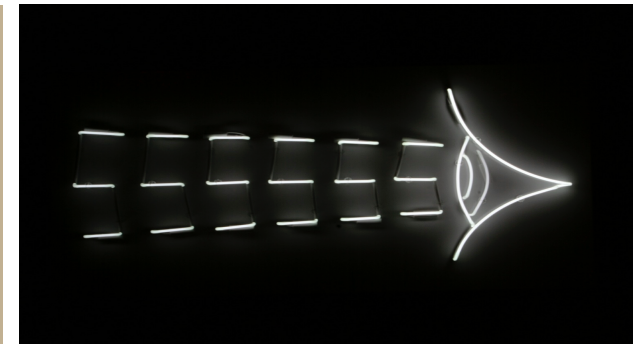
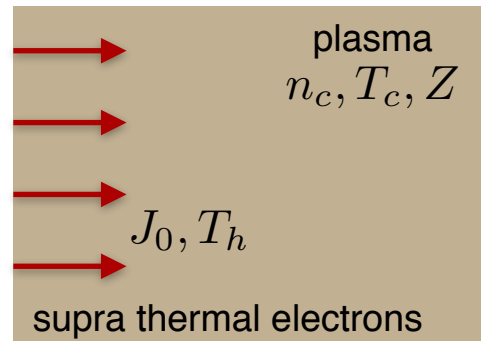
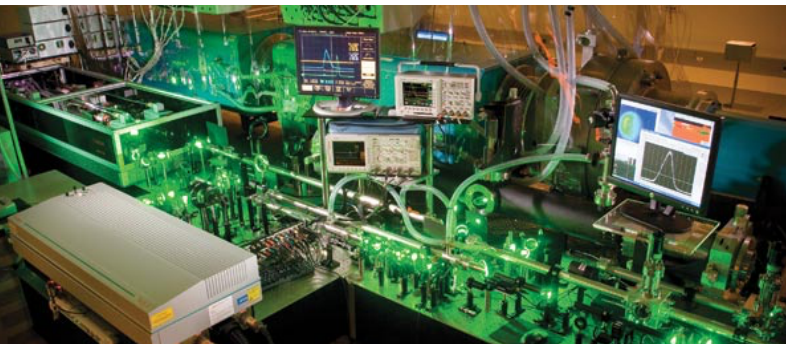


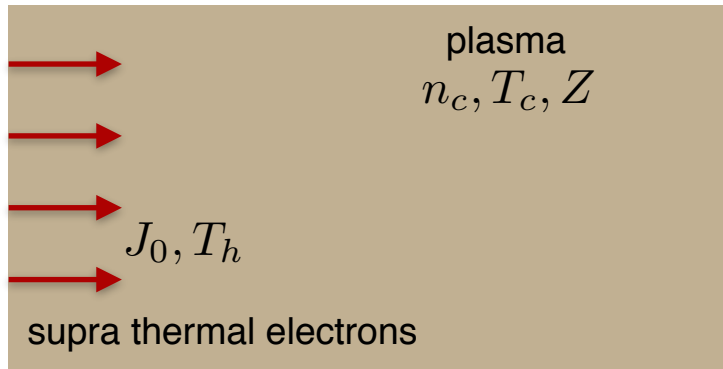
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# Regimes of suprathreshold electron transport\*

