



On Applications of Game-Theoretic Probability and Defensive Forecasting to Agent-based Market Models

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Abstract

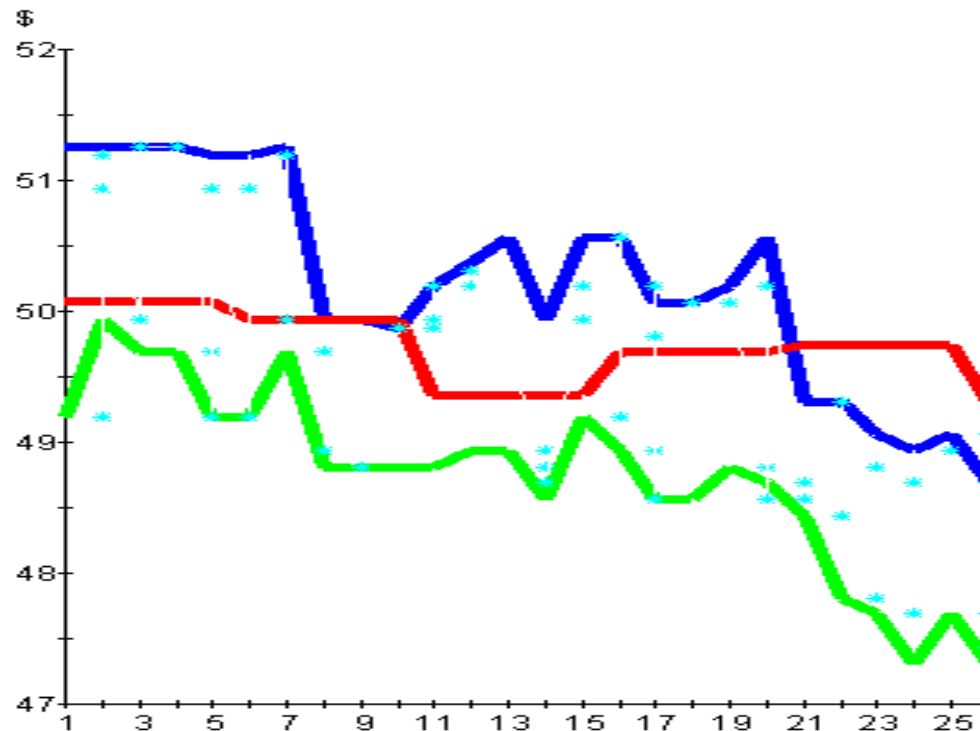
We present an attempt on connecting agent-based modeling with Game-Theoretic Probability (GTP) and defensive forecasting and outline a framework connecting elements of game-theoretic probability with agent-based models. We illustrate this framework on an example of our model of the Nasdaq stock market and on an example of a model of a natural gas market model, and show how game-theoretic probability can be used to test the simulated market price dynamics, the individual agent trading strategies, and the overall agent-based model.

What Is Agent-Based Modeling?

- In ABMs, complex, real-world systems are represented in software as collections of *autonomous decision-making entities, situated in appropriate environment and interaction structure*.
 - Each type of agent executes behaviors appropriate to it and its context.
 - Agents thus produce, consume, trade securities, ship freight, and so forth. Agents are heterogeneous and variable, as in reality.
 - ABM simulations explicitly recognize that real world agents are not independent, but are affected by other agent behaviors, common information sources, and relevant social communities.
- The dynamics of systems *emerge* from large numbers of interactions among many kinds of agents. System behavior thus arises from the bottom up.

Example: Market Maker - Investor Interaction

- Market makers: adjust their quotes
- Investors: submit market orders



Advantages of Agent-Based Models

- Agent-Based Model's (ABM's) bottom-up approach leverages the organization's knowledge of the details
 - Allows modeling of individual behaviors, rules, incentives
 - Allows modeling of complex interactions and interaction structures
 - Allows incorporating of human expertise and AI techniques in building the market's agents
- Intuitive, concrete, easy to understand
- Produces actionable results and counter-intuitive insights at many levels

Advantages of ABMs(cont'd)

- ABMs and traditional statistical methods produce the same results when the assumptions required by traditional methods are valid (e.g. independence, etc.)
- Models can be validated using historical data, but can be applied to *unique situations* that lack history
 - Allows combining both a hindsight and foresight perspective
- Agents can be programmed to *evolve* and *learn*. This permits the emergence of new, unanticipated behaviors and strategies
- A variety of what-if scenarios can be investigated

Key Features of Agent-Based Approaches

- Represent casual structure of the world
- Can be calibrated against individual behaviors and global system behaviors
- Span micro- to macro- gap
- Allow representation of existing emergent properties of the system and of previously unobserved behaviors

Disadvantages of ABMs

- Difficult to calibrate and to validate
- Much of the data is missing (even if trades are observable, the past context is generally not)
- Complete information on individual strategies is rarely available
- Treating the available data as generated by a probabilistic mechanism is problematic:
 - Lack of data
 - Fundamental goal of ABMs is to model causal decisions of agents, based on unique conditions and contexts

Combining GTP with ABMs

- GTP:
 - Forecaster: market, as combination of agent strategies
 - Skeptic: one specific agent strategy
- Strategy development: can an individual strategy generate abnormal returns?
- Model testing:
 - Do individual strategies generate abnormal returns?
- Defensive forecasting:
 - Can market (Forecaster) ensure no abnormal individual strategy returns?

