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Sampling and Streaming Algorithms for Counting Small Patterns in BIG Graphs

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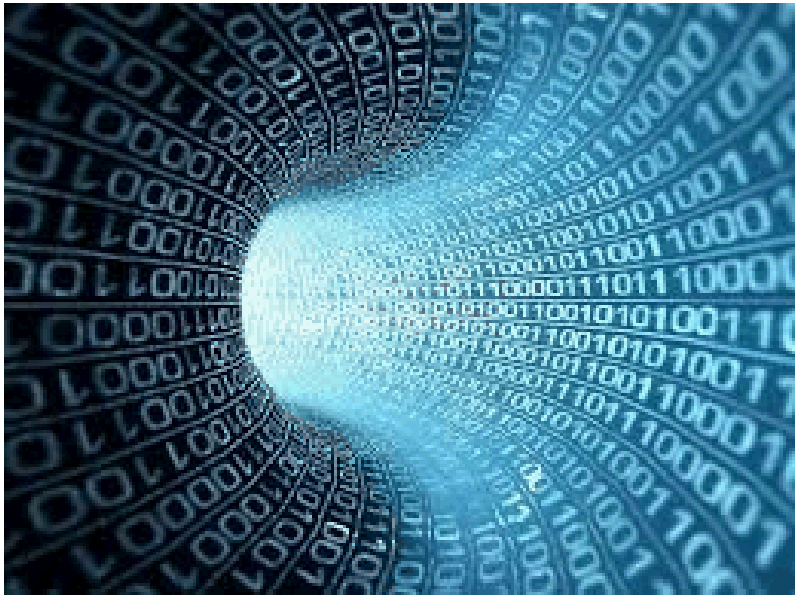
Sampling may be the key to process large data sets

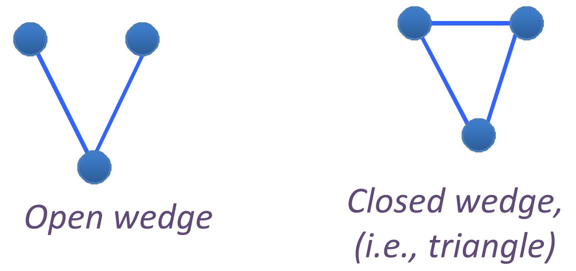


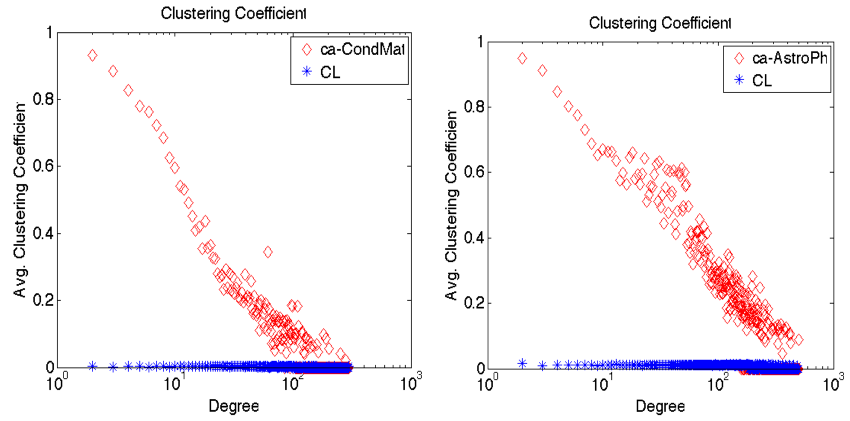
Source: http://www.greenbookblog.org/wp-content/uploads/2012/03/big-data.jpg

- Sizes of the modern data sets redefine the landscape for algorithmic research.
  - Single pass through the data may be a luxury.
- In many applications the speed of data is the challenge.
- Sampling/streaming algorithms can identify general trends in the data.
  - but not find needle in a haystack.
- The goal of sampling is to provide
  - good estimations with error/confidence bounds,
  - by looking at a small portion of the data.
- Sampling is not an alternative to parallelism.
  - They get along well together.

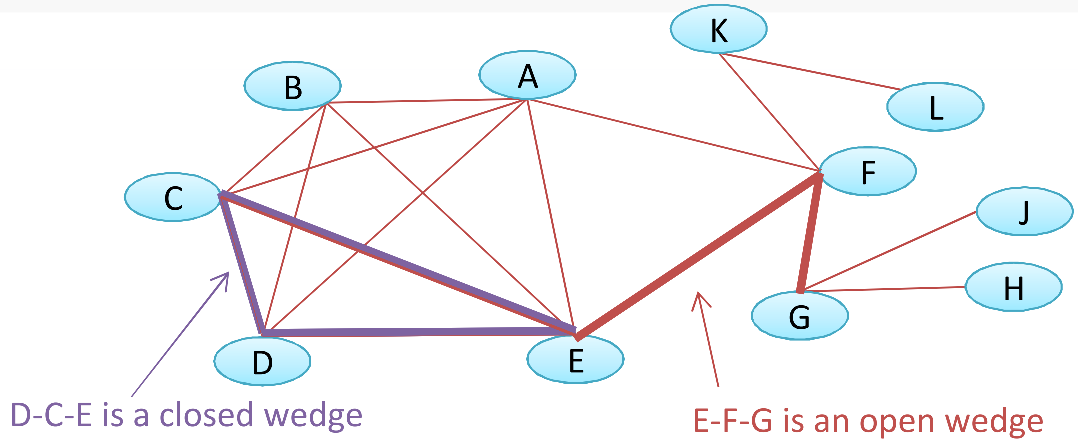
Triangles are critical for graph analysis

- Interpreted in many ways in social sciences.
  - Identifier for bridges between communities.
  - Likelihood to go against norms
- Applied to spam detection
- Used to compare graphs
- Proposed as a guide for community structure.
- Stated as a core feature for graph models [Vivar&Banks11]
  - Cornerstone for Block Two-level Erdos-Renyi (BTER) model
- Rich set of algorithmic results
  - Algorithms, runtime analysis, streaming algorithms, MapReduce, …
  - Enables decomposition into dense blocks
  - Well-defined property of the graph, not an artifact of the algorithm





Algorithms for important metrics: transitivity for large graphs



Enumeration: Find *every* wedge. Check if each is closed.

Transitivity = C = # closed wedges / # wedges

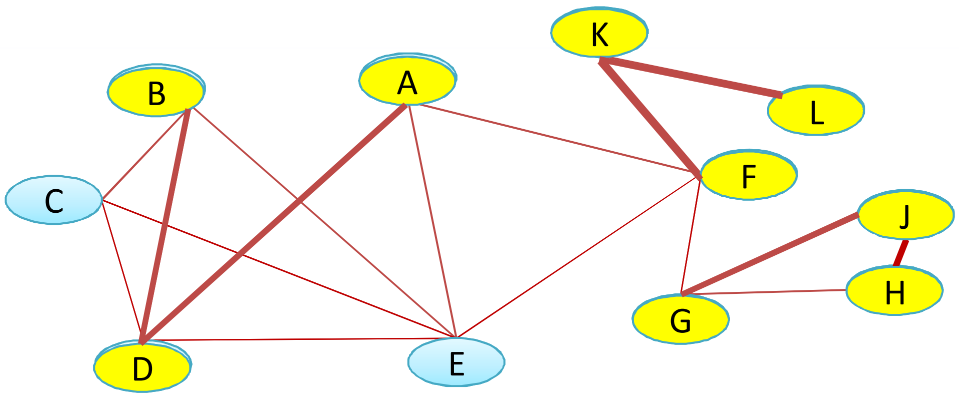
= 3*#triangles/ # wedges

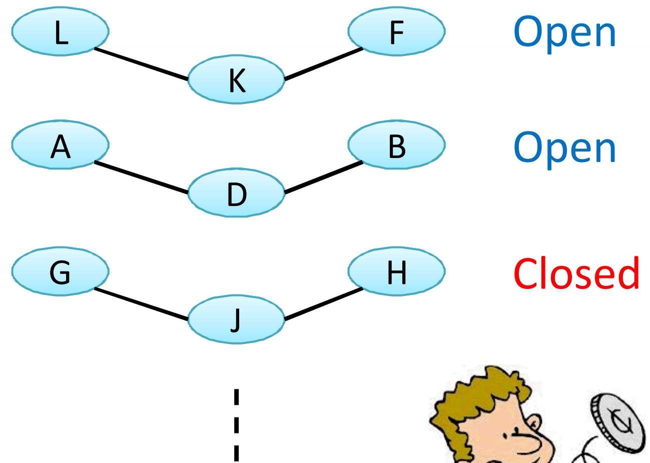
Sampling: Sample a few wedges (uniformly). Check if each is closed.

C = # closed sampled wedges / # sampled wedges

Seshadhri, P., Kolda, *SIAM Intl. Conf. Data Mining* 2013, Best Research Paper award

Wedge sampling to compute transitivity

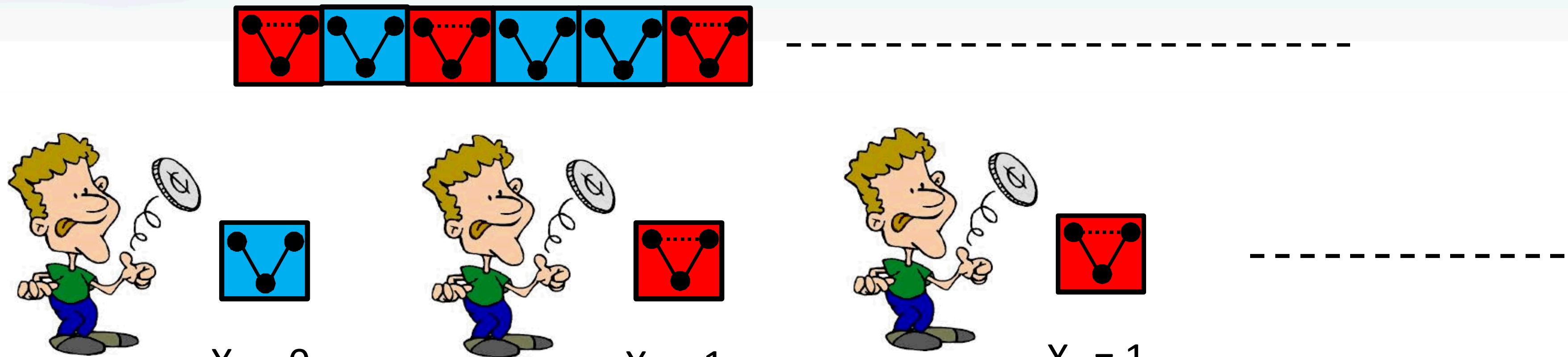




- C = 3T/W = fraction of closed wedges
- Consider list of all wedges, indexed with open/closed
- Pick a uniform random wedge. X = 1 if wedge is closed. Else X = 0
- X is Bernoulli random variable

  and E[X] = fraction of closed wedges = C = 3T/W

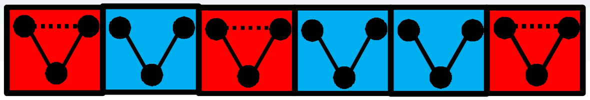
Repeat, repeat, repeat

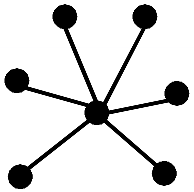


$X_1 = 0$ $X_1 = 1$ $X_2 = 1$

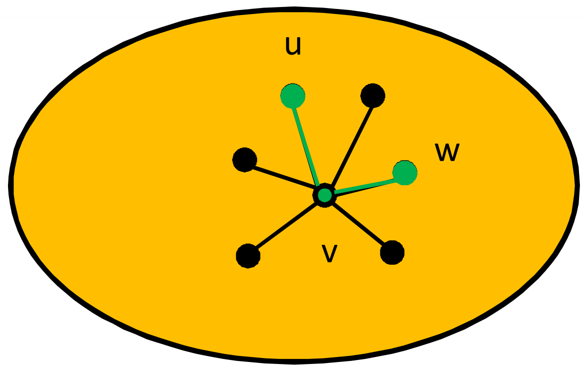
- Perform k independent experiments. Let $Y = (1/k) \sum_i X_i$
  - Y is fraction of closed wedges in sample
  - E[Y] = C. Y converges to C as k grows
- [Chernoff-Hoeffding]: $\Pr[|Y - \tau| > \varepsilon] < e^{-k\varepsilon^2}$
  - $k = \varepsilon^{-2}\log(1/\delta)$. With prob > 1- δ, estimate is accurate within ε
  - With 38K samples, error < 0.01 with prob > 0.999
  - Number of samples independent of graph size

