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Multigrid framework for multiphysics problems

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What this is all about...
A new software framework allows to flexibly assemble multigrid preconditioners for strongly coupled monolithic multiphysics problems.

Motivation & Facts

- Due to the increasing multiphysics coupling, segregated solvers are often no longer sufficient
- Need for efficient monolithic iterative solvers
- Algebraic Multigrid methods among most efficient preconditioning techniques

Challenges

- Design of application-specific monolithic multiphysics preconditioners generally complex and difficult
- Implementation of Algebraic Multigrid preconditioners for monolithic multiphysics problems extremely hard and expensive

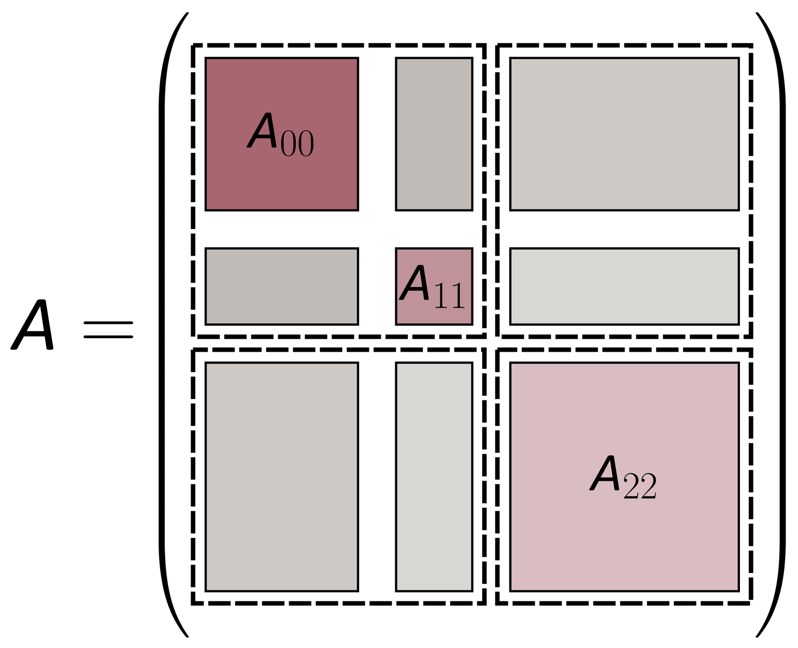
Solution approach

Algebraic multigrid framework for monolithic multiphysics problems

- flexible modular design allows adaptions to concrete applications
- use existing well-tested algorithms instead of reimplementing methods from scratch

Multiphysics problems

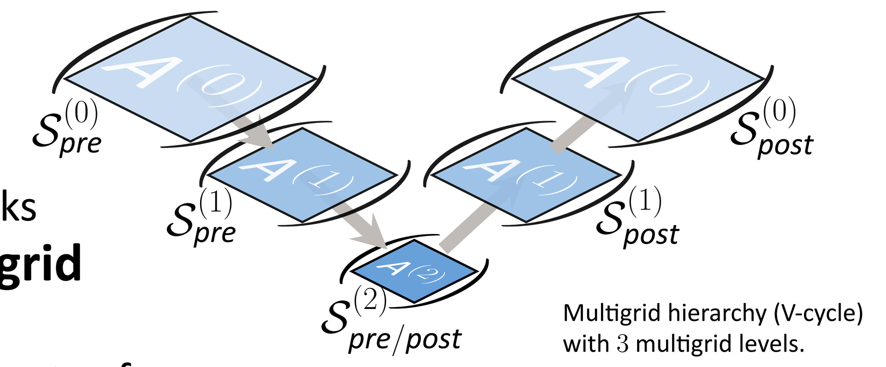
- **Multiphysics problems** consist of
  - Several equation sets describing the different physics/constraints
  - Coupling of the equation sets
- **Algebraic representation:** blocked operators containing the equation sets and the coupling
- **Hierarchical relationship** of equation sets through nesting blocked operators
- **Solution appraoch:** Iterative linear solver with block preconditioners (physics-based)