Shaver and Vovk (2001)

Nasdaq Example: Problem Statement

- Nasdaq had to consider decimalization and its impacts in 1998.
- How reducing the tick size may affect the market behavior? Why should it have any effect?
 - How a change to decimals can be modeled?
 - What is the mechanism through which changed tick size would affect the market?
 - Given specific mechanisms, what other effects may occur?
- Nasdaq decimalization study: an empirical example.
 - Study done during 1998-2000.
 - Decimalization occurred in April 2001.
 - *Darley and Outkin (2007)*

Goals

- Investigate effects of possible policy and environment changes:
 - Ex: Evaluate the effects of changing the tick size (decimalization) and of parasitism
- Evaluate the influence of market rules and structure on market dynamics and strategies
- Demonstrate that simulated market participants and aggregate market parameters are “sufficiently similar” to those in the real world to validate model empirically

Model Implementation

Construct and analyze an agent-based model of the market:

- Populate with agents (investors and market makers).
- Simulate market infrastructure and rules.
- Calibrate with the actual stock market data:
 - To ensure that the simulated distribution of trade sizes, volumes, prices and other statistical parameters is similar to that observed in the real world.
 - To simulate real-world behaviors of and interactions of market makers and investors using data sets of historical quotes and trades.
- Design it to reflect the look and feel of the then existing Level 2 Nasdaq system.

Agent Details

- Market makers
- Investors
- Market Agent Features:
 - Autonomous
 - Adaptive/handcrafted strategies
 - Various levels of sophistication/adaptability/
access to information

Simulation Basics

- Market agents are trading in a single stock
- Investors have a price target which follows a Poisson process, random walk, *etc.*
- Investors:
 - Receive noisy information about this target
 - Decide whether to trade by
 - Comparing this target with available price
 - Incorporating market trends
 - Performing sophisticated technical trading, *etc.*
- Market makers:
 - Receive buy and sell orders
 - Must learn how to set their quotes profitably

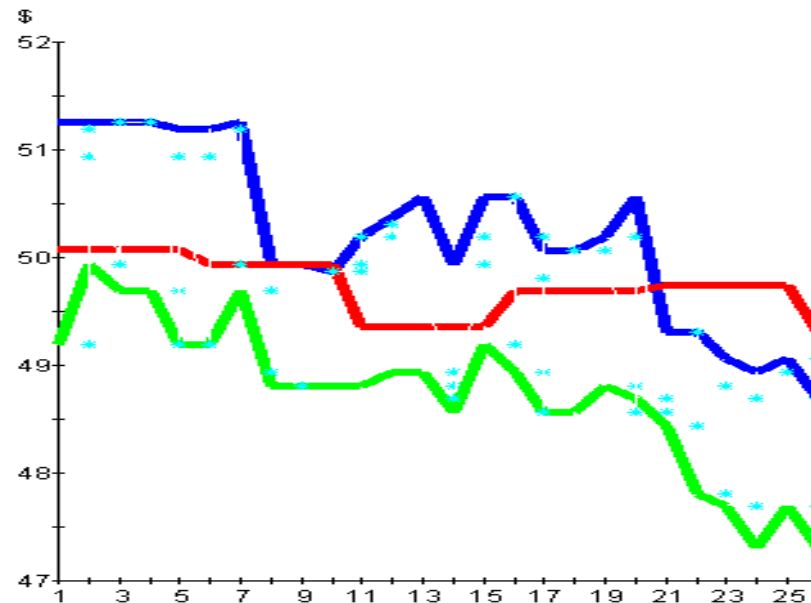
Nasdaq Model and GTP

- Individual strategy testing
 - Basic strategy and variants
 - Parasitic strategies and variants
 - Learning strategies
- Market testing
 - Does market allow abnormal returns?
 - Is market stable against specific strategies

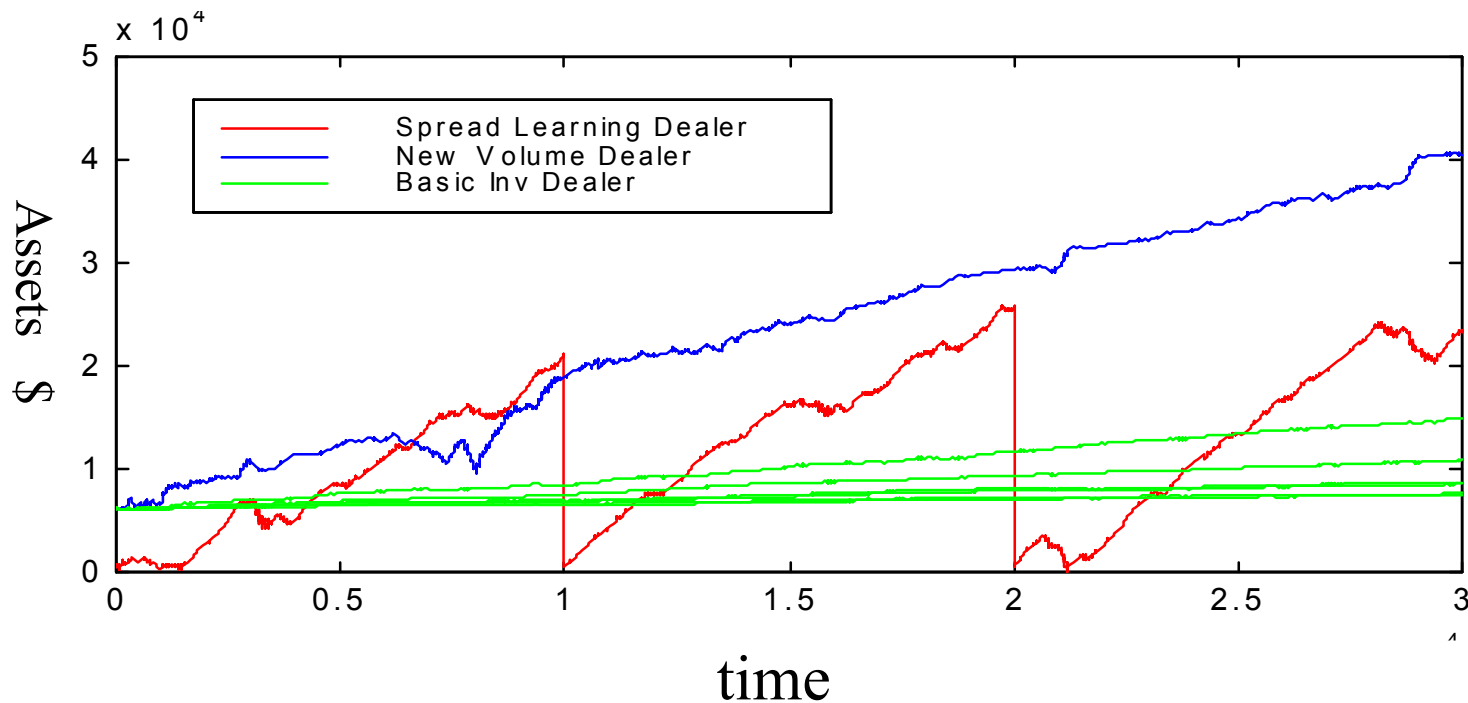
Individual Strategy: Parasitic

Parasitic strategy:

- Attempts to undercut the current bid/offer by a small increment (tick size)
- Is not a major source of liquidity for the market



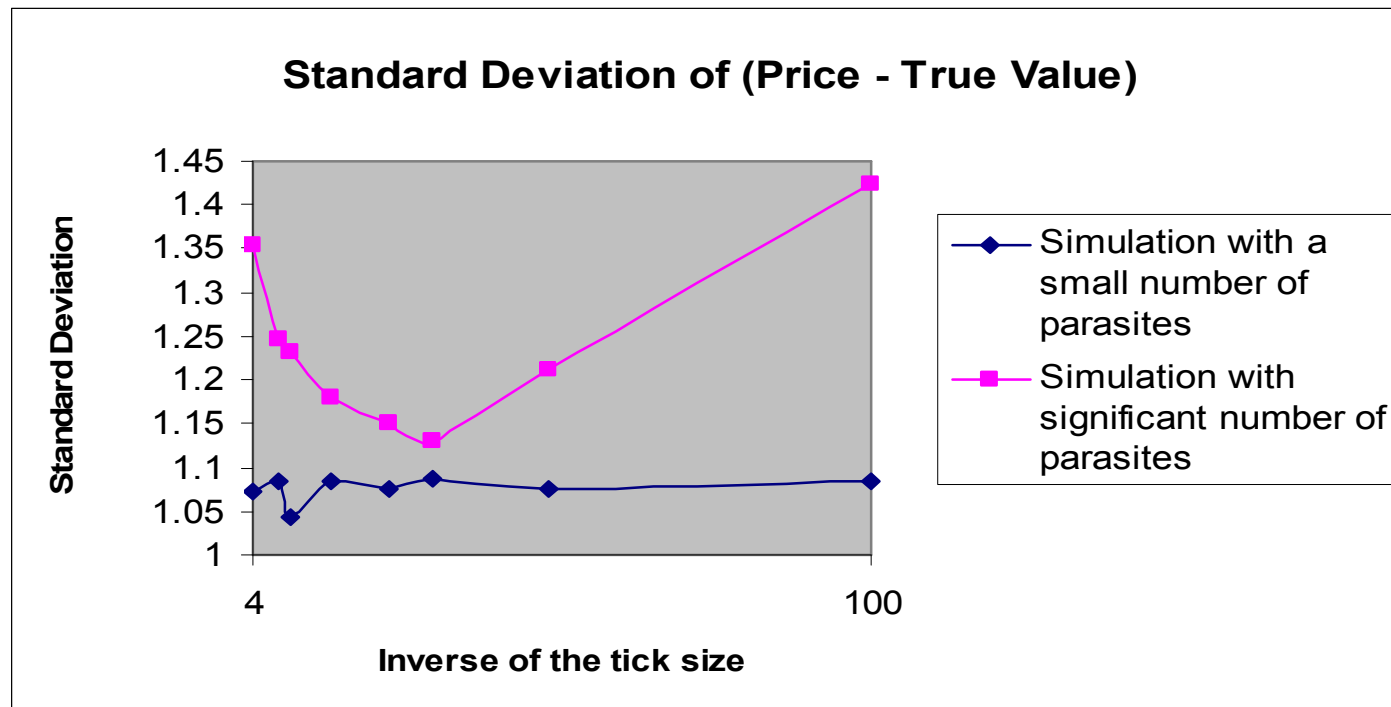
Individual Strategy: Learning



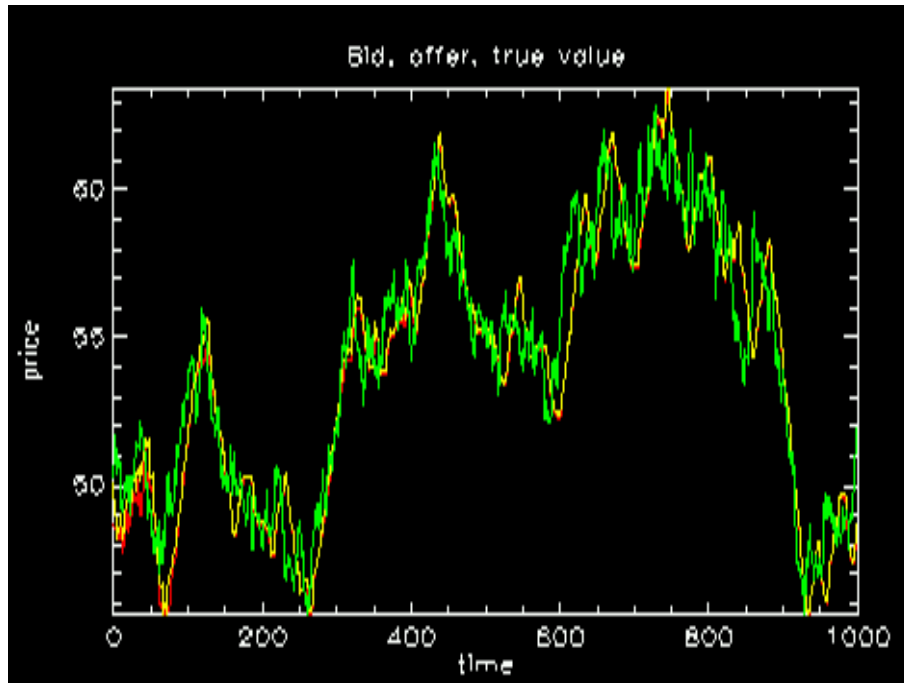
- Spread Learning market maker is the most profitable dealer on the market under many circumstances
- Known exceptions: high volatility, many parasites