We do not need to generate a wedge list





$$W_v = \binom{d_v}{2}$$



- But list of wedges not possible to generate. So how to get random wedge?
- Pick vertex v with probability W_v/W
- Pick two uniform random neighbors of v to get wedge (u,v,w)
  - This is a uniform random wedge
- So simply repeat this many times to get a set of wedges. Output fraction of closed wedges as estimate for C

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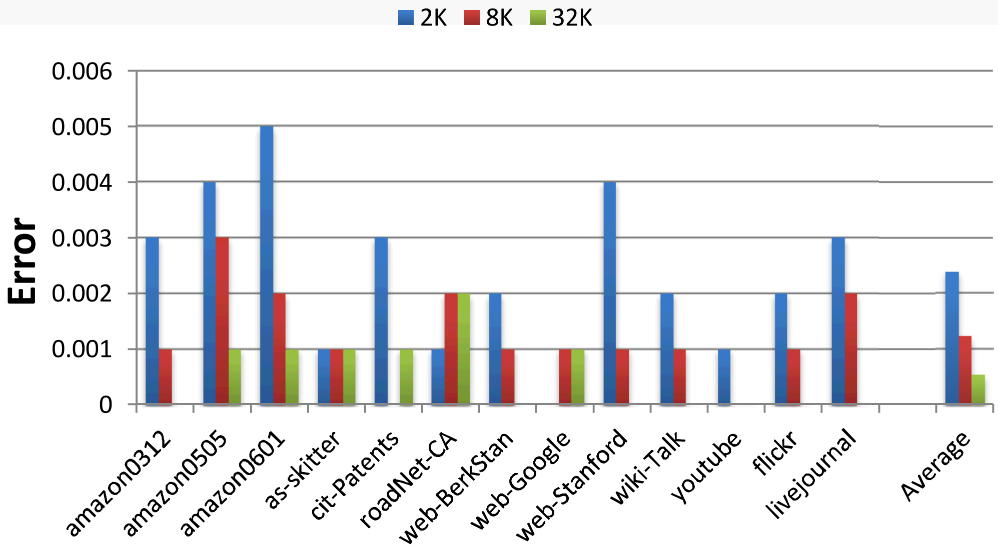
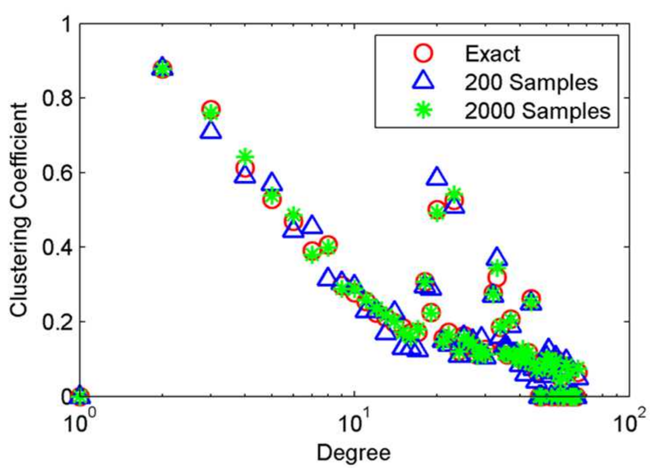
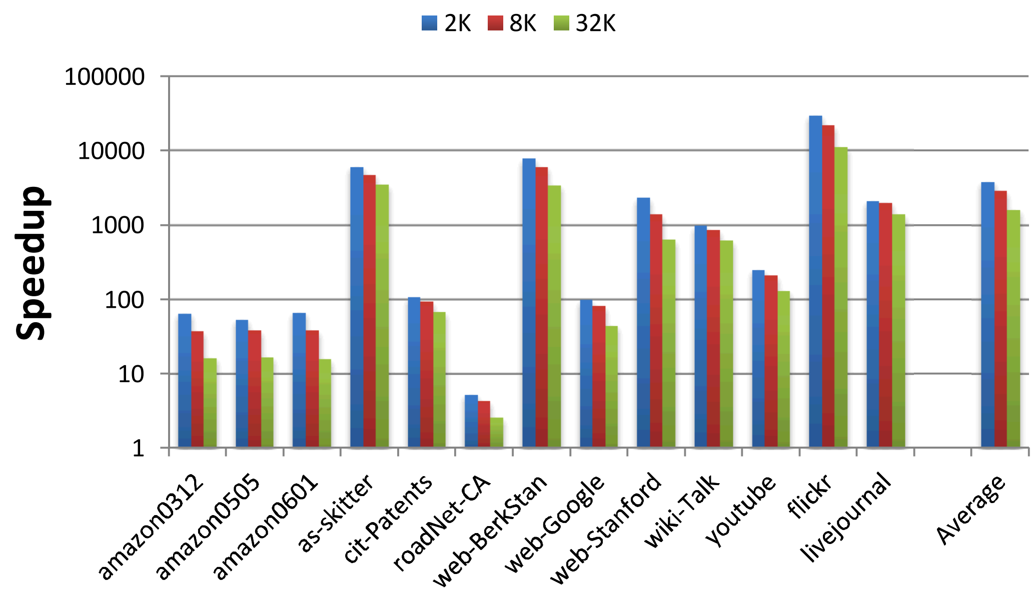
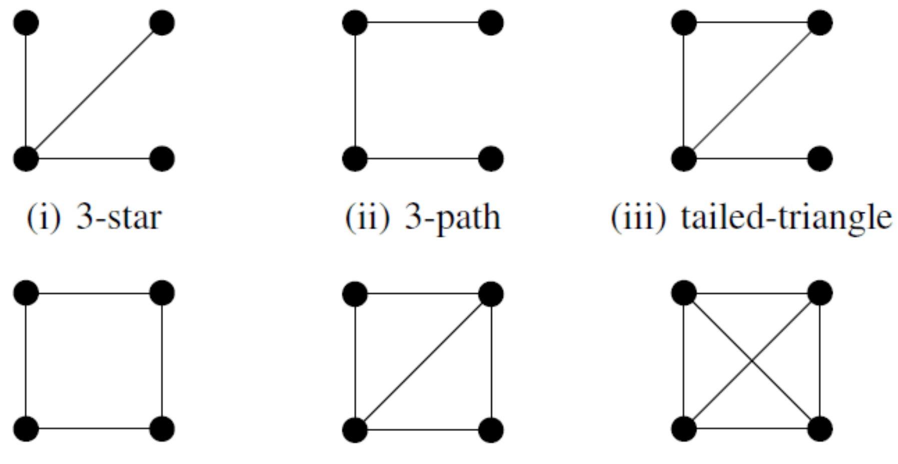
Wedge sampling is effective in practice

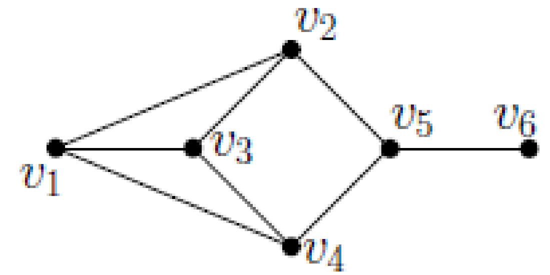






Beyond 3 vertices: how about 4?

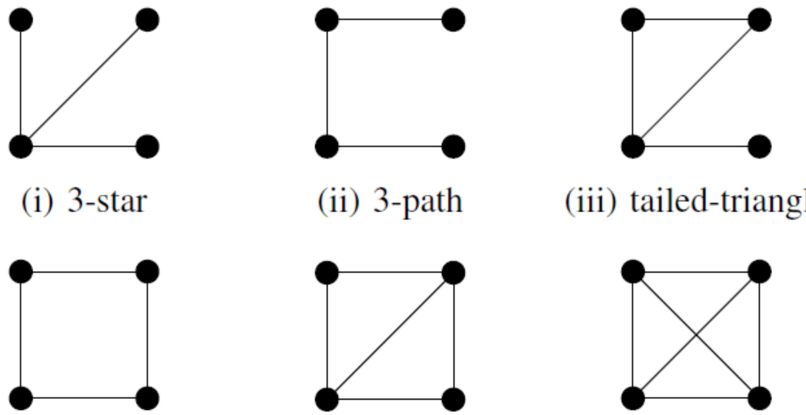

(i) 3-star (ii) 3-path (iii) tailed-triangle (iv) 4-cycle (v) chordal-4-cycle (vi) 4-clique



$$\begin{pmatrix} 1 & 0 & 1 & 0 & 2 & 4 \\ 0 & 1 & 2 & 4 & 6 & 12 \\ 0 & 0 & 1 & 0 & 4 & 12 \\ 0 & 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{pmatrix} = \begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \end{pmatrix}$$

- Much richer set of (connected) patterns
- Induced, C_i, vs. Non-induced, N_i
  - (Vanilla) subgraph: take subset of edges
  - Induced subgraph: take subset of vertices, take all edges in them
  - Getting vanilla counts form induced subgraph counts is not hard

Exact counting is not scalable



(i) 3-star (ii) 3-path (iii) tailed-triangle

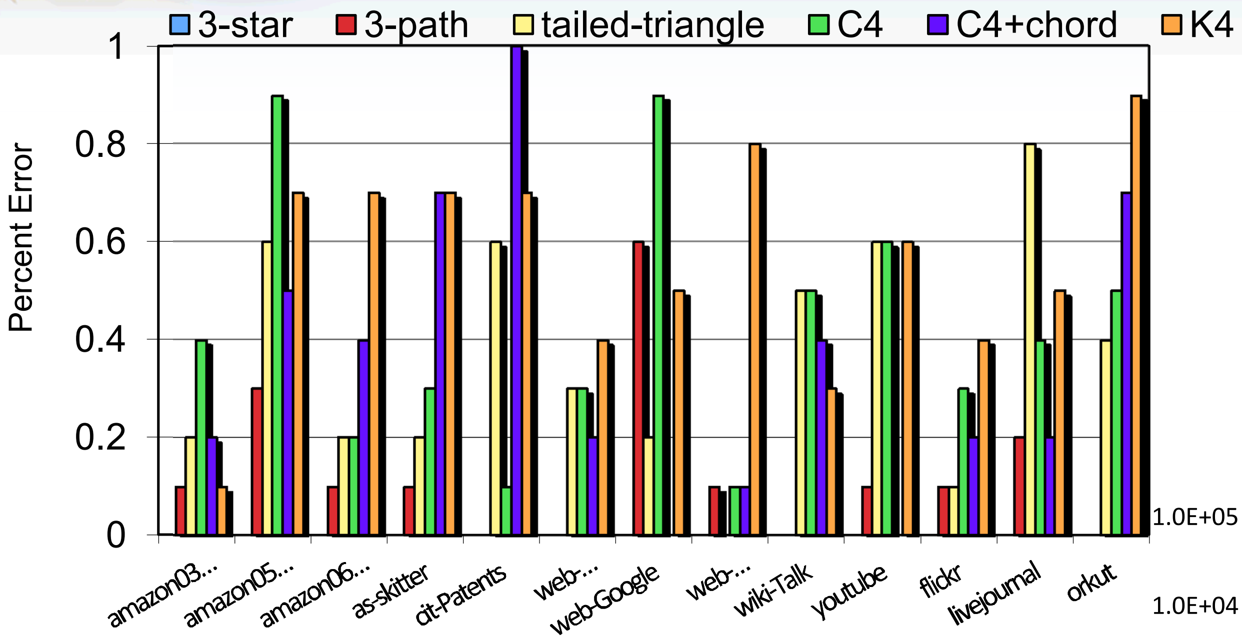
(iv) 4-cycle (v) chordal-4-cycle (vi) 4-clique

