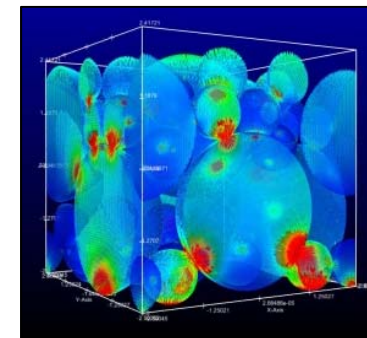
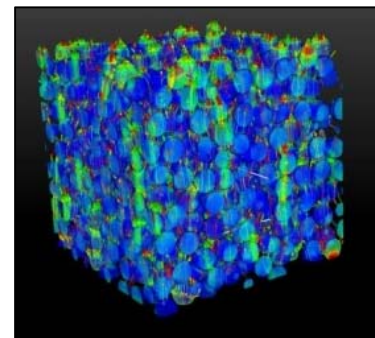
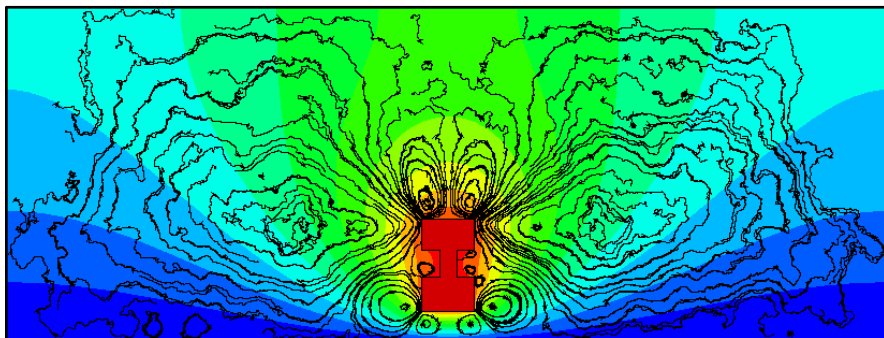


Exceptional service in the national interest



Uncertainty Quantification for Multiscale Thermal Transport Simulations

Leslie M. Phinney, William W. Erikson, and Jeremy B. Lechman

June 2014



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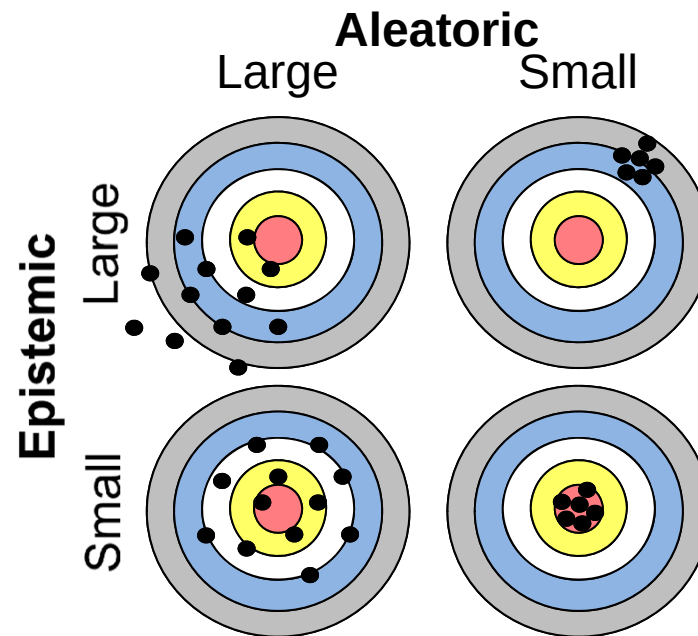
V&V and UQ

Verification ensures that the simulation mathematics and code are correctly computing the solution to the model equations.

Validation is the assessment of the accuracy of a computational simulation by comparison with experimental data.

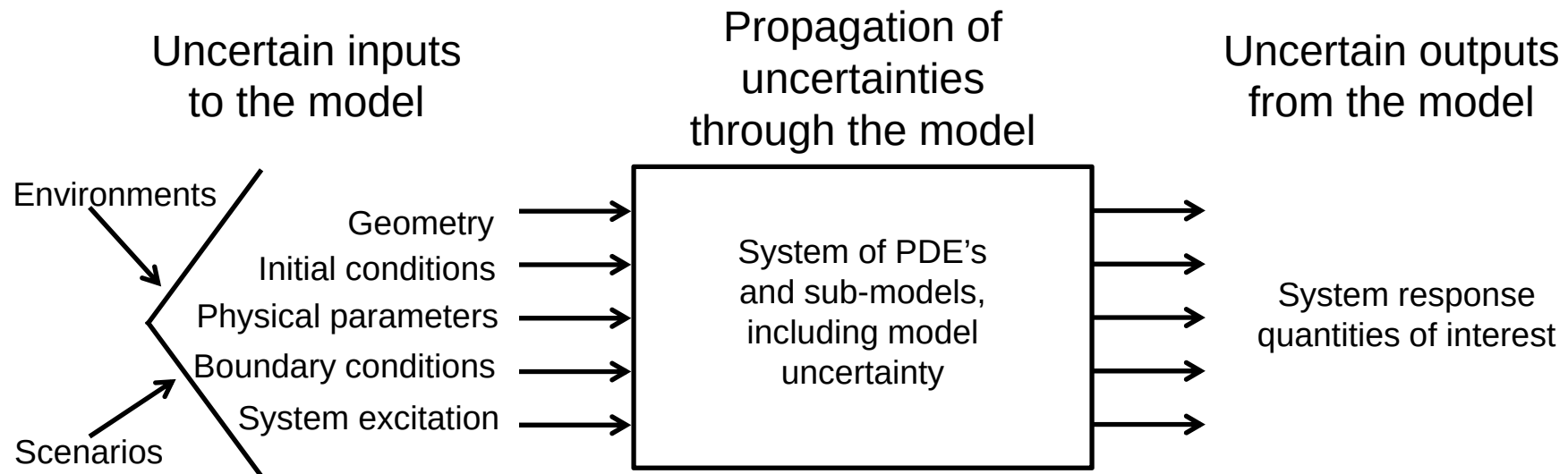
Uncertainty Quantification assesses the statistical range by which a simulation result differs from the true value.

