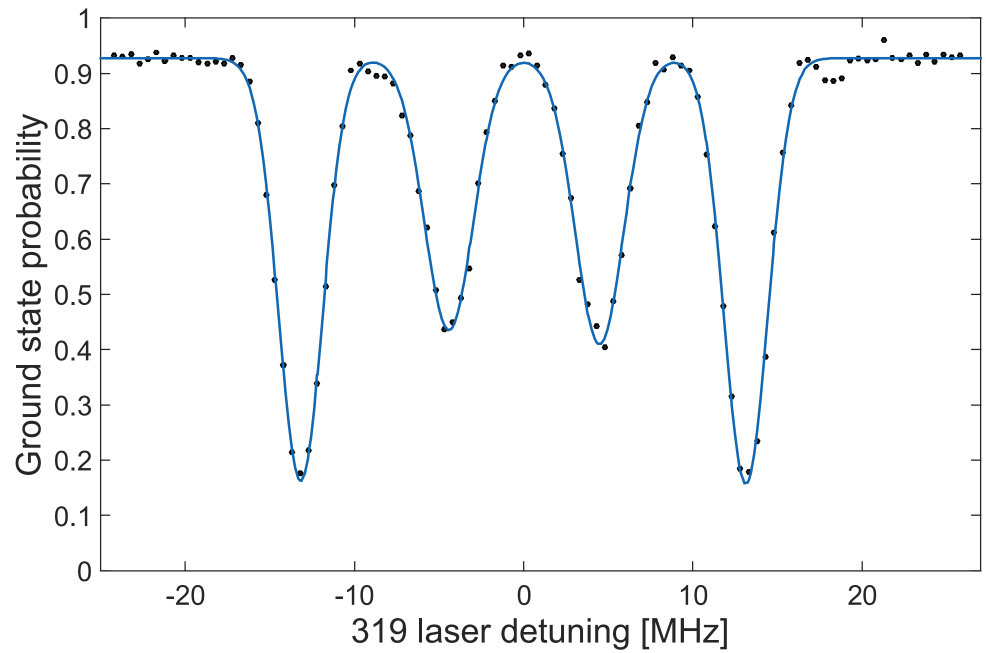




New design

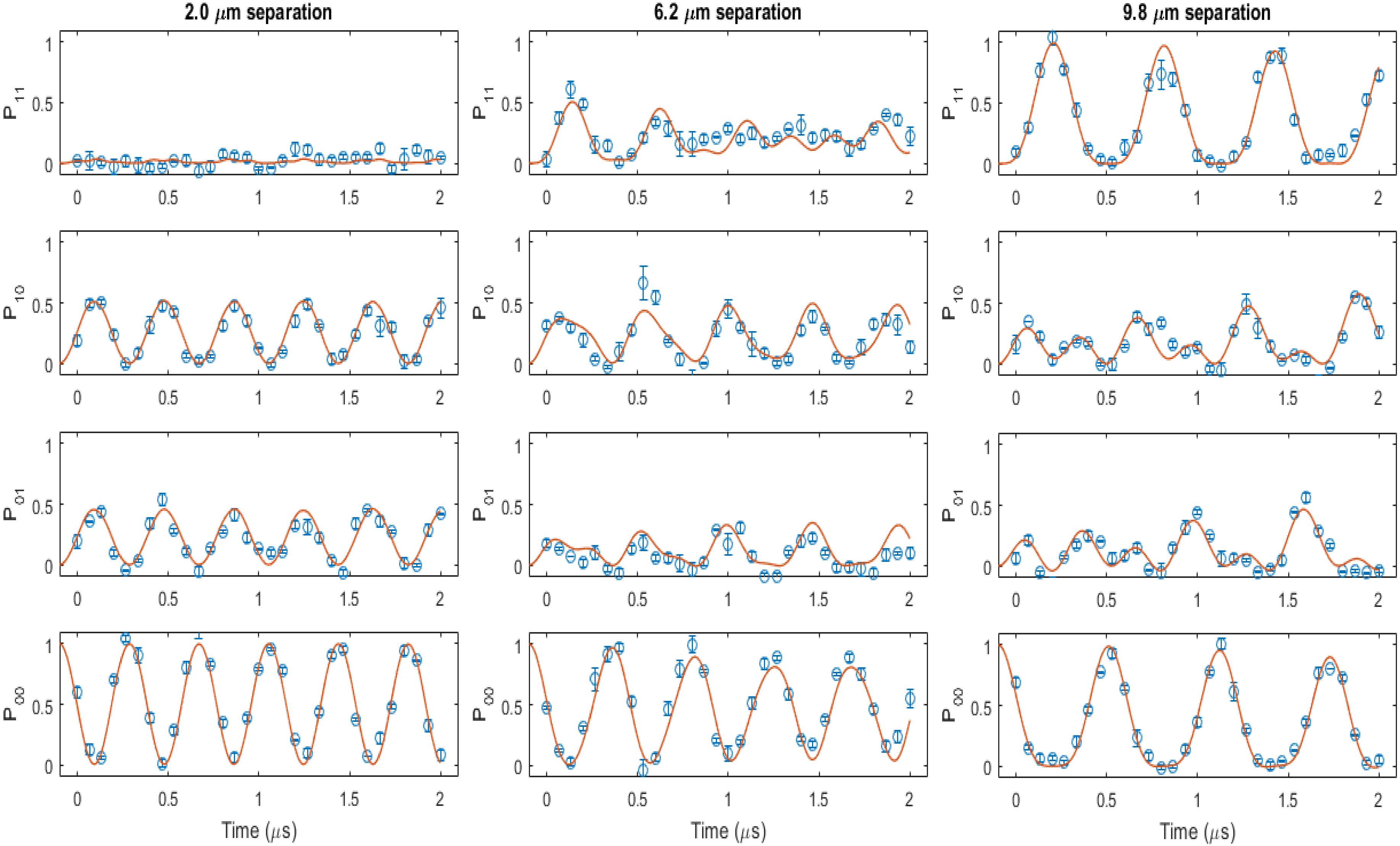
Rydberg state spectrum





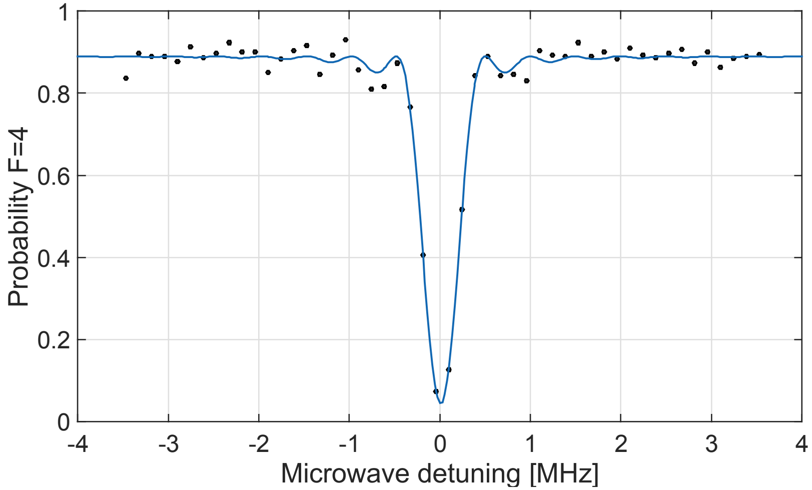
Rydberg blockade

Strong ($U_{RR} > 6$ MHz) | Intermediate ($U_{RR} = 1.9$ MHz) | Weak ($U_{RR} < 100$ kHz)