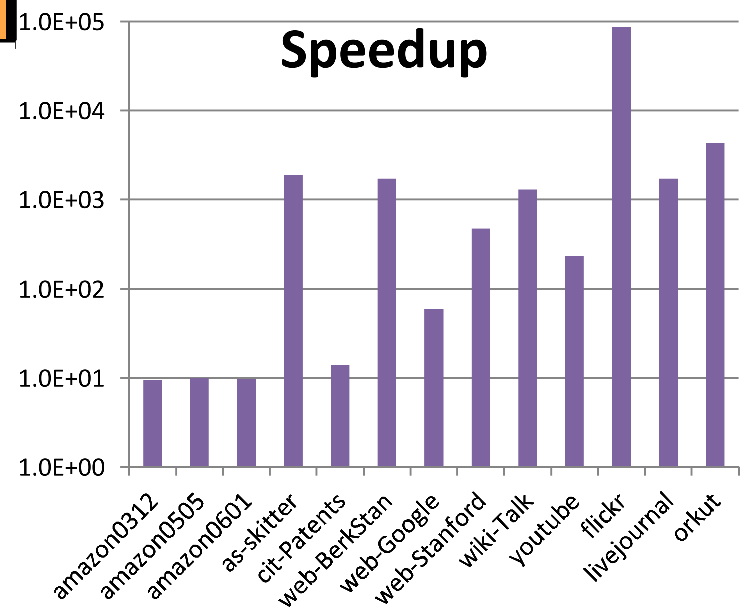
Past approximate counting work does not scale, either

- MCMC methods, color coding, graph sparsification
- No provable methods, accuracies at best ~10%, often need computer clusters
- No results for (say) 100M edges

Graph	n	m	3-path	Tail-tri	4-cycle	4-clique	Time
Web-Berk	600K	6M	10B	400B	20B	1B	2 hrs
Flickr	1M	15M	7T	100M	100B	25B	60 hrs
Orkut	3M	200M	10T	1T	70B	3B	19 hrs

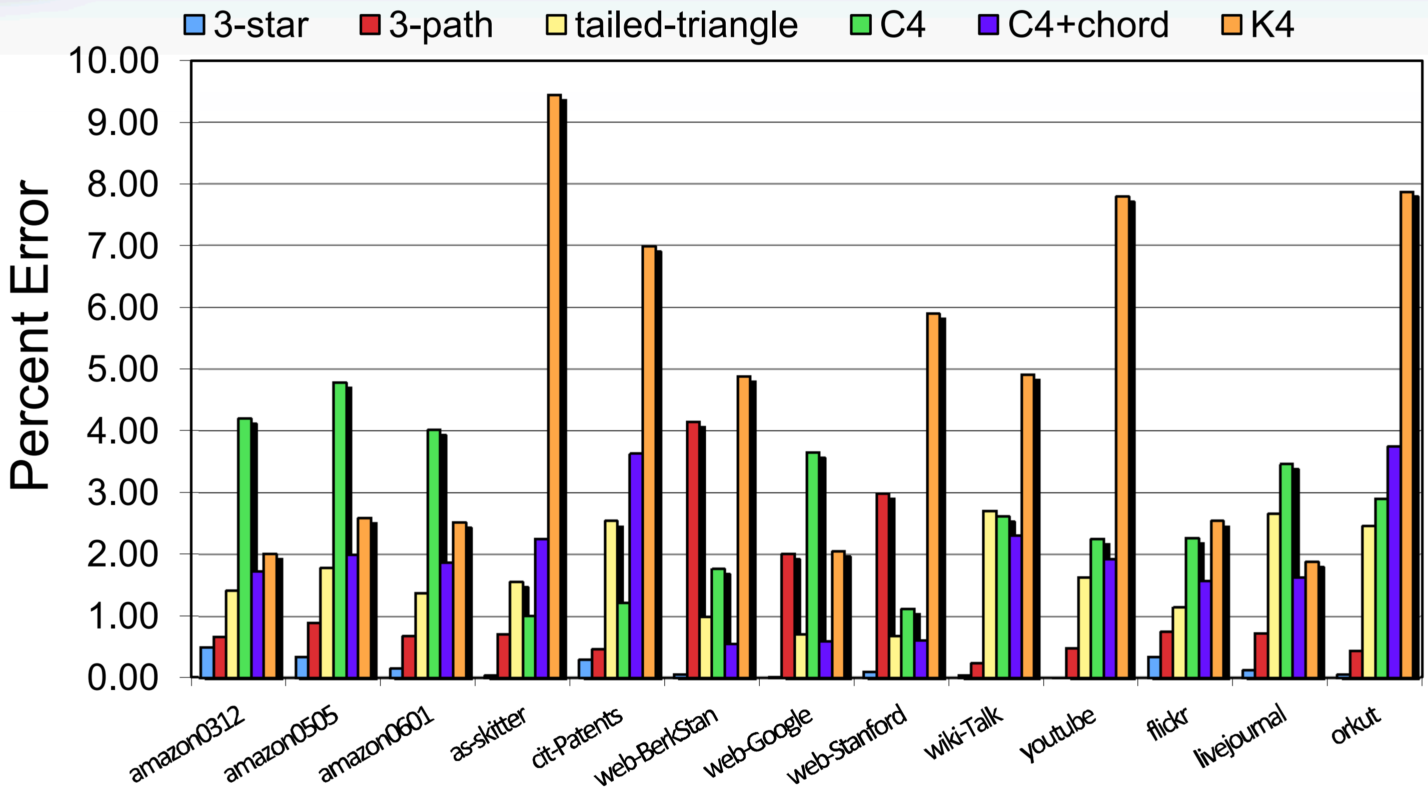
3-path sampling algorithm is fast and accurate





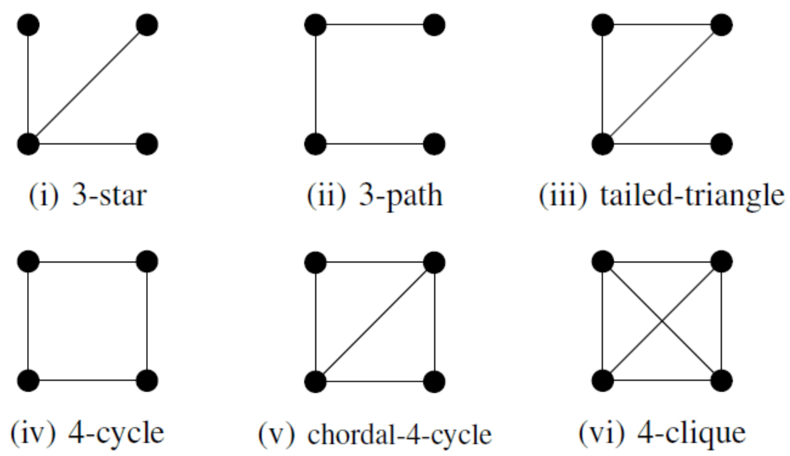
Graph	Time (exact)	Path-sampling
Web-Berk	2 hrs	3 sec
Flickr	60 hrs	2 sec
Orkut	19 hrs	16 sec

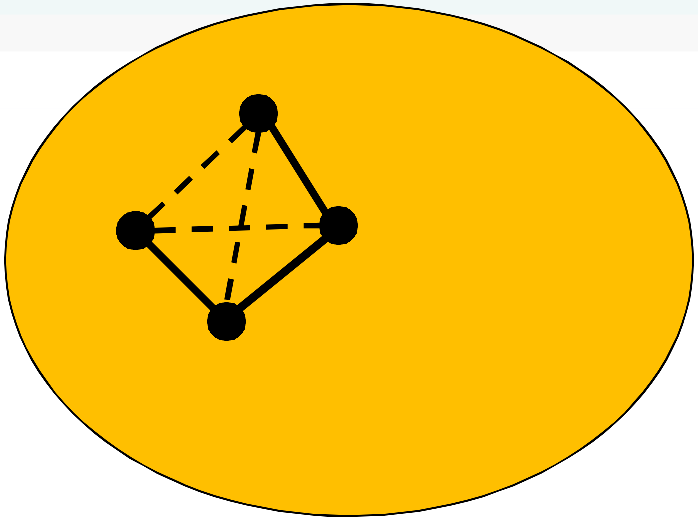
Sampling gives provable accurate results



- Algorithm outputs hard error bounds for any desired confidence
  - "With confidence > 99.9%, the output is within 3% of true answer."
- No assumption on the graph; probability is over the randomness of the algorithm.

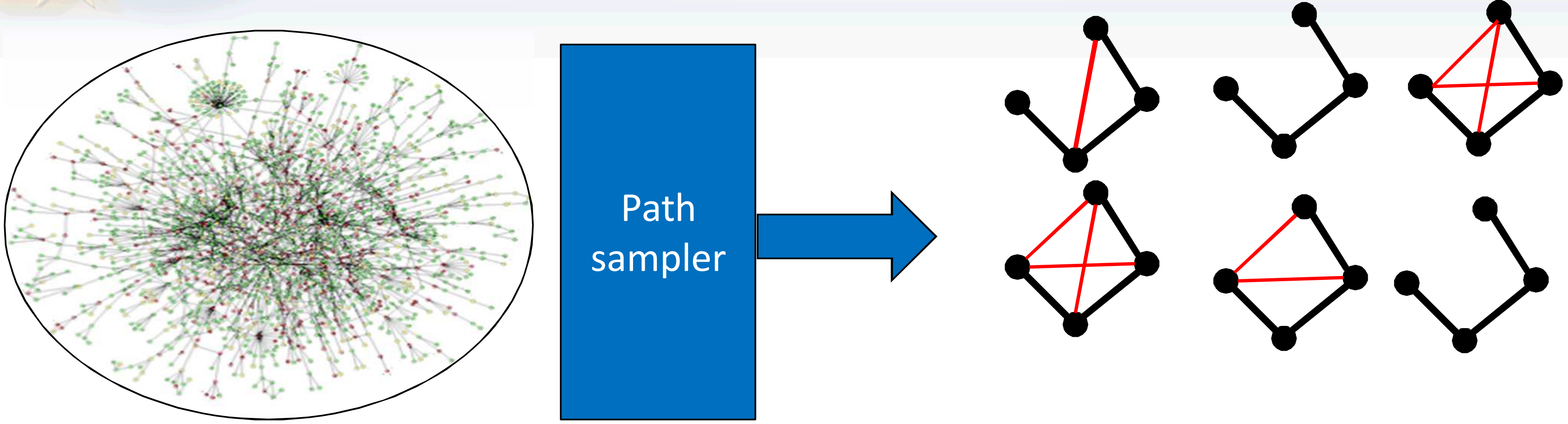
The method





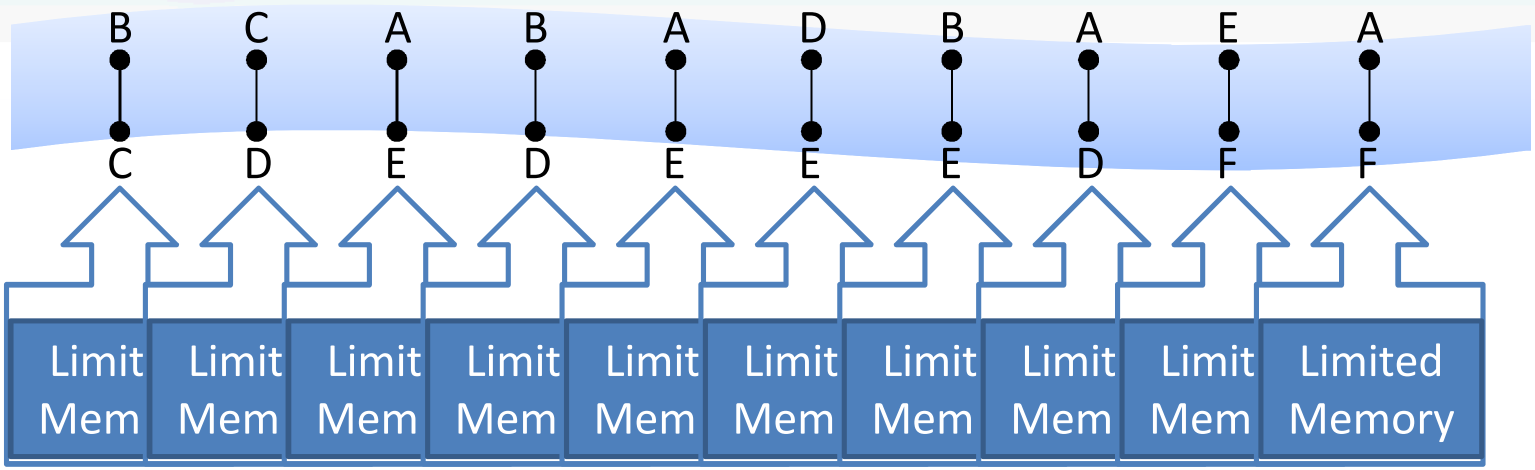
- Except for 3-stars, each pattern contains a 3-path
- Sample set of uniform random 3-path, check the vertices to see what pattern is induced
  - We do not need to generate a full list of 3-paths.
- Extrapolate these counts to get estimates

