*A Sparse Quadrature Approac*SAND2016-6234C
Optimization in Power Grid Models

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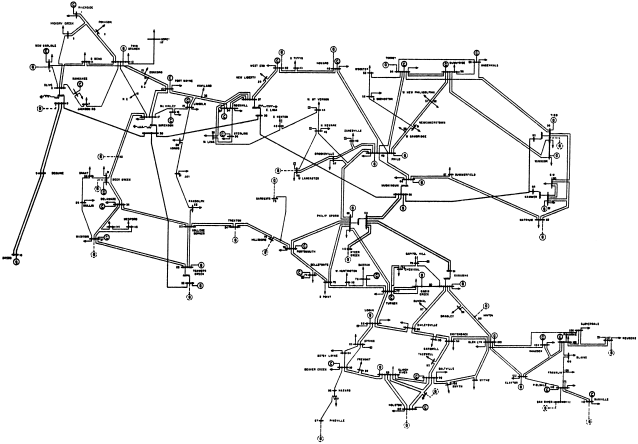
Livermore, CA & Albuquerque, NM, USA

International Conference on Stochastic Programming
Buzios, Brazil

Thanks: Sandia LDRD Program

SAND2016-xxxx

Electric Grid Operations



IEEE 118-bus Model

- Integrate renewable generators in the system
- Propose efficient methods to treat uncertainties for Stochastic Economic Dispatch algorithms

Efficient Operation

- **Unit Commitment/Economic Dispatch** (UC/ED): schedule thermal generating units to minimize overall production costs
  - satisfy forecasted demand for electricity; employ reserve margins to ensure that sufficient capacity is available in case demand is higher
  - respect constraints on both transmission (e.g., thermal limits) and generator infrastructure
- **Stochastic UC/ED model**: typically minimize the expected cost across load scenarios, thus ensuring sufficient flexibility to meet a range of potential load realizations during operations.
  - reliance on reserve margins is reduced, yielding less costly solutions than deterministic UC/ED
  - computationally difficult due to the *large number of samples* needed to achieve “converged” solutions

Outline

- Stochastic Economic Dispatch
- Uncertainties in Wind Power Generation
  - Low-dimensional Representation of Uncertainties in Wind Power via Karhunen-Loeve Expansion (KLE)
- Use Polynomial Chaos Expansion to represent optimal cost
  - Accuracy of Polynomial Chaos Representations
  - Computational Saving Compared to Traditional Approaches
- Summary

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Stochastic Economic Dispatch

$$Q(\boldsymbol{x}, \boldsymbol{\xi}(\omega)) = \min_{\boldsymbol{f},\boldsymbol{p}\geq\boldsymbol{0},\boldsymbol{q}\geq\boldsymbol{0},\boldsymbol{\theta}} \quad \sum_{t\in T}\sum_{g\in G} c_g^P(p_g^t) + \sum_{t\in T}\sum_{i\in N} Mq_i^t$$

s.t.

$$\sum_{r\in R_i} p_r^t(\boldsymbol{\xi}(\omega)) + \sum_{g\in G_i} p_g^t + \sum_{e\in E_{.i}} f_e^t - \sum_{e\in E_{i.}} f_e^t = D_i^t(\boldsymbol{\xi}(\omega)) - q_i^t,$$

$$B_e(\theta_i^t - \theta_j^t) - f_e^t = 0, \quad \forall e = (i,j), t$$

$$\underline{F}_e \leq f_e^t \leq \overline{F}_e, \quad \forall e, t$$

$$\underline{P}_g x_g^t \leq p_g^t \leq \overline{P}_g x_g^t, \quad \forall g, t$$

$$p_g^t - p_g^{t-1} \leq R_g^u x_g^{t-1} + S_g^u(x_g^t - x_g^{t-1}) + \overline{P}_g(1 - x_g^t), \quad \forall g, t$$

$$p_g^{t-1} - p_g^t \leq R_g^d x_g^t + S_g^d(x_g^{t-1} - x_g^t) + \overline{P}_g(1 - x_g^{t-1}), \quad \forall g, t$$

Consider uncertain renewables $p_r^t(\boldsymbol{\xi}(\omega))$ and demand $D_i^t(\boldsymbol{\xi}(\omega))$.

