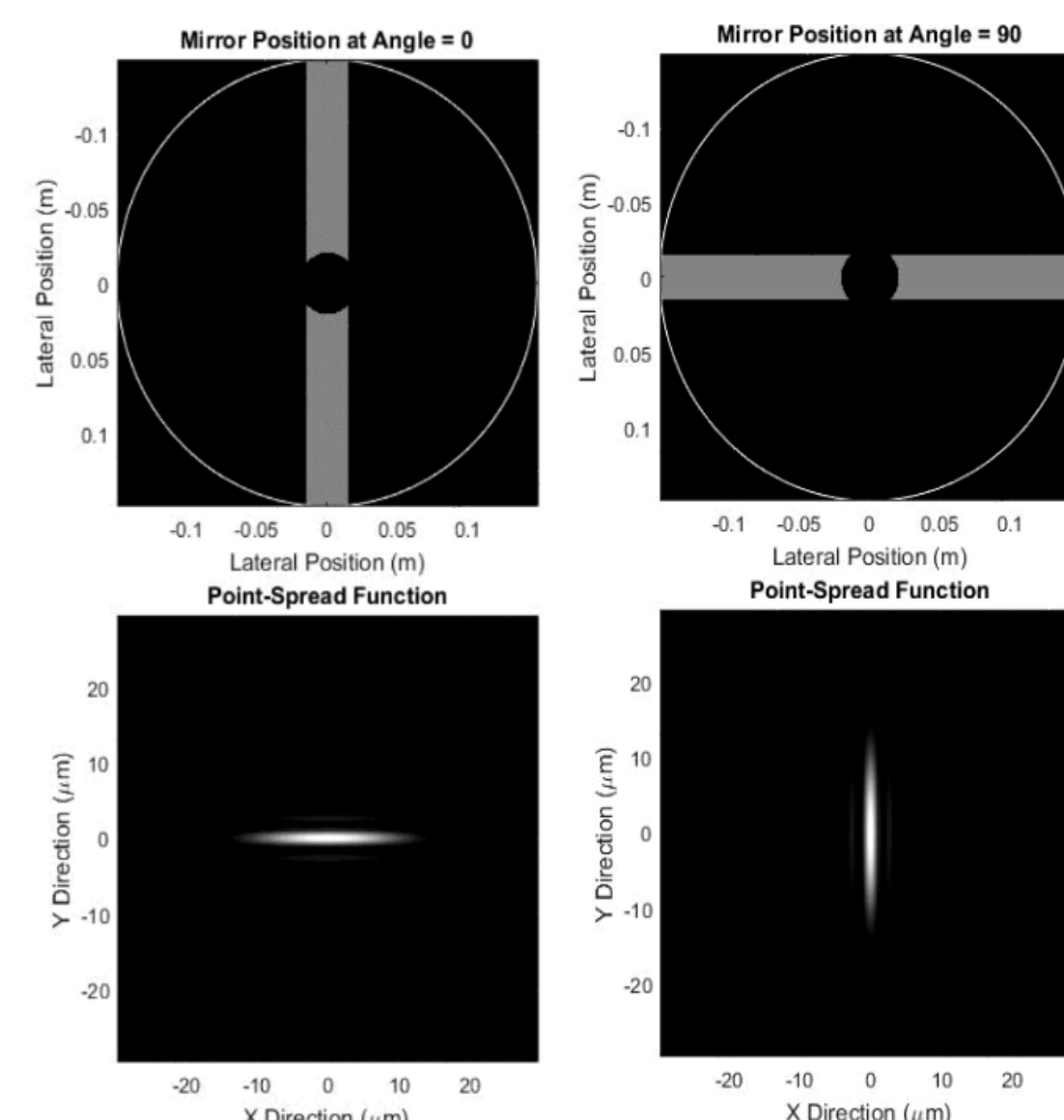
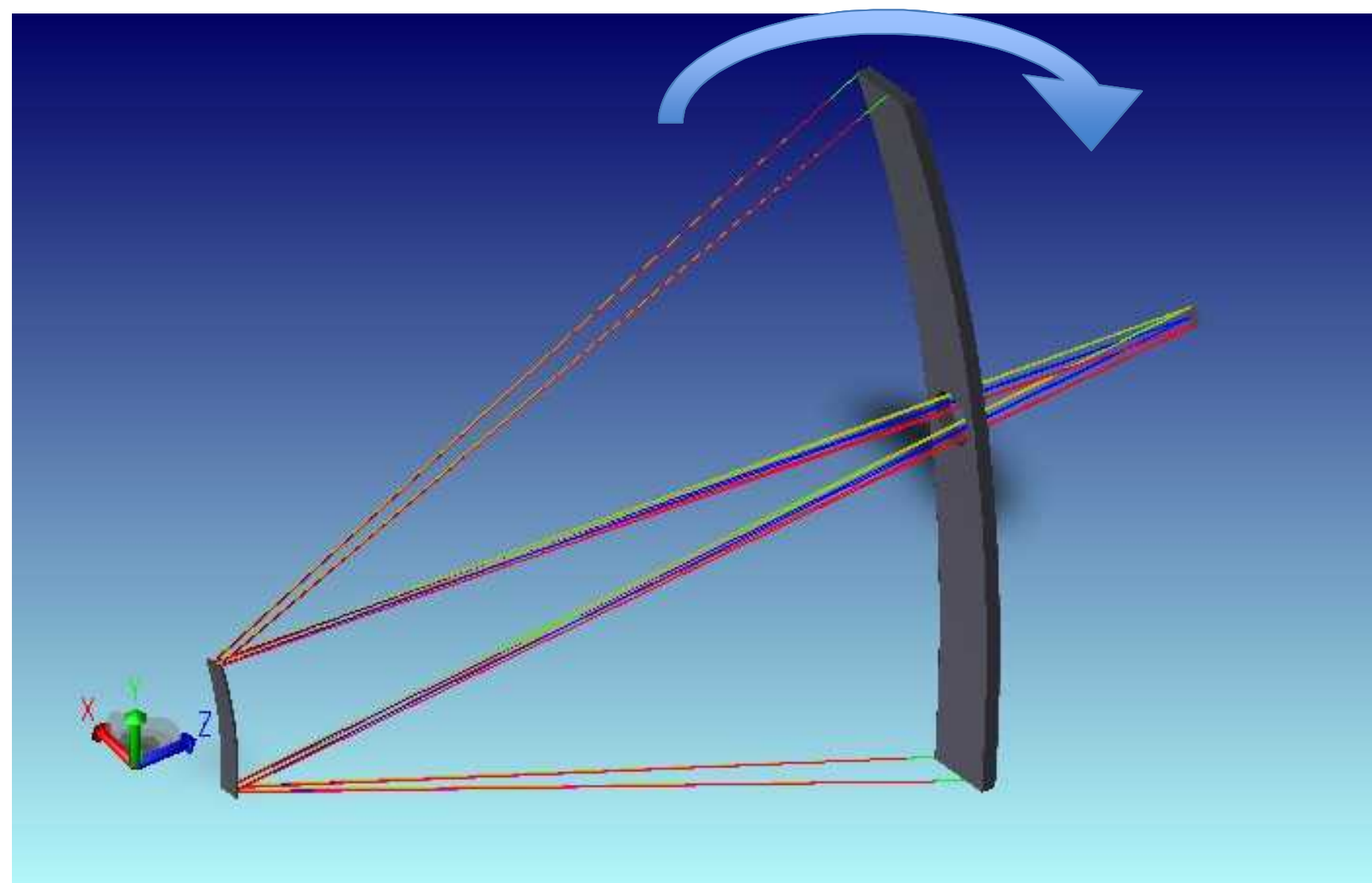