Aleatoric and Epistemic Uncertainties



Adapted from: Chernatynskiy, Phillpot, and LeSar, *Annu. Rev. Mater. Res.*, **43**, pp. 157-182, 2013.

Uncertainty Quantification (UQ)

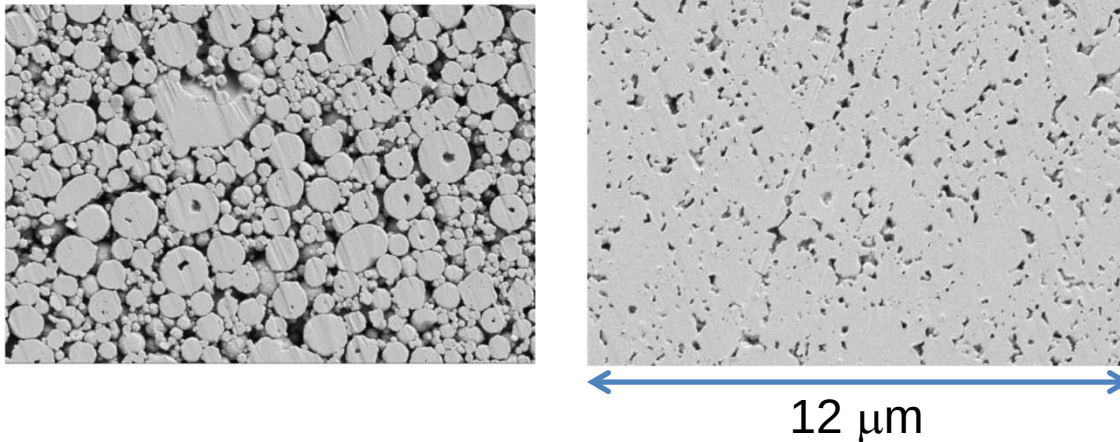


Classification of Uncertainty

- Aleatory – the inherent variation in a quantity that, given sufficient samples of the stochastic process, can be characterized via a probability density function
- Epistemic – uncertainty due to a lack of knowledge by the modelers, analysts conducting the analysis, or experimentalists in validation

Roy and Oberkampf, *Computational Methods in Applied Mechanics and Engineering*, **200**, pp. 2131-2144, 2011.

Thermal Transport in Powder Explosives

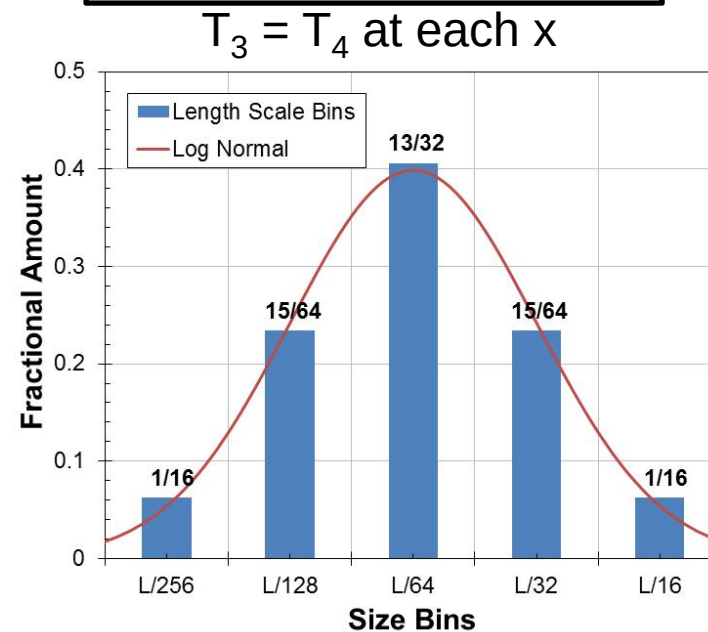
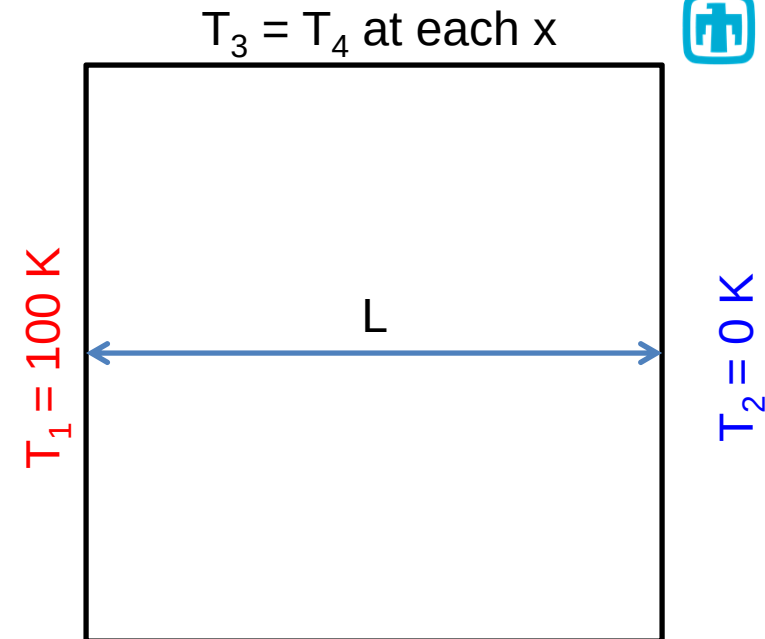


Local hot spots very important in powder explosives – need to know variability.