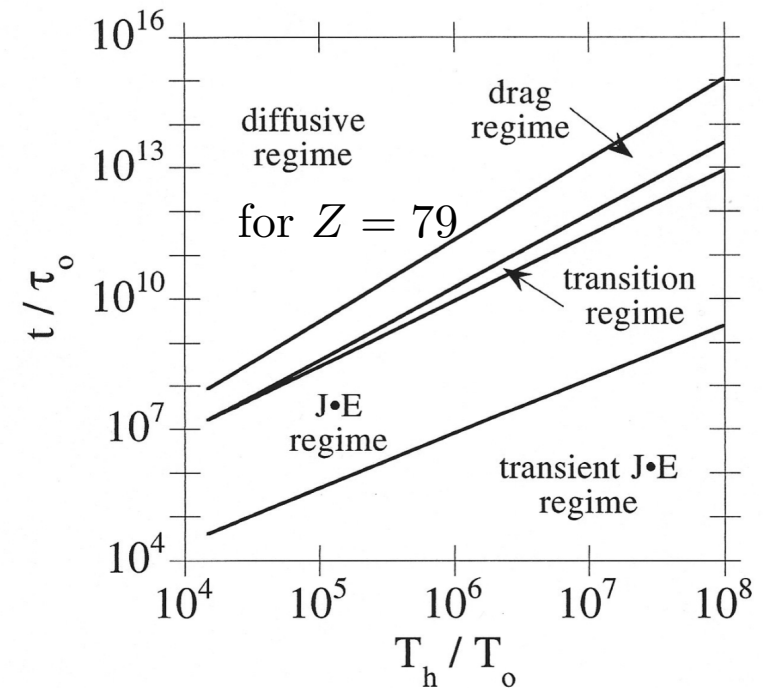
Michael E. Glinsky

# Four simple scaling solutions



$$u_0 \equiv \frac{J_0}{en_c} \quad T_0 \equiv mu_0^2$$

$$b_0 \equiv e^2/T_0 \quad 1/\tau_0 \equiv n_c b_0^2 u_0 Z \ln \Lambda$$



predicts:

- fast ion generation
- magnetic field generation
- self similar plasma conditions
- E-field transport inhibition

Glinsky, Phys. Plasmas **7**, 2796 (1995).

# Outline of presentation

- derivation of basic equations
- five regimes of superthermal electron transport:
  - free streaming, transient J.E
  - J.E
  - J.E to cold electron collisional drag transition
  - cold electron collisional drag
  - diffusive
- applications:
  - fast ion generation
  - magnetic field generation
  - self similar plasma conditions
  - E-field transport inhibition

# Starting point for physics

Start with collisional Vlasov equation

$$\frac{\partial f_{\alpha}}{\partial t} + \vec{v} \cdot \frac{\partial f_{\alpha}}{\partial \vec{r}} + \frac{Z_{\alpha} e}{m_{\alpha}} \vec{E} \cdot \frac{\partial f_{\alpha}}{\partial \vec{v}} = \sum_{\beta} C(f_{\alpha}, f_{\beta})$$

using Fokker-Planck collision operator  
and three species

$$f_i = f_{0i}(v) \quad \text{(ions)}$$

$$f_c = f_{0e}(v) + \hat{v} \cdot \delta \vec{f}(v) \quad \text{(cold electrons)}$$

$$f_h = n_h g(\Omega) \delta(v - \sqrt{2T_h / m}) \quad \text{(hot electrons)}$$



# Full moment equations

## cold electron equations

$$\mathbf{E} = \frac{\mathbf{J}}{\sigma} - \frac{5}{2} \frac{1}{e} \nabla T_c - \frac{\mathbf{R}_h}{en_c} \quad \text{(momentum)}$$

thermoelectric pressure
hot-cold electron collisional drag

$$\frac{3}{2} n_c \frac{\partial T_c}{\partial t} = \nabla \cdot (\kappa \nabla T_c) + \frac{J^2}{\sigma} - \frac{\mathbf{J} \cdot \mathbf{R}_h}{en_c} + Q_h + \frac{3}{2} \frac{1}{e} \mathbf{J} \cdot \nabla T_c + \frac{5}{2} \frac{T_c}{e} \nabla \cdot \mathbf{J}$$

thermal diffusion
J·E heating
hot-cold electron collisional drag heating

(energy)

where

$$\sigma \equiv \frac{4\sqrt{2}}{\pi^{3/2}} \frac{T_c^{3/2}}{Ze^2 m_e^{1/2} \ln \Lambda} \quad \mathbf{R}_h = - \left( \frac{m}{e \tau_{ch}} \right) \mathbf{J}_h$$

$$\tau_{ch} \equiv \frac{3\sqrt{3}}{8\pi} \frac{m_e^{1/2} T_h^{3/2}}{n_c e^4 \ln \Lambda} \quad Q_h = \frac{3}{2} T_h n_h / \tau_{ch}$$

$$\kappa \equiv \frac{16\sqrt{2}}{\pi^{3/2}} \frac{T_c^{5/2}}{Ze^4 m_e^{1/2} \ln \Lambda}$$

## hot electron equations

$$m_e n_h \frac{d\mathbf{u}_h}{dt} + m_e \mathbf{u}_h \left( \frac{\partial n_h}{\partial t} + \nabla \cdot (n_h \mathbf{u}_h) \right) = -\nabla p - \nabla \cdot \Pi - en_h \mathbf{E} - \frac{Z}{2} \frac{n_h \mathbf{u}_h}{\tau_{ch}}$$

