Peridynamics
Ordinary Isotropic Plasticity Model

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Plasticity model**

- Generalization/extension of local Von Mises plasticity to peridynamics
- Continuum and ordinary-state constitutive model
- Isotropic
- Inherits all of the advantages for modeling fracture
- Satisfies 2nd law of thermodynamics
- Single step return algorithm for time integration
- Linearization of discrete return algorithm
- Applicable to implicit/explicit peridynamics codes

***John A. Mitchell. A nonlocal, ordinary, state-based viscoelasticity model for peridynamics. Sandia Report SAND2011-8064, 2011.*

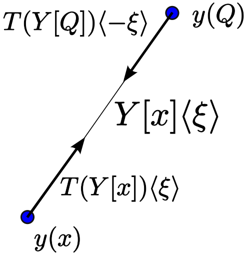
Outline

Talking points

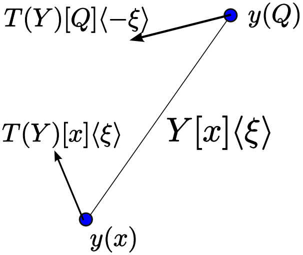
- Review ordinary LPS model
- Summarize perfect plasticity model and discrete return algorithm
- Introduce hardening
- Iterative return algorithm
- Simple numerical examples
- PALS-like treatment for surface effects

Ordinary and Non-ordinary Materials

Ordinary



Non-ordinary



Ordinary Materials
Silling, Epton, Weckner, Xu, and Askari, 2007

Vector force state T:

$$T(Y) = t(Y)M(Y) \quad \text{where} \quad M(Y) = \frac{Y}{|Y|}$$

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Isotropic and elastic material

$$t(Y) = \frac{3k\theta}{m}\omega \underline{x} + \alpha\omega e^d$$

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Isotropic and elastic material

$$t(Y) = \frac{3k\theta}{m}\omega \underline{x} + \alpha \omega e^d$$

Isotropic and elastic-plastic material

$$t(Y) = \frac{3k\theta}{m}\omega \underline{x} + \alpha \omega \underbrace{\left(e^d - e^{dp}\right)}_{elastic}$$

Elastic-Plastic Constitutive Model

Summary of Governing Equations and Constraints

- Additive decomposition of extension state: $e^d = e^{de} + e^{dp}$
- Elastic force state relations: $t(Y) = \frac{3k\theta}{m}\omega \underline{x} + \alpha\omega(e^d - e^{dp})$
- Elastic force states domain defined by a yield surface/function that depends upon the deviatoric force state:
  $f(t^d) = \psi(t^d) - \psi_0 \leq 0$, where $\psi(t^d) = \frac{\|t^d\|^2}{2}$
- Flow rule which describes rate of plastic deformation:
  $\dot{e}^{dp} = \lambda \nabla^d \psi$
- Loading/un-loading conditions (Kuhn-Tucker constraints):
  $\lambda \geq 0, \;\; f(t^d) \leq 0, \;\; \lambda f(t^d) = 0$
- Consistency condition: $\lambda \dot{f}(t^d) = 0$

Exact Solution to Associated Backward Euler Approximation

- Compute trial deviatoric force state: $t^d_{trial} = \alpha\omega(e^d_{n+1} - e^{dp}_n)$
- if $f(t^d_{trial}) \leq 0$, then step is elastic, $\Delta\lambda = 0$, and $t^d_{n+1} = t^d_{trial}$
- else

$$t^d_{n+1} = \sqrt{2\psi_0}\,\frac{t^d_{trial}}{\|t^d_{trial}\|}, \quad e^{dp}_{n+1} = e^{dp}_n + \frac{1}{\alpha}\left[\frac{\|t^d_{trial}\|}{\sqrt{2\psi_0}} - 1\right] t^d_{n+1}$$

Practical value** for ψ_0

$$\begin{aligned}\psi_0 &= \frac{1}{2}\left[\frac{15\mu}{m}\right]^2 \|e^d\|^2 \\ &= \frac{75}{8\pi}\,\frac{E_y^2}{\delta^5}\end{aligned}$$

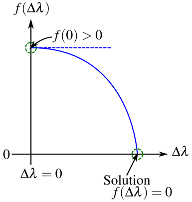
where E_y is the shearing yield stress and δ is the *horizon*

***John A. Mitchell. A nonlocal, ordinary, state-based viscoelasticity model for peridynamics. Sandia Report SAND2011-8064, 2011.*

Outline of Newton Algorithm
Implicit Update of Plasticity State

Initialize yield function

Let $f(\Delta\lambda_k) = \frac{1}{2}\|t^d(\Delta\lambda_k)\|^2 - (\psi_0 + H\|\beta(\Delta\lambda_k)\|)$