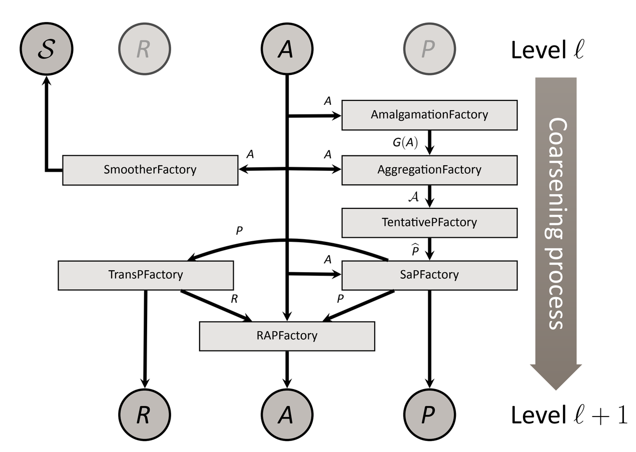
Monolithic blocked 3×3 operator consisting of a nested 2×2 blocked operator, a 1×1 operator A_{22} and the corresponding 2×1 and 1×2 off-diagonal blocks. This operator design allows to use nested block smoothers (e.g., Schur-complement approach for the inner 2×2 blocked operator with an outer Block Gauss-Seidel loop).

Multigrid framework

- **Software framework:**
  - Flexible definition of application-specific preconditioners
  - Reuse existing well-tested building blocks
- **MueLu: the next-generation multigrid framework in Trilinos**
  - Software design based on modern concepts of software architecture (factory pattern, prototype class, facade classes,...)
  - Description of preconditioner layout through XML files using existing ready-to-use building blocks



Multigrid hierarchy (V-cycle) with 3 multigrid levels.



Exemplary dependency tree of algorithmic factories in MueLu to produce the transfer operators P and R, the multigrid level smoother $\mathcal{S}$ and the coarse level matrix A for level $\ell+1$ using the fine level matrix A from level ℓ. This coarsening process can be repeated to set up the full multigrid hierarchy.

Multigrid framework for monolithic multiphysics problems

General idea:

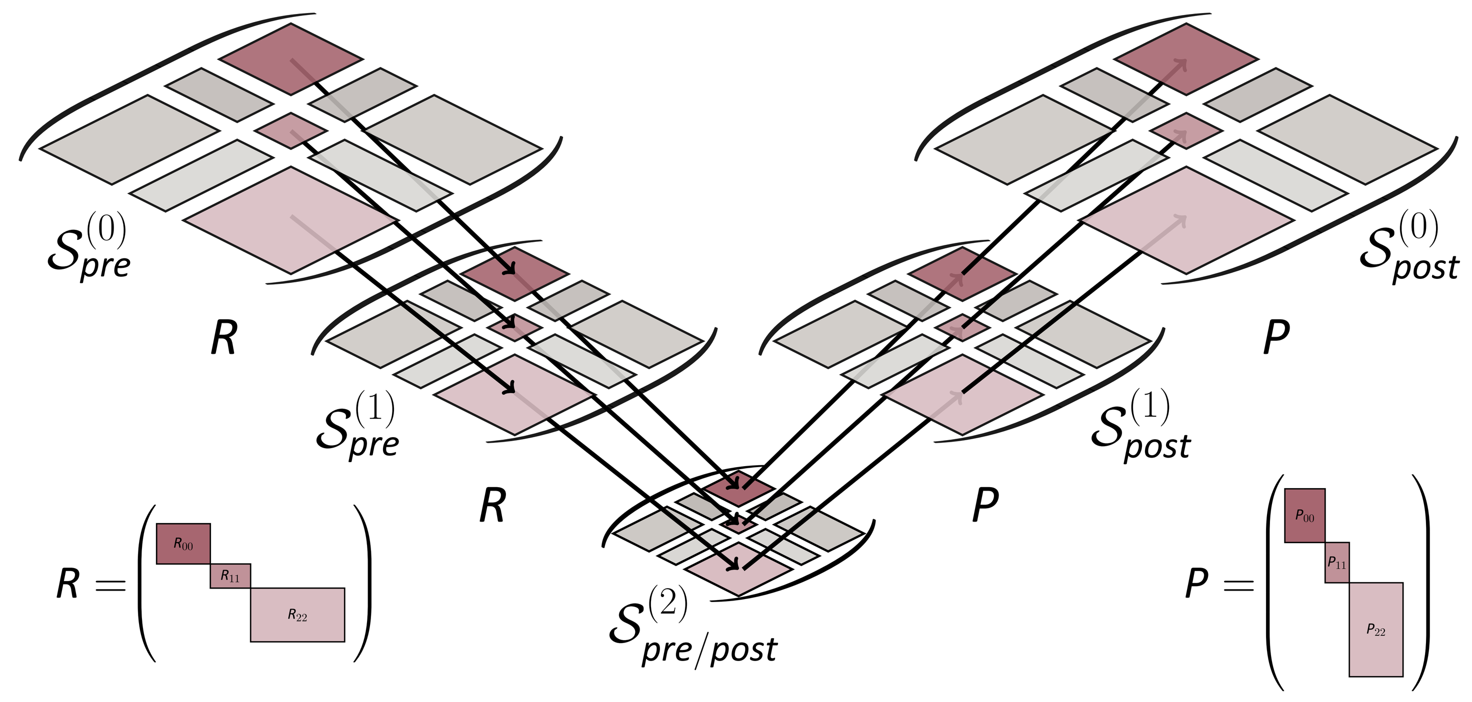
- Apply monolithic multigrid to full monolithic multiphysics problem
- Preserve the block structure on all multigrid levels