(momentum)

$$\frac{3}{2} T_h \left( \frac{\partial n_h}{\partial t} + \nabla \cdot (n_h \mathbf{u}_h) \right) = -en_h \mathbf{E} \cdot \mathbf{u}_h - Q_h \quad \text{(energy)}$$

where

$$p \equiv m_e n_h \langle (\mathbf{v} - \mathbf{u}_h)^2 / 3 \rangle$$

$$\Pi \equiv m_e n_h \left\langle \mathbf{v} \mathbf{v} - \mathbf{I} \frac{(\mathbf{v} - \mathbf{u}_h)^2}{3} \right\rangle$$

# Reduced set of equations

now assume:

1.  $n_c = \text{constant} \gg n_h$
2.  $T_h \gg T_c$
3.  $J = -J_h$  (quasi-neutrality)
4.  $u_h \ll \sqrt{3T_h/m_e}$

which gives the following set of 1D equations:

$$E = \frac{en_h u_h}{\sigma(T_c)} \quad (\text{cold electron momentum, Ohm's law})$$

$$\frac{3}{2}n_c \frac{\partial T_c}{\partial t} = \frac{\partial}{\partial x} \left[ \kappa(T_c) \frac{\partial T_c}{\partial x} \right] + \frac{(en_h u_h)^2}{\sigma(T_c)} + \frac{3}{2}n_h T_h / \tau_{ch} \quad (\text{cold electron energy})$$

$$0 = T_h \frac{\partial n_h}{\partial x} + en_h E + \frac{Z}{2} n_h m_e u_h / \tau_{ch} \quad (\text{hot electron momentum})$$

$$\frac{\partial n_h}{\partial t} + \frac{\partial}{\partial x} (n_h u_h) = -\frac{(en_h u_h)E}{3T_h/2} - n_h / \tau_{ch}, \quad (\text{hot electron energy})$$

$$u_h \leq \sqrt{3T_h/m_e} \quad (\text{hot electron flux limit})$$

# Dimensionless equations

$$\frac{\partial T_c}{\partial t} = C \frac{\partial}{\partial x} \left( T_c^{5/2} \frac{\partial T_c}{\partial x} \right) + \left( \frac{2A}{3} \right) T_c^{-3/2} (n_h u_h)^2 + \frac{2B}{Z} n_h T_h^{-1/2}$$

$$\frac{\partial n_h}{\partial t} = - \frac{\partial}{\partial x} (n_h u_h) - \left( \frac{2A}{3} \right) \frac{T_c^{-3/2}}{T_h} (n_h u_h)^2 - \frac{2B}{Z} n_h T_h^{-3/2}$$

two coupled 1D nonlinear  
parabolic partial differential  
equations

$$n_h u_h^* = -T_h \frac{\partial n_h}{\partial x} / (B T_h^{-3/2} + A n_h T_c^{-3/2})$$

$$u_h = \frac{u_h^* \sqrt{3T_h}}{u_h^* + \sqrt{3T_h}}$$

using these variables to scale the quantities:

$$\begin{aligned} u_0 &\equiv \frac{J_0}{en_c} & T_0 &\equiv m u_0^2 & A &\equiv \pi^{3/2} / 4\sqrt{2} \\ b_0 &\equiv e^2 / T_0 & 1/\tau_0 &\equiv n_c b_0^2 u_0 Z \ln \Lambda & B &\equiv 4\pi / 3\sqrt{3} \\ & & & & C &\equiv 32\sqrt{2} / 3\pi^{3/2} \end{aligned}$$

and boundary conditions:

$$\frac{\partial T_c(x=0)}{\partial x} = \frac{\partial T_c(x=\ell)}{\partial x} = \frac{\partial n_h(x=\ell)}{\partial x} = 0 \quad n_h u_h(x=0) = 1$$