The big picture



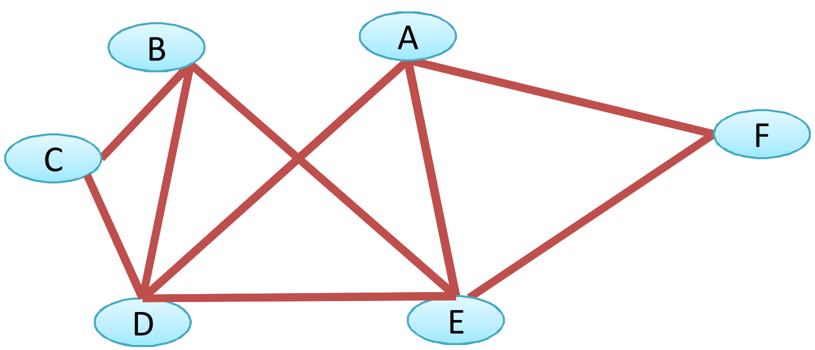
- Use pattern counts from samples to estimate true count
- Not hard to argue that our output is unbiased estimator of true count
- No assumption on graph, probability over randomness of algorithm
- How many samples needed to get accurate estimates?
  - For better results, we sample "centered 3-paths"

Streaming Triangle Counting



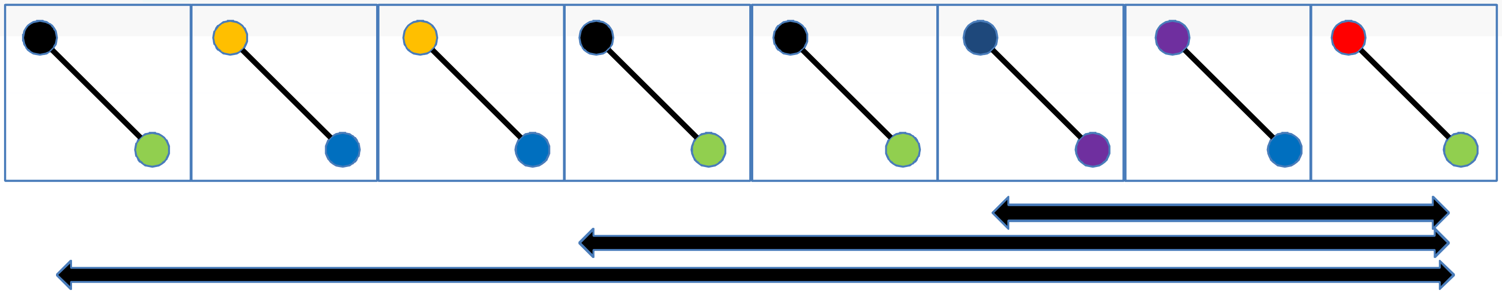
Triangles so far: 4

Graph seen so far:



- Data streams important for situational awareness
  - Streaming algorithms also useful for large data sets
- Algorithmically
  - See each edge only once
  - Either take action or lose that piece of information forever

Real-world messiness



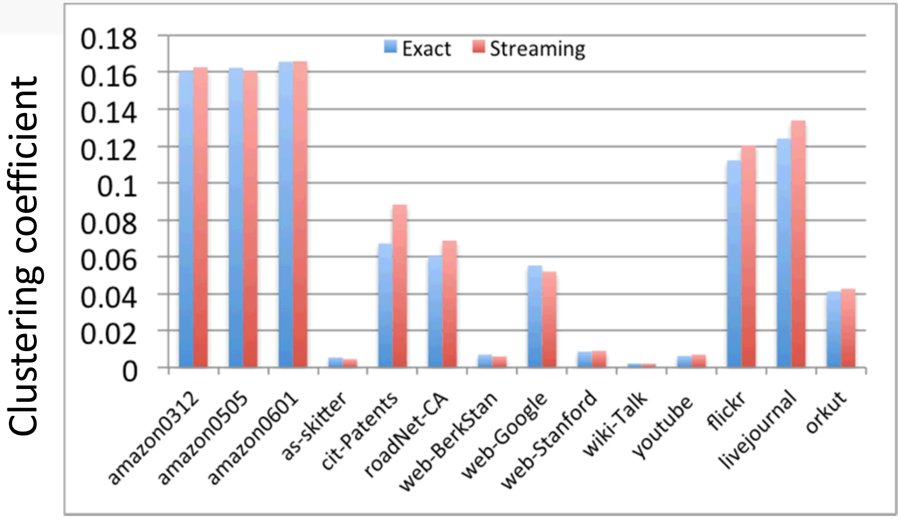
- Real-world streams are multigraphs: edges can be repeated
- There is no “true” graph. It depends on how you aggregate

Standard approaches and their drawbacks

- There are no repeats. Assume graph is simple
  - Removing repeated edges requires extra pass over edges
  - Assumption of no repeats is expensive to enforce
- Aggregate every edge seen. The “window” is all of history
  - Not clear how to store information of various time-windows simultaneously

We can analyze streams of edges

- Approximating triangle counts and transitivity in graph stream with repeated edges
  - No preprocessing.
    Works with raw stream
- Information on multiple time windows with same data structures
  - Potential solution to the problem of how much data to store

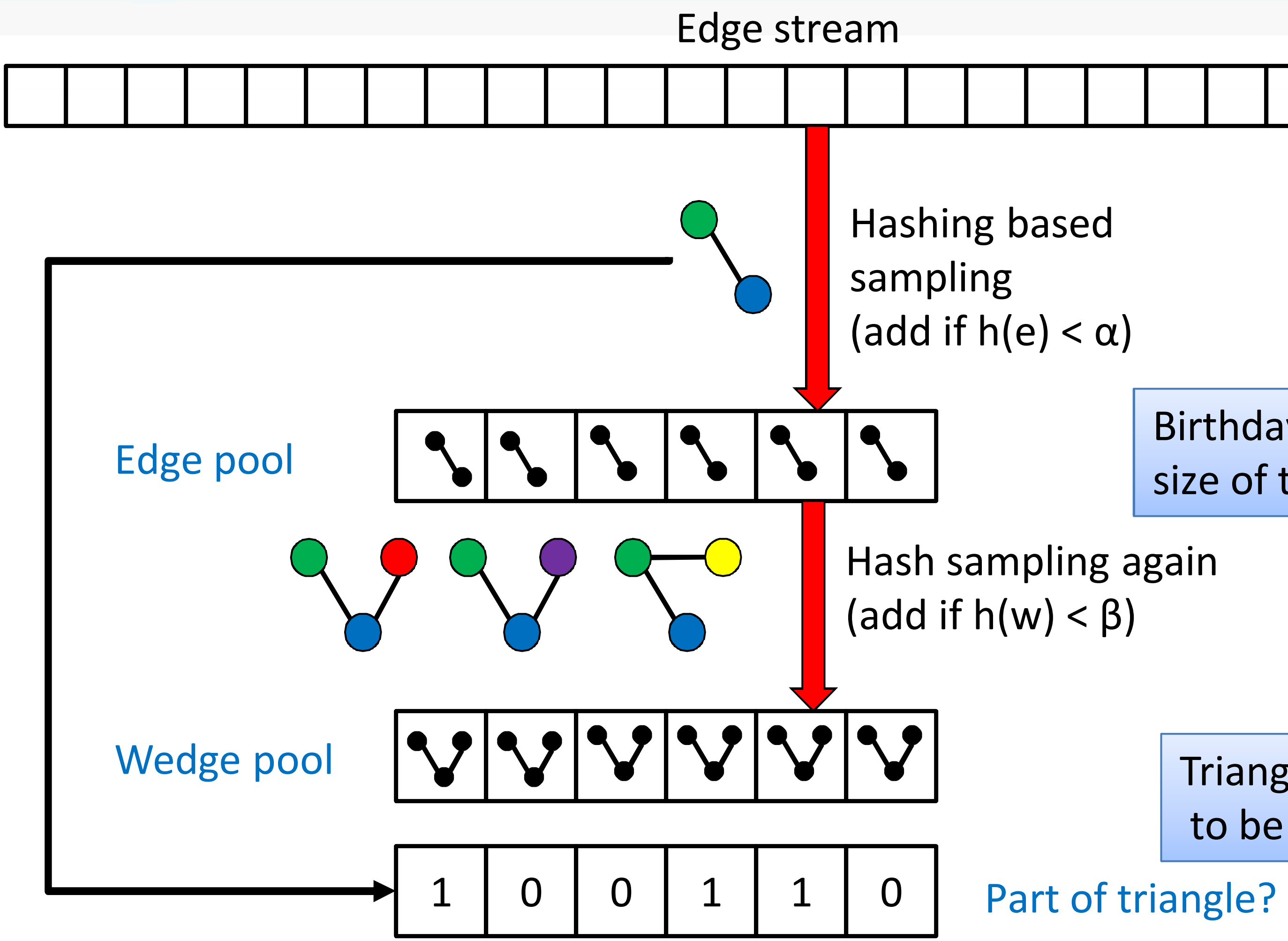


- Edge pool size: 20K; Wedges pool size is: 20K
- Jha, Seshadhri, P. KDD13, Best Student Paper award

- Provable bounds on accuracy, excellent empirical behavior
- Based on methods in [Jha-Seshadhri-Pinar13], but needs new ideas to overcome issues

Core Idea:

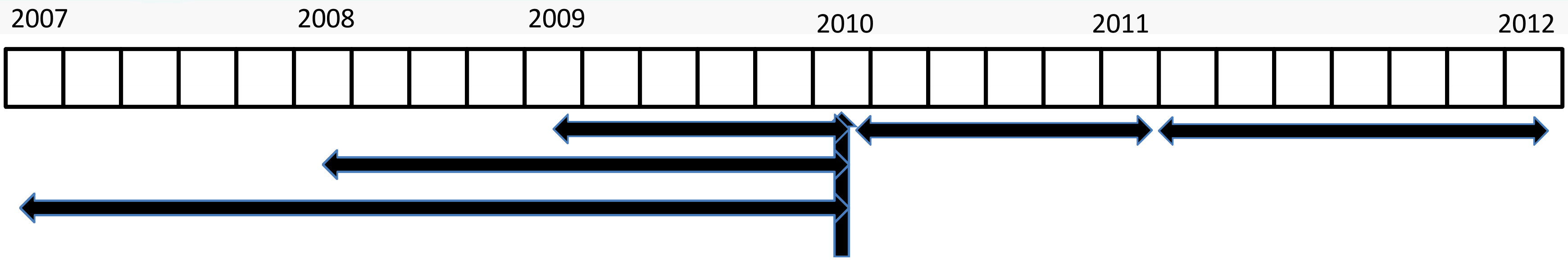
Wedge sampling on a stream



Birthday Paradox tells the size of the hedge pool

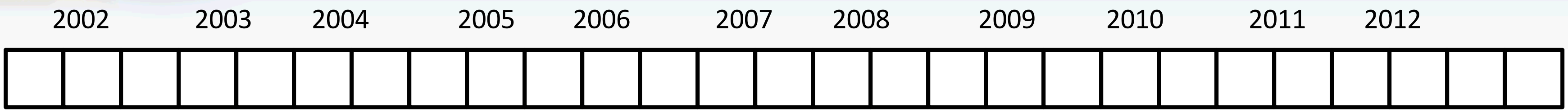
Triangle counts need to be debiased

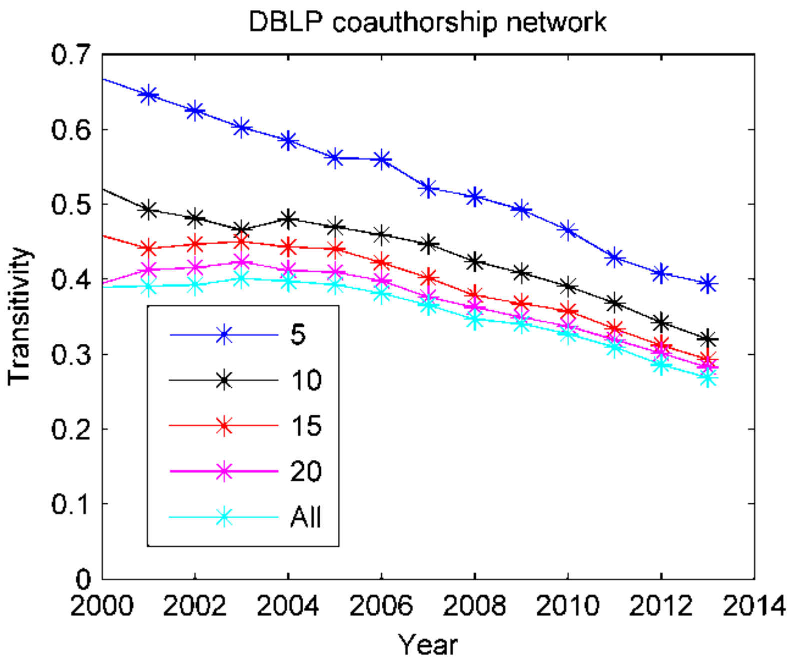
Case study: DBLP graph

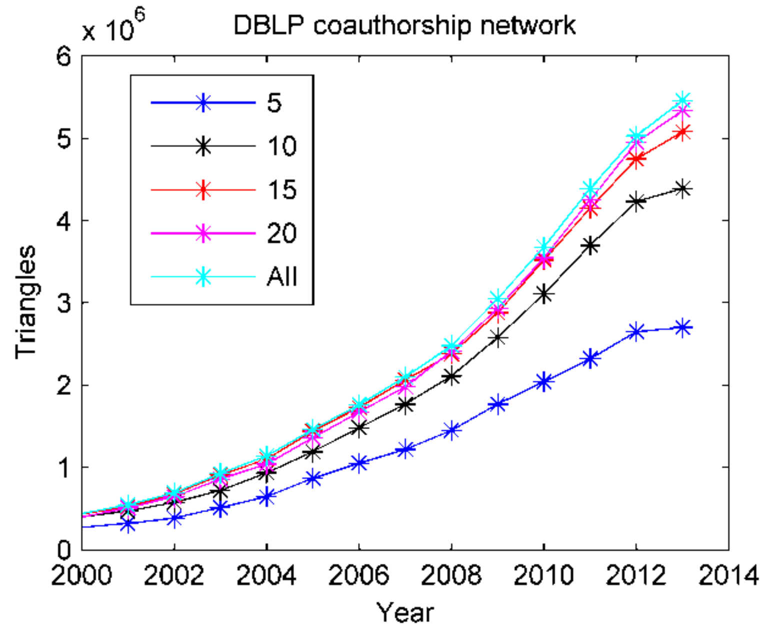


- DBLP co-authorship graph: all paper records over 50 years gives graph stream
  - Naturally repeated edges. Colleagues work together for many papers
  - Size = 3600K, non-repeated edges = 254K
- For graph G[t:t+Δt], there is associated transitivity and triangle count
  - How does this vary with t and Δt?

Triangle trends in DBLP graph

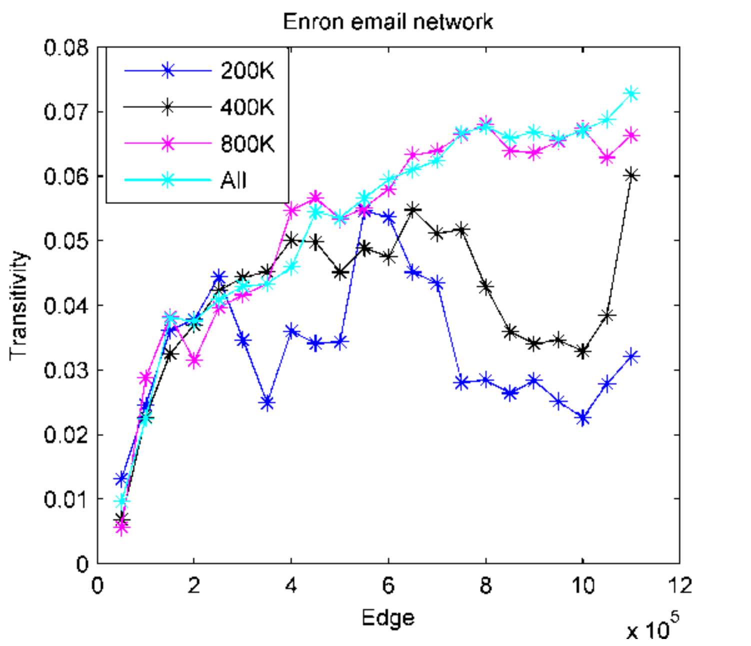


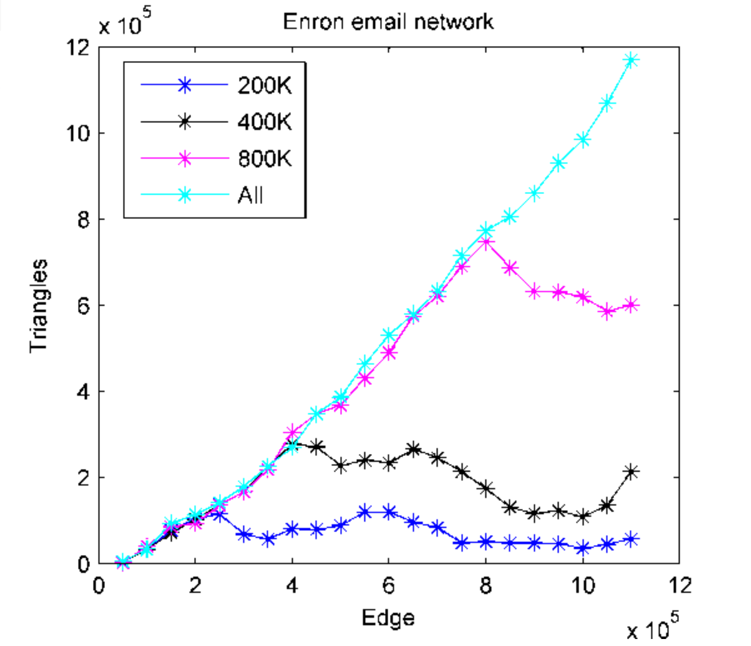




- Size = 3600K, non-repeated edges = 254K
- Results obtained with storing 30K edges

Triangle trends in Enron graph

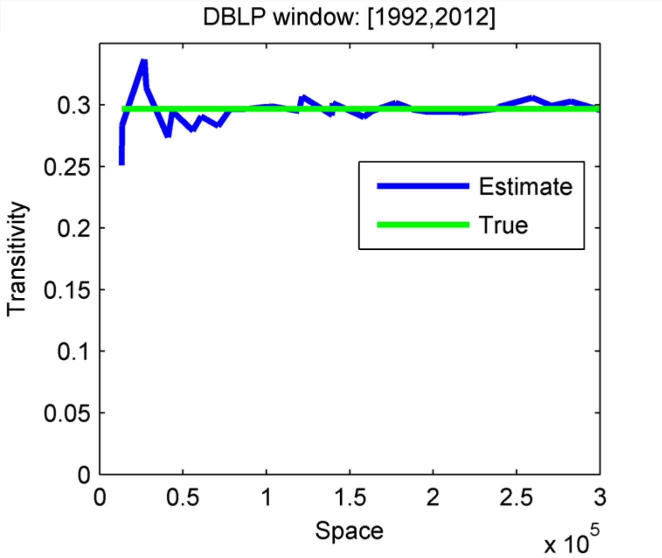




- Enron email network: stream size 1100K, non-repeated 300K
- Storage used = 8K
- Trends “opposite” to DBLP graph

Streaming Algorithm Features

- Only two parameters α, β
  - No knowledge of graph required
- Provable guarantee on expectation
  - Provable variance bound (though not useful in practice)
- Space around 1% of total stream
- Accuracy always within 5%



Conclusions

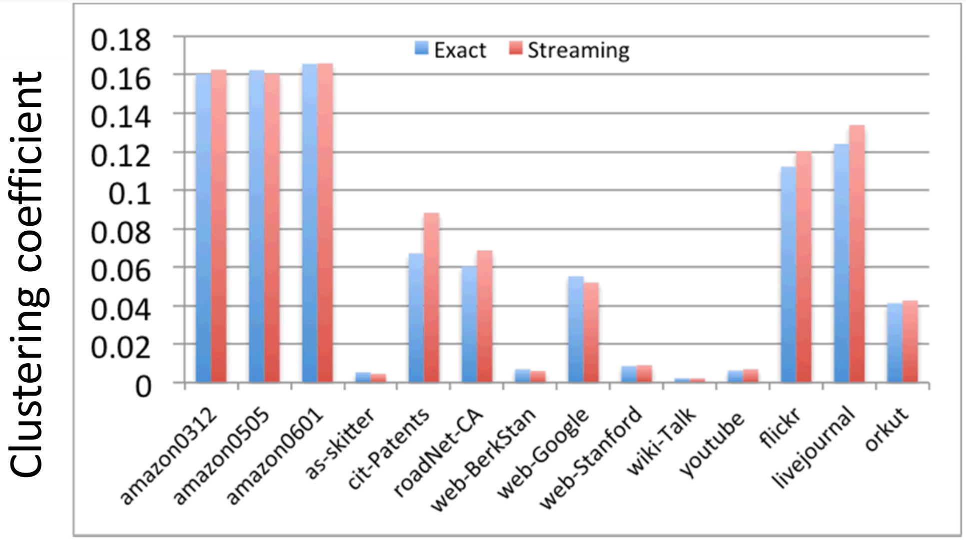
- If you need the *counts* of small patterns on a large graph, use sampling (streaming).
- If you need a list of small patterns,
  - If the output size is small, enumerate!
  - If not, the list should be an input to another process, and let's talk about the full process.
- Wedge sampling enables efficient computation of many triadic measures
  - Has provable error/confidence bounds
  - Amenable to handling distributed data
  - Extended to streaming analysis
    - Can handle repeated edges and different time windows
- Similar techniques can be used for 4-vertex patterns
  - Used 3-path sampling instead of wedge sampling

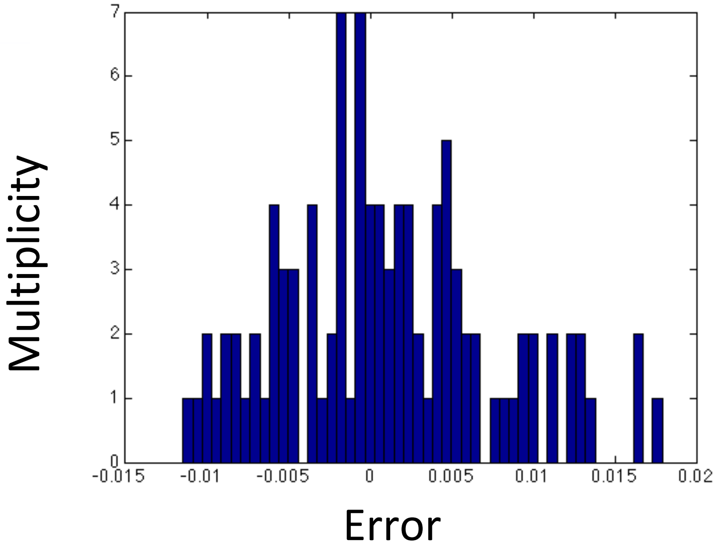
References

- Wedge Sampling: C. Seshadhri, A. Pinar and T. G. Kolda, ***Triadic Measures on Graphs: The Power of Wedge Sampling***, Proc. SIAM Intl. Conf. on Data Mining (SDM'13), Apr 2013 (preprint: arXiv:1202.5230).
- Wedge Sampling MapReduce: T. G. Kolda, A. Pinar, T. Plantenga, C. Seshadhri, and C. Task, ***Counting Triangles in Massive Graphs with MapReduce***, arXiv:1301.5887, to appear SIAM Scientific Computing.
- Wedge Sampling for directed patterns: C. Seshadhri, A. Pinar and T. G. Kolda, ***Wedge Sampling for Computing Clustering Coefficients and Triangle counts for Large Graphs,*** arXiv:1309.3321, Stat. Analysis & Data Mining, 7(4), pages: 294–307.
- Streaming Algorithm for triangles: M. Jha, C. Seshadhri, and A. Pinar, ***A Space Efficient Streaming Algorithm for Triangle Counting using the Birthday Paradox***," Proc. ACM SIGKDD Conf. on Knowledge Discovery and Data Mining (KDD'13) August 2013 (preprint; arXiv:1212.2264).
- Streaming Algorithm for multigraphs: M. Jha, A.Pinar, and C. Seshadhri, ***Counting Triangles in Real-World Graph Streams: Dealing with Repeated Edges and Time Windows***, arXiv:1310.7665.
- Counting 4-vertex patterns: M. Jha, A. Pinar, and C. Seshadhri, ***Path Sampling: A Fast and Provable Method for Estimating 4-Vertex Subgraph Counts***, submitted for conference publication.