Newton steps for calculation of $\Delta\lambda_k$

while $f(\Delta\lambda_k) > 0 \quad \{$

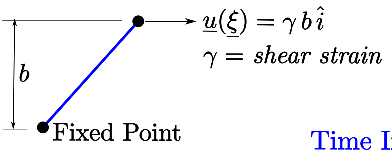
- Solve: $0 = f(\Delta\lambda_k) + Df(\Delta\lambda_k)[u]$ for u
  $Df(\Delta\lambda_k)[u] = -f(\Delta\lambda_k)$
- Increment: $\Delta\lambda_{k+1} = \Delta\lambda_k + u$
- Evaluate: $f(\Delta\lambda_{k+1}) = \frac{1}{2}\|t^d(\Delta\lambda_{k+1})\|^2 - (\psi_0 + H\|\beta(\Delta\lambda_{k+1})\|)$

$\}$

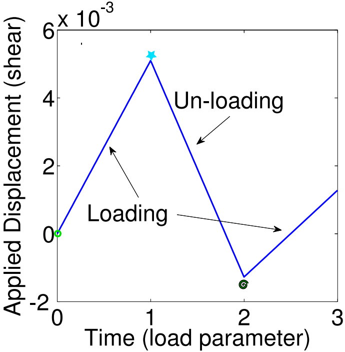
Where

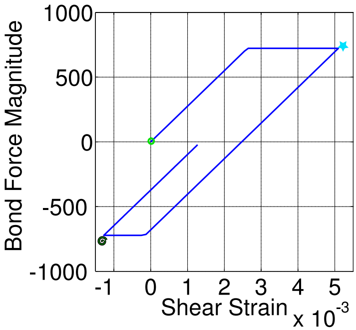
$Df(\Delta\lambda_k)$ is Fréchet derivative of $f @ \Delta\lambda_k$

Elastic-Plastic Constitutive Model

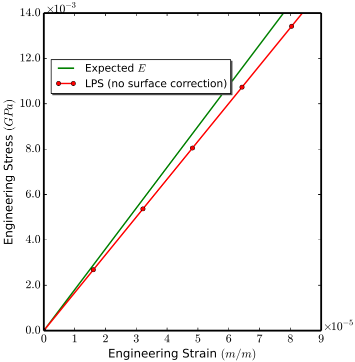


Time Integration of Single Bond



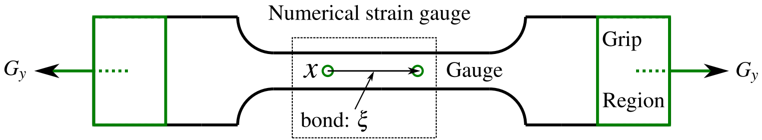


What is the *Dreaded Surface Effect*?
Example: Isotropic-Ordinary Model (LPS)



The following related aspects contribute to mismatch.

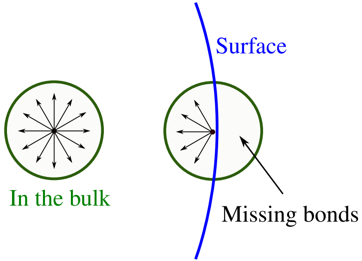
- Geometric surface effects
- Nonlocal model kinematics
- Nonlocal model properties
- Discretization error



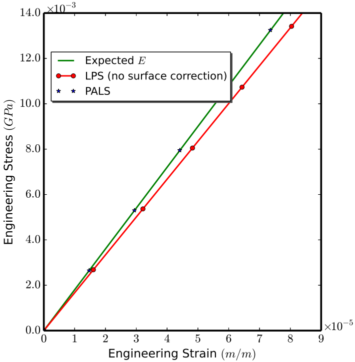
Ordinary peridynamic models
Dreaded Surface Effect

Causes relate to material points near surface

- Mathematical models assume all points are in the *bulk*
  - Points near surface are *missing bonds*
  - *Missing bonds* imply and induce incorrect material properties
  - In the bulk mathematical models are consistent
- Isotropic ordinary materials have a *dilatation defect* at the surface

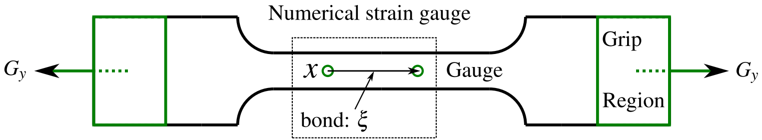


Re-visit tensile test using PALS and LPS
Influence functions: $\underline{\omega}^0 = \underline{\sigma}^0 = 1$