# Simple scalings

$$\frac{\partial T_c}{\partial t} \sim O\left(\frac{T_c^{7/2}}{L_T^2}\right) + O(T_c^{-3/2}) + O\left[\frac{2B}{Z} n_h T_h^{-1/2}\right],$$

thermal diffusion
J•E heating
hot-cold electron drag heating

$$\frac{\partial n_h}{\partial t} \sim O\left(\frac{1}{L_n}\right) - O\left(\frac{T_c^{-3/2}}{T_h}\right) - O\left[\frac{2B}{Z} n_h T_h^{-3/2}\right],$$

advection
J•E heating
drag heating

$$L_n \sim \min\left[\frac{n_h T_h^{5/2}}{B}, T_h T_c^{3/2}, n_h (3T_h)^{1/2} L_n\right].$$

hot electron resistivity
cold electron resistivity
flux limit

where the density and thermal scale lengths are defined as:

$$L_n \sim n_h / (\partial n_h / \partial x)$$

$$L_T \sim T_c / (\partial T_c / \partial x)$$

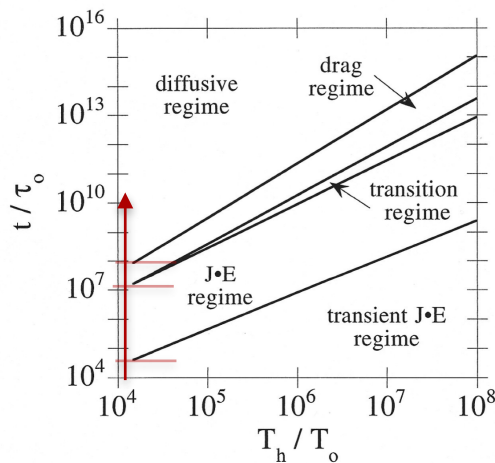
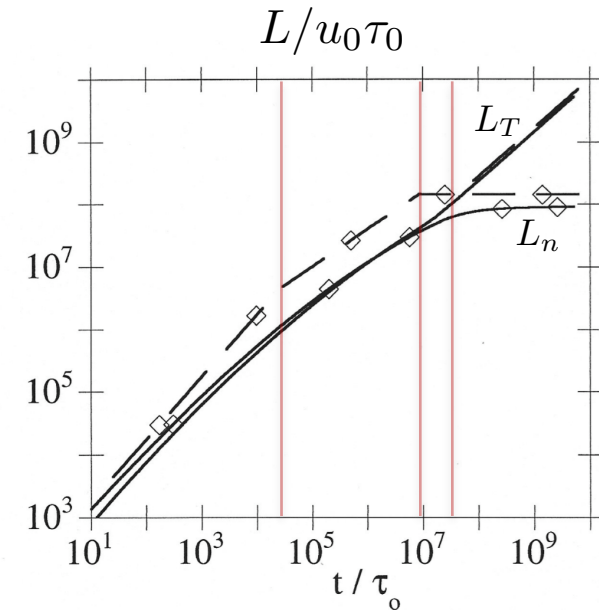
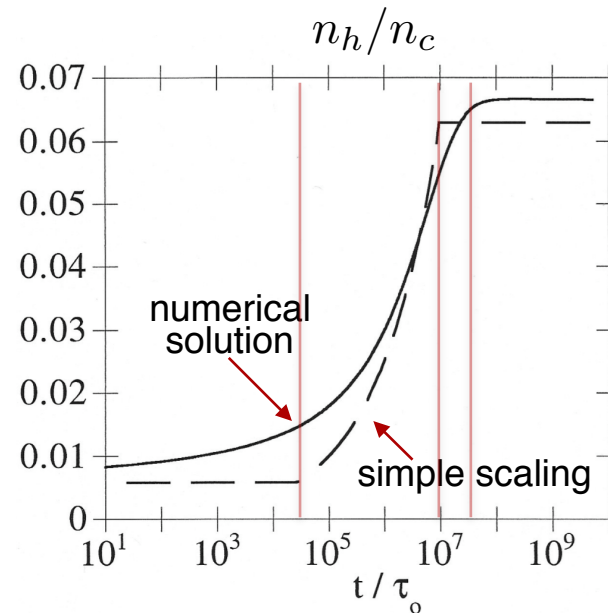
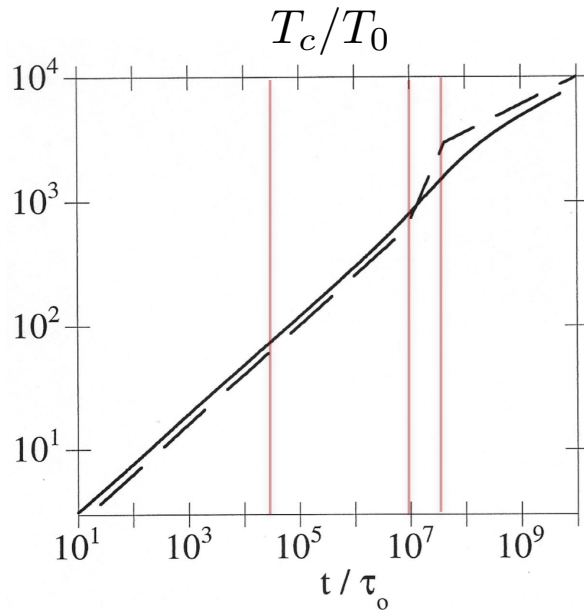
# Regimes determined by which terms dominate