# Super-Resolution Reconstruction of Images from a System with an Asymmetric Spinning Aperture

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**Abstract:** Asymmetric spinning aperture imaging systems have been proposed to minimize the cost and volume of spaced-based telescopes. Previous work on such systems has been constrained to the case where the imaging system is spatially Nyquist sampled. However, there are advantages to designing undersampled systems, in which case the spatial resolution is limited by the focal plane pixel size rather than diffraction. In such systems multi-frame super-resolution techniques can be employed to improve the spatial resolution of the system. Results from applying modern multi-frame super-resolution techniques to imagery from undersampled, asymmetric spinning aperture systems are presented.

## System Concept



Deploying large, circular apertures in space presents a variety of problems. The weight and size of the mirror can significantly increase launch costs. Large apertures also require significant volume within the host.

Spinning asymmetric apertures have been proposed to mitigate such effects. By combining images collected as the asymmetric mirror is rotated about the optical axis, the resolution of a full-aperture system can be obtained at significantly reduced weight and volume.

## Image Reconstruction

### Deconvolution-Based Reconstruction

An initial reconstruction is performed prior to an optimization-based super-resolution reconstruction. The images are first registered and summed. The point-spread functions (PSFs) are summed as well. Let  $I(x,y)$  be the actual image,  $I_{sum}(x,y)$  be the sum of the registered images, and  $P_{sum}(x,y)$  be the sum of the PSFs. We have:

$$I_{sum}(x,y) = I(x,y) * P_{sum}(x,y)$$

where  $*$  denotes convolution. Hence deconvolution techniques can be used to obtain an initial estimate of the image.