Multigrid components:

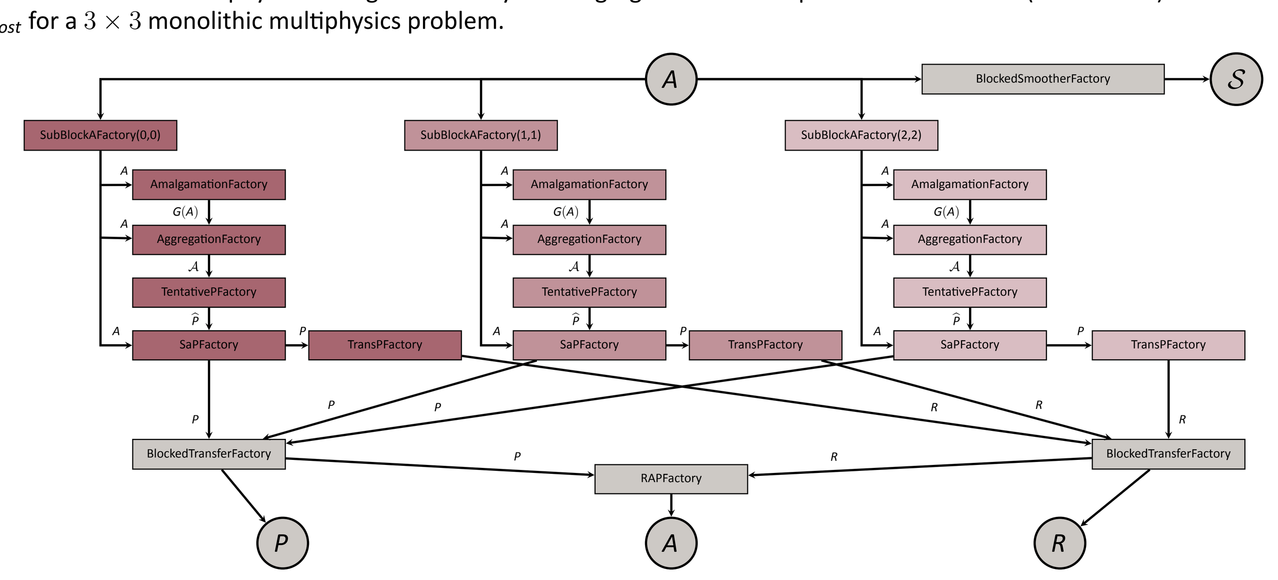
- Segregated transfer operators
  - Preserve block structure on all multigrid levels
  - Sub-problem specific transfer operators
- Block level smoothers
  - Problem-specific field coupling methods
  - Nested block smoothers

Software framework:

- Define preconditioner layout through flexible XML format
  - Use standard building blocks to define segregated transfer operators
  - Provide easy access to (nested) block smoothers (e.g., Block-Jacobi, Block Gauss-Seidel, Simple, ...)
- Extensible modular framework



Three-level monolithic multiphysics multigrid hierarchy with segregated transfer operators P and R and (non-nested) block smoothers $\mathcal{S}_{pre}$ and $\mathcal{S}_{post}$ for a 3×3 monolithic multiphysics problem.

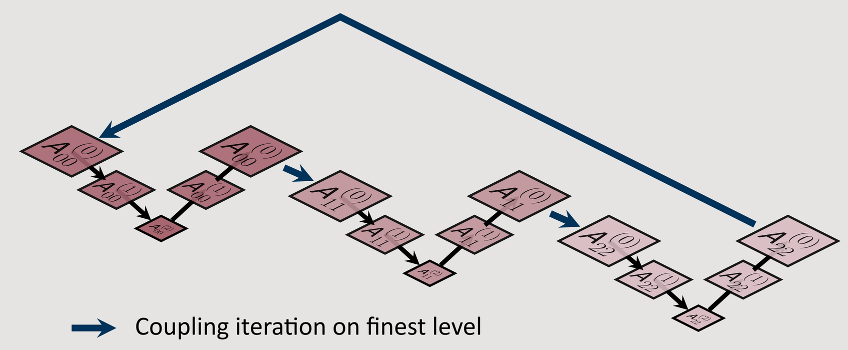


Exemplary factory layout for defining segregated transfer operators between multigrid level ℓ and $\ell+1$ for a 3×3 monolithic multiphysics problem. Each sub-problem has its own pipeline to generate P_{ii} and R_{ii} ($i\in\{1,2,3\}$) consisting of standard building blocks.