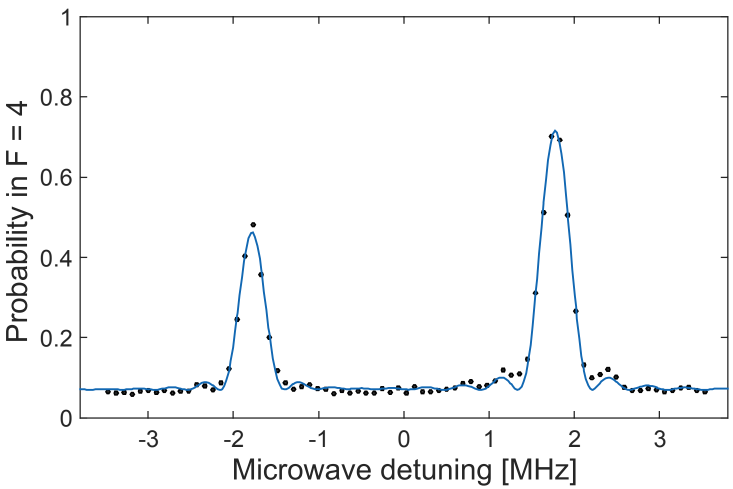


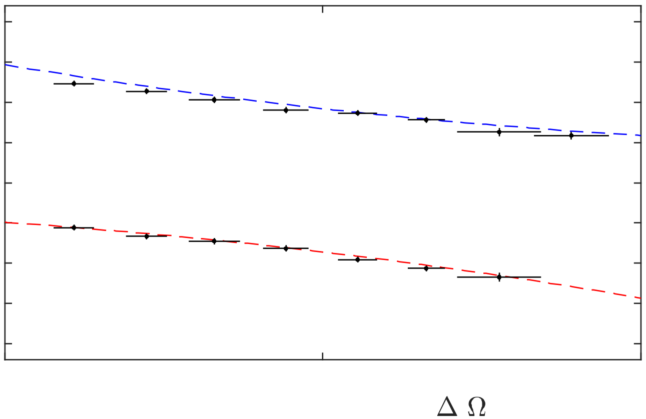
Rydberg-Dressed states

Bare atom

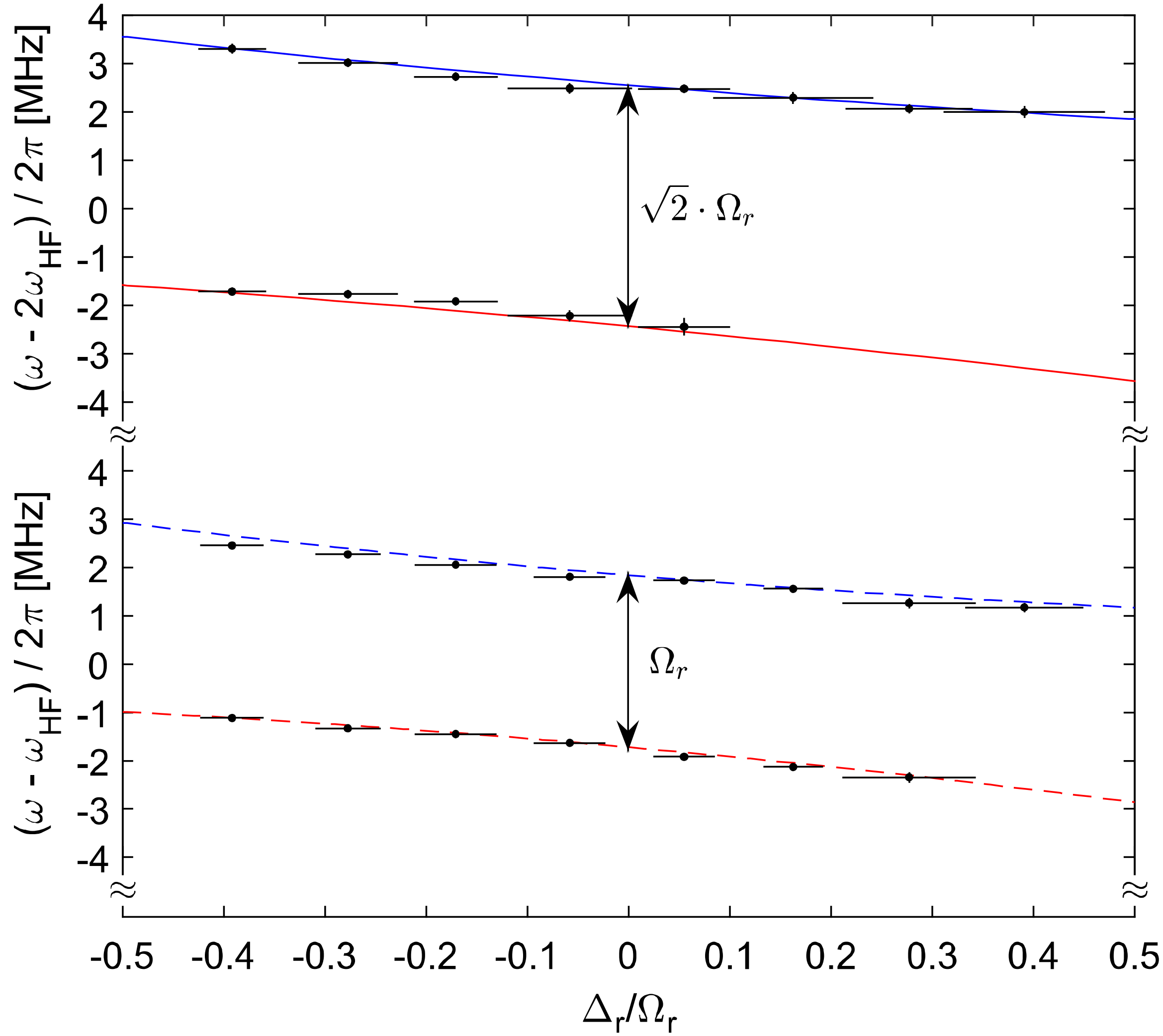


Dressed F=4 state



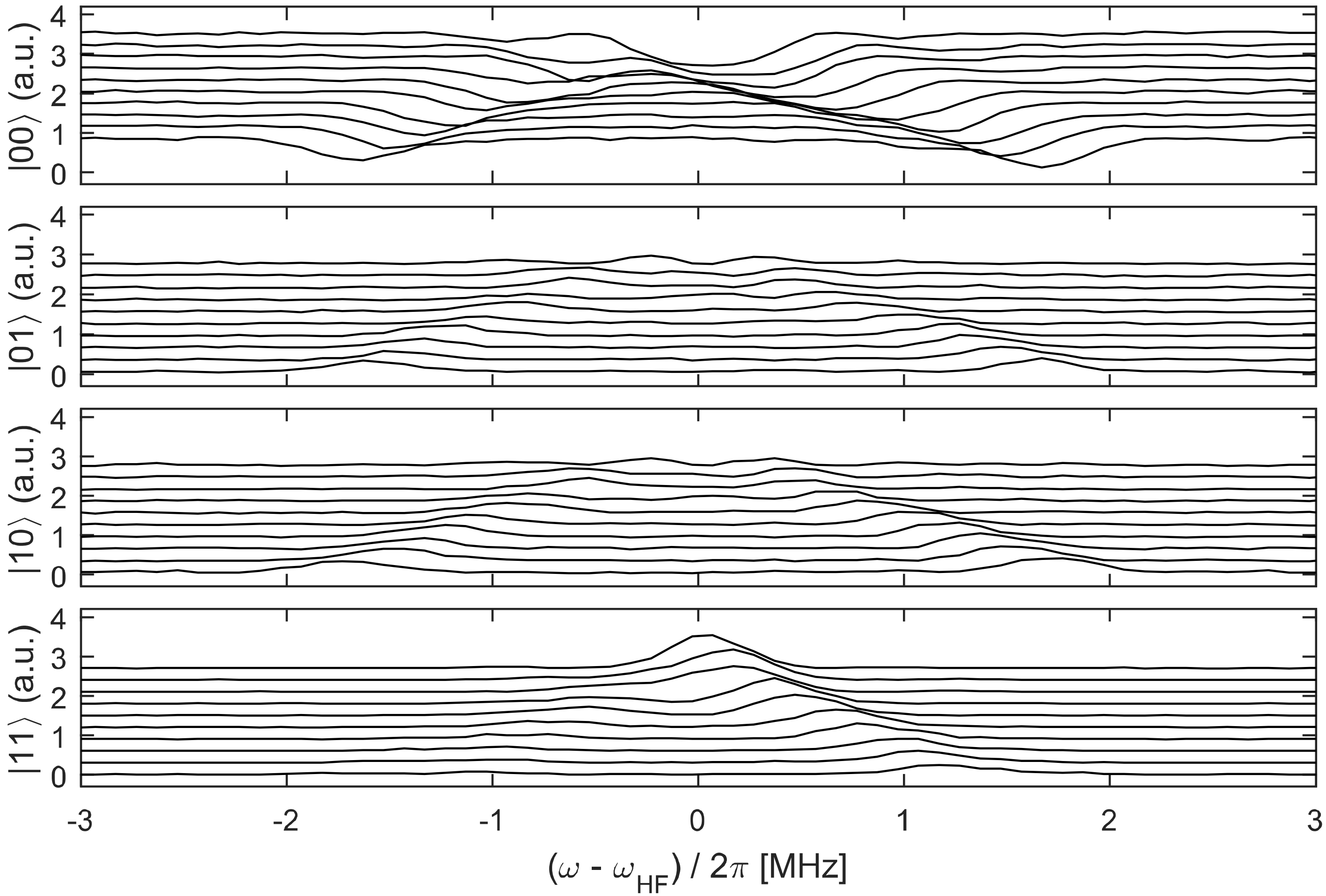


