On Applications of Game-Theoretic Probability and Defensive Forecasting to Agent-based Market Models

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Abstract

We present an attempt on connecting agent-based modeling with Game-Theoretic Probability (GTP), and defensive forecasting and outline a framework connecting elements of game-theoretic probability with agent-based models. We illustrate this framework on an example of our model of the Nasdaq stock market and of a natural gas market model, and show how game-theoretic probability can be used to test the simulated market price dynamics, the individual agent trading strategies, rule changes, and the overall agent-based model.

What Is Agent-Based Modeling?

- In ABMs, complex, real-world systems are represented in software as collections of *autonomous decision-making entities, situated in appropriate environment and interaction structure*.

  - Each type of agent executes behaviors appropriate to it and its context.

  - Agents thus produce, consume, trade securities, ship freight, and so forth. Agents are heterogeneous and variable, as in reality.

  - ABM simulations explicitly recognize that real world agents are not independent, but are affected by other agent behaviors, common information sources, and relevant social communities.

- The dynamics of systems *emerge* from large numbers of interactions among many kinds of agents. System behavior thus arises from the bottom up.

Market Maker - Investor Interaction

- Market makers: adjust their quotes

- Investors: submit market orders
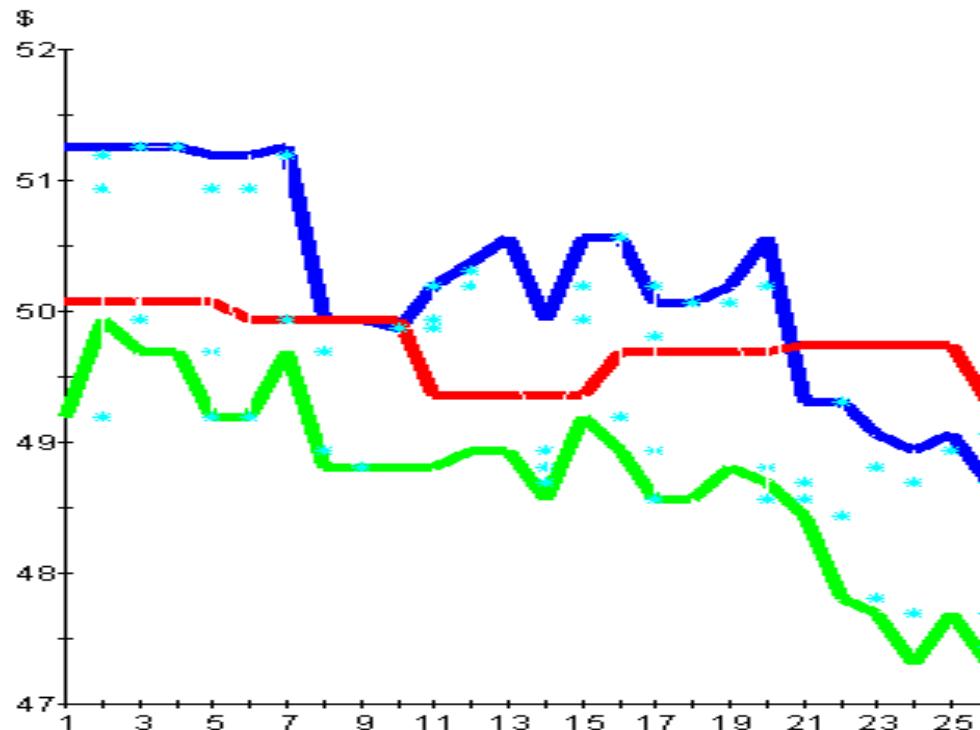
Advantages of Agent-Based Models

- Agent-Based Model's (ABM's) bottom-up approach leverages the organization's knowledge of the details

  - Allows modeling of individual behaviors, rules, incentives

  - Allows modeling of complex interactions and interaction structures

  - Allows incorporating of human expertise and AI techniques in building the market's agents

- Intuitive, concrete, easy to understand

- Produces actionable results and counter-intuitive insights at many levels

Advantages of ABMs(cont'd)

- ABMs and traditional statistical methods produce the same results when the assumptions required by traditional methods are valid (e.g. independence, etc.)

- Models can be validated using historical data, but can be applied to *unique situations* that lack history
  - Allows combining both a hindsight and foresight perspective

- Agents can be programmed to *evolve* and *learn*. This permits the emergence of new, unanticipated behaviors and strategies

- A variety of what-if scenarios can be investigated

Key Features of ABMs

- *Represent casual structure of the world*
- Can be calibrated against individual behaviors and global system behaviors
- Span micro- to macro- gap
- Allow representation of existing emergent properties of the system and of previously unobserved behaviors

Disadvantages of ABMs

- Difficult to calibrate and to validate
- Much of the data is missing (even if trades are observable, the past context is generally not)
- Complete information on individual strategies is rarely available
- Treating the available data as generated by a probabilistic mechanism is problematic:
  - Lack of data
  - Fundamental goal of ABMs is to model causal decisions of agents, based on unique conditions and contexts

Combining GTP with ABMs

GTP - conceptual, quantitative foundation for ABMs

- Forecaster: market, as combination of agent strategies
- Skeptic: one specific agent strategy

- Strategy development

  - Can an individual strategy generate abnormal returns?

- Rule changes:

  - Can individual strategy generate abnormal returns under new rules?
  - If so, the *new* system may not be stable. What are plausible new strategies? (learning)

- Defensive forecasting:

  - Can market (Forecaster) ensure no abnormal strategy returns?

Shaver and Vovk (2001), Takemura and Suzuki (2005), Wu and Shafer (2007), Shafer (2007), Vovk (2011)

Nasdaq Example

- Nasdaq had to consider decimalization and its impacts in 1998.

- How reducing the tick size may affect the market behavior? Why should it have any effect?
  - How a change to decimals can be modeled?
  - What is the mechanism through which changed tick size would affect the market?
  - Given specific mechanisms, what other effects may occur?