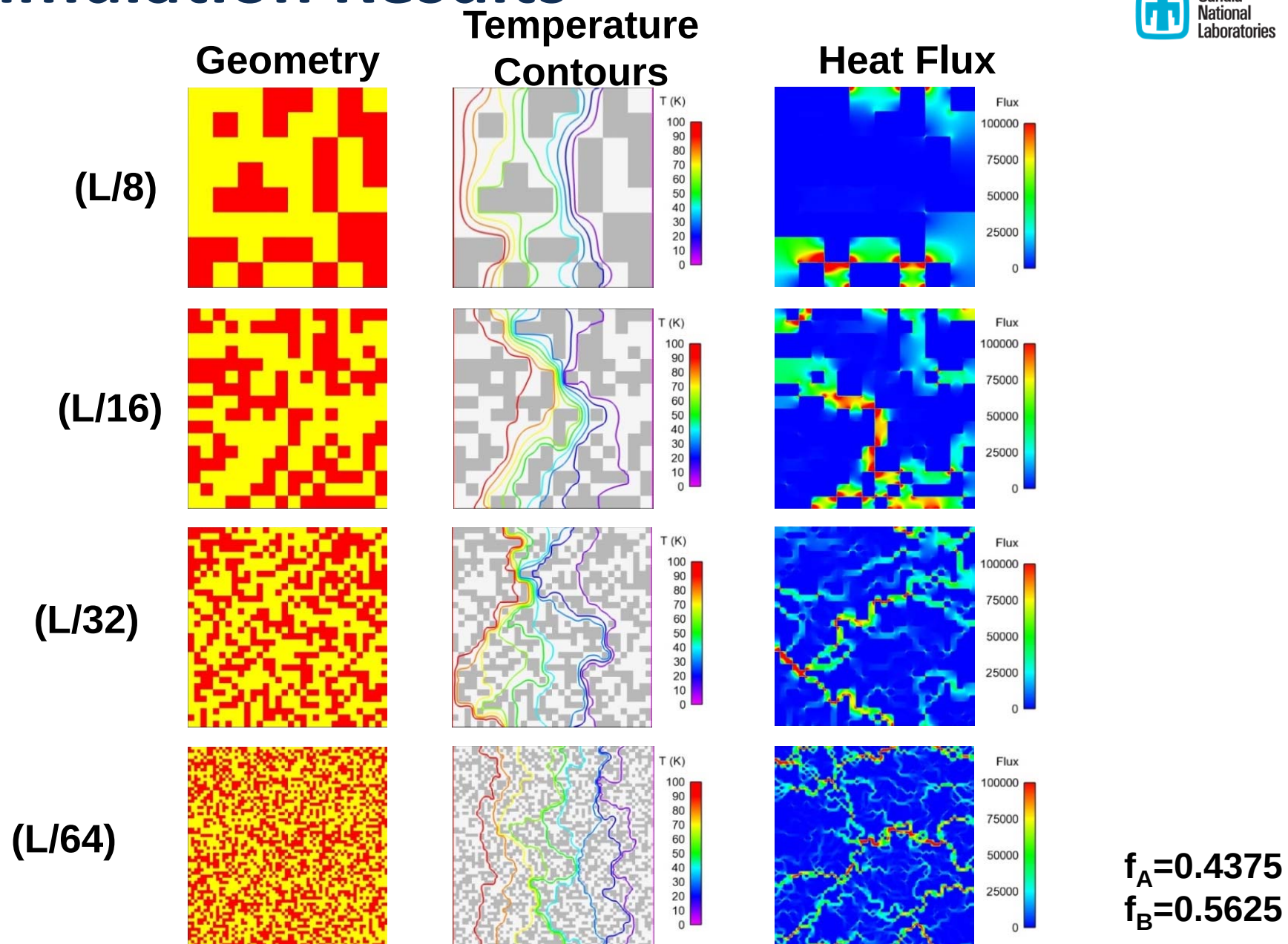
How does the microstructure affect the uncertainty in thermal transport simulations for heterogeneous materials?

Simulations

- Steady state simulations
$$T(x) = T_1 \left(1 - \frac{x}{L}\right)$$
- $L = 5.12$ cm, grid 512×512 elements, grid size is 0.01 cm
- Material A: high density, high thermal conductivity
- Material B: low density, low thermal conductivity
- Feature sizes varied from $L/8$ to $L/256$, one log-normal mixture
- Volume fractions varied from all A to all B
- Used SNL's Sierra-Aria code



Simulation Results



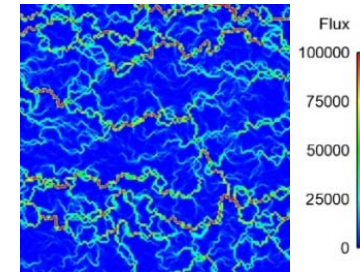
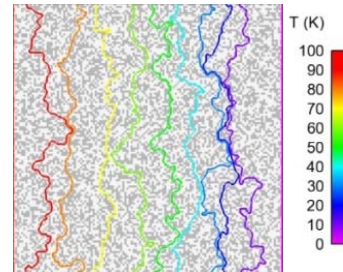
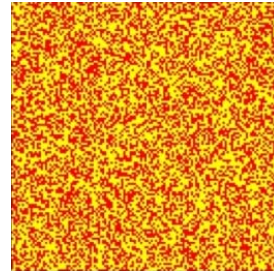
Simulation Results

Geometry

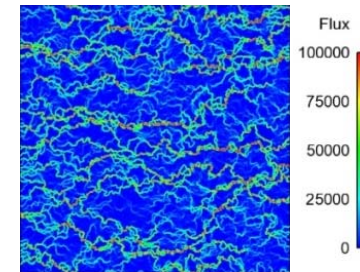
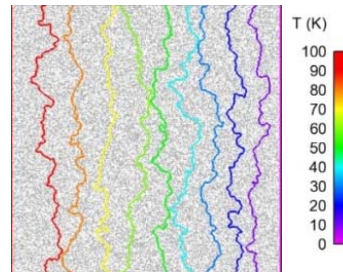
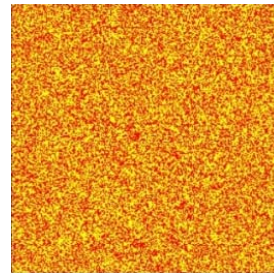
Temperature
Contours