### Super-Resolution Reconstruction

The super-resolution objective function  $J$  is given by<sup>1</sup>:

$$J(z, \Theta) \stackrel{\text{def}}{=} \sum_{k=1}^K \mathcal{N}(\text{DC}_k(\theta_k)z - d_k) + \mathcal{R}(z)$$

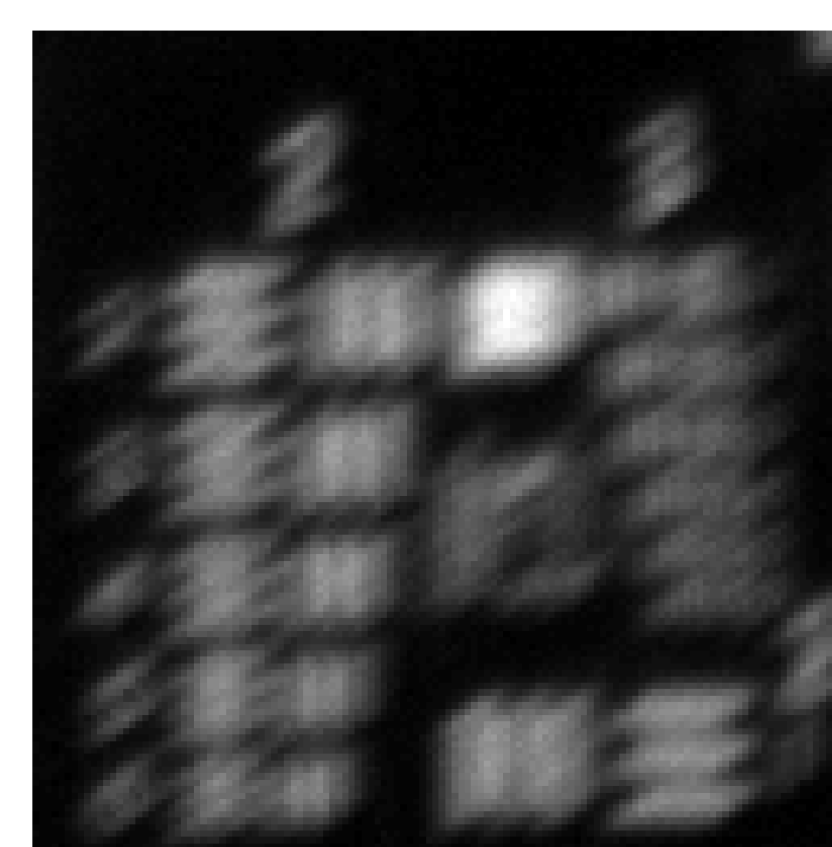
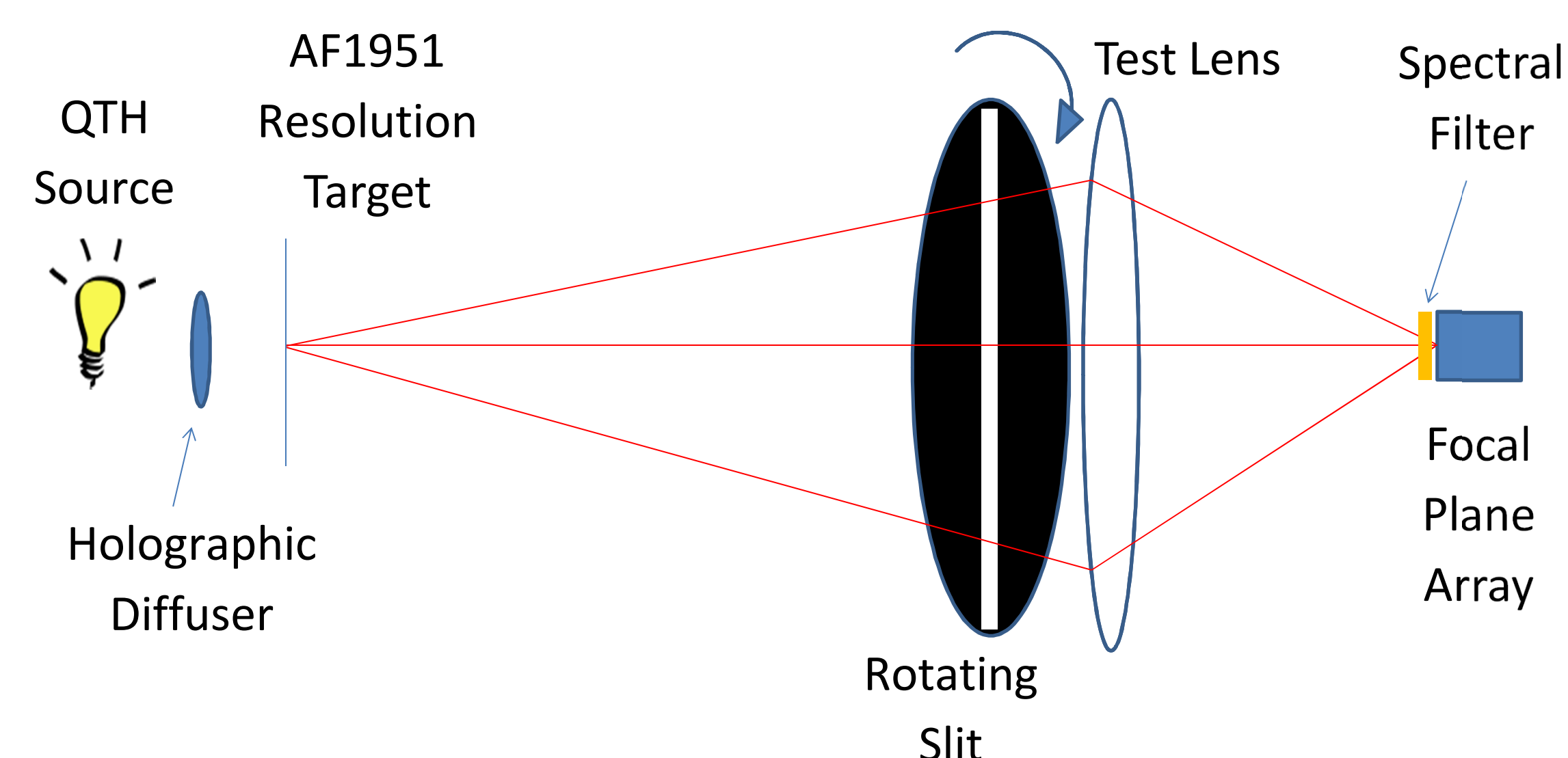
where