Questions

Streaming algorithm is effective in practice

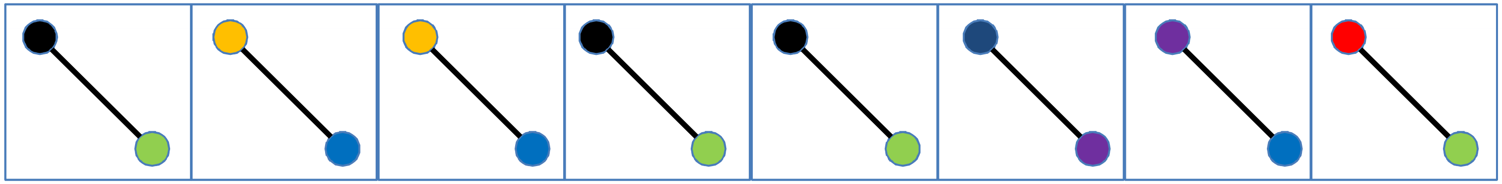




- Experiments on public data sets
- Edge pool size: 20K; Wedges pool size is: 20K
  - Pool sizes are independent of the graph size.
- The estimates are accurate.
- The variance is small.

Jha, Seshadhri, P. KDD 2013, Best Student Paper award

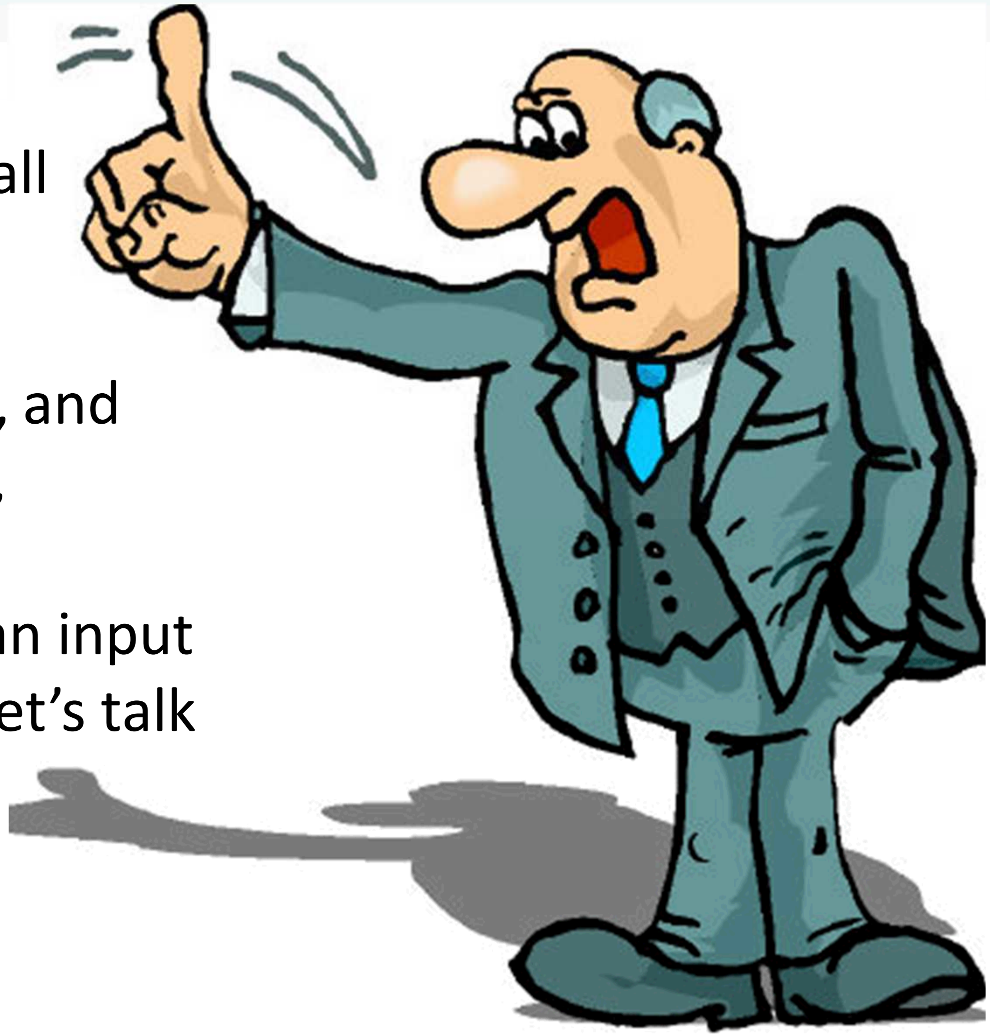
Drawbacks of ignoring repeats



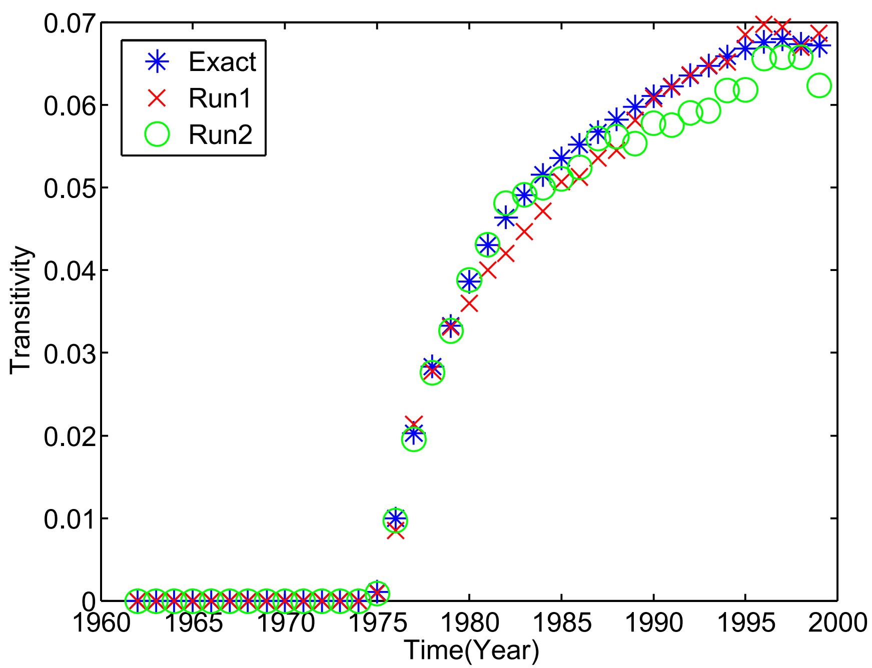
- Assumptions useful for algorithmic progress, but avoids real-world complexities
  - Algorithms cannot be deployed in "wild"
- Removing repeated edges requires extra pass over edges
  - Assumption of no repeats is expensive to enforce
- Not clear how to store information of various time-windows simultaneously

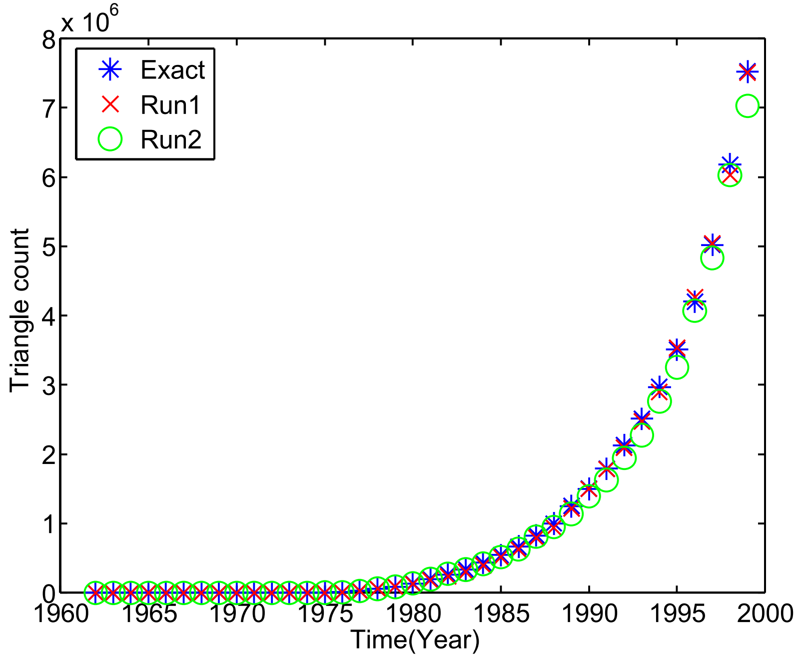
Take home lesson

- If you need the *counts* of small patterns on a large graph, use sampling.
- If you need a list of patterns, and
  - if the output size is small, enumerate.
  - If not, the list should be an input to another process, and let's talk about the full process.



Streaming algorithm provides a running estimate

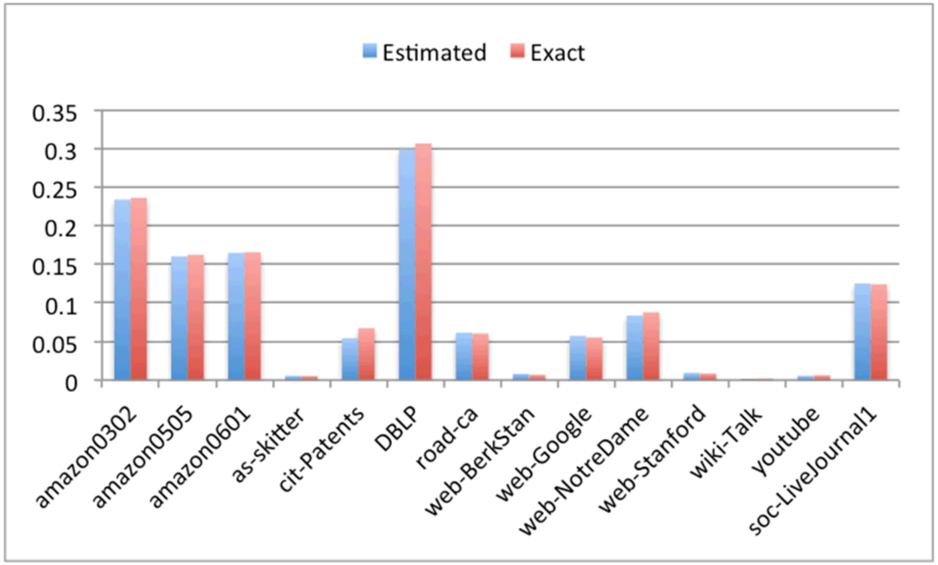




- Results on the patent citation network
  - 3.8M vertices, 16.5M edges.
- The algorithm provides accurate running estimates.

Next step: Streaming multigraphs

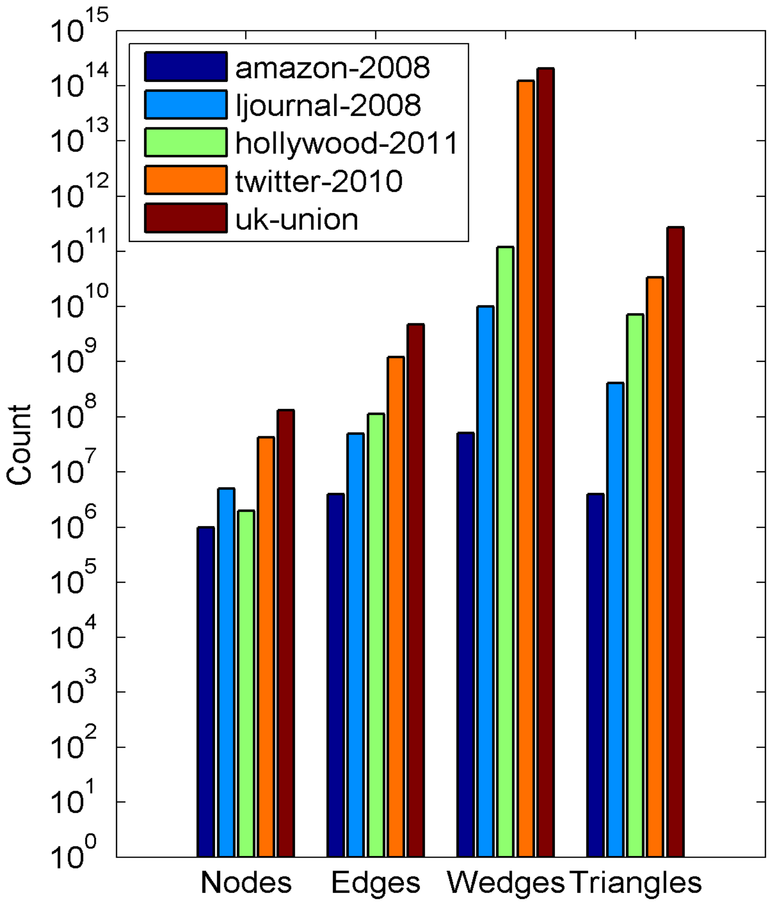
- Many graphs are a stream of *repeated* edges. (Emails, data transfers, co-authorship etc.)
- Generalized our algorithm for multigraphs.
  - Used random hashing to detect multiple instances.
  - Devised an unbiasing technique to avoid stream order sensitivity.
    - aaabbbccc vs. abcabcabc
- Processed the DBLP raw data
  - |V|=1.2M, |E|=5.1M,
    9.0 repeated edges,
    11.4M triangles transitivity= 0.174
  - Estimate with 30K edges
    and 30K wedges
    11.3M triangles transitivity= 0.173