Heat Flux

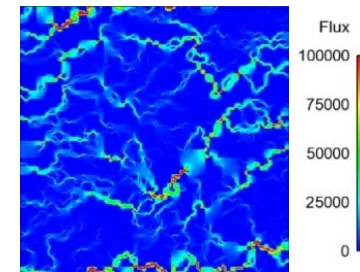
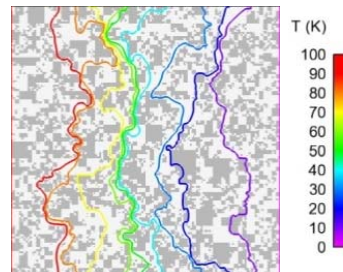
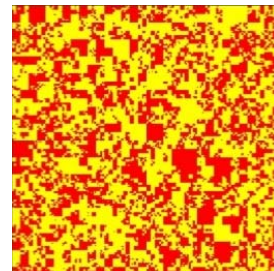
(L/128)



(L/256)



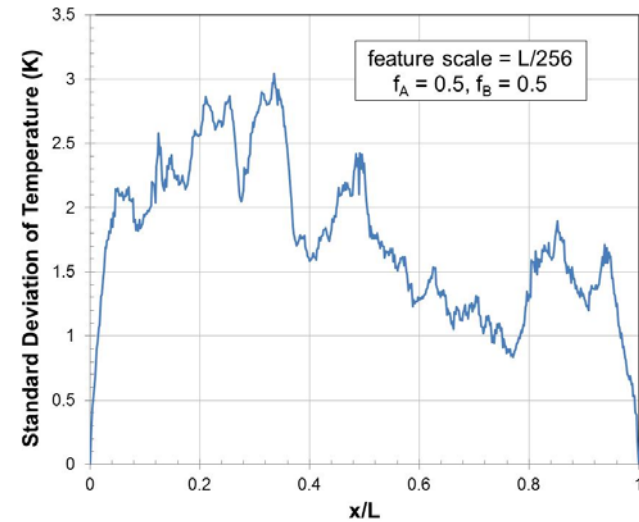
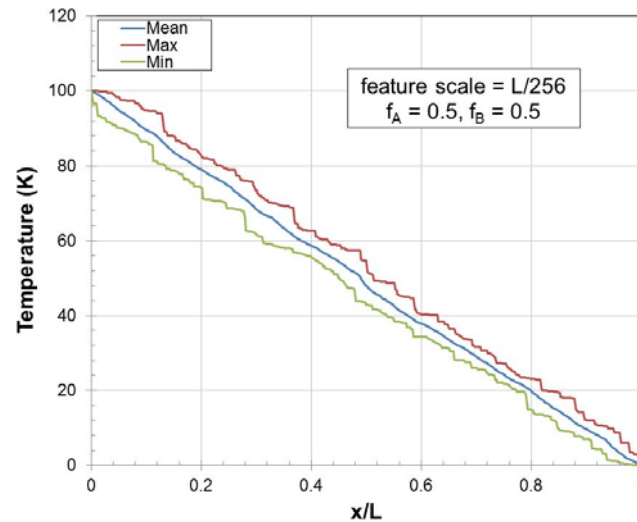
(Log-normal
mixture)



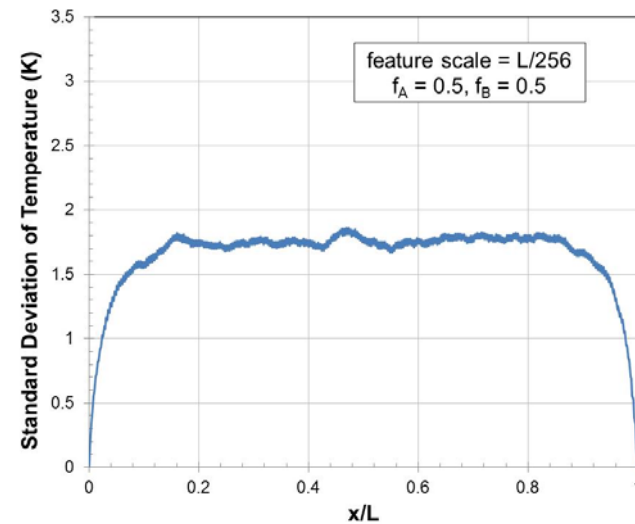
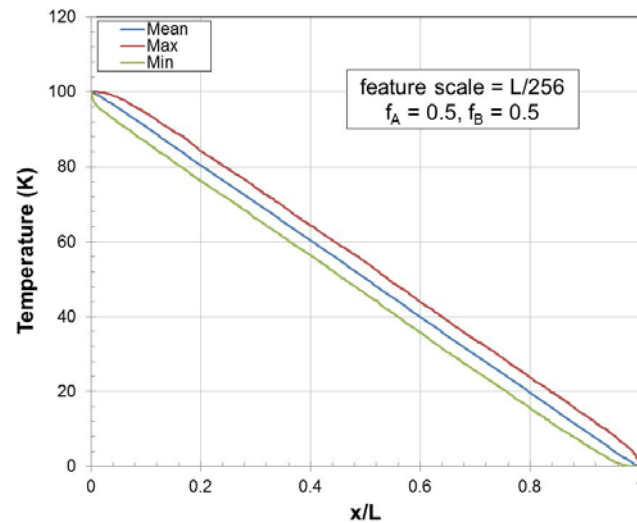
$f_A=0.4375$
 $f_B=0.5625$

