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The Use of Safety Factors to Bound Uncertainty in Data

Sandia National Laboratories

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Objective

- **Understand cases where uncertainty is not quantified**

Problem

- **How to develop safety factors and probabilistic distributions that bound consequences by a required probability of $O(10^{-6})$ or $O(10^{-9})$**

Go/No-Go Testing (No Distribution)

Location	Energy	Reaction
A	33.9	No-Go
B	56.9	No-Go
B	61.4	No-Go
B	82.5	No-Go
B	91.1	No-Go
A	37.0	No-Go
A	59.3	No-Go
B	62.9	No-Go
B	83.1	No-Go
B	89.6	No-Go
A	94.0	No-Go

Unable to develop a distribution. We applied a safety factor of 1.5 to an energy of 60.

How do we associate a probability with this safety factor?

Go/No-Go Testing (No Distribution)

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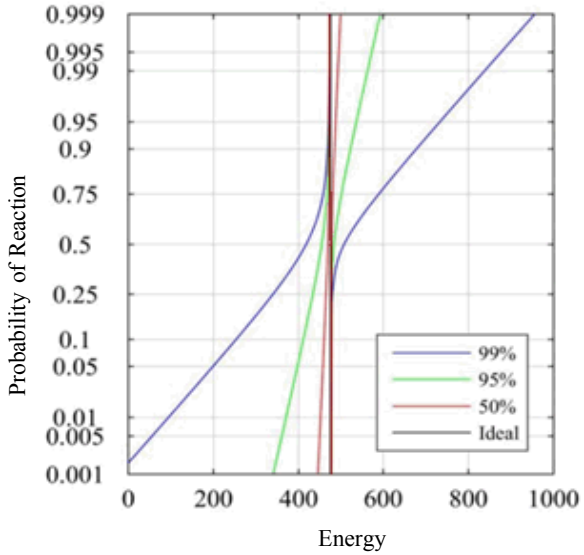
Table 2. Observed behavior of tests listed in terms of increasing energy

Energy	Observed Behavior
126.9+2.7 [93.6+2.0]	No Reaction
304.1+6.8 [224.3+5.0]	No Reaction
489.6+10.6 [361.2+7.8]	No Reaction
596.6+13.5 [440.1+9.9]	No Reaction
629.8+14.3 [464.6+10.5]	No Reaction
642.2+14.9 [473.7+11.0]	No Reaction
649.1+14.3 [478.8+10.6]	Reaction
651.3+14.3 [480.4+10.6]	Reaction
651.9+15.0 [480.8+11.1]	Reaction
695.9+16.0 [513.3+11.8]	Reaction

Go/No-Go Testing (Distribution)



How do we chose a confidence bound?

Figure 6. Results of the Likelihood Ratio analysis plotting the critical energy and associated 2-sided confidence regions.

The mean and standard deviation values of the two-sided confidence regions plotted in Figure 6 are given in Table 3. For a 1% probability of reaction (99% probability of no reaction), the 95% confidence region is 371.9 and the 99% confidence region is 95.9.

Table 3. Mean and standard deviation of two-sided confidence regions shown in Figure 6.

99%	419.6 + 0.0	508.8 + 156.9
95%	455.9 + 0.0	484.8 + 40.2
50%	470.2 + 0.0	479.4 + 8.4

10^{-6} probability of reaction at 95% confidence level: 275.