- Nasdaq decimalization study: an empirical example.
  - Study done during 1998-2000.
  - Decimalization occurred in April 2001.
  - *Darley and Outkin (2007)*

Goals

- Investigate effects of policy and environment changes:
    - Evaluate the effects of changing the tick size (decimalization) and of parasitism

- Represent the influence of market rules and structure on market dynamics and strategies

- Demonstrate that that simulated market participants and aggregate market parameters are "sufficiently similar" to those in the real world to validate model empirically

Nasdaq Model

- Agents: investors and market makers
  - Rule of thumb, data-driven, expert knowledge, learning, experimentation
- Market infrastructure and rules:
  - Tick size,
  - Priority rules
- Calibration: audit trail data
  - Ensure simulated distribution of trade sizes, volumes, prices is similar to that observed in the real world
  - Represent individual market maker strategies

Nasdaq Model Basics

- Single stock
- **Investors**
  - Receive noisy information on fundamental value / price target
  - Decide whether to trade by
    - Comparing this target with available price
    - Incorporating market trends …
- **Market makers**
  - Receive buy and sell orders
  - Must learn how to set their quotes profitably

Nasdaq Model and GTP

- Individual strategy testing
  - Basic strategy (reactive)
  - Parasitic strategies (active, undercuts)
  - Learning strategies (learns undercutting, …)
- Market testing
  - Does market allow abnormal returns?
  - Is market stable against specific strategies?

Individual Strategy: Parasitic

Parasitic strategy:

- Attempts to undercut the current bid/offer by a small increment (tick size)

- Is not a major source of liquidity for the market
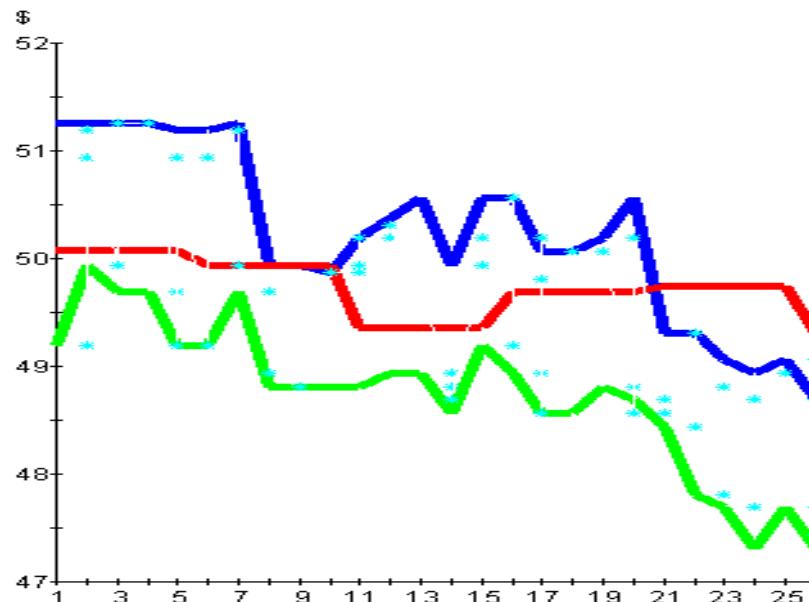
Undercutting (parasitic) Strategy
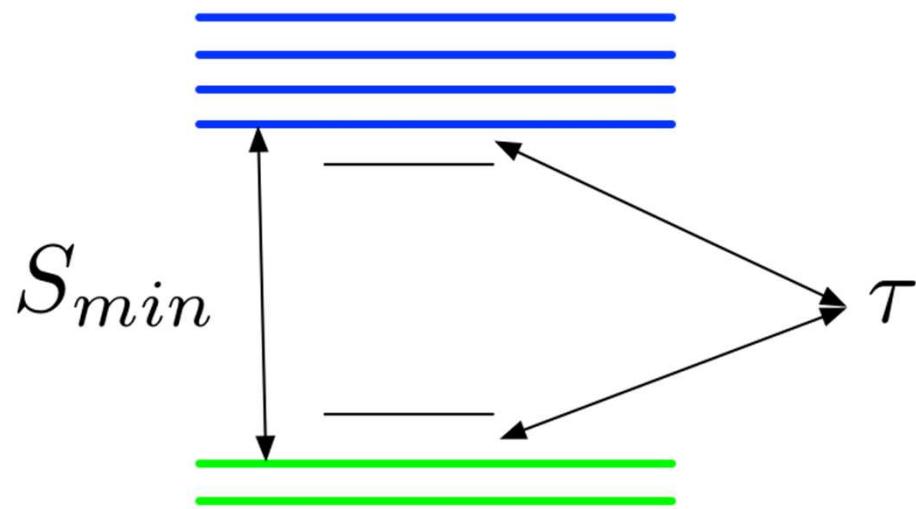
Contrived GTP Protocol: Parasitic vs. Basic Dealers

Protocol for Parasitic Strategy

$K_0 = 1$:

For $n = 1, 2, ...$:

$S_{min} >> \tau$

$n - 1$:

Skeptic: decide if undercut by 2τ and buy or sell one share

Market: move quotes by at most τ

n:

Skeptic: if undercutting successful

$n + 1$:

Skeptic: close the position by undercutting on other side by 2τ

$K_{n+1} = K_n - 1 + S_{min} - 4\tau$.

