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Optomechanical Spring Effect Readout in Resonant Micro-Optical Sagnac Gyroscopes

Design and Scaling Analysis

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***Abstract*—We propose and theoretically analyze a new cavity optomechanical oscillator gyroscope. Mechanical frequency acts as a sensitive readout of rotation through the optomechanical spring and Sagnac effects. Remarkably, reducing device size improves scale factor.**

Keywords—Optomechanical Oscillator; Gyroscope; Optical Microresonator; Sagnac Effect; Optomechanical Spring

I. Introduction

Gyroscopes are used for navigation, guidance, and control. They are particularly necessary when guidance by GPS isn't possible in which case the gyro can be combined with an accelerometer to independently determine position.

Inherent to all known gyroscopes is the principle that larger device size improves the sensitivity (output/input) and resolution. However, larger device size also leads to increased system cost, size, weight and power (CSWaP). We propose a new Optomechanical Oscillator gyroscope (OMOG) to readout rotation rate. The OMOG is unique in that the scale factor improves with smaller device size. This is due to the scaling of the optomechanical spring effect which acts as an intermediary between the rotation-induced Sagnac shift and oscillation frequency. We also analyze the signal to noise ratio which is fundamentally limited by mechanical Brownian motion.

II. OMOG Operating Principles

A. Overview

Fig. 1 illustrates the proposed gyroscope comprised of a waveguide that is evanescently coupled to free-standing optical cavity. The cavity can be made free standing by using thin spokes [1] or by partially releasing a disk. If the laser is blue detuned with respect to the cavity resonance and is greater than the threshold power, regenerative oscillation will ensue [1,2]. In the absence of noise and rotation, the oscillation frequency will occur at a precise mechanical angular frequency, Ω_m and the output light may consist of mechanical sidebands located at integer multiples of Ω_m [3].

Upon rotation, the resonant frequency of the optical cavity and thus the detuning, shifts by an amount ω' due to the Sagnac effect and results in a corresponding shift in mechanical oscillation frequency, Ω' due to the optomechanical spring effect. By comparing the oscillation frequency of two OMOGs with light launched in opposite directions, the signal is doubled and common mode noise is reduced. In monitoring the N^{th} order harmonic, the rotation induced frequency shift is multiplied by a factor of N and is advantageous when the height of the harmonic is greater than the fundamental [3].

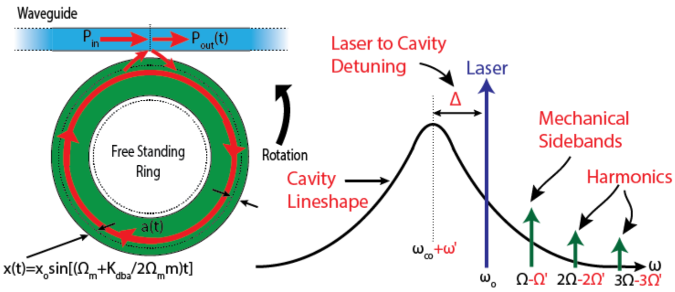

Figure 1: OMOG Device concept. Left: The device is pumped with laser light and oscillates harmonically. Rotation of the cavity changes the sinusoidal oscillation frequency. Right: Frequency domain representation. Blue-detuned laser light is modulated by the oscillating cavity resulting in mechanical sidebands and harmonics. The detuning shift due to rotation results in a shift in the sideband frequencies.

B. Intuitive Model

We make use of Fig. 2 where the optomechanical cavity is understood as a movable mirror attached to a rigid anchor through a spring-damper system.. The radiation pressure force applied to the mirror is proportional to the photon flux impinging on the mirror and is inversely proportional to the mirror position, x. The optomechanical stiffness induced by dynamical back-action is then given by $k_{dba} = \partial F/\partial x \propto 1/L^2$, or $\propto 1/R^2$ in a disk of radius, R.

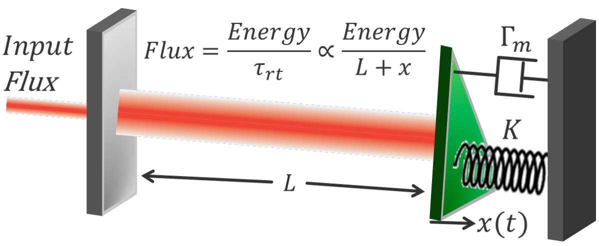

Fig. 2: Model for OMOG. The cavity is understood as a movable mirror attached to a spring damper system.