Alternative approach:

Segregated multigrid method:

- Use an outer coupling iteration (e.g., Block Jacobi, Block GS,...)
- Solve sub-problems sequentially using multigrid



Advantages of monolithic multigrid:

- Explicit multiphysics coupling on all multigrid levels through level smoothers
  - ⇒ Faster convergence
  - ⇒ Increased stability
- Inherently scalable approach rather than sequential solution of segregated sub-problems

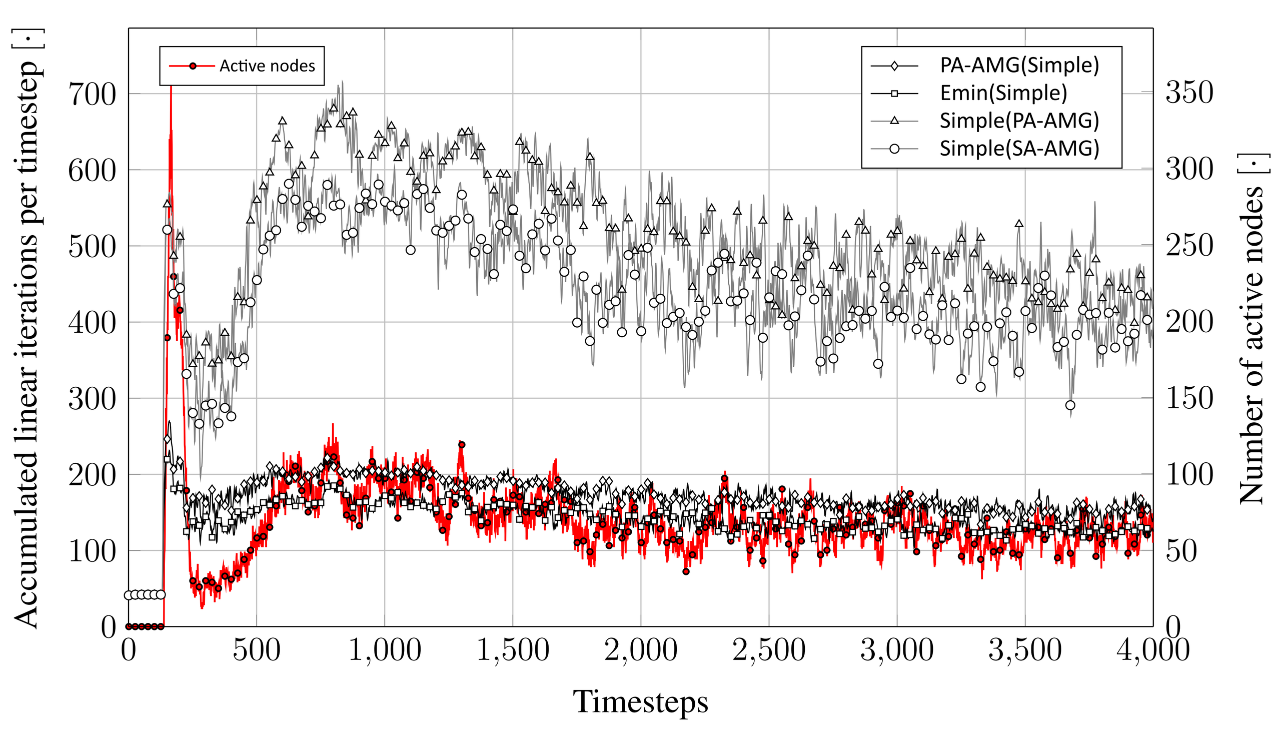
Example: Contact mechanics

Problem description:

- Nonlinear elastodynamics + contact constraints (Lagrange multipliers)
- 4,000 time steps with approx. 22,000 linear systems to solve
- 2×2 saddle-point problems of size $>300{,}000$ DOFs
- GMRES + segregated or monolithic preconditioning (3-level AMG)

Findings:

- Significant reduction of linear iterations with monolithic coupling
- Monolithic coupling nearly independent of number of active nodes
- Setup costs for monolithic multigrid same as for segregated approach



Information

Find the poster and more information here:

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