Simulated Strategies Testing
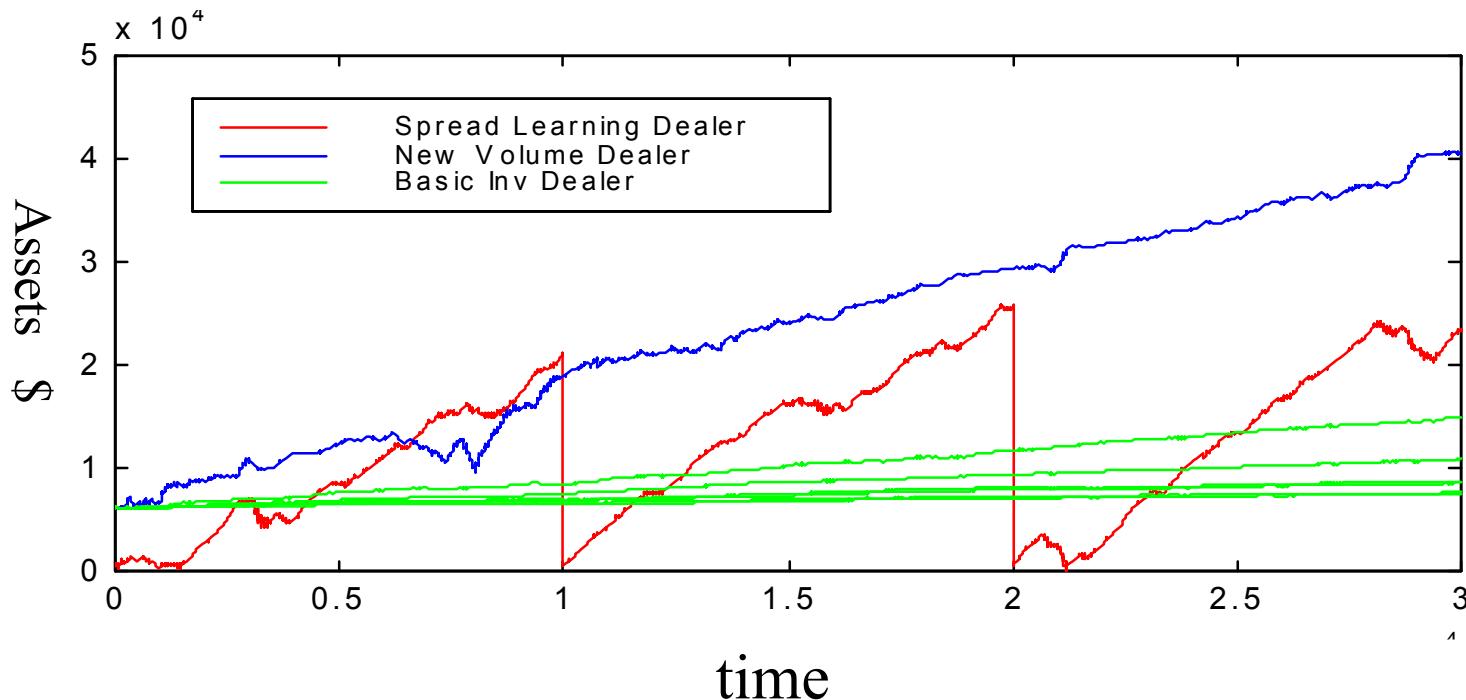
- Spread Learning market maker is the most profitable dealer on the market under many circumstances

- Known exceptions: *high volatility, many parasites*

Market Testing: Tick size effects

As tick size is reduced, parasitic strategies increasingly impede price discovery / market's ability to generate useful information

Standard Deviation of (Price - True Value)
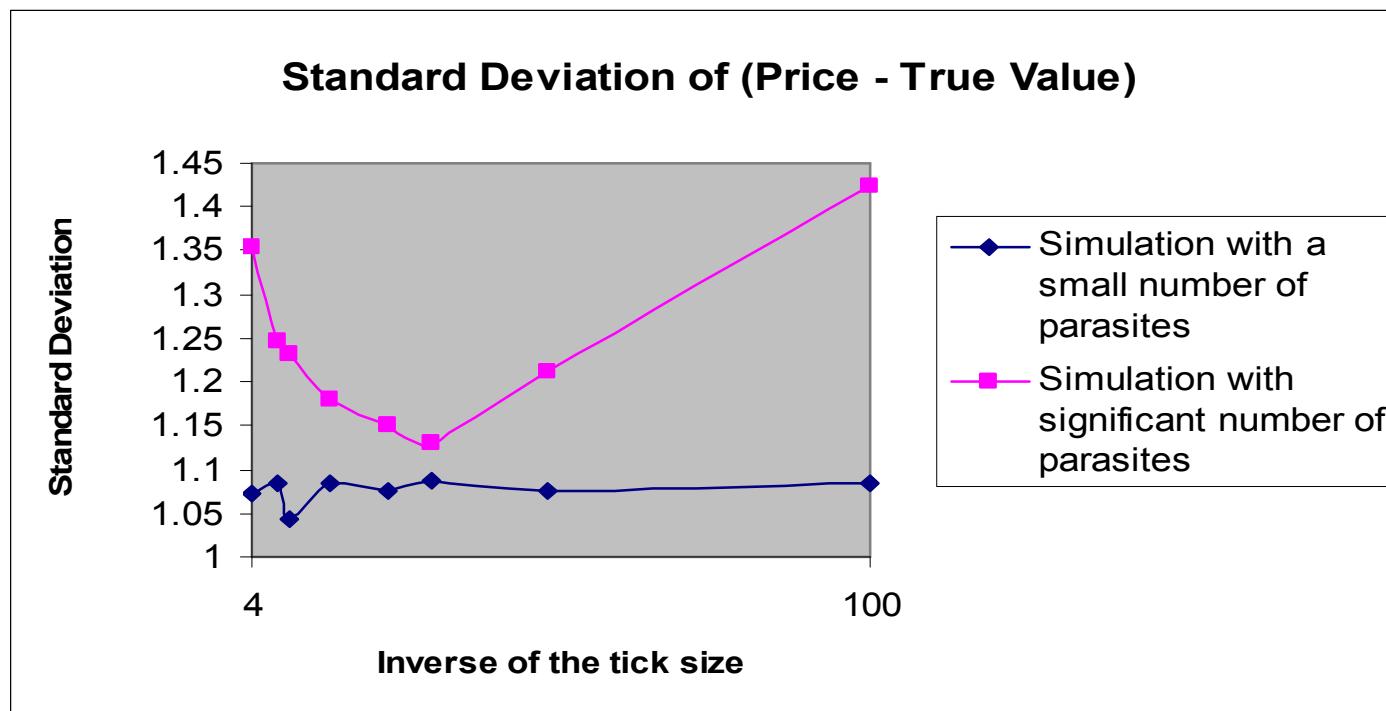
Tick Size Effects, Many Parasites
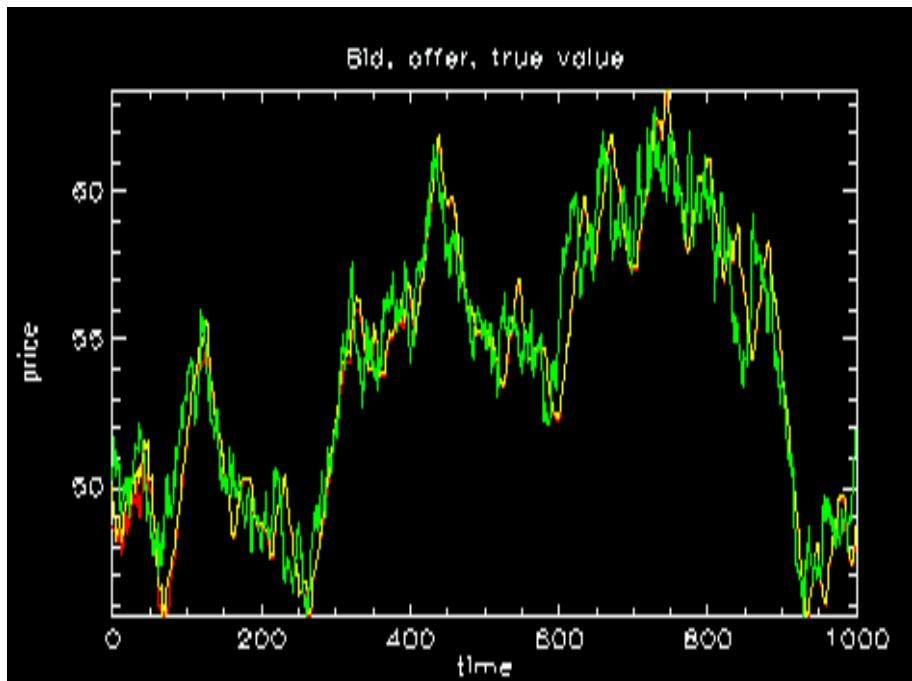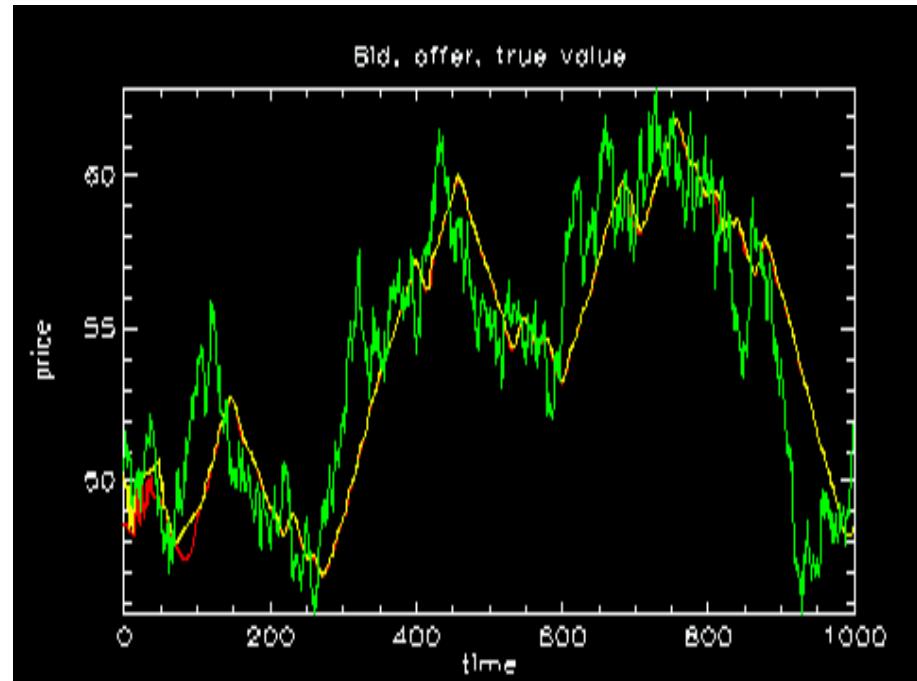
Tick size 1/16   Tick size 1/100