Market Testing: Tick size effects

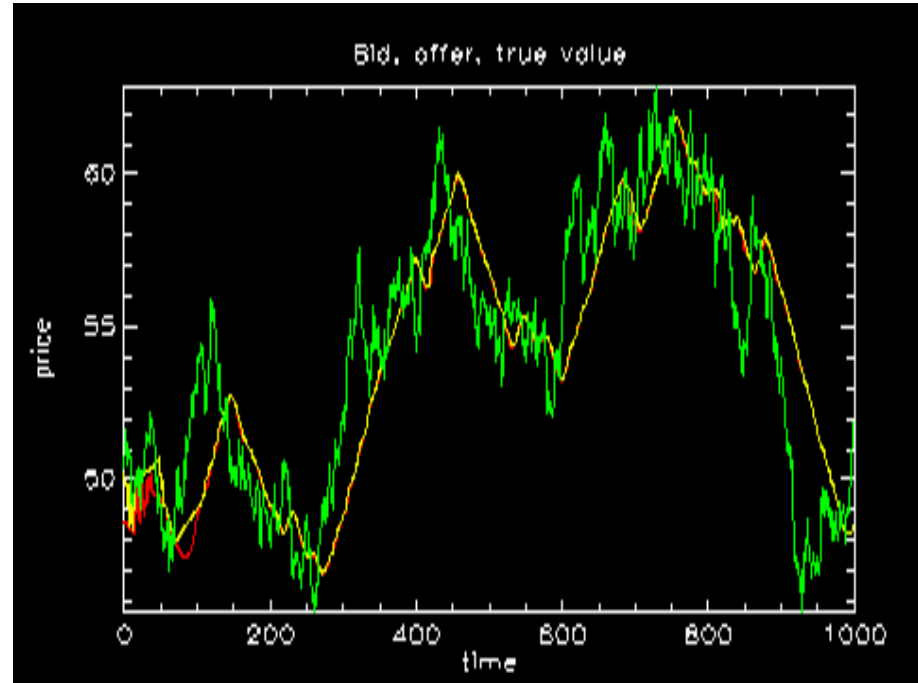
As tick size is reduced, parasitic strategies increasingly impede price discovery / market's ability to generate useful information



Tick Size Effects, Many Parasites



Tick size 1/16



Tick size 1/100

Comparison to Original Model Calibration

- Calibrated the model to
 - Individual strategies
 - Aggregate market parameters
- Simulated strategies are able to replicate the real-world ones (with precision up to 60-70%)
- Tested against existence of real-world patterns, such as presence of fat tails and spread clustering
- Created self-calibrating software to use data as it comes in

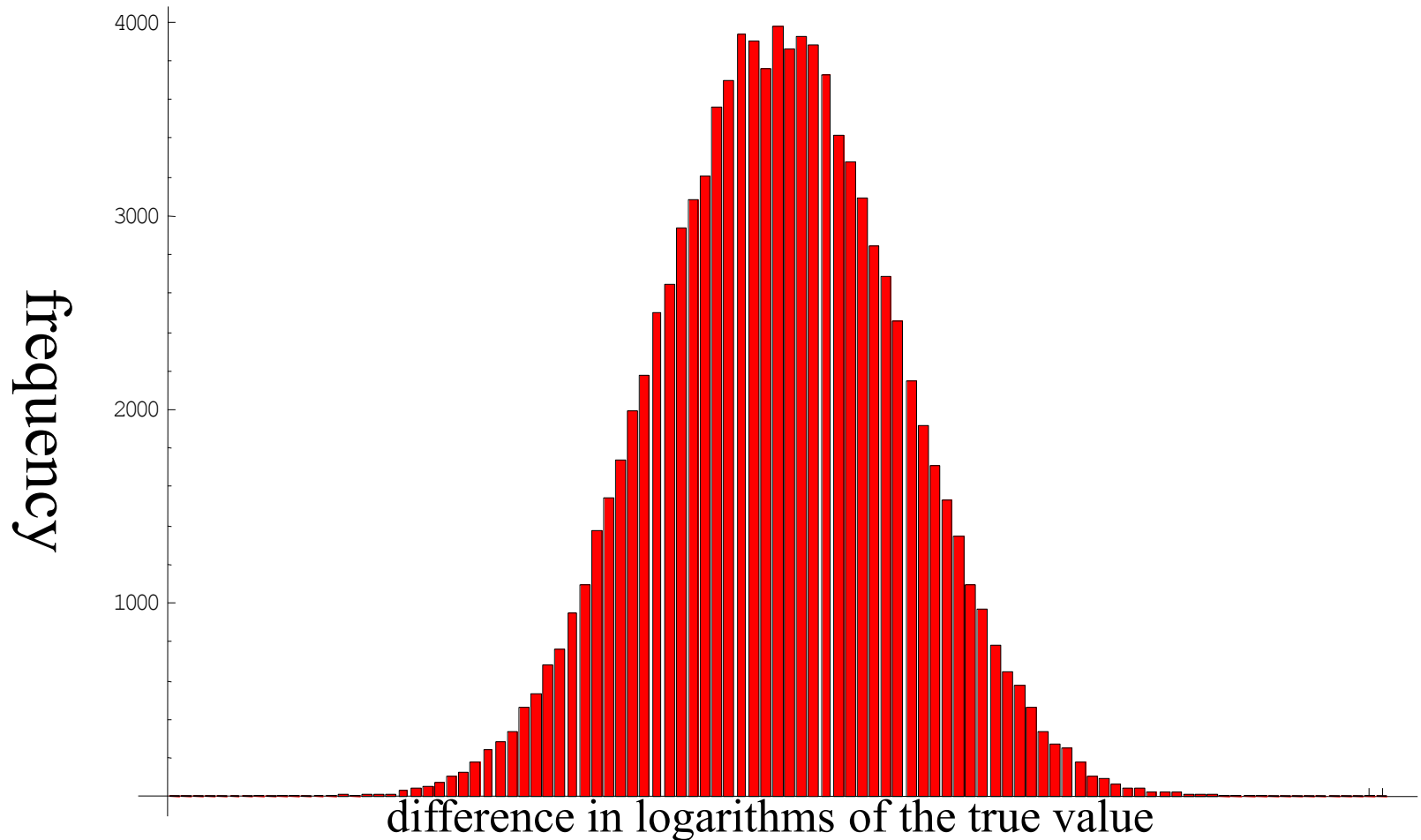
Questions Investigated

- Effects of tick-size changes and parasitism
- Market dynamics effects:
 - Presence and origin of “fat tails”
 - Spread clustering and its causes
- Effects of market maker and investor learning and strategy evolution