Rydberg-Dressed states under blockade

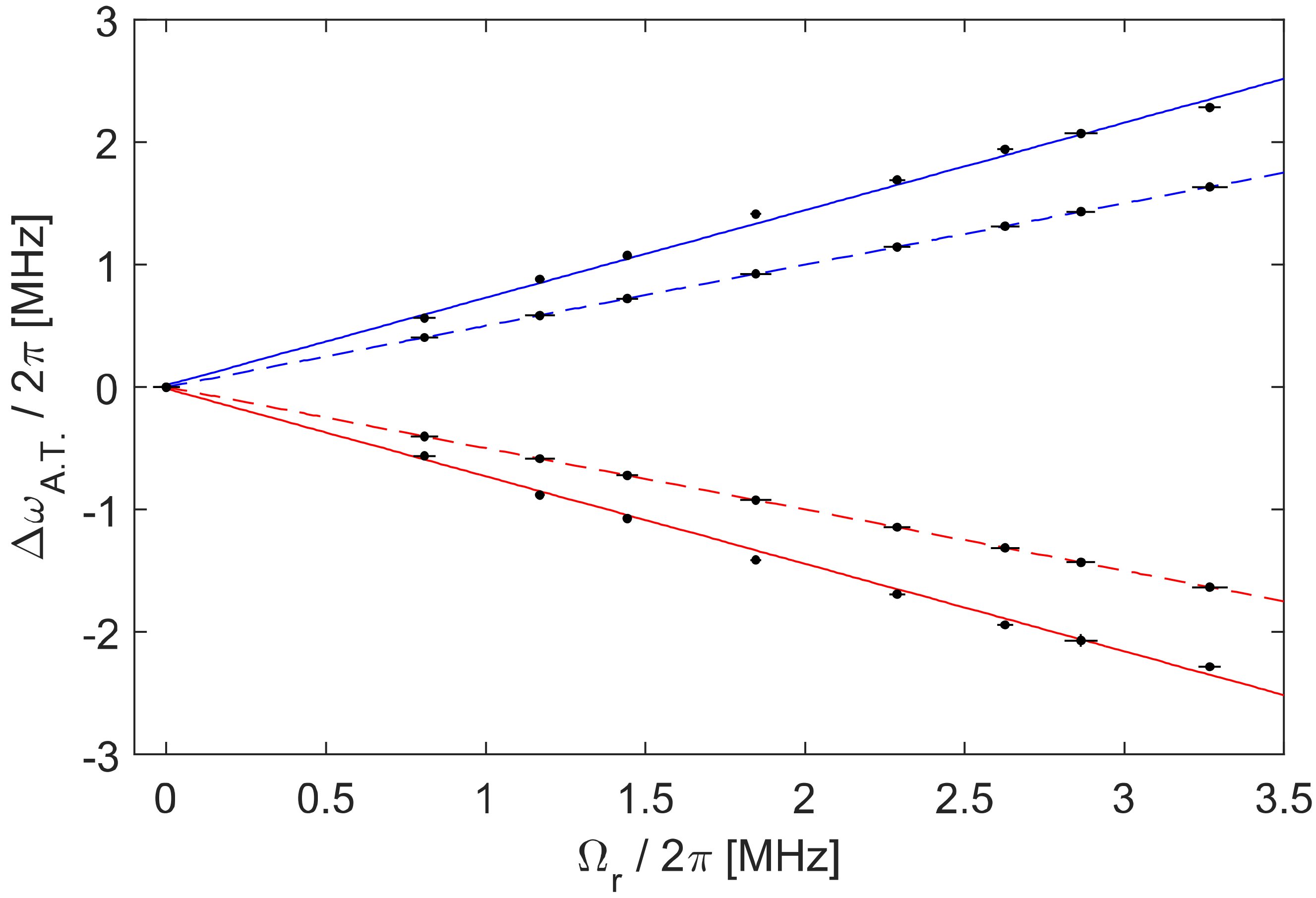


$\sqrt{N}$ dependence

Sandia National Laboratories



$\sqrt{N}$ dependence



Extending the JC ladder

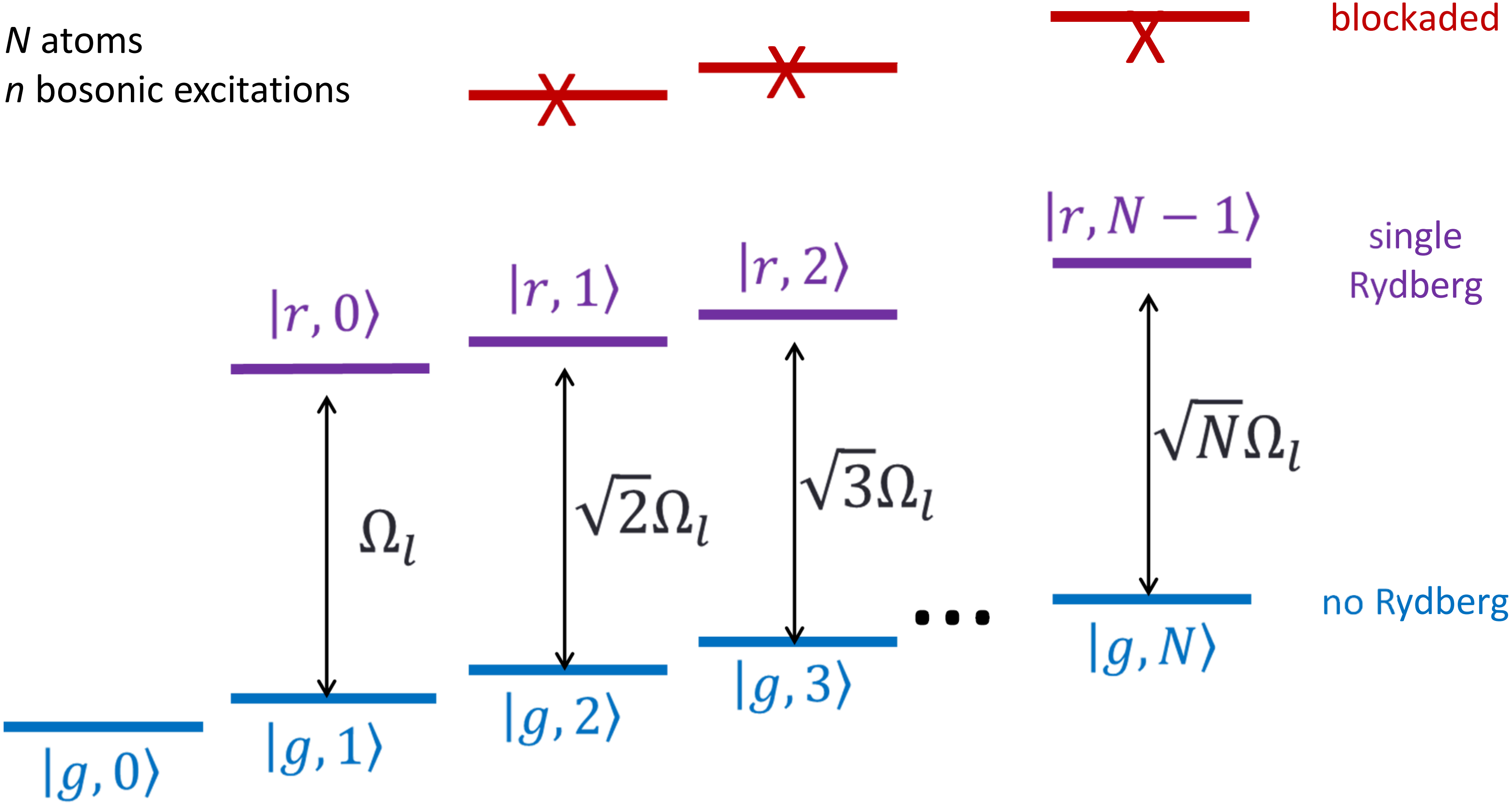
<u>notation</u>

$$|g, n\rangle \equiv \text{Sym.}\left(|0\rangle^{\otimes N-n}|1\rangle^{\otimes n}\right)$$

$$|r, n\rangle \equiv \text{Sym.}\left(|r\rangle|0\rangle^{\otimes N-n-1}|1\rangle^{\otimes n}\right)$$