Market Testing: Strategy Phase Transitions, Tick Size Reduction

Market Maker Behavior Regression Coefficients

Blue dots - May data
Red dots - June data   1997
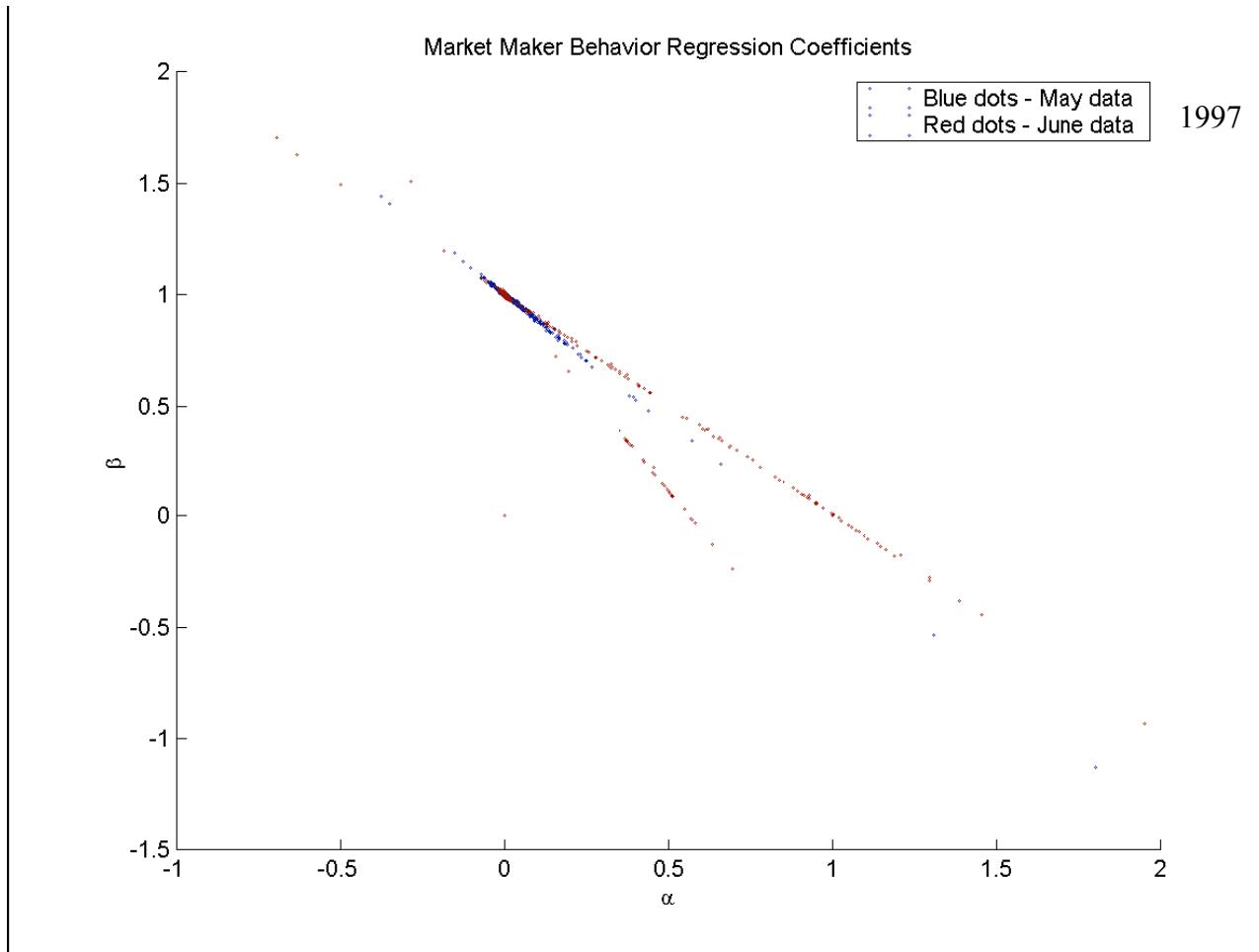
GTP+ Orig. Model Calibration