Temperature and Standard Deviation

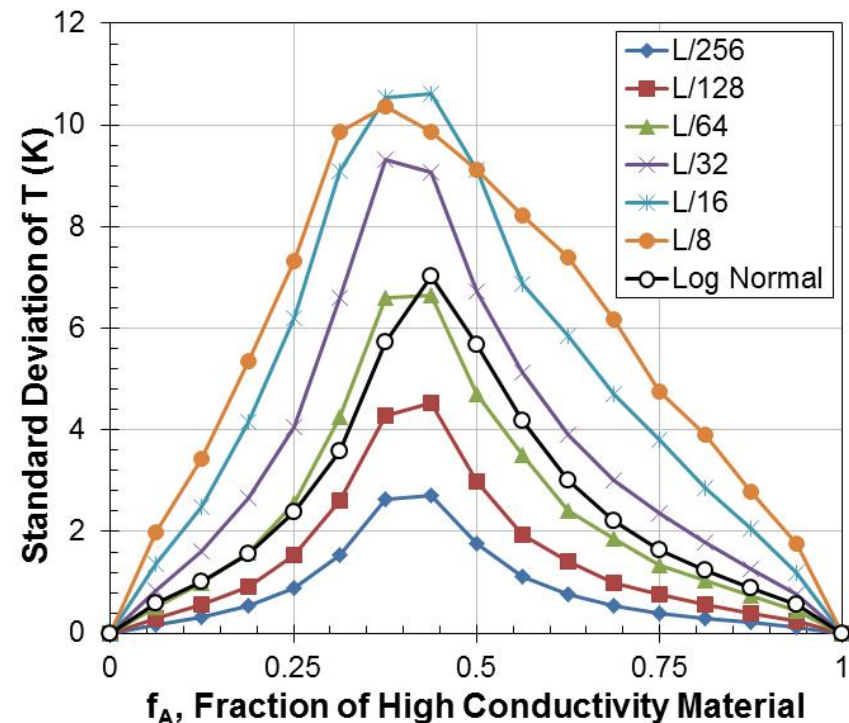
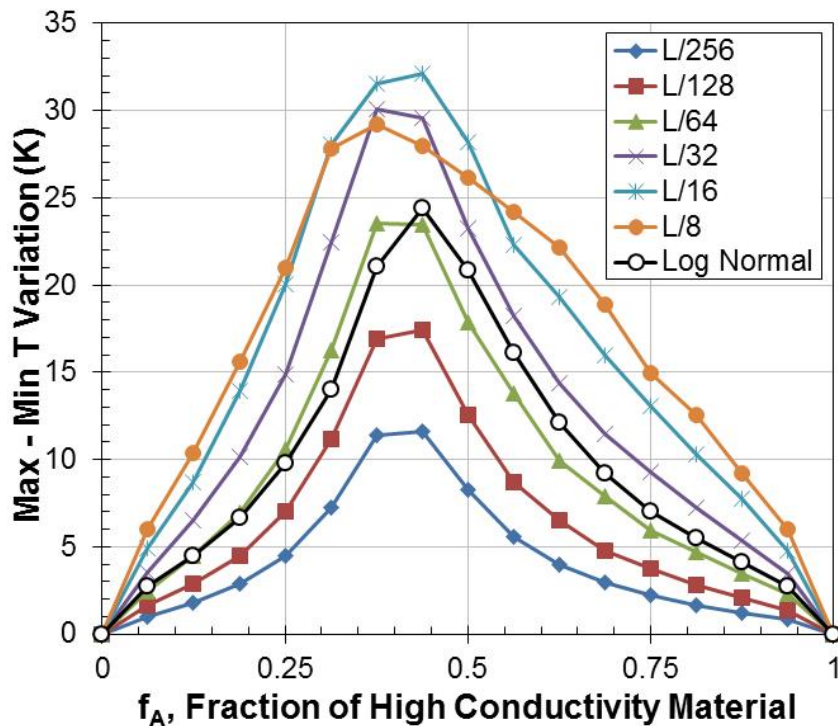
1 realization



100
realizations



Variation as a Function of Volume Fraction and Feature Length Scale



Effective Medium Theory Expressions

Symmetric
$$f_A \frac{(\lambda_A - \lambda_{\text{mix}})}{(\lambda_A + (d-1)\lambda_{\text{mix}})} + f_B \frac{(\lambda_B - \lambda_{\text{mix}})}{(\lambda_B + (d-1)\lambda_{\text{mix}})} = 0$$

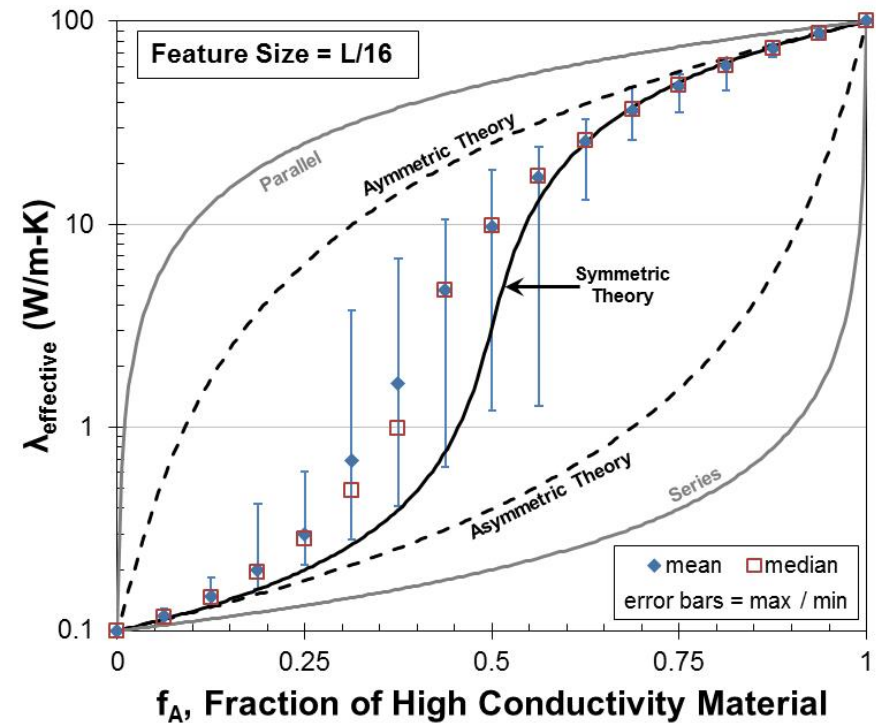
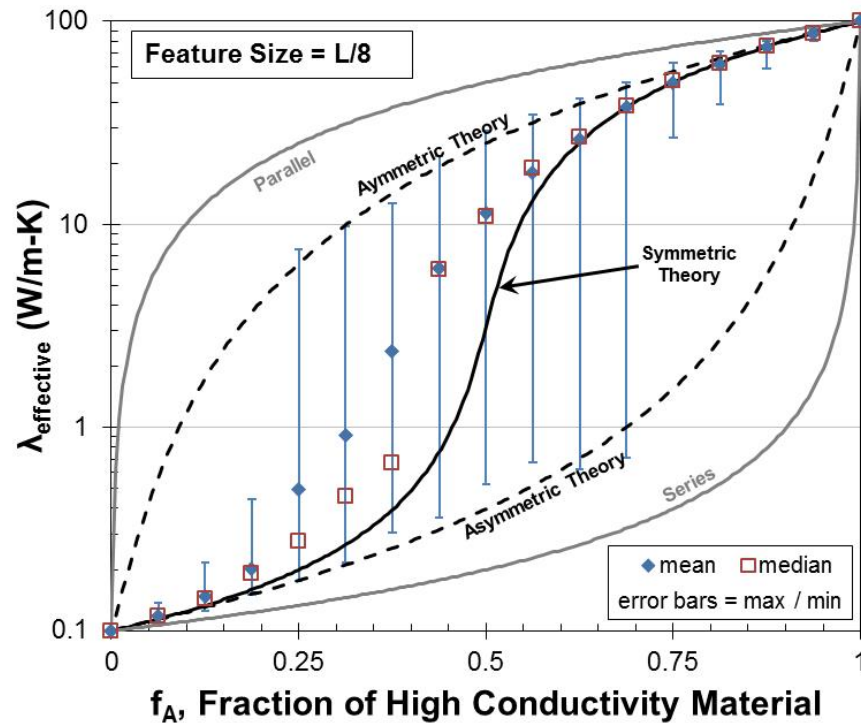
Asymmetric
$$\frac{(\lambda_{\text{mix}} - \lambda_A)^d}{\lambda_{\text{mix}}} = \frac{(1-f_A)^d (\lambda_B - \lambda_A)^d}{\lambda_B}$$