Jha, Seshadhri, P., *arxiv 1310.7665*

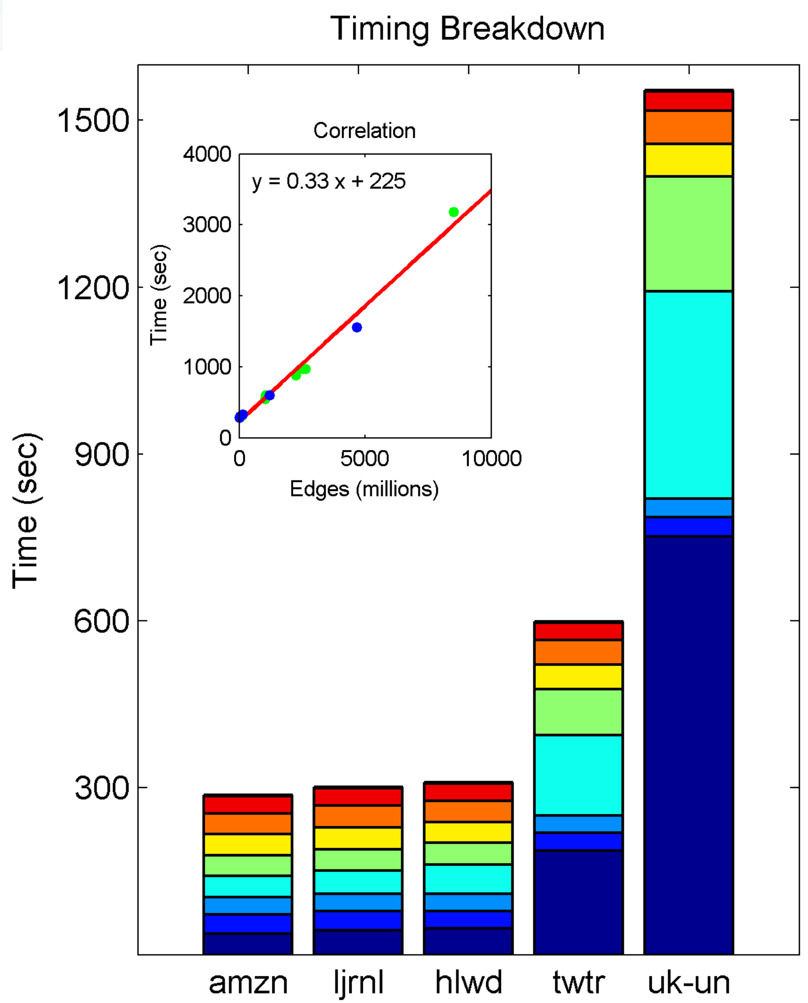
How about even bigger graphs?

- Wedge sampling can be executed when the data is distributed.
- We proposed a Hadoop implementation.
  - Key to success: data movement is minimal.
- 5 real-world networks
  - Source: Laboratory for Web Algorithms
  - Largest: 132M nodes, 4.6B edges
- Distributed Server: 32-Node Hadoop Cluster
  - 32 Intel 4-Core i7 930 2.8GHz CPU
  - 32 x 12GB = 384GB memory



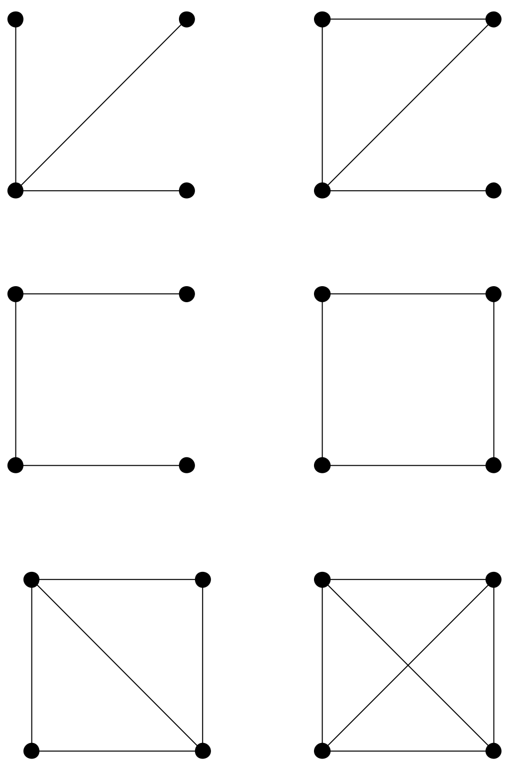
Wedge Sampling for BIG Graphs

- 32-node Hadoop cluster results using wedge sampling to compute degree wise clustering coefficients
  - Logarithmic bins; 2000 samples per bin
- Compare twitter times
  - Sampling: 10 mins on 32-node Hadoop cluster
  - Enumeration: 483 mins on 1636-node Hadoop cluster
    - Suri & Vassilvitskii, 2011
  - Enumeration: 180 mins on 32-core SGI, using 128GB RAM
    - by Jon Berry, 2013
- No comparisons for uk-union due to its size

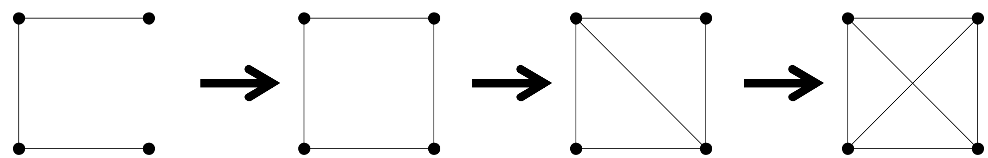


Kolda, P., Plantenga, Seshadhri, Task, arXiv:1301.5886, 2013 to appear in SISC

Counting 4-vertex patterns



- Our sampling approach can be generalized to count 4- vertex patterns.
- Algorithm
  - Count the number of 3-paths
  - Sample 3-paths and count how many of them other patterns

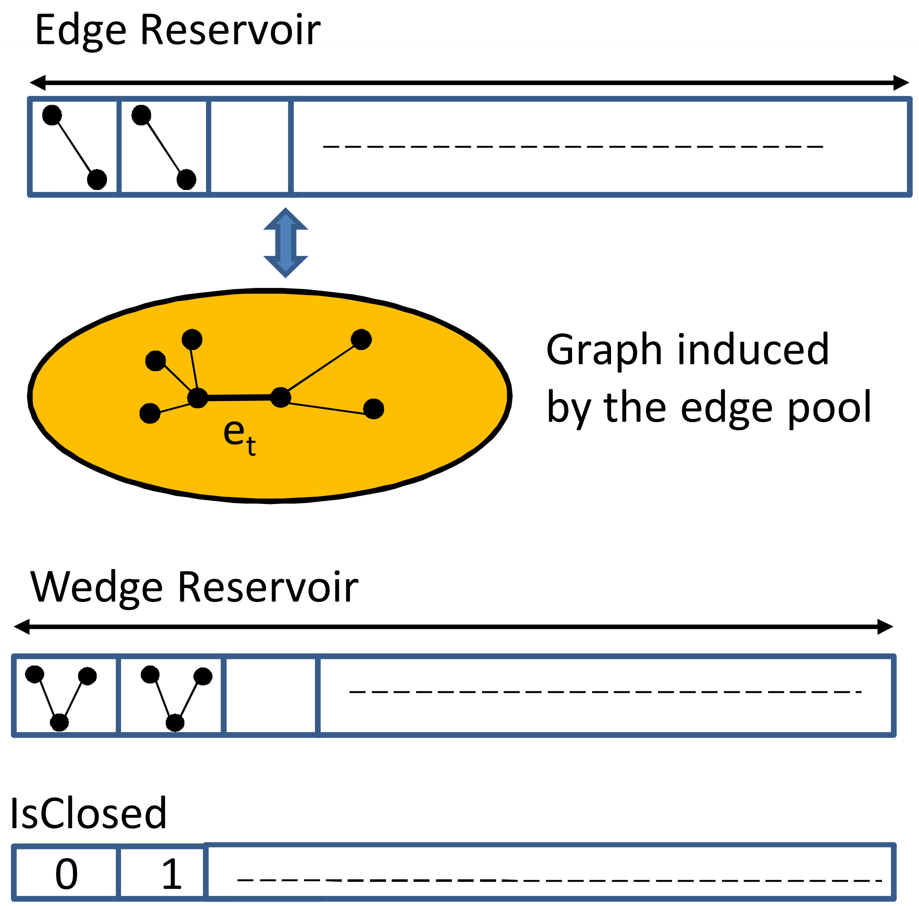


- Experiments show >1K speedups, with <%1 error using 160K samples.

Jha, Seshadhri, P., *coming soon*

Wedge sampling in a streaming world

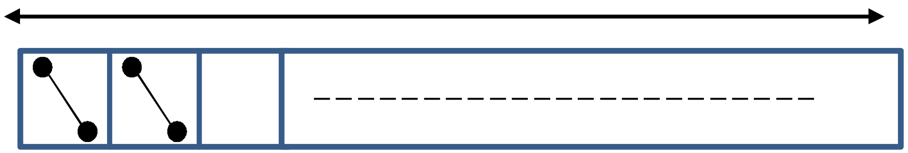
- Keep a random sample of the edges using the reservoir sampling.
- Keep a random sample of the wedges generated by the edges in the edge reservoir.
- Track whether the wedges are closed or not.
- The clustering coefficient is 3*ratio of closed wedges.



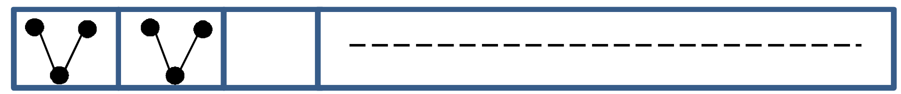
Jha, Seshadhri, P., *KDD* 2013, Best Student Paper award

Birthday paradox to the rescue

Edge reservoir



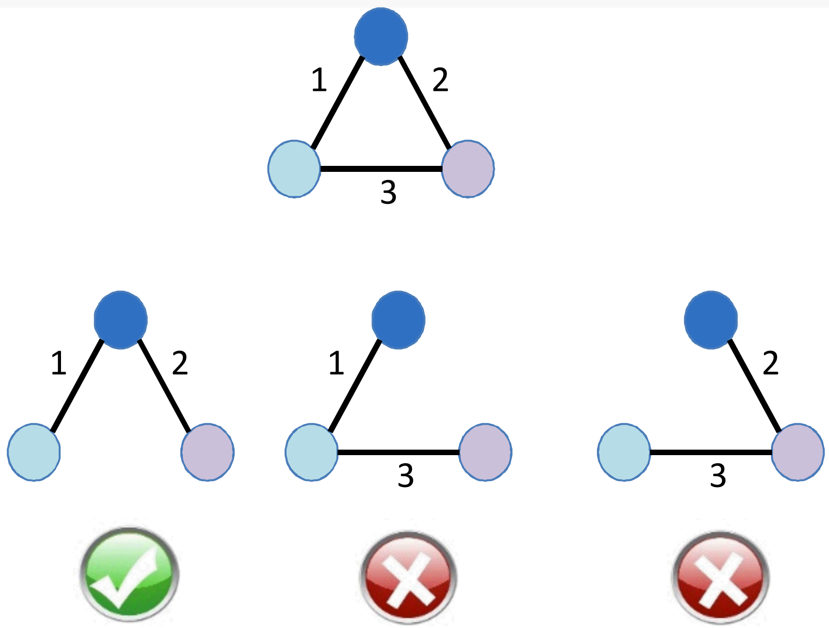
Wedge Reservoir



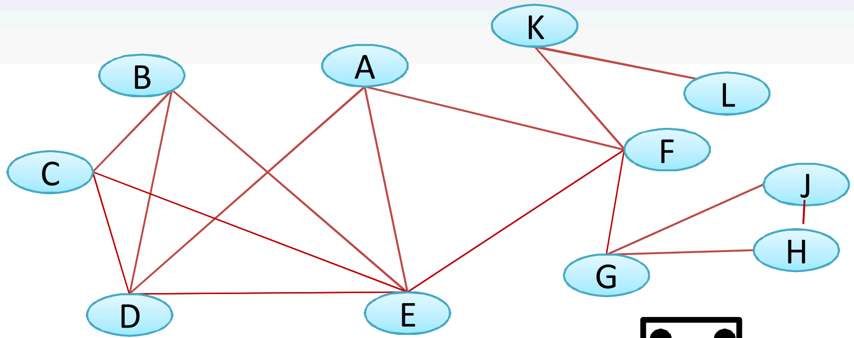
- A wedge is formed by two edge with the same birthday.
- Birthday paradox: O(√n) edges are sufficient to generate a wedge.
  - O(k√n) edges will produce $O(k^2)$wedges.
- Idealized algorithm: Maintain a separate edge reservoir for each wedge
  - Needs O(|S|√n)storage for |S| samples.
  - Has provable bounds; but not as effective in practice.
- Practical algorithm: Maintain a single and slightly bigger edge pool
  - Needs O(√(|S|n) storage
  - Wedge samples are biased, but in practice so enough wedges are generated to unbias the sample.
  - Effective in practice