- Calibrated the model to
    - Individual strategies
    - Aggregate market parameters
- Simulated strategies are able to replicate the real-world ones (with precision up to 60-70%)
- Tested against existence of real-world patterns, such as presence of fat tails and spread clustering
- Created self-calibrating software to use data as it comes in

Fat Tail Results

- "Fat tails":
  - A large probability of extreme events by comparison with a Gaussian distribution
- Origins are uncertain
  - Herd effects, other?
- Our model generates fat tails with no herd effects

Original Gaussian Distribution

frequency

difference in logarithms of the true value
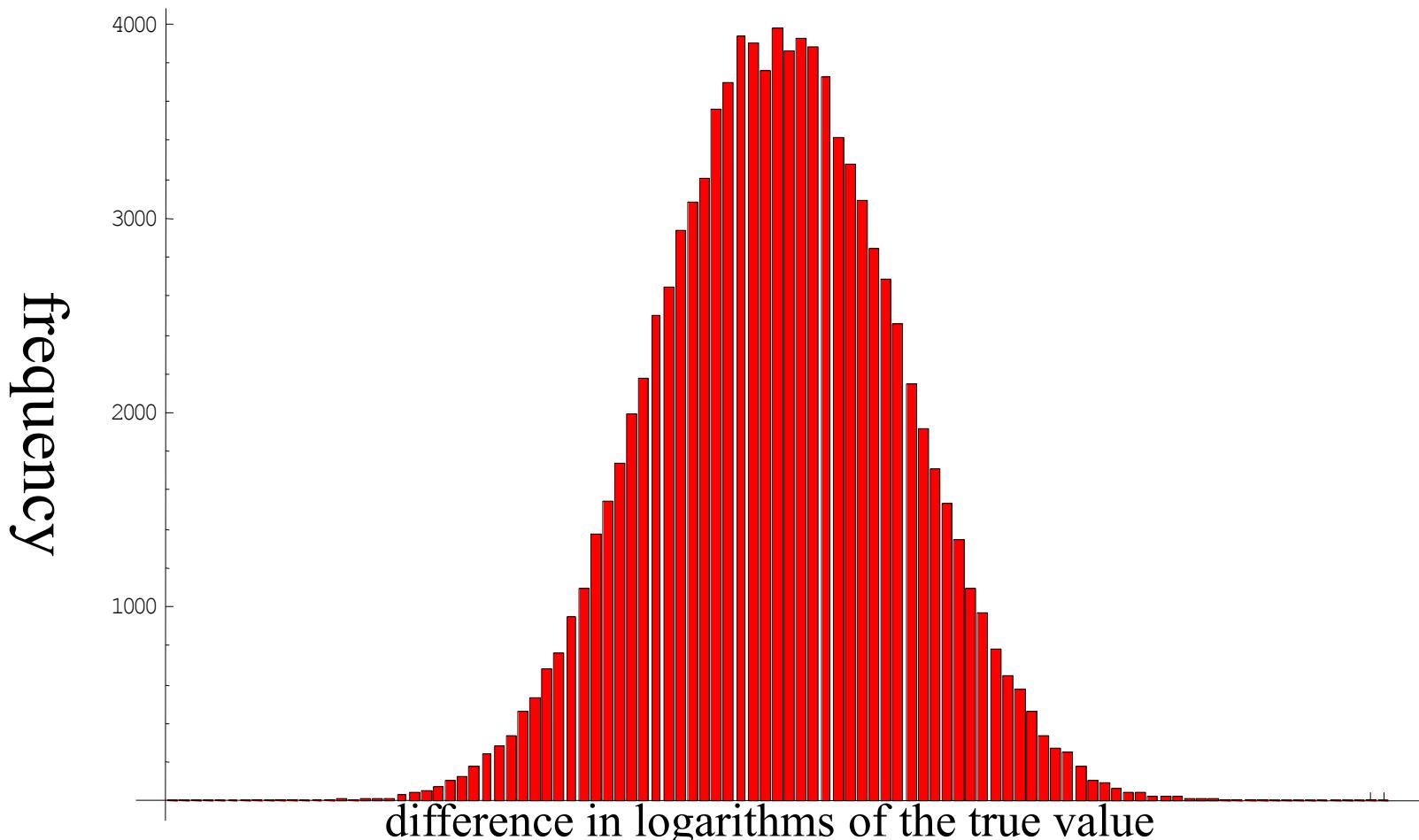
Fat Tails in Simulated Average Price Dynamics