Stochastic Unit Commitment

$$\min_{\boldsymbol{x}} \quad c^u(\boldsymbol{x}) + c^d(\boldsymbol{x}) + \overline{Q}(\boldsymbol{x})$$
$$\text{s.t.} \quad \boldsymbol{x} \in \mathcal{X},$$
$$\boldsymbol{x} \in \{0,1\}^{|G|\times|T|}$$

- G and T: index sets of generating units and time periods
- $\mathcal{X}$ and $\boldsymbol{x}$: set of unit commitment constraints and vector of unit commitment decisions
- $c^u(\boldsymbol{x})$ and $c^d(\boldsymbol{x})$: generating unit start-up and shut-down costs
- $\overline{Q}(\boldsymbol{x})$: the expected generation cost

Classical approach, compute

$$\overline{Q}(\boldsymbol{x}) = \langle Q(\boldsymbol{x},\boldsymbol{\xi})\rangle \approx \frac{1}{|\mathcal{S}|}\sum_{s=1}^{|\mathcal{S}|} Q(\boldsymbol{x},\boldsymbol{\xi}_s)$$

using a finite number of renewable generation and load realizations (i.e., scenarios) $s \in \mathcal{S}$

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Random Fields (RFs)

- A random field $W(x, \omega)$ is a function on a product space $D \times \Omega$
  - a random variable (RV) at any $x \in D$
  - an infinite dimensional object
- In many physical systems, uncertain field quantities, described by RFs, have an underlying *smoothness* due to correlations
  - Can be represented with a small number of stochastic degrees of freedom
- ℓ_2-Optimal representation – second-order statistics
  - Karhunen-Loève expansion (KLE)

Random Fields Representation – KLE

- KLE for a RF with a continuous covariance function

$$W(x,\omega) = \mu(x) + \sum_{k=1}^{\infty} \sqrt{\lambda_k} f_k(x)\xi_k(\omega)$$

- $\mu(x)$ is the mean of $W(x,\omega)$ at x
- λ_k and $f_k(x)$ are the eigenvalues and eigenfunctions of the covariance

$$\Sigma(x_1,x_2) = \langle [W(x_1,\omega) - \mu(x_1)][W(x_2,\omega) - \mu(x_2)] \rangle$$

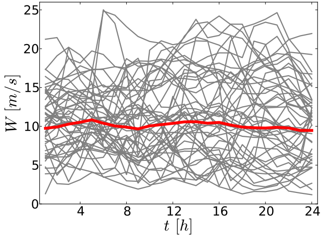
- The ξ_k are uncorrelated zero-mean unit-variance RVs

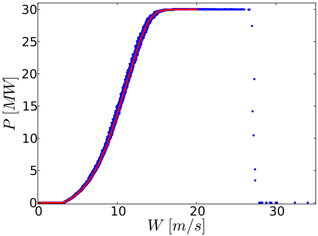
$$\xi_k(\omega) = \frac{1}{\sqrt{\lambda_k}} \int_D W(x,\omega) f_k(x) \mathrm{d}x$$

Uncertainties in Wind Power Generation

Typical daily wind profiles

- site #15414 (NREL Western Wind Dataset)
- Jan 2004-2006 (93 days)



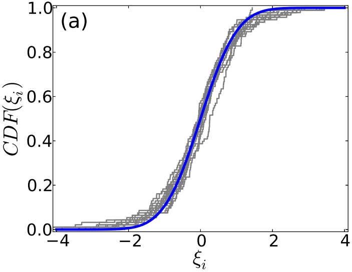


Rated power output at the same site

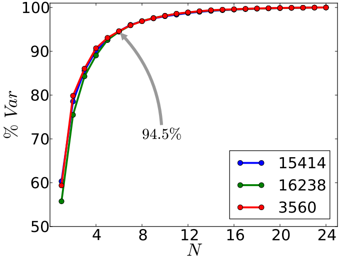
Low-dimensional Representation of Uncertainties in Wind Power via Karhunen-Loeve Expansion (KLE)

$$W_L(t,\omega) = \log(W) = \langle W_L(t,\omega)\rangle + \sum_{k=1}^{\infty} \sqrt{\lambda_k} f_k(t)\xi_k(\omega)$$