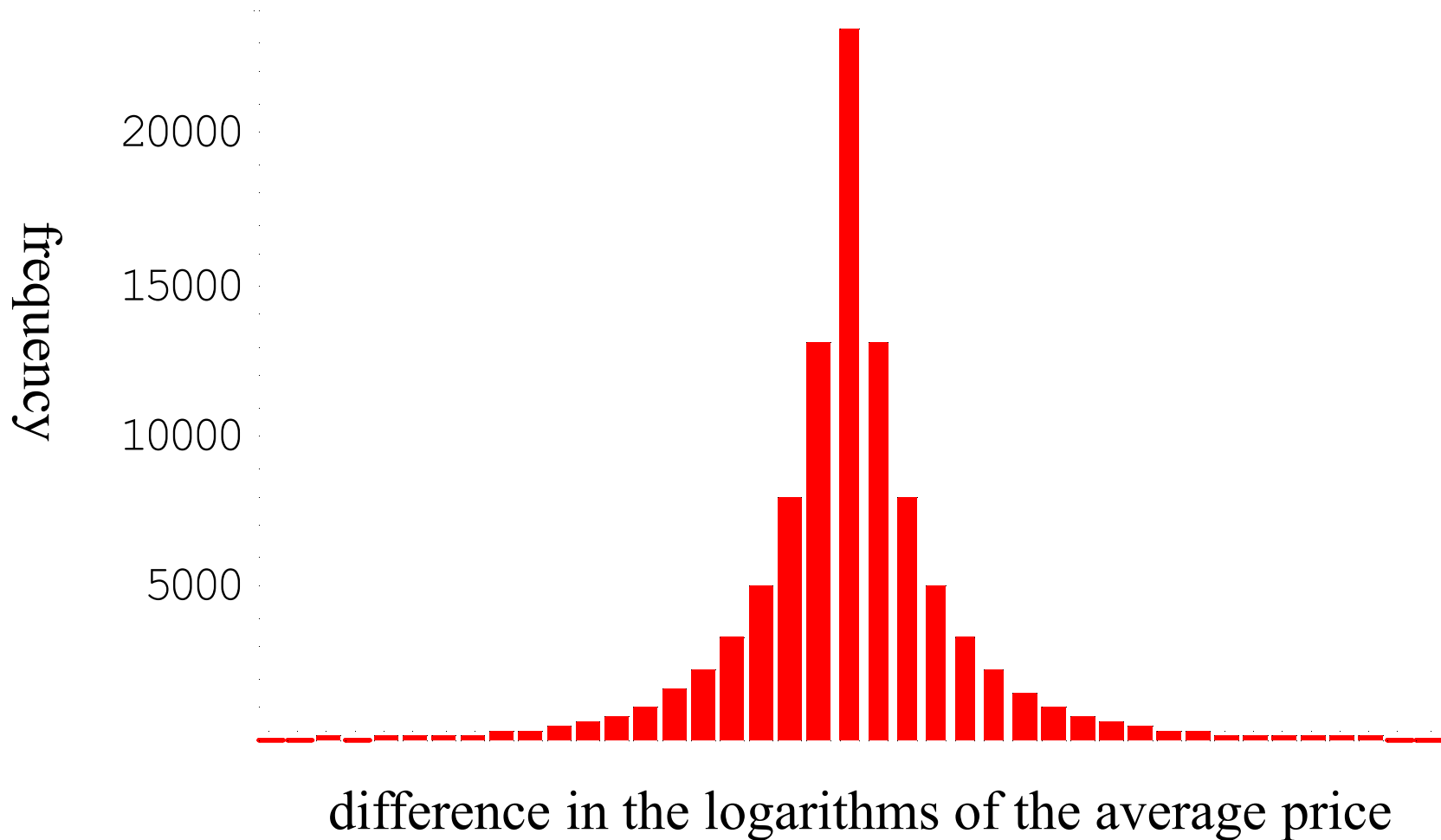
Fat Tail Results

- “Fat tails”:
 - A large probability of extreme events by comparison with a Gaussian distribution
- Origins are uncertain
 - Herd effects, other?
- Our model generates fat tails with no herd effects

