

Assessing List-Mode Observer Performance in Classification Tasks when Imaging Nuclear Inspection Objects



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Introduction

This project leverages advanced nuclear-medicine techniques to perform tasks useful for arms-control treaty applications. We developed and studied observer models that avoid aggregating sensitive information by classifying sources using list-mode data. The long term goals of this project are:

- Effectively classify objects using observer models that account for nuisance parameters such as variable object position, orientation, and radioactivity.
- Develop a range of observer models that store a variety of information about the object. The end goal is to develop observer models that avoid the need for sensitive-information barriers.

Theory

Ideal Observer

The recorded detector data consists of the counts N hitting a detector and list-mode data A_n for the n^{th} detection event. Under signal-known-exactly (SKE) conditions, where the count rate and spectra are known, the ideal observer is

$$A_n = \{\text{pixel, energy, particle type}\}$$

$$\gamma_j = \{\text{orientation, location, material age for source } j, \text{ etc}\}$$

$$\Lambda_{\text{SKE}}(\{A_n\}, N | \gamma_1, \gamma_2) = \frac{pr(\{A_n\}, N | \gamma_2, H_2)}{pr(\{A_n\}, N | \gamma_1, H_1)} \\ = \frac{Pr(N | \bar{N}_2) \prod_{n=1}^N pr(A_n | h_2)}{Pr(N | \bar{N}_1) \prod_{n=1}^N pr(A_n | h_1)}$$

The first set of terms is the ratio of the Poisson pdf values for the sample counts N given the known count rates in H_2 and H_1 . On the right is the ratio of the spectra pdf values for each observed energy. Data for each detection event is read/acquired, processed, and forgotten.

Generalizing the ideal observer with nuisance parameters improves performance:

$$\Lambda(\{A_n\}, N) = \frac{\int pr(\{A_n\}, N | \gamma_2, H_2) pr(\gamma_2) d\gamma_2}{\int pr(\{A_n\}, N | \gamma_1, H_1) pr(\gamma_1) d\gamma_1}$$

This expression can be calculated via Monte Carlo methods in some circumstances. In others [see future J.O.S.A paper], we derive a form that is an integral over the SKE ideal observer.

$$\Lambda(\{A_n\}, N) = \int \Lambda_{\text{SKE}}(\{A_n\}, N | \gamma_1, \gamma_2) pr(\gamma_1 | \{A_n\}, N, H_1) d\gamma_1 \dots \\ pr(\gamma_2 - \gamma_1) d(\gamma_2 - \gamma_1)$$

Two examples:

- Ideal observer averaging over two orientations

$$\Lambda_\theta(\{A_n\}, N) = \frac{pr(\{A_n\}, N | H_2, \theta = 0^\circ) + pr(\{A_n\}, N | H_2, \theta = 45^\circ)}{pr(\{A_n\}, N | H_1, \theta = 0^\circ) + pr(\{A_n\}, N | H_1, \theta = 45^\circ)}$$

- Ideal observer averaging over activity distribution

$$\Lambda_{\bar{N}_1, \bar{N}_2, \bar{N}_b}(\{A_n\}, N) = \int \Lambda_{\text{SKE}}(\{A_n\}, N | \bar{N}_1, \bar{N}_2) pr(\bar{N}_2) d\bar{N}_2 \dots \\ pr(\bar{N}_1, \bar{N}_B | \{A_n\}, N, H_1) d\bar{N}_B d\bar{N}_1$$

Hotelling Observer

Defining a data vector \mathbf{g} in terms of an operator acting on list-mode data $\{A_n\}$

$$g_m = \sum_{n=1}^N T_m(A_n)$$

The Hotelling observer uses only the mean $\bar{\mathbf{g}}$ and covariance K_g of the imaging data vectors. It is the linear discriminant \mathbf{w} that maximizes the SNR for the test-statistic distributions under H_2 and H_1 .

$$\mathbf{w} = K_g^{-1} \Delta \bar{\mathbf{g}}$$

$$t = \mathbf{w}^T \mathbf{g} = \sum_{n=1}^N \sum_{m=1}^M w_m T_m(A_n)$$

We examined two forms of \mathbf{g} :

- g_m is the detected counts in a given energy-pixel bin
- g_m is the sum of energies in a given pixel

Imaging System

Object

Classification tasks were performed on inspection objects developed by Idaho National Lab (INL). Object 8 is plutonium shielded by depleted uranium while object 9 is plutonium shielded by highly enriched uranium. Both assemblies are supported by an aluminum framework inside an 8" by 8" aluminum box.