frequency

difference in the logarithms of the average price
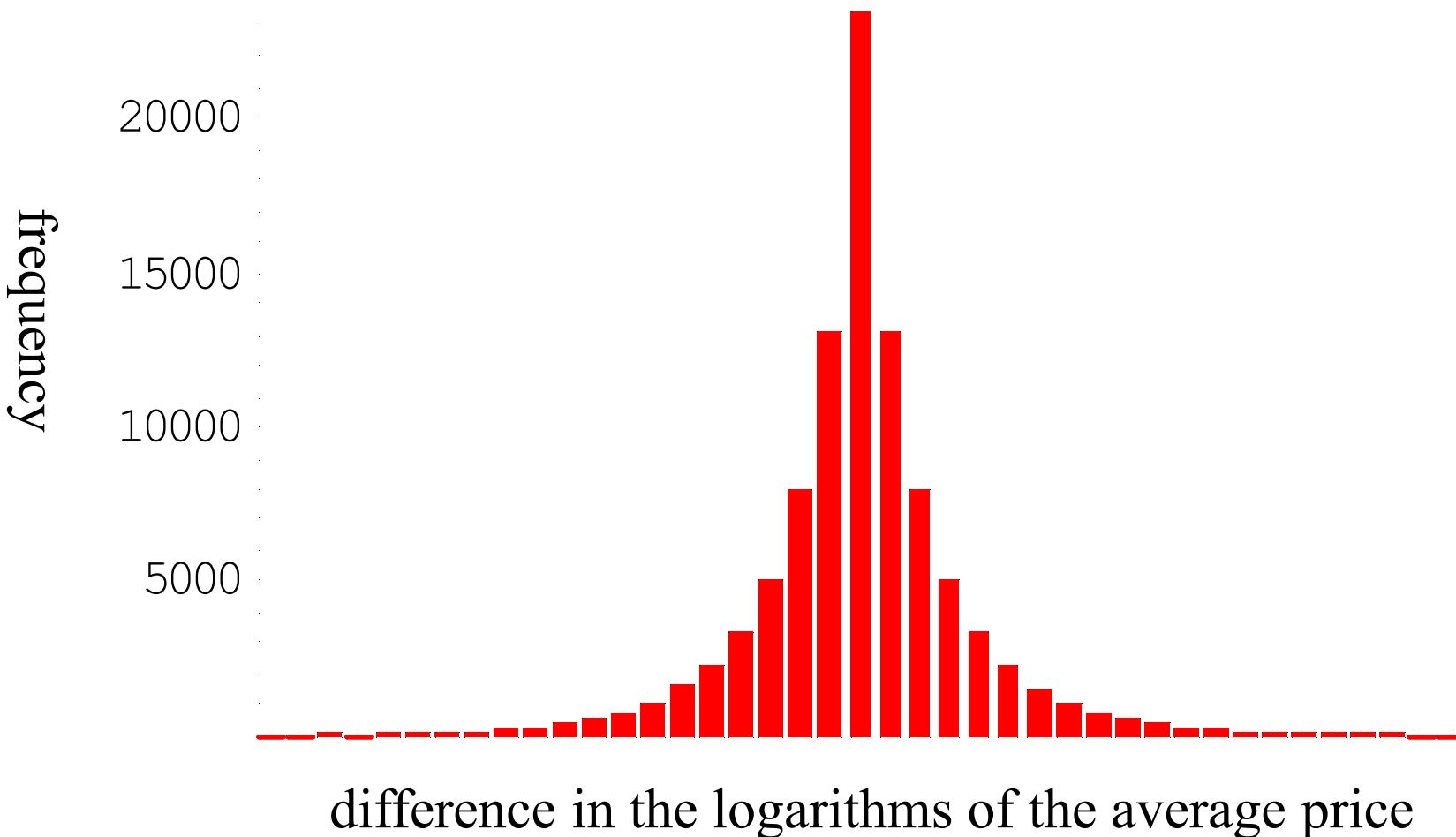
Time Correlations and Fat Tails

The fat tails seem to disappear when the data points are taken far apart (50 periods here)

frequency

difference in the logarithms of the average price
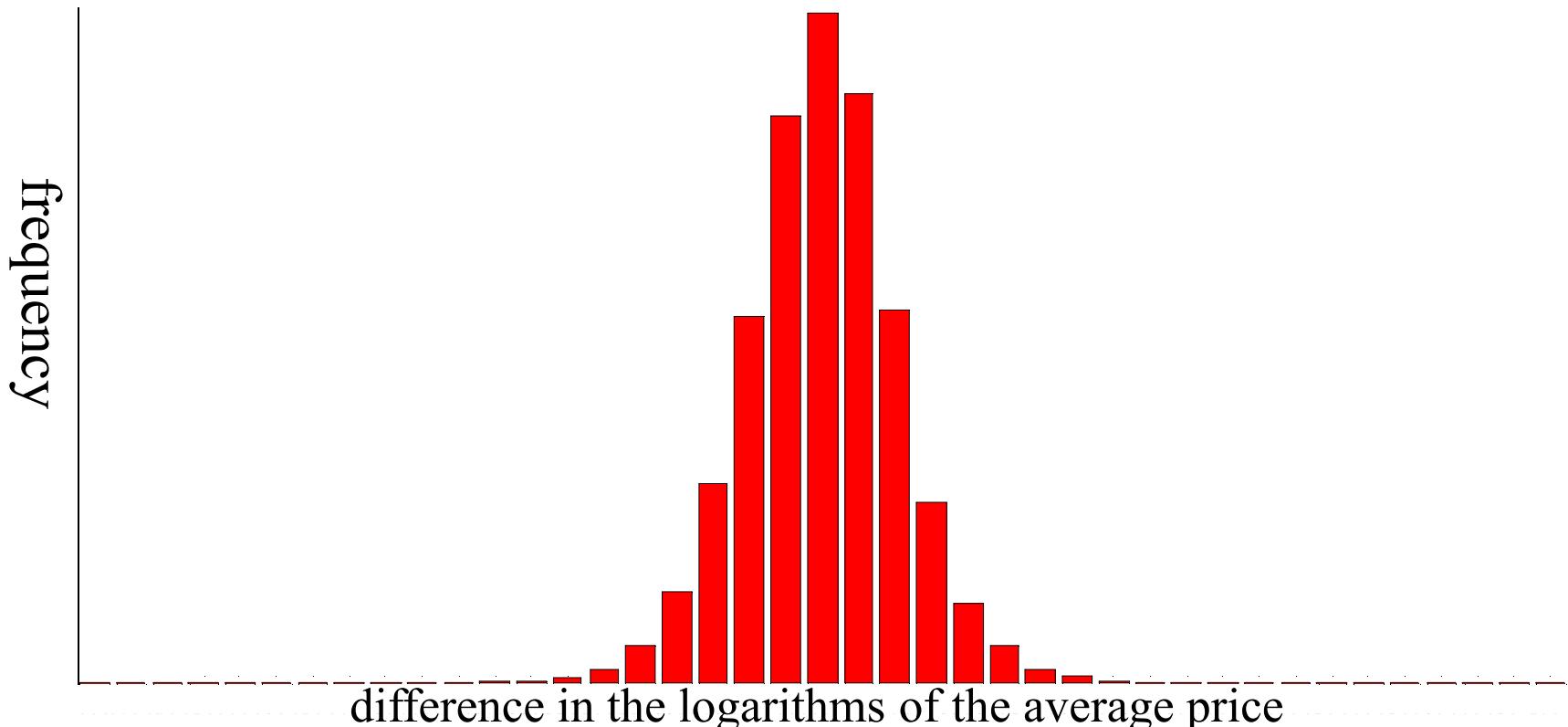
Why Fat Tails in the Simulation?