Original Gaussian Distribution

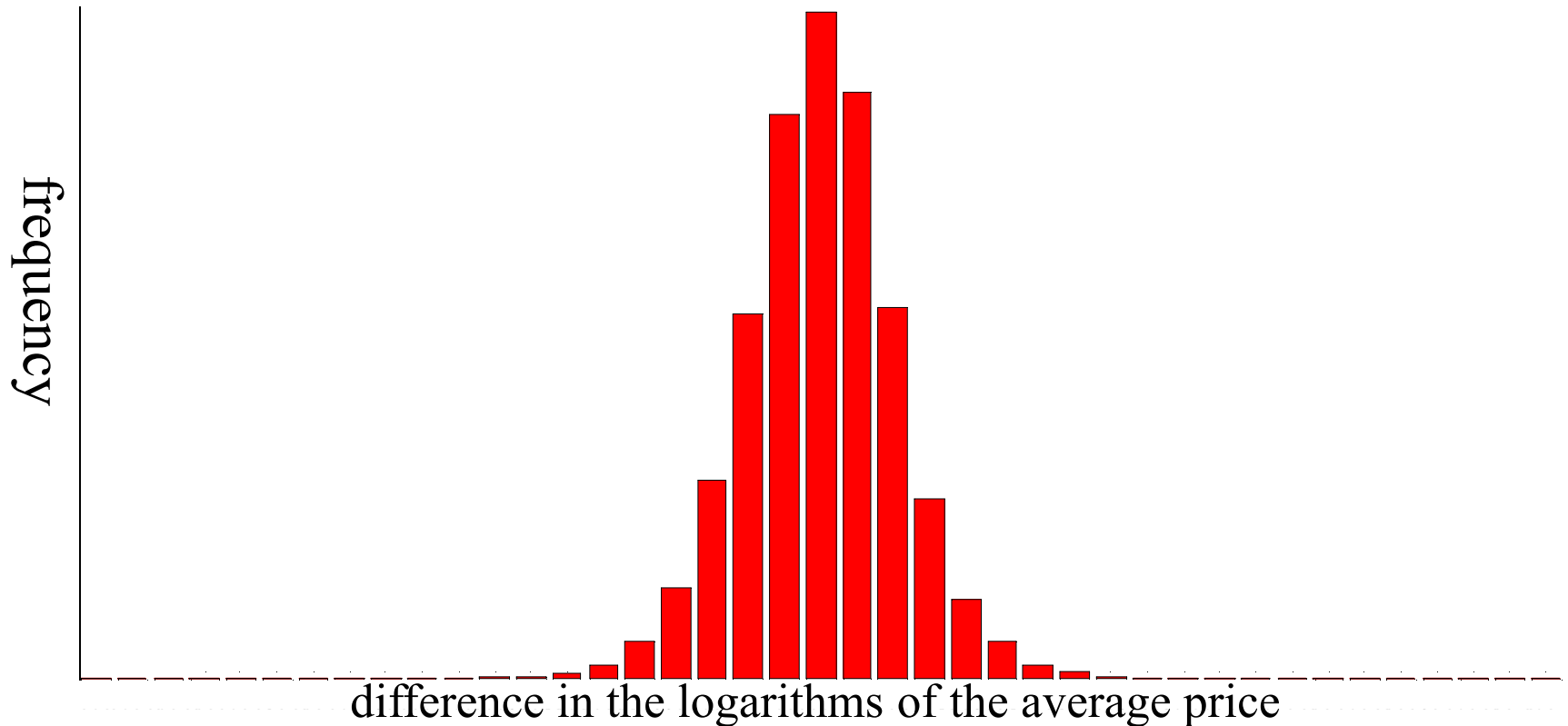


Fat Tails in Simulated Average Price Dynamics



Time Correlations and Fat Tails

The fat tails seem to disappear when the data points are taken far apart (50 periods here)



Why Fat Tails in the Simulation?

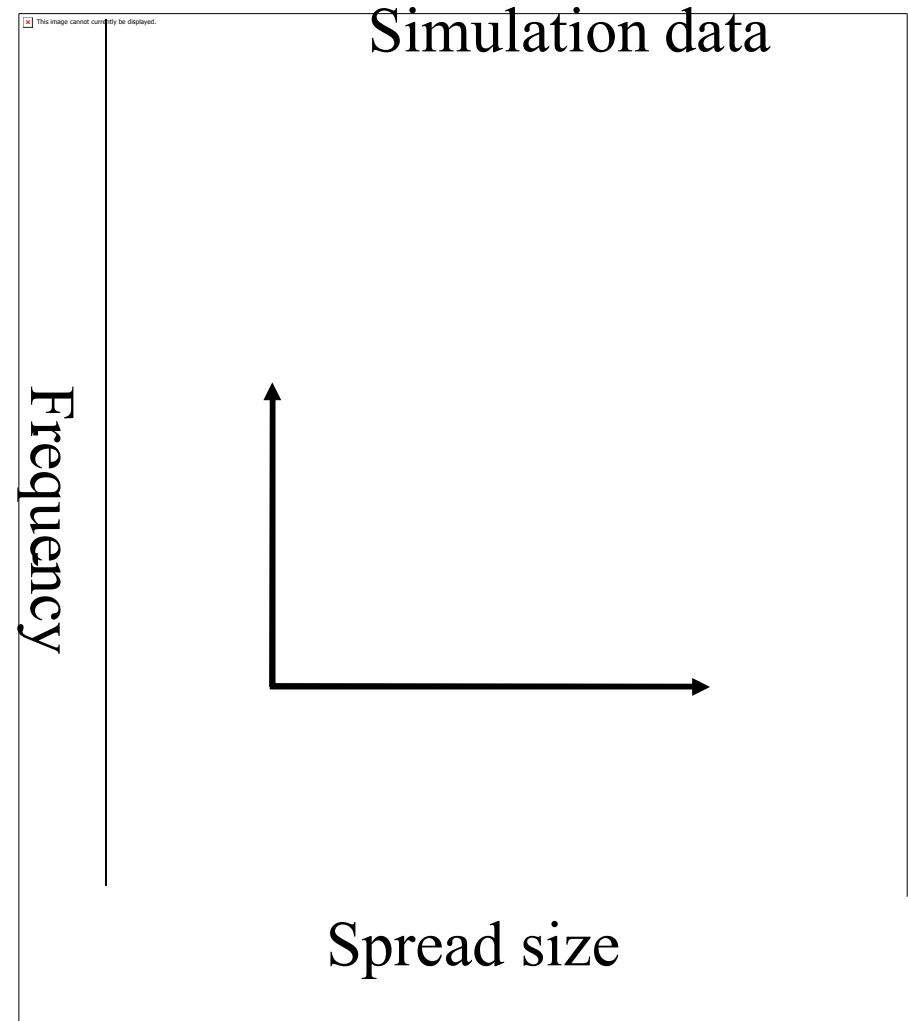
- Possible explanations:
 - Interaction and self-interaction through price
 - Existence of spread
 - Memory of traders, investors, etc.
- No explicit “herd” effects included