Go/No-Go Testing (Distribution)

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Table 3: 2016 Test Data

Shot	Param1	Param2	Result	Shot	Param1	Param2	Result
1	2,000	9.0	Go	**13**	370	7.3	Go
2	2,000	7.0	Go	**14**	370	7.9	Go
3	2,000	5.0	Go	**15**	370	7.1	No-Go
4	370	7.0	No-Go	**16**	370	7.8	No-Go
5	370	9.3	Go	**17**	370	8.1	Go
6	370	8.2	Go	**18**	370	7.1	No-Go
7	370	7.6	No-Go	**19**	370	8.0	Go
8	370	6.5	No-Go	**20**	370	7.2	No-Go
9	370	8.9	Go	**21**	370	7.3	Go
10	370	8.6	Go	**22**	370	8.0	Go
11	370	7.9	Go	**23**	370	7.1	No-Go
12	370	7.1	No-Go				

Energy α Param1 $(\text{Param2})^2$

Go/No-Go Testing (Distribution)

	Energy	Tot Energy
Normal		
Mean	9.18363	10.51330
St Dev	0.87541	0.96243
1E-03 percentile	6.47842	7.53918
99% tb	3.71868	4.79381
1E-06 percentile	5.02245	5.93847
99% tb	0.90136	1.81611
1E-09 percentile	3.93311	4.74085
99% tb	-1.22064	-0.42981

	Energy	Tot Energy
Lognormal		
Location	2.21332	2.34812
Scale	0.09470	0.09133
1E-03 percentile	6.82555	7.89239
99% tb	5.07829	6.07176
1E-06 percentile	5.83089	6.78019
99% tb	3.75541	4.57990
1E-09 percentile	5.18271	6.05183
99% tb	2.99186	3.70259

Interpreting the Results in the Table

The first two rows of the tables, in white, show the defining parameters of the estimated distributions. The colored rows show estimated lower 1 X 10^{-3}*th,* and 1 X 10^{-9}*th*, percentiles with their associated lower 99% tolerance (confidence) bounds.

In the leftmost table, blue rows, middle column, for example, the value 6.47842 represents the estimated lower 1 X 10^{-3}*th* percentile of the energy input threshold distribution. The number below it is the lower 99% tolerance bound on this percentile. It is interpreted as follows: *We are 99% confident that the probability of a reaction at on energy input level of 3.71868 is 1 X 10^{-3} or less.*

Mitigation (Material Failure)

Test Number	Energy
2	30.5
3	28
5	24.4
6	28.4
7	28.1
8	28.7
9	28.7
10	29.0
11	28.7
12	27.8
Mean	28.4
Standard Deviation	0.4

95% Confidence Bounds

	Probability	Energy	Safety Factor
Mean (95% confidence)	0.5	28.1	1.01
Bound (95% confidence)	0.01	27.1	1.05
Bound (95% confidence)	0.001	26.8	1.06
Bound (95% confidence)	0.000001	26.2	1.09

The highlighted value in the table above is the value that will be used. It is the energy that could cause failure through the component one time in a million, with 95% certainty.

Mitigation (Material Failure)

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Table 1: Test Results

Test	Energy
Test 1 (2-2)	67.1
Test 2 (2-3)	63.1
Test 3 (2-1)	65.1
Test 4 (1-1)	65.1
Test 5 (1-2)	61.8
Test 6 (1-3)	62.9
Test 7 (1-4)	62.9
Mean	64.0
Standard Dev.	1.69

The table below displays the energy at the 95th percentile and at a variety of probability levels.

Table 2: Measurement of Uncertainty

Measure	Probability	Energy
Mean (95% confidence)	0.5	62.4
Bound (95% confidence)	10^{-2}	58.5
Bound (95% confidence)	10^{-3}	57.2
Bound (95% confidence)	10^{-6}	54.4
Bound (95% confidence)	10^{-9}	52.3

Material Failure (Simulation)

- Material failure simulations bound material failure energies
- Many cases typically involve multi-physics phenomena such as the mechanical initiation of energetic material which is not well quantified
- In most cases threshold is bounded by the energy required for the case to fail or sufficient deformation of the case
- A safety factor of 2.0 is applied to these simulated material failure energies

How do we associate a probability with this safety factor?

Simulations with Known/Assumed PDFs

- An adequately small value is set at five standard deviations below the mean energy
  - This is slightly less than 50% of the threshold energy
  - Assuming a normal distribution this corresponds to a probability of 3×10^{-7}