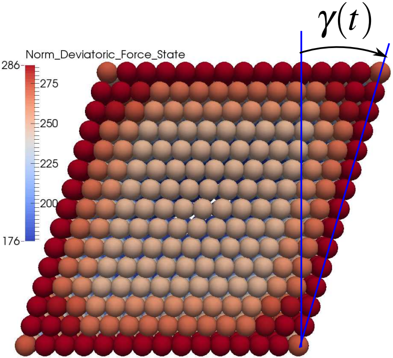
Results

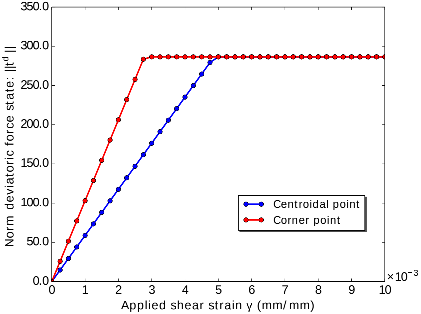
- PALS sharply reduces error



Elastic-Plastic Constitutive Model

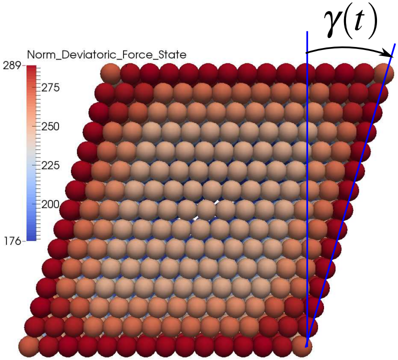
- Perfect plasticity yield function: $f(t^d) = \frac{\|t^d\|^2}{2} - \psi_0$
- Position sensitive yield (*undesirable*) but fixable using PALS-like approach

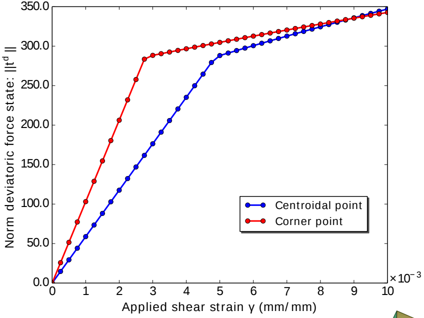




Elastic-Plastic Constitutive Model

- Plasticity w/hardening yield function
  $$f(t^d) = \frac{\|t^d\|^2}{2} - (\psi_0 + H\|\beta\|)$$
- Position sensitive yield (*undesirable*) but fixable using PALS-like approach

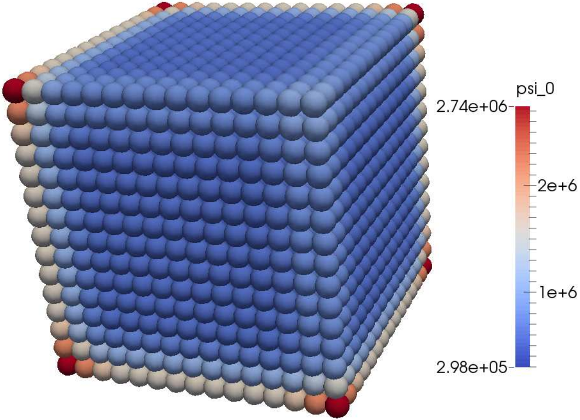




Position Aware Yield Condition: Status

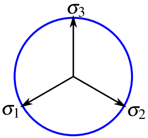
PALS-like correction for yield value ψ_0

↬ Artifact showing but confident this is fixable.



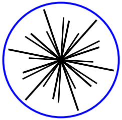
Brief Comparision (food for thought)
VonMises Model with Nonlocal Model

- Both models principally apply to metals
- Plastic deformations volume preserving, independent of pressure
- Yield functions are quite different



Octahedral/deviatoric plane

- $f(\sigma) = \sqrt{3J_2} - \sigma_0$
- Principal values $\sigma_1, \sigma_2, \sigma_3$
- Yield defined @ point
- Two-dimensional yield surface



Schematic of bonds @ point

- $f(t^d) = \frac{\|t^d\|^2}{2} - \psi_0$
- Neighborhood of a point
- Yield defined @ point via collective
- Yielding occurs on bond
- Infinite dimensional aspect due to bonds
- Length scale is native to the model

Closing remarks

Summary

- Presented non-local peridynamics plasticity model
- Non-local equivalence to Von Mises model
- Non-local model has far more information embedded
- PALS-like surface correction underway

THANK YOU

Questions?