Spread Clustering

- Nasdaq dealers collusion accusations - Christie and Schulz (1994)
- SEC investigation into quoting behavior on Nasdaq (1996) and subsequent settlement
- Clustering in various financial markets - Hasbrouck (1998)

Spread Clustering

- Spread = difference between smallest offer and largest bid
- Spread clustering occurs when some spread values occur much more frequently than others

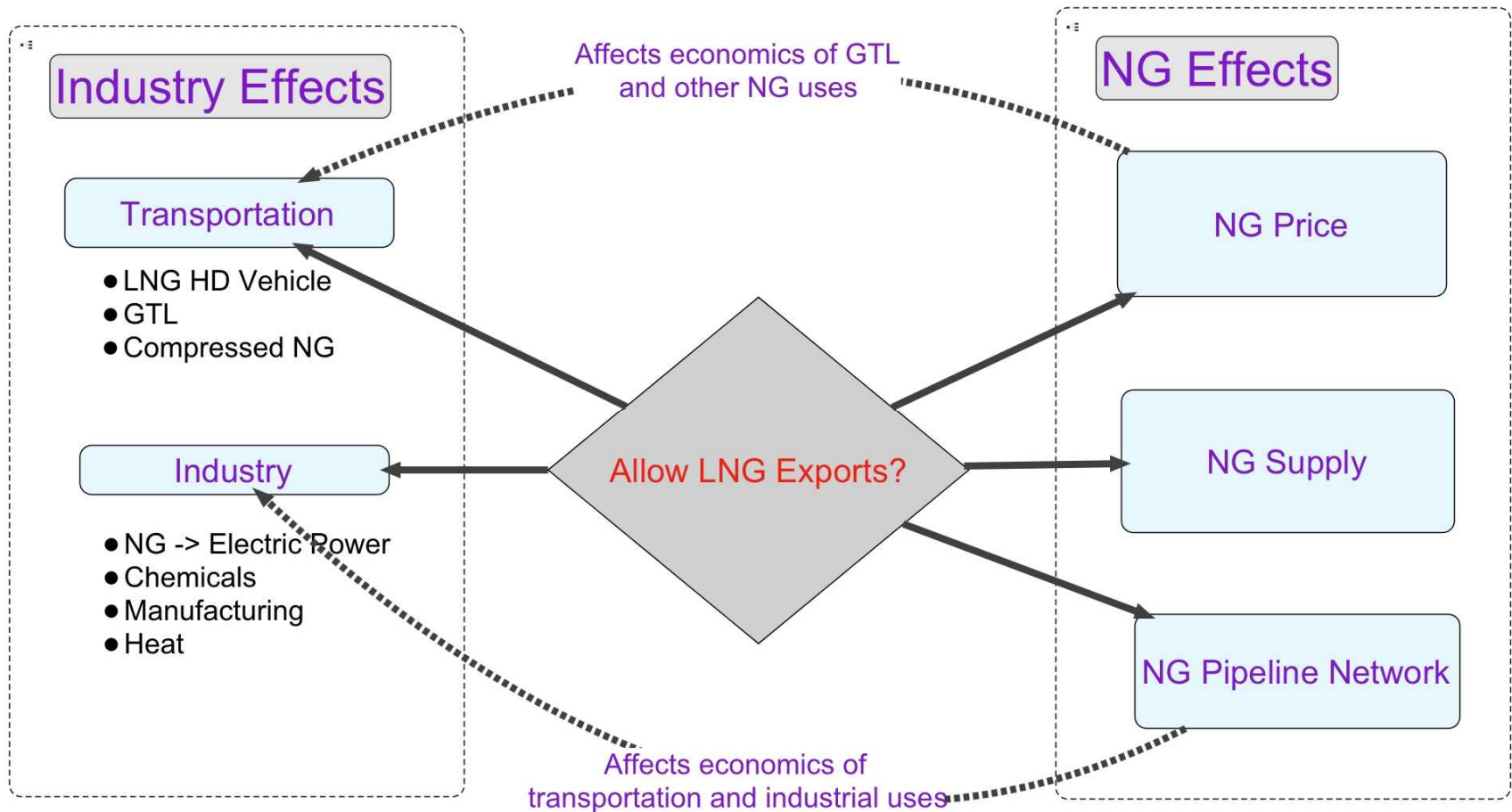


Importance of Spread Clustering

- Emergent property in the simulation: no collusion is present, yet the spread clustering occurs
- Real-world issue: Nasdaq, Forex

Applications to Energy Markets

Natural Gas (NG)



NG Prices for Calibration

Natural Gas Prices

(Dollars per Thousand Cubic Feet, except where noted)

Area: Period:

Download Series History Definitions, Sources & Notes								
Show Data By:	Graph	Jun-13	Jul-13	Aug-13	Sep-13	Oct-13	Nov-13	View History
<input checked="" type="radio"/> Data Series <input type="radio"/> Area	Clear							
Wellhead Price	<input type="checkbox"/>	NA	NA	NA	NA	NA	NA	1973-2013
Imports Price	<input type="checkbox"/>	3.90	3.41	3.17	3.48	3.4	3.78	1989-2013
By Pipeline	<input type="checkbox"/>	3.73	3.37	3.01	3.01	3.34	3.77	1997-2013
As Liquefied Natural Gas	<input type="checkbox"/>	8.65	4.59	7.42	9.96	5.79	4.62	1997-2013
Exports Price	<input type="checkbox"/>	4.22	3.94	3.75	3.88	3.88	4.17	1989-2013
By Pipeline	<input type="checkbox"/>	4.22	3.93	3.75	3.88	3.88	3.92	1997-2013
As Liquefied Natural Gas	<input type="checkbox"/>	13.38	12.89	13.25	13.53	13.09	14.21	1997-2013
Citygate Price	<input type="checkbox"/>	5.74	5.53	5.23	5.20	4.88	4.77	1973-2013
Residential Price	<input type="checkbox"/>	14.97	16.30	16.44	15.69	12.48	10.10	1973-2013
Percentage of Total Residential Deliveries included in Prices	<input type="checkbox"/>	94.9	94.9	94.8	94.9	95.2	95.5	2002-2013
Commercial Price	<input type="checkbox"/>	9.09	8.99	9.07	8.80	8.34	7.95	1973-2013
Percentage of Total Commercial Deliveries included in Prices	<input type="checkbox"/>	59.3	57.9	57.0	57.4	61.3	66.2	1983-2013
Industrial Price	<input type="checkbox"/>	4.91	4.50	4.34	4.38	4.39	4.63	2001-2013
Percentage of Total Industrial Deliveries included in Prices	<input type="checkbox"/>	16.3	16.0	16.2	16.6	16.9	17.2	2001-2013
Electric Power Price	<input type="checkbox"/>	4.56	4.34	4.03	4.19	4.26	4.36	2002-2013