N atoms

n bosonic excitations



Extending the JC ladder

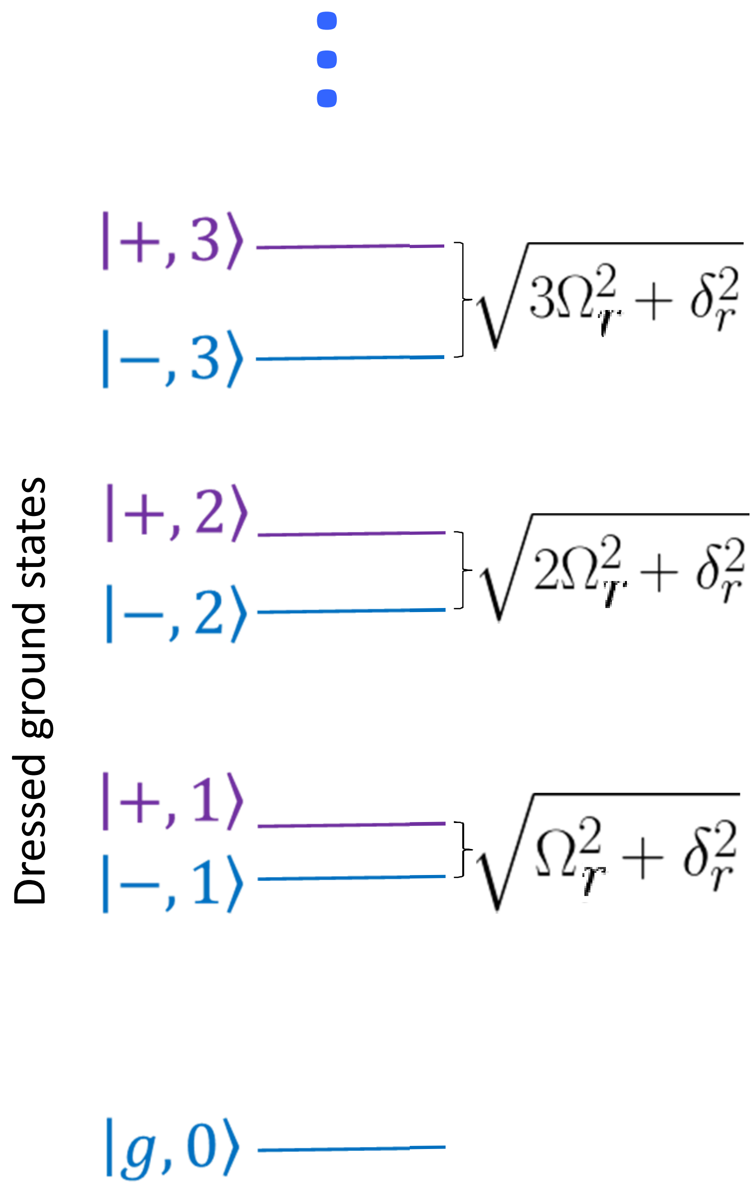
notation

$$|g, n\rangle \equiv \text{Sym.}\left(|0\rangle^{\otimes N-n}|1\rangle^{\otimes n}\right)$$

$$|r, n\rangle \equiv \text{Sym.}\left(|r\rangle|0\rangle^{\otimes N-n-1}|1\rangle^{\otimes n}\right)$$

N atoms

n bosonic excitations



Isomorphism with Jaynes-Cummings

Hyperfine splitting (energy per 1)

Rydberg detuning (energy for Rydberg excitation)

Rydberg laser power

Rydberg: $$\hat{H} = E_{HF}\hat{a}_1^\dagger \hat{a}_1 + \frac{\delta_r}{2}\sigma_z + \frac{\Omega_r}{2}\left(\hat{a}_1\sigma_+ + \hat{a}_1^\dagger \sigma_-\right)$$

Cavity JC: $$\hat{H} = \omega \hat{a}^\dagger \hat{a} + \frac{\omega_0}{2}\sigma_z + \frac{\Omega_r}{2}\left(\hat{a}\sigma_+ + \hat{a}^\dagger \sigma_-\right)$$

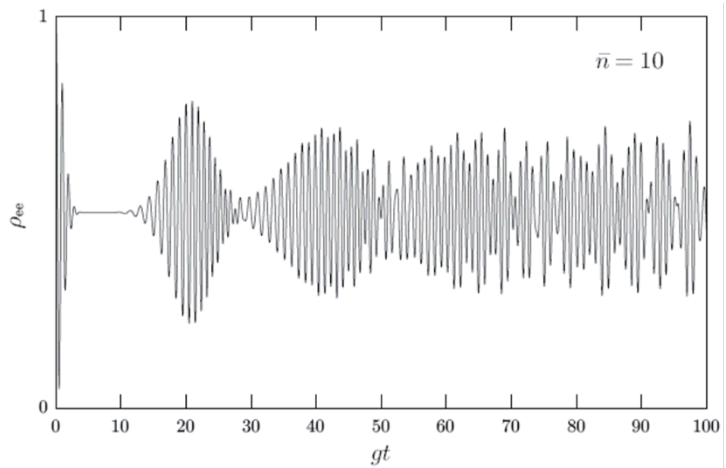
Cavity frequency (energy per photon)

Qubit splitting (energy for spin excitation)