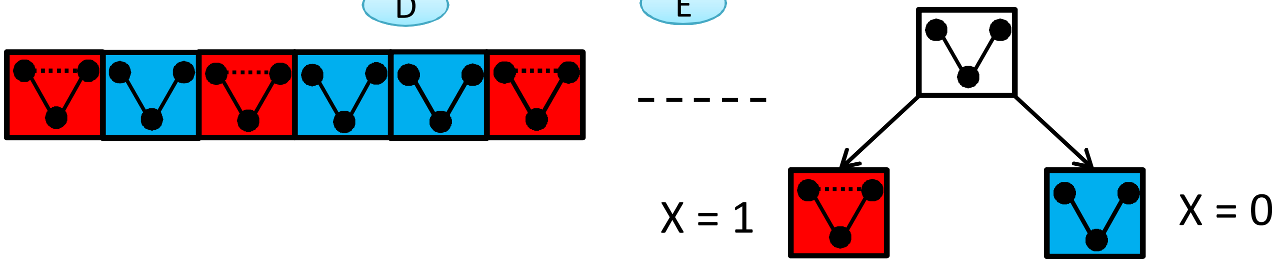
Making up for wedges closed by earlier edges

- Each triangle comprises of 3 wedges.
- In the original wedge sampling, we were able to detect any wedge as closed.
- In the streaming algorithm, we can only detect 1 of the 3 as closed.
- Since wedges are selected randomly, the expected closure rate is 3* the closure rate of the wedge pool.



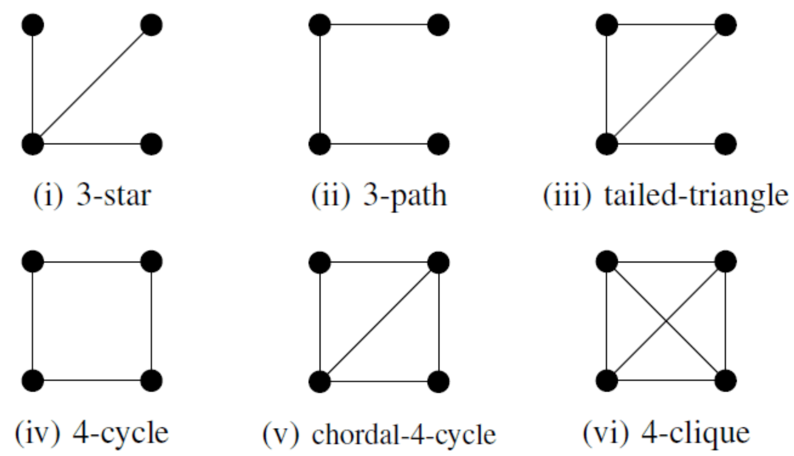
Wedge sampling for τ

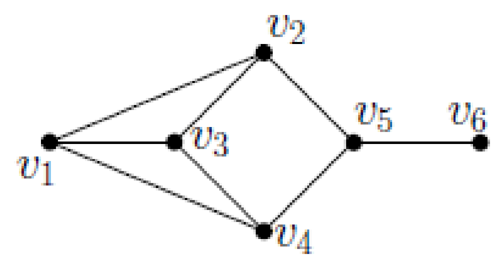




- C = 3T/W = fraction of closed wedges
- Consider list of all wedges, indexed with open/closed

Induced vs non-induced

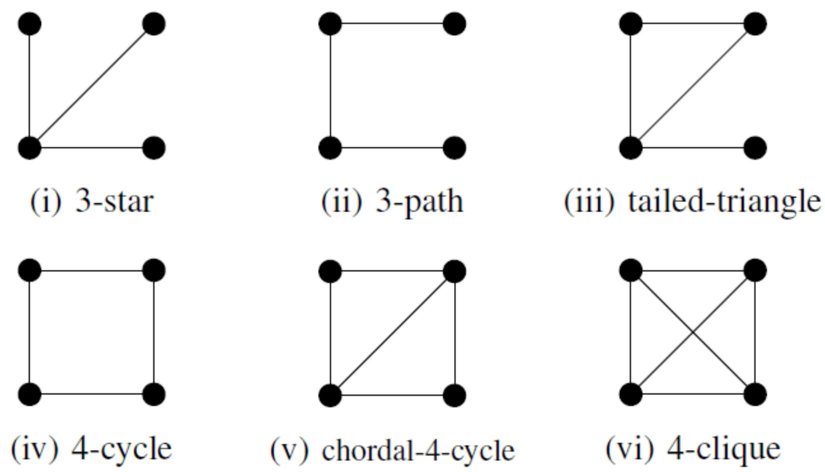




$$\begin{pmatrix} 1 & 0 & 1 & 0 & 2 & 4 \\ 0 & 1 & 2 & 4 & 6 & 12 \\ 0 & 0 & 1 & 0 & 4 & 12 \\ 0 & 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{pmatrix} = \begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \end{pmatrix}$$

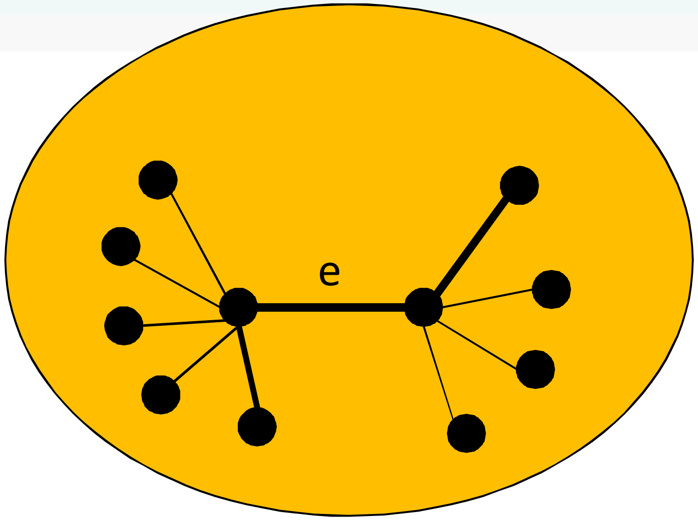
- (Vanilla) subgraph: take subset of edges
- Induced subgraph: take subset of vertices, take all edges in them
- Let C_i is induced count of pattern i
  - Getting vanilla counts not hard

Past art does not scale either



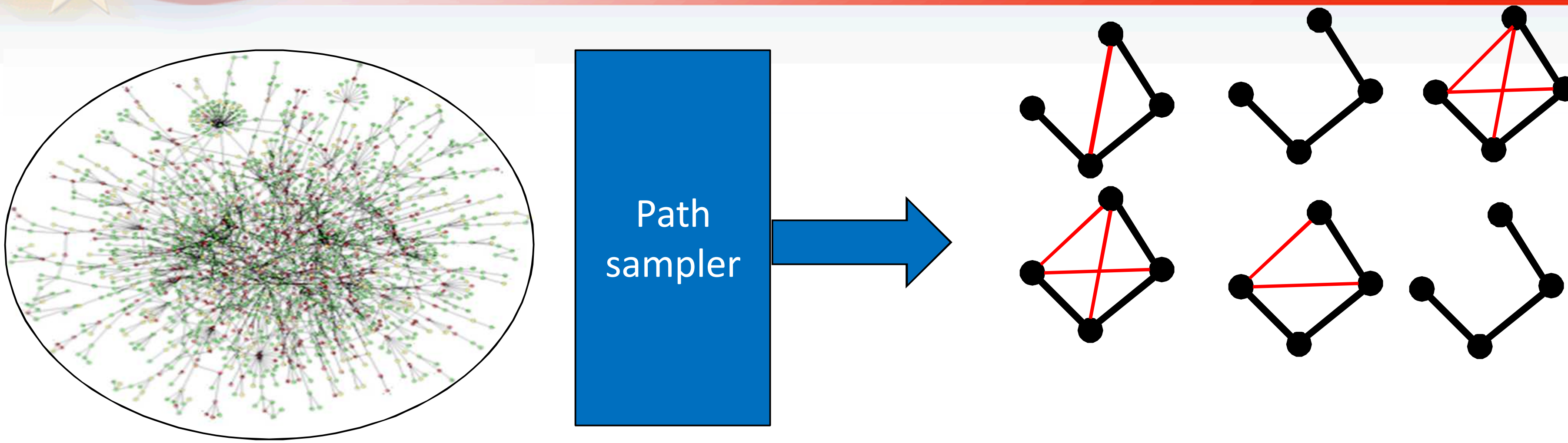
- MCMC methods, color coding, graph sparsification
- No provable methods, accuracies at best ~10%, often need computer clusters
- Nothing tailored for 4 vertices
- No results for (say) 100M edges

Sampling random 3-paths



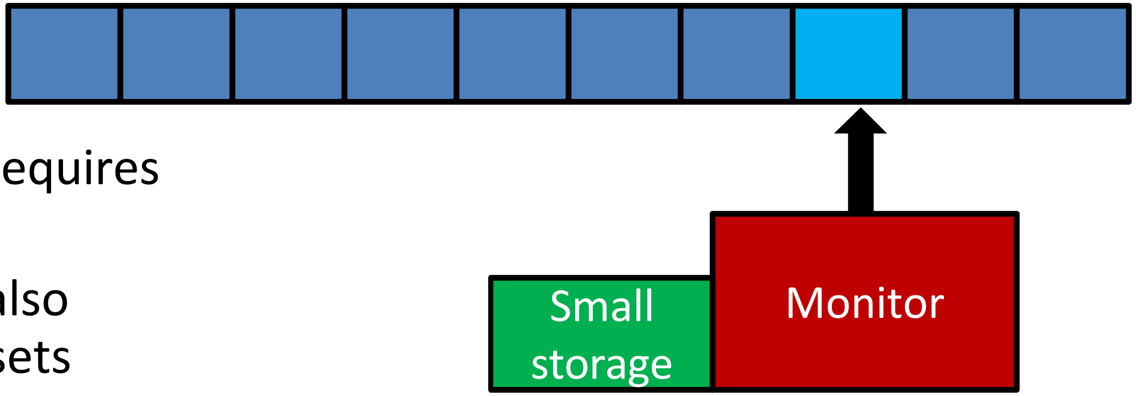
- First set for all edges, $W_{u,v} = (d_u - 1)(d_v - 1)$.
- Pick edge e = (u,v) with probability prop. to $W_{u,v}$
- Pick uniform random neighbor of u and of v
- If output is 3-path, guaranteed to be uniform random

The devilish details



- Works, but (provable) accuracy is not great
- Design methods to reduce samples
- Can give provable bounds: “for s samples, with 99.9% confidence, the true count is within 1% of answer”

What if we observe the data as a stream of edges?



- Many data analysis problems deal with data streams.
  - Situational awareness requires real time analysis.
- Streaming algorithms are also used to analyze large data sets with limited memory.
  - Multiple passes may be feasible.
- Algorithmically
  - We see each data point only once.
  - We either take action, or forever hold our peace.
- Not all problems are amenable to streaming analysis.
  - We cannot find needle in a haystack
  - But we can count frequent items, such as triangles