- $k$  enumerates the measured low-resolution images
- $d_k$  denotes the measured low-resolution images
- $z$  denotes the high-resolution image estimate
- $\theta_k$  denotes the lateral shifts for the  $k$ -th low-resolution image in the high-resolution domain
- $\text{DC}_k(\theta_k)$  converts a high-resolution image into a low-resolution image via translation, convolution with the  $k$ -th PSF, and downsampling
- $\mathcal{N}$  is commonly chosen to be either the  $\ell^1$  or  $\ell^2$ -norm function
- $\mathcal{R}(z)$  is an image regularization function

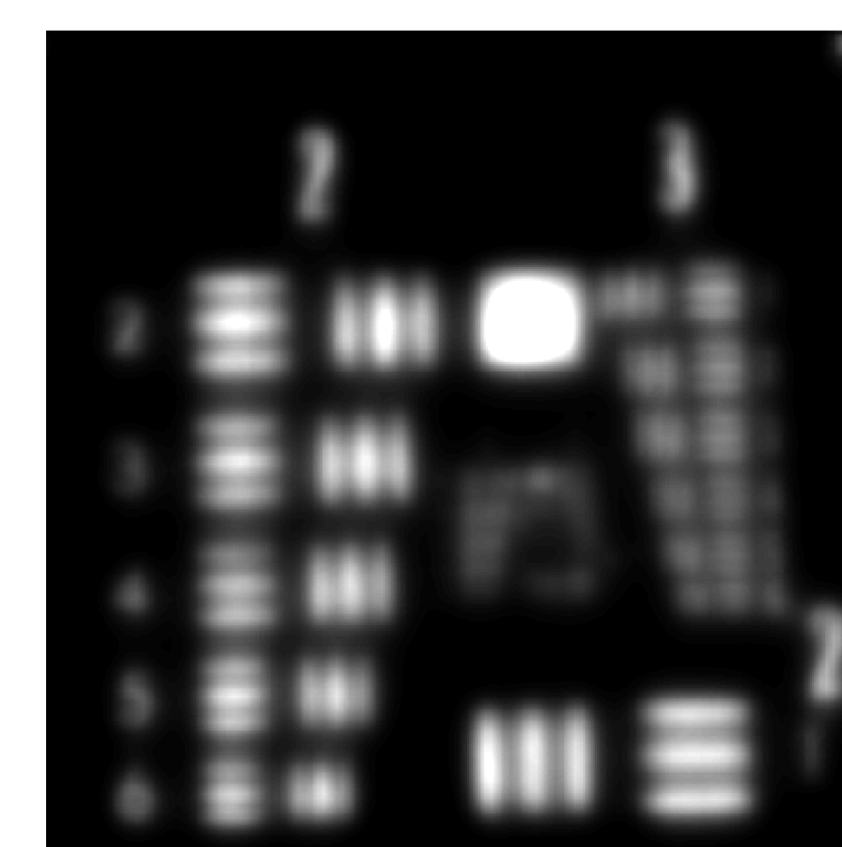
This super-resolution reconstruction algorithm provides two significant benefits. Many imaging systems are spatially undersampled for a variety of reasons<sup>2</sup>. This technique allows resolution up to the diffraction limit to be recovered.