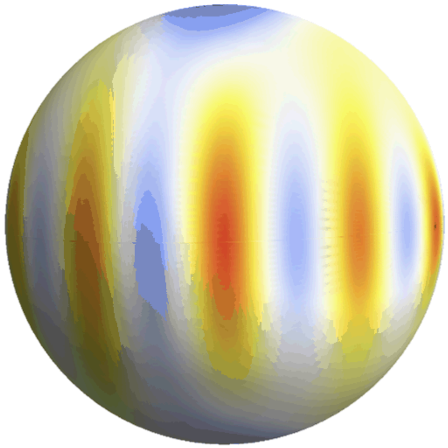
Cavity coupling strength

Controlling the System

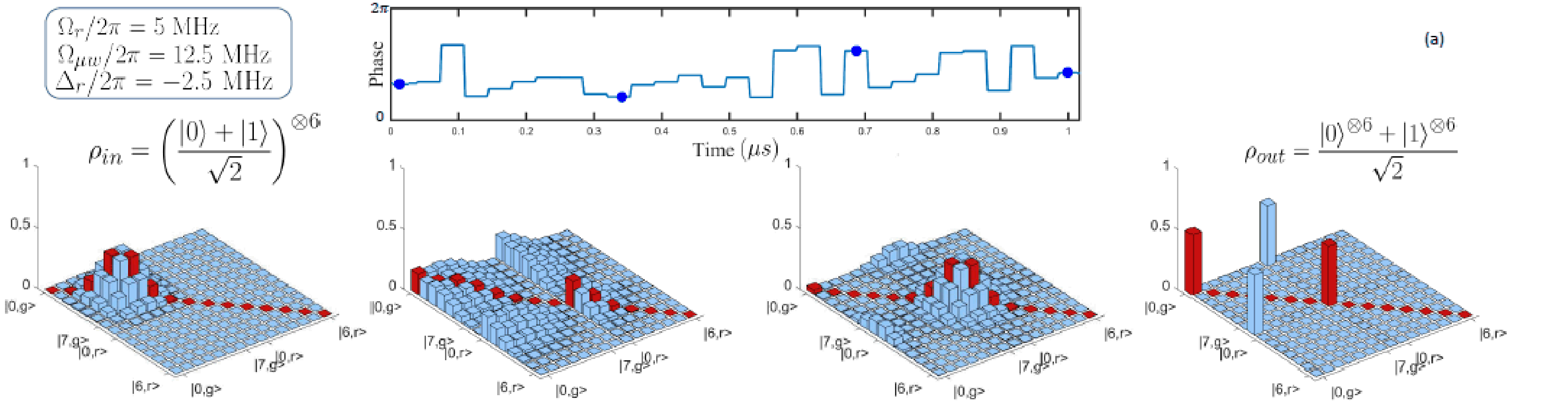
- Port over entanglement protocols from other JC platforms
  - Collective spin squeezing: Tomáš Opatrný and Klaus Mølmer. *Phys. Rev. A* **86**, 023845 (2012)
  - Collapse and revival: I. I. Beterov *et al. Phys. Rev. A* **90**, 043413 (2014)