	transient J•E	J•E	drag–J•E transition	drag	diffusion
$T_c \sim$	$t^{2/5}$	$t^{2/5}$	$t^{2/5}$	$T_h^{-1} \left( \frac{Z}{2B^2} \right)^{-1/2} t$	$T_h^{4/9} t^{2/9}$
$L_n \sim$	$(3T_h)^{1/2} t$	$T_h^1 t^{3/5}$	$T_h^1 t^{3/5}$	$T_h^2 \left( \frac{Z}{2B^2} \right)^{1/2}$	$T_h^2 \left( \frac{Z}{2B^2} \right)^{1/2}$
$L_T \sim$	$(3T_h)^{1/2} t$	$T_h^1 t^{3/5}$	$T_h^1 t^{3/5}$	$T_h^2 \left( \frac{Z}{2B^2} \right)^{1/2}$	$T_h^{5/9} t^{7/9}$
$n_h \sim$	$(3T_h)^{-1/2}$	$T_h^{-1} t^{2/5}$	$\left( \frac{Z}{2B} \right) T_h^{1/2} t^{-3/5}$	$\left( \frac{Z}{2} \right)^{1/2} T_h^{-1/2}$	$\left( \frac{Z}{2} \right)^{1/2} T_h^{-1/2}$
$t_{\min} \sim$	0	$(T_h / 3)^{5/4}$	$T_h^{3/2} \left( \frac{Z}{2B} \right)$	$T_h^{5/3} \left( \frac{Z}{2B^2} \right)^{5/6}$	$T_h^{13/7} \left( \frac{Z}{2B^2} \right)^{9/14}$

$\frac{\partial T_c}{\partial t} \sim$	J•E	J•E	drag	drag	diffusion
$\frac{\partial n_h}{\partial t} \sim$	advection	advection	advection vs. drag	advection vs. drag	advection vs. drag
$L_n \sim$	flux limit	cold electron resistivity	cold electron resistivity	hot electron resistivity	hot electron resistivity

# Example solutions

for  $Z = 79$  and  $T_h/T_0 = 10^4$



$$(n_h)_{\max} = (Z/2)^{1/2} T_h^{-1/2} \left( \frac{T_h}{B^4 Z/2} \right)^{1/10} \gg (Z/2)^{1/2} T_h^{-1/2}$$