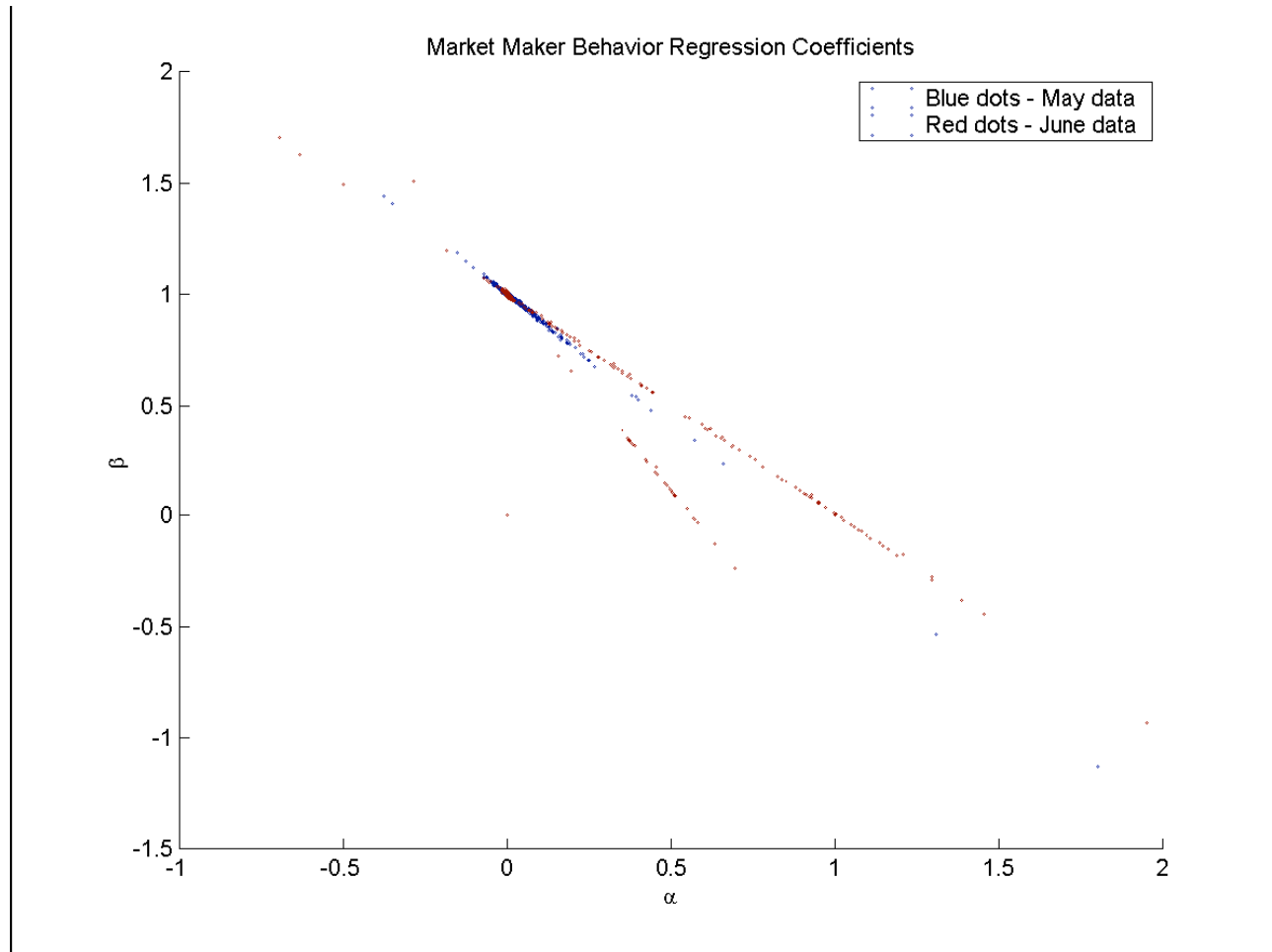
Source: EIA

Conclusions

- GTP can help in individual strategies development and provide conceptual foundations for strategies when data is unavailable or is non-probabilistic.
- Defensive forecasting can be used to test the overall market behavior.
- Can be used for financial, energy, and other markets.



Phase Transitions



Summary of Findings

1. Decimalization (tick size reduction) will negatively impact the price discovery process.
2. Ambiguous investor wealth effects may be observed. (Investors' average wealth may actually decrease in the simulation, but the effect is not statistically significant).
3. Phase transitions will occur in the space of market-maker strategies.
4. Spread clustering may be more frequent with tick size reductions.
5. Parasitic strategies may become more effective as a result of tick size reductions.
6. Volume will increase, potentially ranging from 15% to 600%.

Comparisons with Data

Tick size was officially reduced from a $1/16^{\text{th}}$ to \$.01 (in phases) in March, 2001.

Nasdaq economists captured actual data from this transition and put the findings in their Economic Research study report.

BiosGroup compared our model's results with the findings from the Nasdaq report.

Comparisons with Data (Cont.)

5 of the 6 likely outcomes actually occurred.

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Nasdaq Book