- Possible explanations:
  - Interaction and self-interaction through price
  - Existence of spread
  - Memory of traders, investors, etc.
- No explicit "herd" effects included

Spread Clustering

- Nasdaq dealers collusion accusations - Christie and Schulz (1994)

- SEC investigation into quoting behavior on Nasdaq (1996) and subsequent settlement

- Clustering in various financial markets - Hasbrouck (1998)

Importance of Spread Clustering

- Emergent property in the simulation: no collusion is present, yet the spread clustering occurs

- Real-world issue: Nasdaq, Forex

Applications to Energy Markets Natural Gas (NG)

Industry Effects

Transportation
- LNG HD Vehicle
- GTL
- Compressed NG

Industry
- NG -> Electric Power
- Chemicals
- Manufacturing
- Heat

Affects economics of GTL and other NG uses

Affects economics of transportation and industrial uses

Allow LNG Exports?

NG Effects

NG Price

NG Supply

NG Pipeline Network
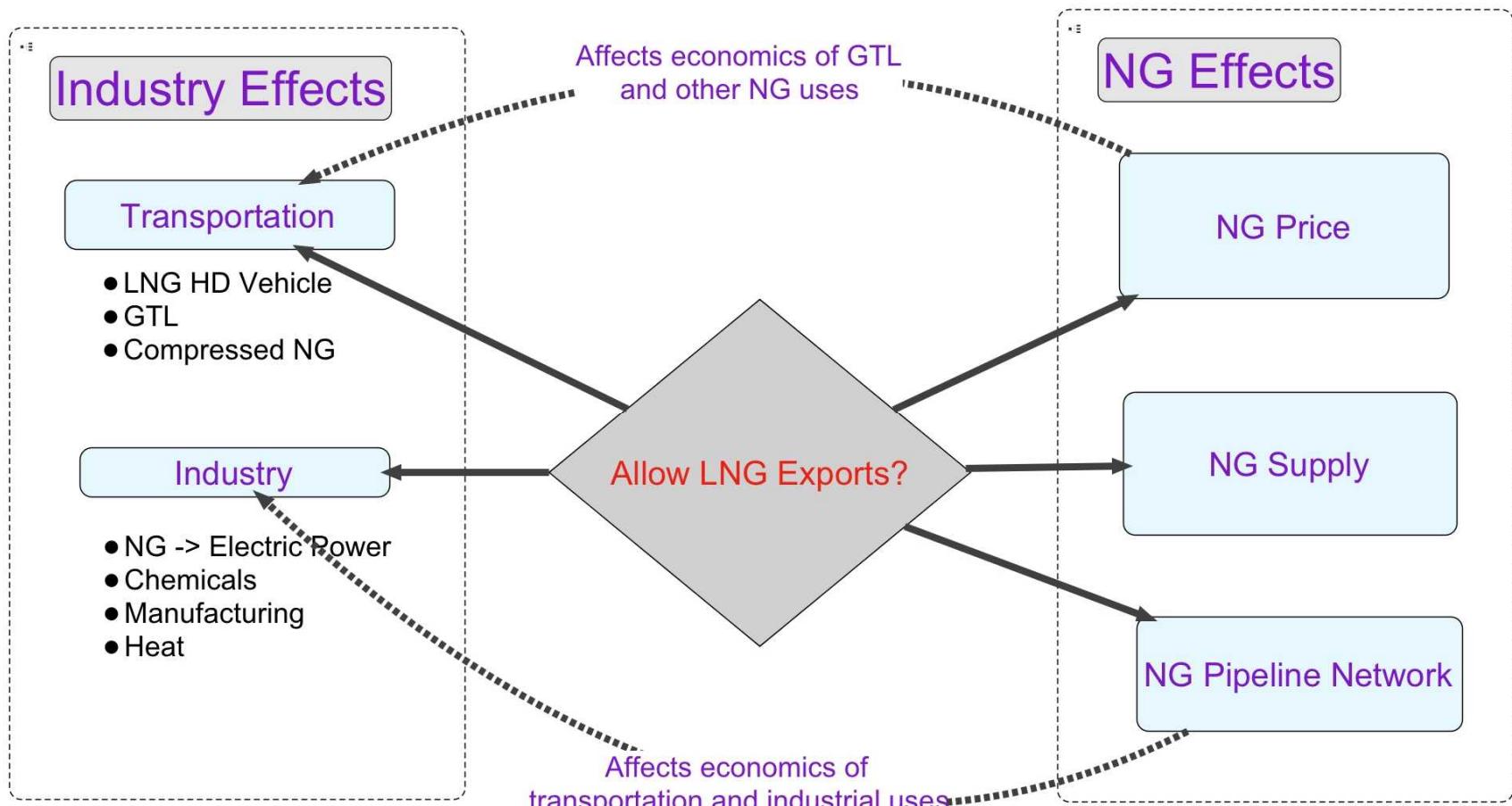
NG Prices for Calibration