from D.A. Steck, *Quantum and Atom Optics*



Arbitrary control

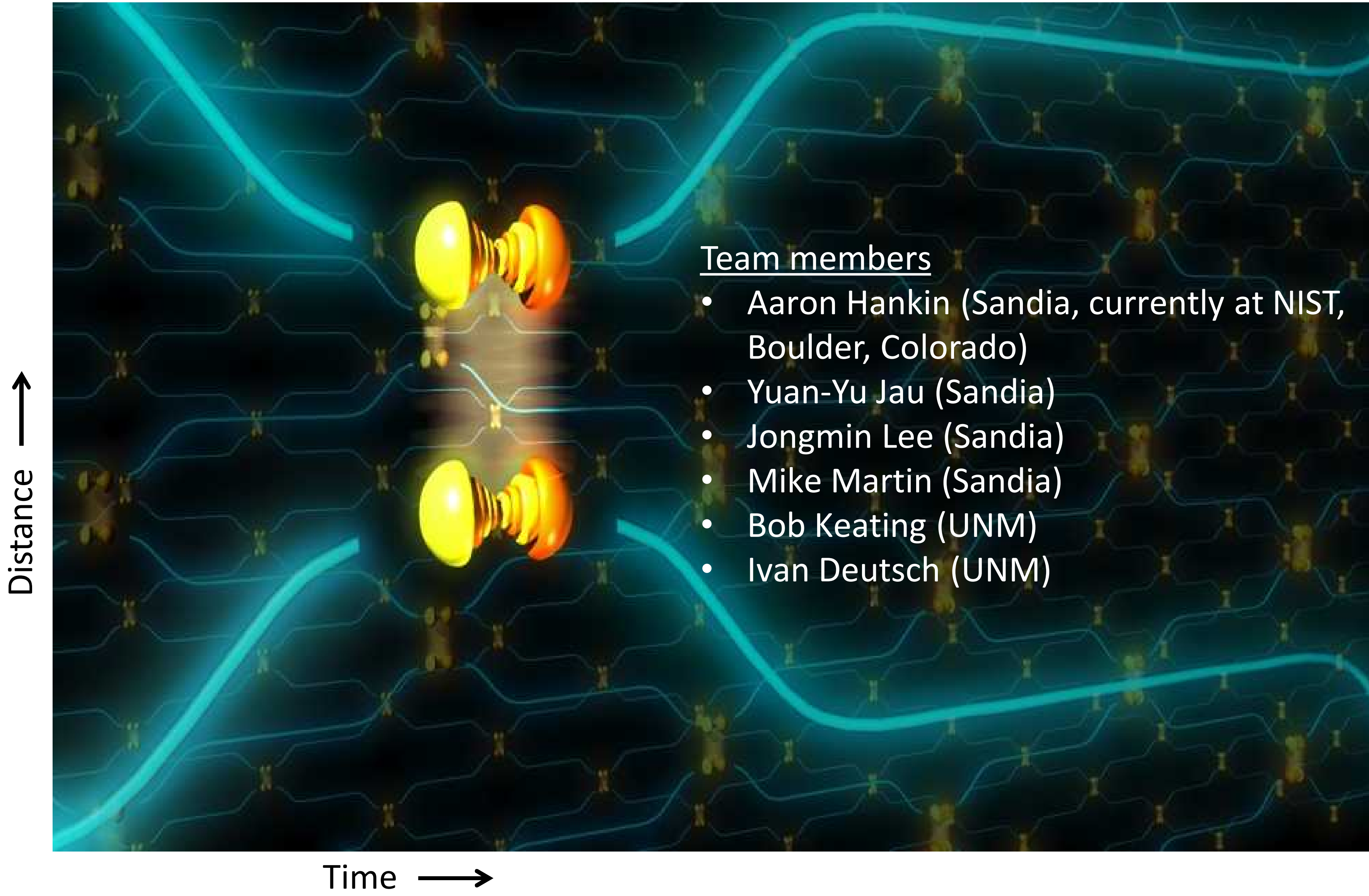


$$\hat{H}_c[\phi_{\mu w}(t)] = \frac{\hbar\Omega_{\mu w}}{2}\sum_{i=1}^{N}\left(e^{-i\phi_{\mu w}(t)}|1\rangle_i\langle 0| + e^{+i\phi_{\mu w}(t)}|0\rangle_i\langle 1|\right) = \hbar\Omega_{\mu w}(\cos[\phi_{\mu w}(t)]\hat{J}_x + \sin[\phi_{\mu w}(t)]\hat{J}_y)$$

control
$\phi_{\mu w}(t)$

The nonlinearity of the JCM, together with externally applied fields makes the system fully