Asymmetric
$$\frac{(\lambda_{\text{mix}} - \lambda_B)^d}{\lambda_{\text{mix}}} = \frac{(1-f_B)^d (\lambda_A - \lambda_B)^d}{\lambda_A}$$

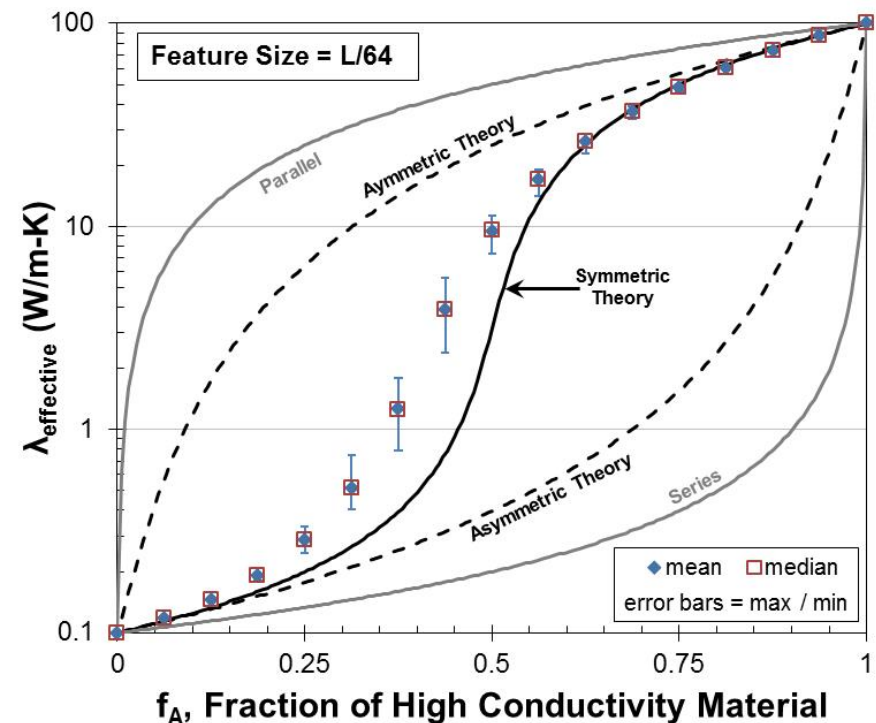
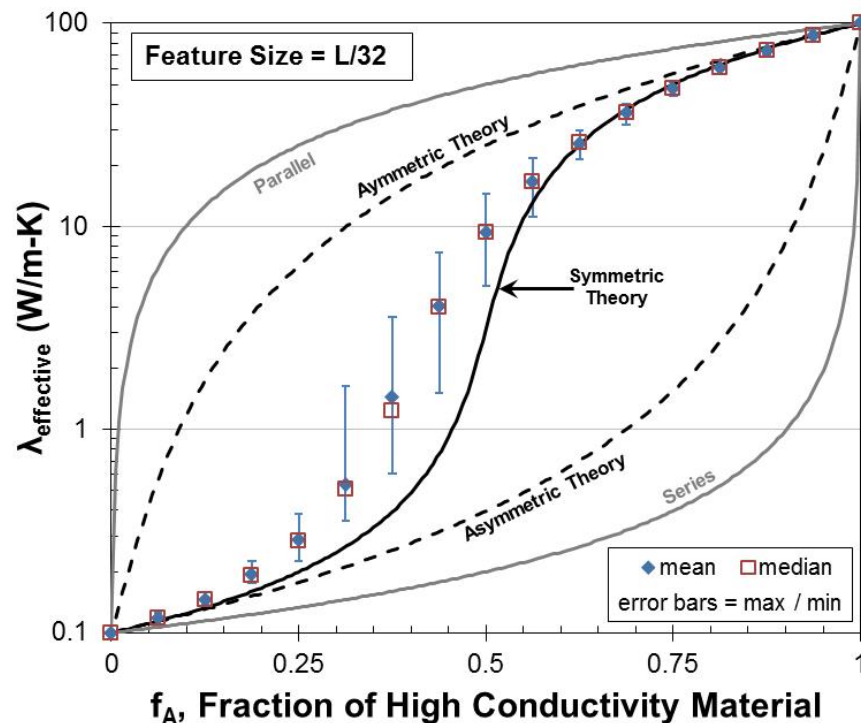
Parallel
$$\lambda_{\text{mix}} = f_A \lambda_A + f_B \lambda_B$$