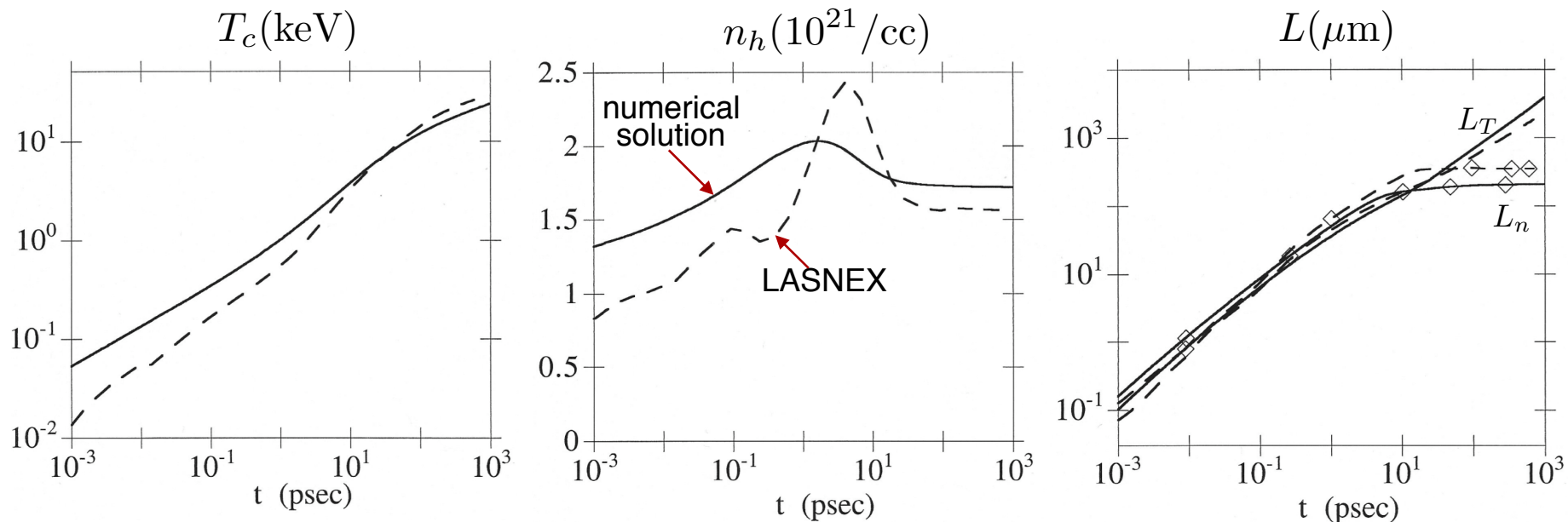
diffusive limit

$$L_n \leq \sqrt{Z/2B^2} T_h^2 \ll \sqrt{3T_h} t$$

free streaming  
limit

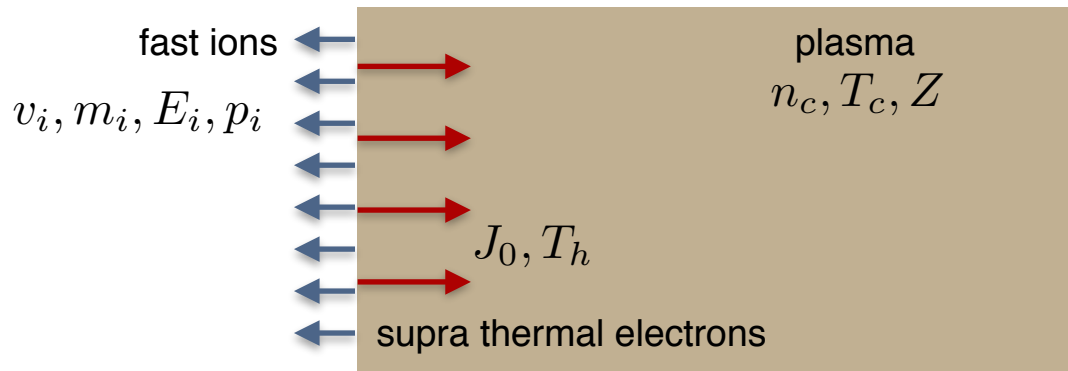
# Comparison to LASNEX

$$3 \times 10^{17} \text{W/cm}^2, T_h = 100 \text{keV}, n_c = 0.24 \text{gm/cc}$$



1D LASNEX with multigroup hot electron diffusion

# APP: fast ion generation



momentum and energy exchange at surface:

$$\frac{1}{\text{area}} \frac{dp_i}{dt} = (n_h v_h) (m_e v_h)$$

$$\frac{1}{\text{area}} \frac{dE_i}{dt} = v_i \frac{1}{\text{area}} \frac{dp_i}{dt} \sim n_h T_h v_i$$

ion velocity from two expressions for ion energy:

$$\frac{n_h}{Z} (m_i v_i^2) d \sim \int_0^d \frac{1}{\text{area}} \frac{dE_i}{dt} dt \sim n_h T_h d$$

$$v_i \sim \left( \frac{Z m_e}{m_i} \right)^{1/2} \left( \frac{T_h}{T_o} \right)^{1/2}$$

compare to rate of hot electron production:

$$\frac{dE_i / dt}{dE_h / dt} \sim \left( \frac{n_h}{n_c} \right) \left( \frac{T_h}{T_o} \right)^{1/2} \left( \frac{Z m_e}{m_i} \right)^{1/2}$$

use of  $n_h$  scaling gives:

$$\left( \frac{dE_i / dt}{dE_h / dt} \right)_{\text{max}} \sim \left( \frac{Z}{2} \right)^{1/2} \left( \frac{Z m_e}{m_i} \right)^{1/2} \left[ \left( \frac{T_h}{T_o} \right) \frac{1}{B^4 Z / 2} \right]^{1/10}$$