Natural Gas Prices
(Dollars per Thousand Cubic Feet, except where noted)

Area: U.S.   **Period:** Monthly

Download Series History   Definitions, Sources & Notes

Show Data By: ○ Data Series ○ Area	Graph / Clear	Jun-13	Jul-13	Aug-13	Sep-13	Oct-13	Nov-13	View History
Wellhead Price	☐	NA	NA	NA	NA	NA	NA	1973-2013
Imports Price	☐	3.90	3.41	3.17	3.48	3.4	3.78	1989-2013
By Pipeline	☐	3.73	3.37	3.01	3.01	3.34	3.77	1997-2013
As Liquefied Natural Gas	☐	8.65	4.59	7.42	9.96	5.79	4.62	1997-2013
Exports Price	☐	4.22	3.94	3.75	3.88	3.88	4.17	1989-2013
By Pipeline	☐	4.22	3.93	3.75	3.88	3.88	3.92	1997-2013
As Liquefied Natural Gas	☐	13.38	12.89	13.25	13.53	13.09	14.21	1997-2013
Citygate Price	☐	5.74	5.53	5.23	5.20	4.88	4.77	1973-2013
Residential Price	☐	14.97	16.30	16.44	15.69	12.48	10.10	1973-2013
Percentage of Total Residential Deliveries included in Prices	☐	94.9	94.9	94.8	94.9	95.2	95.5	2002-2013
Commercial Price	☐	9.09	8.99	9.07	8.80	8.34	7.95	1973-2013
Percentage of Total Commercial Deliveries included in Prices	☐	59.3	57.9	57.0	57.4	61.3	66.2	1983-2013
Industrial Price	☐	4.91	4.50	4.34	4.38	4.39	4.63	2001-2013
Percentage of Total Industrial Deliveries included in Prices	☐	16.3	16.0	16.2	16.6	16.9	17.2	2001-2013
Electric Power Price	☐	4.56	4.34	4.03	4.19	4.26	4.36	2002-2013

Source: EIA

Conclusions

- GTP can help in individual strategies development and provide conceptual foundations for strategies when data is unavailable or is non-probabilistic.

- Defensive forecasting can be used to test the overall market behavior.

- ABMs can be used to simulate GTP protocols.

- Can be used for financial, energy, and other markets.

Summary of Findings

1. Decimalization (tick size reduction) will negatively impact the price discovery process.

2. Ambiguous investor wealth effects may be observed. (Investors' average wealth may actually decrease in the simulation, but the effect is not statistically significant).

3. Phase transitions will occur in the space of market-maker strategies.

4. Spread clustering may be more frequent with tick size reductions.

5. Parasitic strategies may become more effective as a result of tick size reductions.

6. Volume will increase, potentially ranging from 15% to 600%.

Comparisons with Data

Tick size was officially reduced from a 1/16th to $.01 (in phases) in March, 2001.

Nasdaq economists captured actual data from this transition and put the findings in their Economic Research study report.

BiosGroup compared our model's results with the findings from the Nasdaq report.

Comparisons with Data (Cont.)

5 of the 6 likely outcomes actually occurred.

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Nasdaq Book

A NASDAQ MARKET SIMULATION

INSIGHTS ON A MAJOR MARKET FROM
THE SCIENCE OF COMPLEX ADAPTIVE SYSTEMS
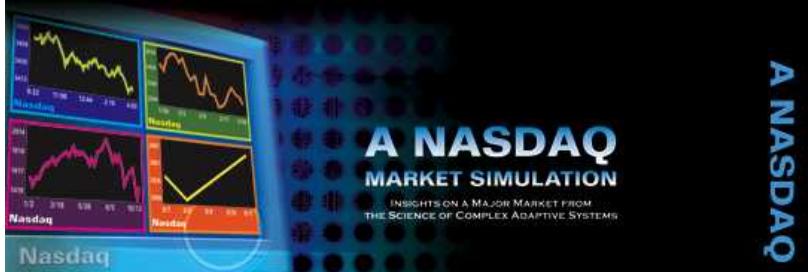
"In a telling story of science and politics, Vince and Sasha provide a glimpse of a hopeful future for market regulation. In 1998, they conceived a model to help regulators predict the consequences of proposed new "decimalization" rules for our markets. Pooh-poohed as "politically naive," their work was ignored. Eight years later, overwhelming evidence points to its uncanny prescience. Their short book is a "must read" for every thoughtful market regulator, those who question if America is ceding its role as the world's financial market to others, and all who delight in the progress of science."

Michael Brown
former Chairman of the NASDAQ Stock Market and former CFO of Microsoft

"I watched Vince Darley and Sasha Outkin develop and mature the agent based model of the NASDAQ stock market in our company, BiosGroup, in 1998. The model was carefully crafted and, as noted by the then President of NASDAQ, offered the first quantitative insights into the expected consequences of regulatory change, here for "decimalization rules" for the market. What Darley and Outkin have accomplished foretells an entire new era using agent based models in business, government, the military, and other domains. This book is a critical read for a very wide range of readers attempting to understand the emergent behavior of complex systems that only agent based models can capture. These new tools will prove essential in ways we cannot yet know."

Stuart Kauffman
Founder of BiosGroup and Director of Institute of Biocomplexity and Informatics
The University of Calgary, Alberta, Canada

This pioneering book describes the applications of agent-based modeling to financial markets. It presents a new paradigm for finance, where markets are treated as complex systems whose behavior emerges as a result of interactions of market participants, market institutions, and market rules. This includes both a presentation of the conceptual model and its software implementation. It also summarises the result of the profound research on the successful practical application of this new approach to answer questions regarding the Nasdaq Stock Market's decimalization that was implemented in 2001.

The book presents conceptual foundations for modeling markets as complex systems. It describes the agent-based model of the Nasdaq stock market, including strategies used by market-makers and investors, market participants interactions, and impacts of rules and regulations. It includes analyses of simulation behavior, comparision with the behaviors observed in the real-world markets (existence of fat tails, spread clustering, etc.), and predictions about possible outcomes of decimalization. A framework for calibrating the market behavior and individual market-makers strategies to historical data is also presented.

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THE SCIENCE OF COMPLEX ADAPTIVE SYSTEMS

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