Series
$$\lambda_{\text{mix}} = \frac{1}{f_A/\lambda_A + f_B/\lambda_B}$$

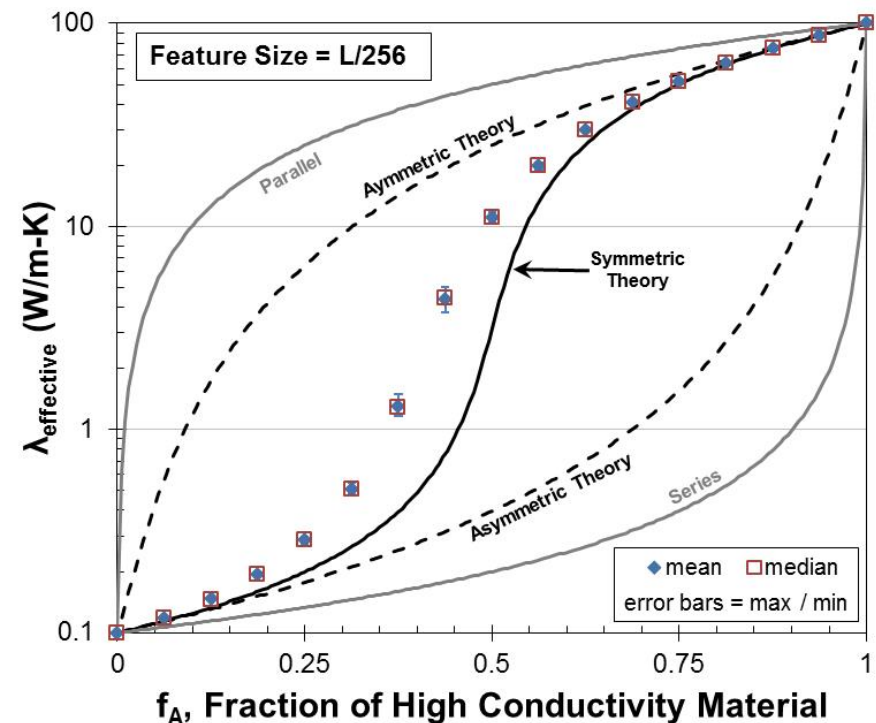
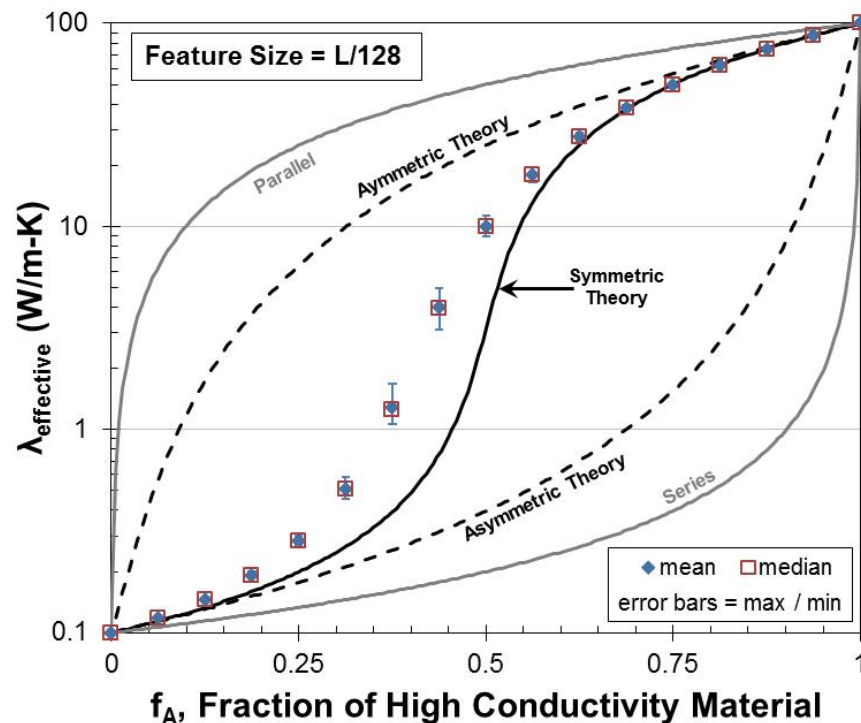
Effective Thermal Conductivity as a Function of Fraction of the Material A