KLE RVs are approx. normally distributed



Reconstruct daily samples via truncated KLE



- Wind sites that are geographically close can employ the same stochastic coefficients for the main modes

Propose RF Model for Wind Predictions

- We examined the opportunity of using KLE to represent 24-h wind samples
  - random variables are quasi-normal
  - reduced dimensionality: approx. 6 modes are sufficient to represent 95% of variance
- Magnitude of variance in generated samples is consistent with historical data rather than with day-ahead weather forecasts

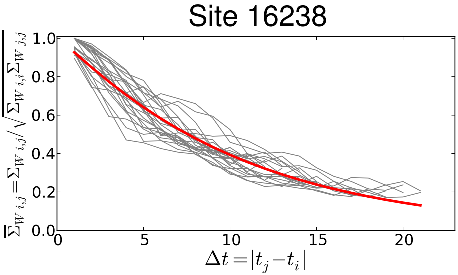
Proposed Model

- Fit normalized covariance models through wind data at each site
- Generate day-ahead wind forecasts and estimate associated uncertainties
- Scale covariance model with uncertainties in day-ahead forecasts and generate RF samples using KLE.

Extract Covariance Structure from Daily Wind Profiles

Propose a Matérn covariance model:

$$\overline{\Sigma}_W(\Delta t) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\frac{\sqrt{2\nu}\Delta t}{l_t}\right) K_\nu \left(\frac{\sqrt{2\nu}\Delta t}{l_t}\right)$$

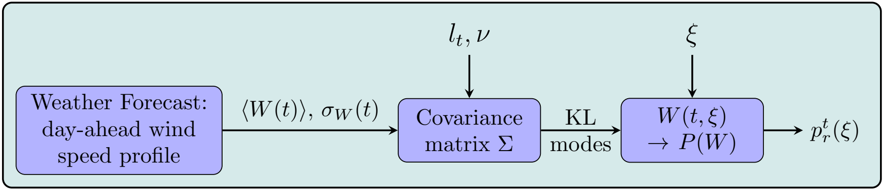


Estimate parameters via regression

Wind Site	l_t	ν
15414	11.40	0.56
16238	11.15	0.57
3560	9.79	0.78

- The covariance matrix parameters are similar for sites that are geographically close.

Generate Wind Power Realizations Consistent with Historical Data



- We employed data available for download from the Belgium Electricity Grid Operator ELIA
- Typical errors between predicted day-ahead wind power profiles and actual values are about $\sigma_P = 35\%$.
  - errors are independent of the time of the day.

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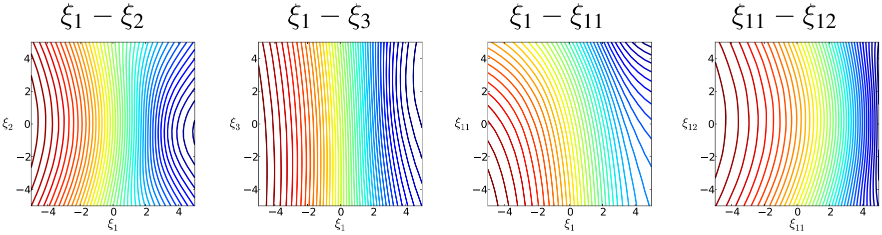
$\ldots$

- Need to compute $\langle Q(\boldsymbol{x},\boldsymbol{\xi})\rangle$ (and other higher moments) more efficiently and more accurately than MC aproaches.

Dependence of Q on ξ



- Weather forecast: $\langle W\rangle, \sigma_W \rightarrow p_r(\xi) \rightarrow p_g(\xi) \rightarrow Q(\xi)$
- Tests show a smooth dependence $\xi - Q$ for the range of uncertainties explored in this study
  - will pursue a Polynomial Chaos approximation for Q.