controllable on the whole Hilbert space, i.e., we can generate an arbitrary superposition state.

Thank you!



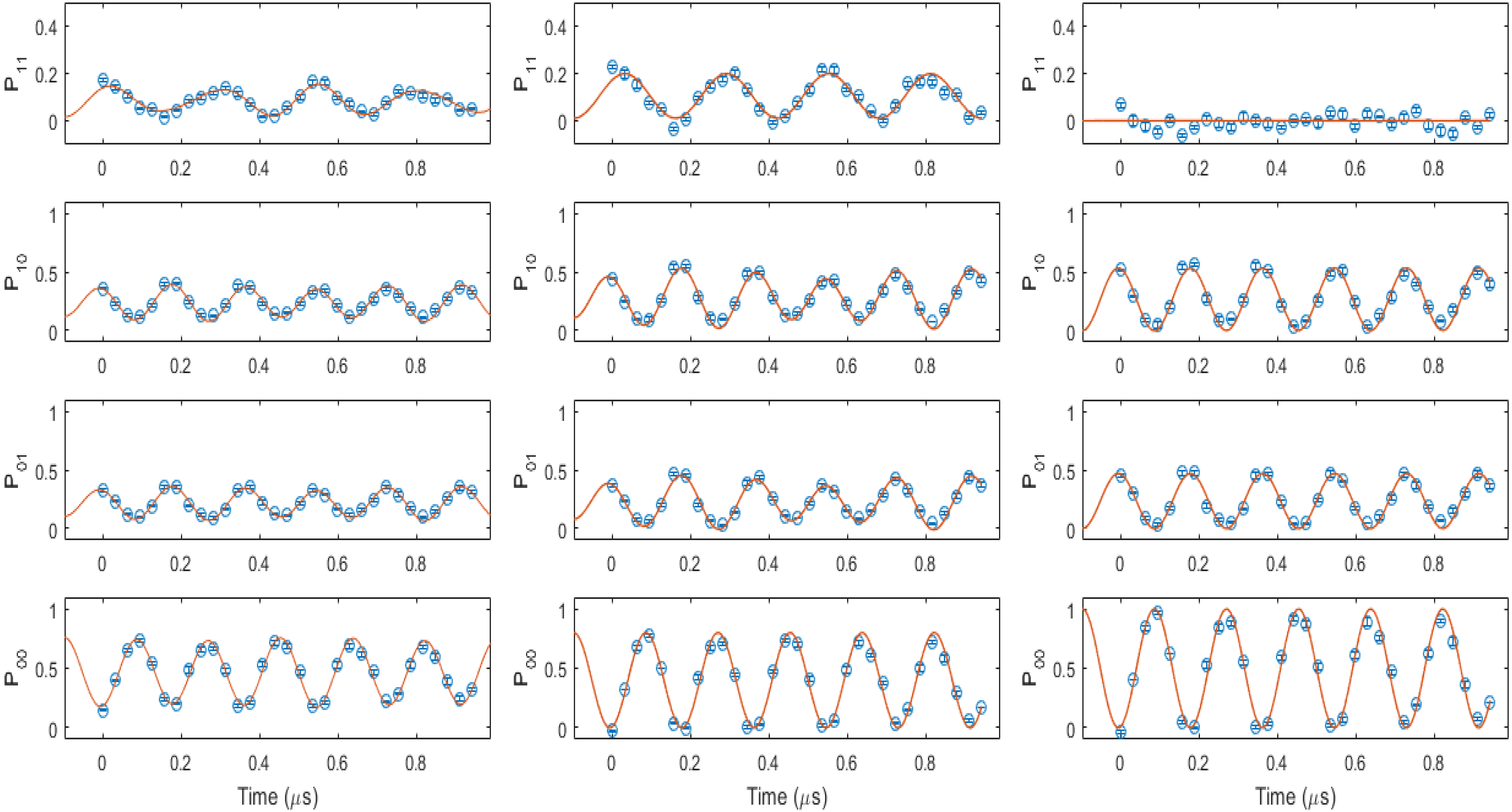
Team members

- Aaron Hankin (Sandia, currently at NIST, Boulder, Colorado)
- Yuan-Yu Jau (Sandia)
- Jongmin Lee (Sandia)
- Mike Martin (Sandia)
- Bob Keating (UNM)
- Ivan Deutsch (UNM)

Raw data and fit

Correct for state detection errors

Remove contributions from all non two-atom loading events and re-normalize



Two-qubit microwave resonances