This technique also allows for correction of errors in the initial image registration. Sub-pixel registration of the measured images is challenging due to the blurred nature of the PSFs. Optimizing these parameters provides robustness against registration errors.

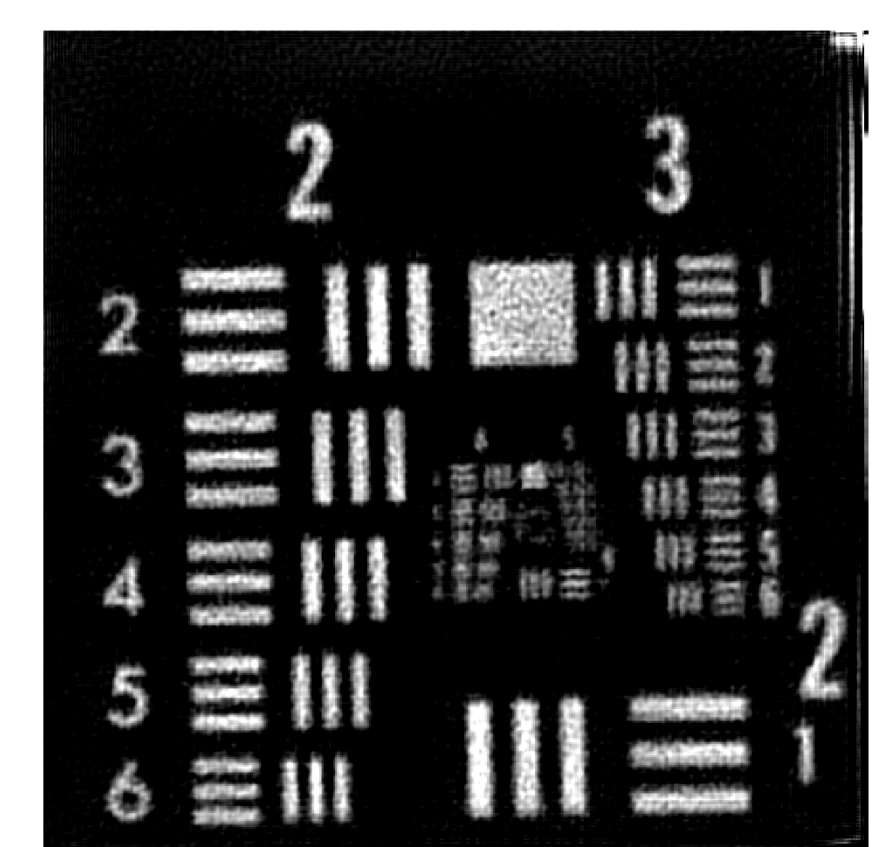
## Experimental Results



Measured Low-Resolution, Blurred Image



Deconvolution Reconstruction



Super-Resolution Reconstruction

Significant resolution improvement is obtained.

- 18 measured images were collected at angles ranging from 0 to 170° at 10° increments.
- Optimizations were performed in C++ using ArrayFire<sup>3</sup> and the Rapid Optimization Library<sup>4</sup> within Trilinos.

### References

- <sup>1</sup> D. P. Kouri and E. A. Shields, "Efficient multiframe super-resolution for imagery with lateral shifts," *Applied Optics* **53**(24), F1-F9 (2014)
- <sup>2</sup> R. D. Fiete, "Image quality and  $\lambda$ FN/p for remote sensing systems," *Optical Engineering* **38**(7), 1229-1240 (1999).
- <sup>3</sup> <https://www.arrayfire.com>
- <sup>4</sup> <https://trilinos.org/packages/rol/>