Effective Thermal Conductivity as a Function of Fraction of the Material A



Effective Thermal Conductivity as a Function of Fraction of the Material A



Summary

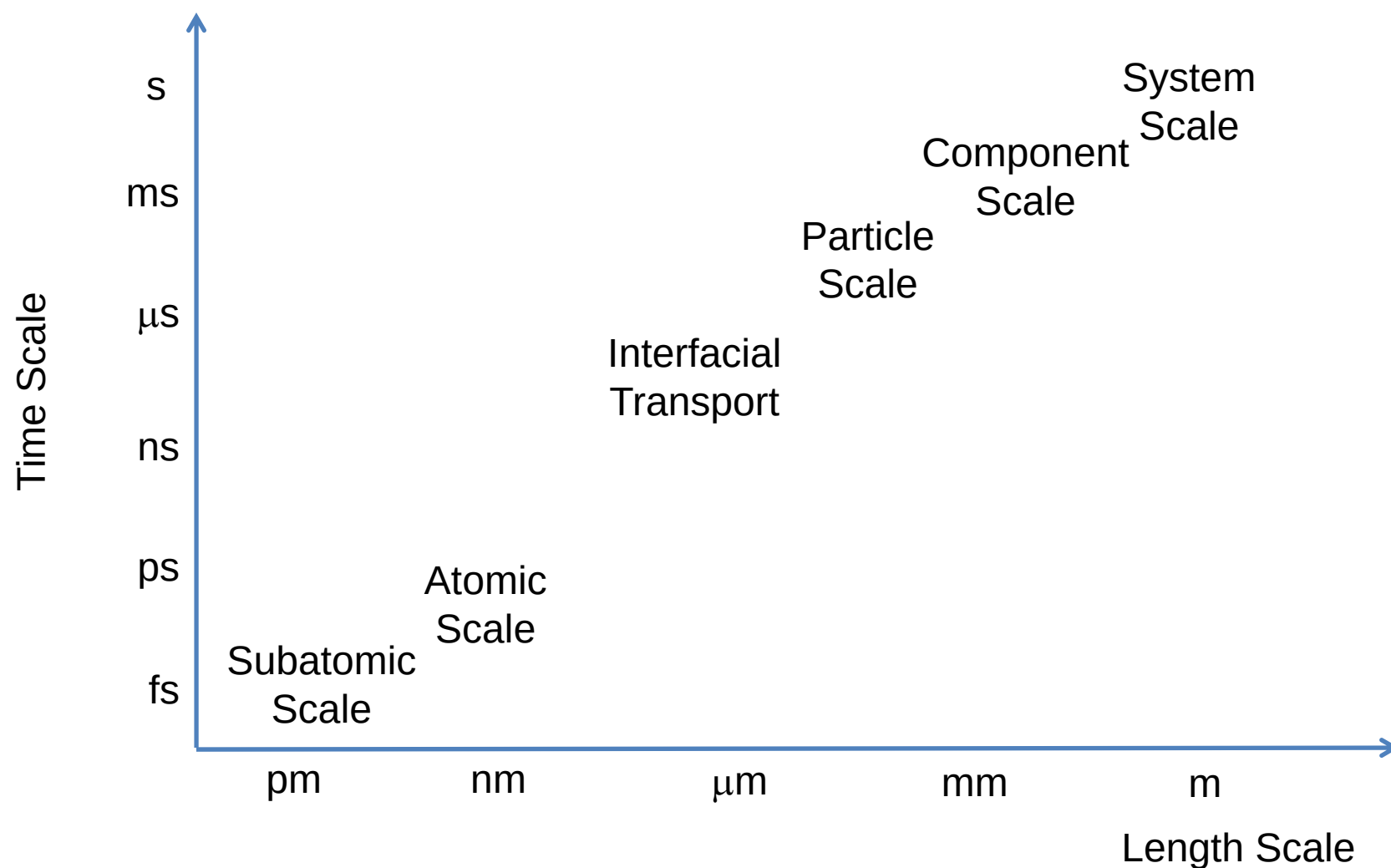
- Uncertainty quantification techniques suitable for multiscale heat transfer simulations are needed.
- Steady state heat transfer simulations of a two component provide insight into the aleatoric uncertainty inherent in heterogeneous materials.
- A peak in the uncertainty was observed near the percolation threshold of $f_A \sim 0.4$ and decreased by a factor of 2 from the peak for f_A 's greater than 0.75 or less than 0.25.
- Uncertainties in max.-min. T 's are around 30K for coarser structures: $L/8$, $L/16$, and $L/32$, and as much as 10K for the finest feature of $L/256$.
- Statistical representations are needed for the aleatoric uncertainty in heterogeneous materials that can be retained and passed between scales.

Acknowledgments

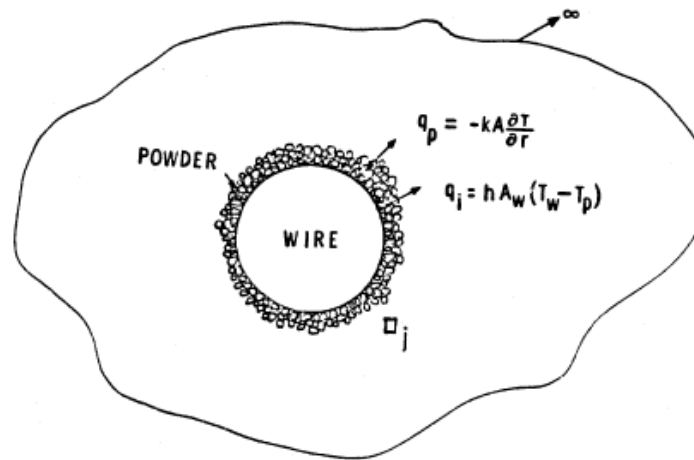


- Sandia National Laboratories Laboratory Directed Research and Development (LDRD) funding.

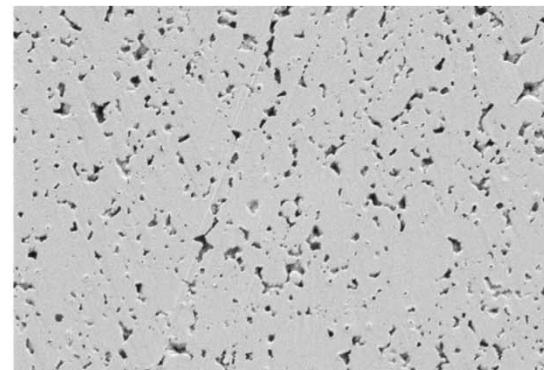
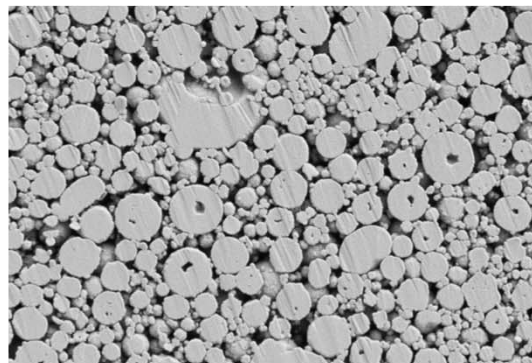
Multiscale Modeling



Transport in Heterogeneous Powders



$$\underbrace{\iint_V (I^2 R)_j dV}_{\text{HEAT GENERATION RATE}} = \underbrace{\iint_V (\rho C_p \frac{\partial T}{\partial t})_j dV}_{\text{INTERNAL ENERGY INCREASE RATE}} + \underbrace{\int_S \vec{q} \cdot d\vec{S}}_{\text{HEAT LOSS TO SURROUNDINGS}}$$



12 μm