In the presence of optomechanics, the instantaneous mechanical frequency is,

$$\Omega = \sqrt{\Omega_m^2 + \left(\frac{k_{dba}}{m_{eff}}\right)^2} \cong \Omega_m + \frac{k_{dba}}{2\Omega_m m_{eff}} \qquad (1)$$

where m_{eff} is the effective mass. For disk cavities, $m_{eff} \propto R^2$ and $\Omega_m \propto 1/R$ so that the shift in mechanical oscillation frequency, $\Omega' = k_{dba}/2\Omega_m m_{eff} \propto 1/R^3$. Thus, the optomechanical spring effect is highly sensitive to radius.

C. Calculations

The Sagnac-induced shift in optical resonant frequency due to rotation is given by,

$$\omega' = -\frac{2\pi R}{\lambda_{c0} n_{eff}} \Omega_{rot} = -\omega_{c0} \frac{R\Omega_{rot}}{c} \qquad (2)$$

Where λ_{c0} and ω_{c0} are the resonant wavelength and frequency in the absence of rotation, c is the speed of light and Ω_{rot} is the angular rotation rate.

Using equations (1) and (2), and assuming k_{dba} in the unresolved sideband regime from [4], the shift in mechanical resonant frequency due to rotation can be written as:

$$\Omega' \cong \frac{k_{dba}}{2\Omega_m m_{eff}} = \frac{\omega_{c0}^2 P_{in} \kappa_{ex} \left(\left(\frac{\kappa}{2}\right)^2 - 3\Delta_0^2 \right)}{m_{eff} \Omega_m R c \left(\left(\frac{\kappa}{2}\right)^2 + \Delta_0^2 \right)^3} \Omega_{rot} \qquad (3)$$

where P_{in} is the power, κ is the loaded optical linewidth (rad/s), κ_{ex} is the waveguide coupling , and Δ_0 is the initial detuning. Assuming $Q_{optical}$ is independent of radius which is roughly true in silicon nitride for R>10µm, and glass for R>40µm, the scale factor (Ω'/Ω_{rot}) for the OMOG should scale with $1/R^2$ in disks and ~1/R in rings. The $1/R^2$ dependence is confirmed for the disk calculated in Fig. 3.

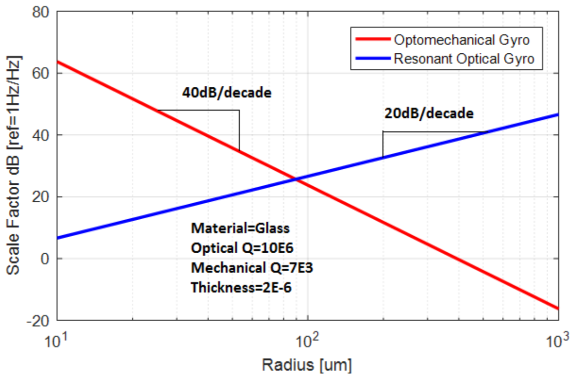

Figure 3: Calculated scale factor for OMOG and optical Sagnac gyro

For small radii in Fig. 3, the OMOG scale factor exceeds the purely optical scale factor in equation (2). Also important is the $1/\kappa^3$ scale factor dependence in equation (3) which emphasizes the need for high optical Q.

D. Brownian-Limited Noise

To assess the fundamental noise limits, we consider the oscillation linewidth, Γ_m limited by thermomechanical Brownian motion which follows a Schawlow-Townes narrowing [4],

$$\Gamma_m = \frac{1}{Q_m} \frac{k_B T}{m_{eff} \Omega_m x^2} \qquad (4)$$

We found through simulations at constant input power, the oscillation amplitude, $x \propto R$. Note that the resulting time-averaged frequency uncertainty is found from the Allan Deviation, $\sigma_y(\tau)$

$$\Delta f(\tau) = \Omega_m \sigma_y(\tau) \propto \sqrt{\frac{\Gamma_m}{2\tau}} \qquad (5)$$

With averaging time, τ. Thus, disk designs are expected to have a $1/\sqrt{R}$ dependence on signal to noise ratio ($\Omega'/\Delta f(\tau)$) for the fundamental Brownian noise contribution.

III. Conclusions

An Optomechanical Oscillator Gygroscope is proposed having interesting scaling behavior. Notably, the expected scale factor improves with reductions in radius. Likewise, the Angle Random Walk-limited resolution due to Brownian motion improves with smaller radius for the modeled disk.

Upcoming research includes experimental realizations and calculations including additional input noise sources. We will also analyze behavior in the resolved sideband regime where the oscillation frequency exceeds the optical linewidth.

Based on models, the proposed OMOG is a promising candidate for low CSWaP gyroscopes. Reductions in device size have numerous additional benefits which include better immunity to shock/vibration, smaller radiation cross-section, lack of temperature gradients, and availability of chip-space for additional functionality.

Acknowledgment

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