Accurate Representation of $Q(x, \xi)$ using Polynomial Chaos Expansions (PCE)

$$Q(\boldsymbol{x}, \boldsymbol{\xi}) \approx \sum_{k=0}^{P} c_k(\boldsymbol{x}) \Psi_k(\boldsymbol{\xi})$$

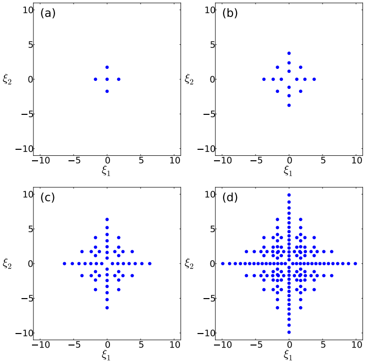
PCE coefficients are evaluated via Galerkin projection:

$$c_k(\boldsymbol{x}) = \frac{\langle Q \Psi_k \rangle}{\langle \Psi_k^2 \rangle} = \frac{1}{\langle \Psi_k^2 \rangle} \int_{\mathfrak{R}^n} Q(\boldsymbol{x}, \boldsymbol{\xi}) \Psi_k(\boldsymbol{\xi}) p(\boldsymbol{\xi})\, d\boldsymbol{\xi}.$$

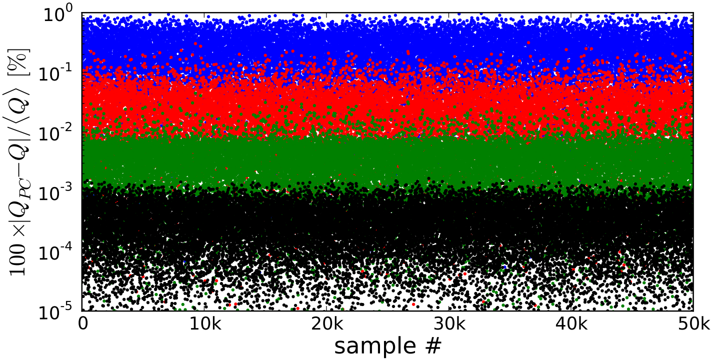
Given the PCE representation, the expected cost is:

$$\langle Q(\boldsymbol{x}, \boldsymbol{\xi}) \rangle = c_0$$

Compute PCE coefficients via sparse quadrature



Validate PCE Approximation

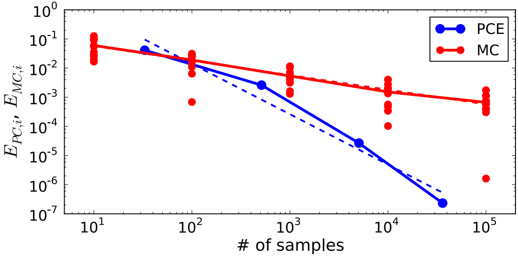


- IEEE 118-bus test system augmented with the three wind generators
- 16-dimensional PCE
- Tested 1st through 4th order approximations (shown with blue,red,green,black).

Expected Cost: PCE vs MC

$$E_{PC,i} = |c_{0,i} - c_{0,i+1}| / c_{0,i+1}$$

$$E^j_{MC,i} = |\overline{Q}^j_i - \overline{\overline{Q}}_{i+1}| / \overline{\overline{Q}}_{i+1}$$



- Relative error E_* is estimated with respect to the adjacent higher accuracy value of the same type.
- The PCE approach shows superior accuracy compared to MC for the same number of samples.

Summary

We present methods for efficient representation of uncertainty in power grids, with emphasis on the Stochastic Economic Dispatch (SED) problem.

- We model wind uncertainty via Karhunen-Loeve (KL) expansions.
  - Dimensionality of the stochastic space can be further reduced for wind sites that are geographically close.
- We represent the dependency of the SED solution on the uncertainty in renewables via Polynomial Chaos expansion (PCE) models.
  - For the examples considered here, if superior accuracy is needed, the PCE approach is significantly cheaper compared to MC

Future

- Build PCE's adaptively (adaptive quadrature, optimized quadrature rules)
- Extend formulation to UC/SED problem