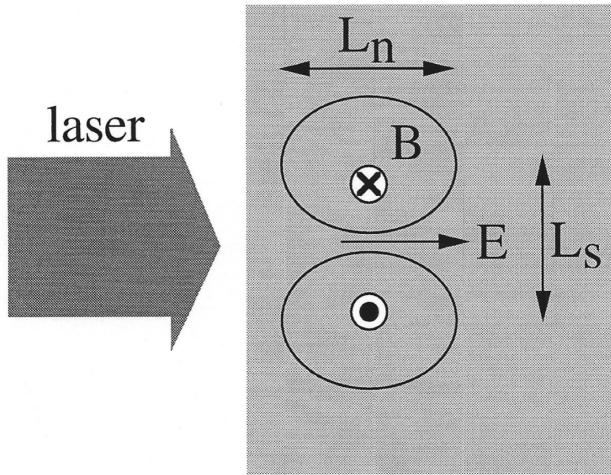
1ps,  $10^{17} \text{ W/cm}^2$ , 80 keV,  $Z^* = 25$  for Au

	simple model	LASNEX
hydrogen	4 %	4 %
solid gold	12 %	11 %
hydrogen on solid gold	18 %	18 %

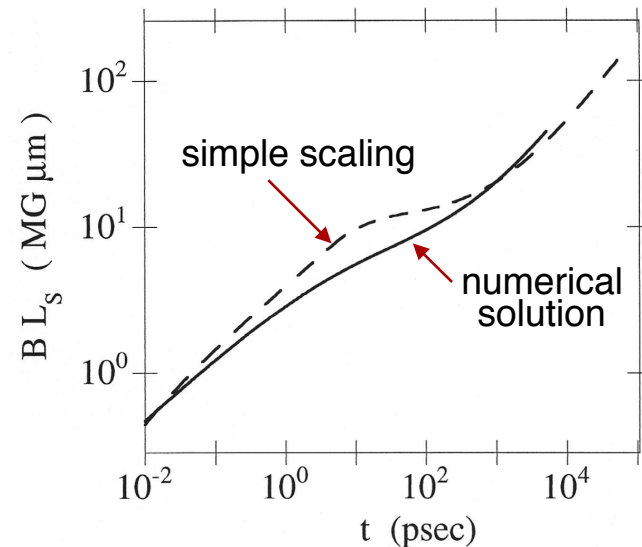
500Å



# APP: magnetic field generation



$$3 \times 10^{17} \text{ W/cm}^2, T_h = 100 \text{ keV}, n_c = 0.24 \text{ gm/cc}$$



from Maxwell's equations:

$$\dot{B} = -c \nabla \times E \sim cE/L_s$$

from Ohm's law for cold electrons:

$$E = B_0 (AZ \ln \Lambda) (T_c/T_0)^{-3/2}$$

$$B_0 \equiv e^3 n_c / T_0$$

# APP: similar plasma conditions

●  $I = 10^{19} \text{ W/cm}^2$ ,  $\lambda = 0.1 \mu\text{m}$ ,  $\tau_{\text{pulse}} = 1 \text{ ps}$ ,  $n_c = 10^{26} / \text{cc}$

●  $I_s = 10^{15} \text{ W/cm}^2$ ,  $\lambda_s = 1 \mu\text{m}$ ,  $\tau_s = 100 \text{ ps}$ ,  $n_c = 10^{23} / \text{cc}$

from Kruer & Eastbrook:

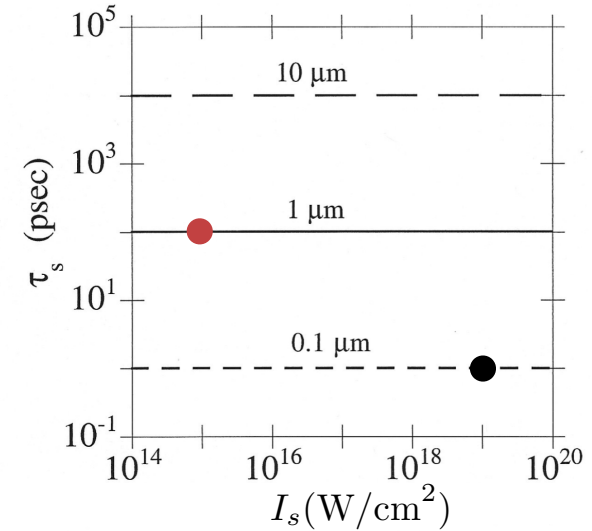
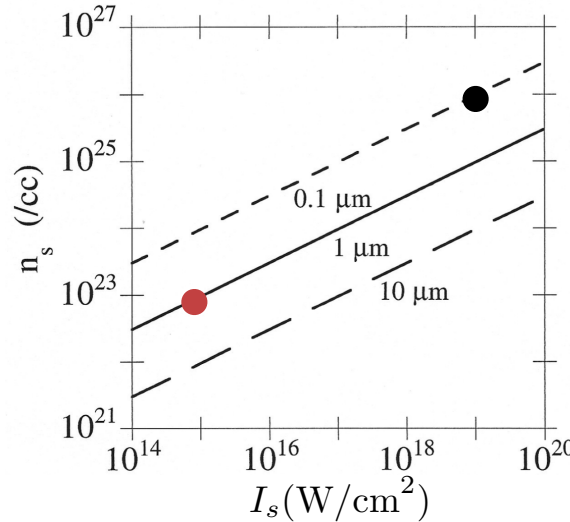
$$T_h \sim (I\lambda^2)^{1/3}$$

given base conditions, find:

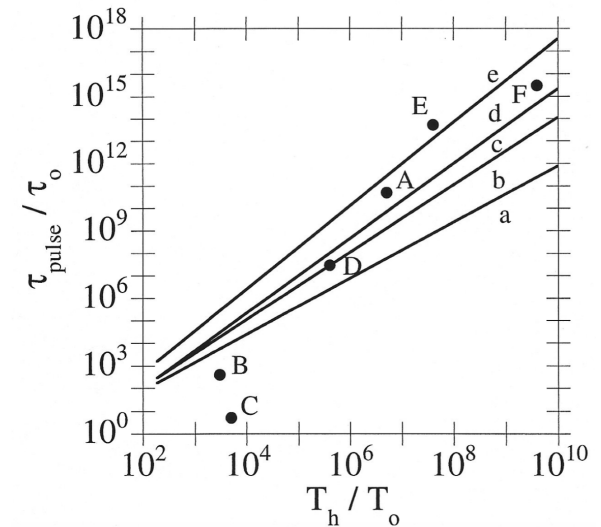
$$I_s, \lambda_s \rightarrow n_s, \tau_s$$

$$n_s = n_c \left( \frac{I_s}{I} \right)^{1/2} \left( \frac{\lambda_s}{\lambda} \right)$$

$$\tau_s = \tau_{\text{pulse}} \left( \frac{\lambda_s}{\lambda} \right)^{-1}$$

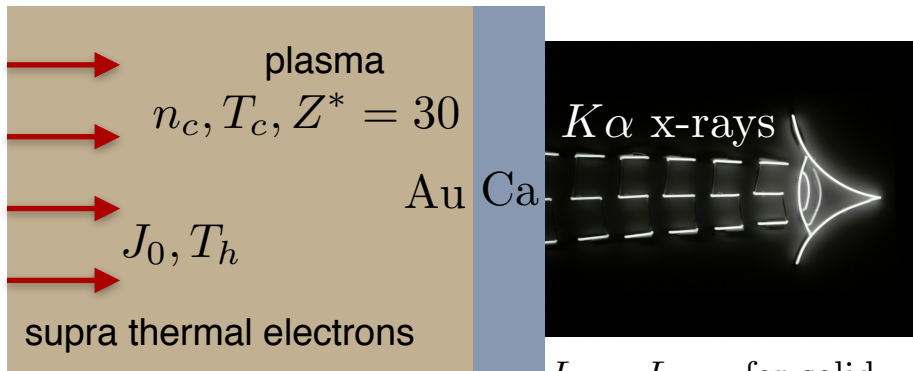


condition	I (W/cm <sup>2</sup> )	t <sub>pulse</sub> (psec)	n <sub>c</sub> (/cc)	T <sub>h</sub> (keV)
A	10 <sup>16</sup>	750	10 <sup>22</sup>	200
B	10 <sup>18</sup>	1	10 <sup>23</sup>	200
C	10 <sup>19</sup>	0.1	10 <sup>23</sup>	1000
D	10 <sup>19</sup>	1	10 <sup>26</sup>	100
E	10 <sup>15</sup>	100	10 <sup>24</sup>	10
F	10 <sup>16</sup>	750	10 <sup>24</sup>	200



# APP: E-field transport inhibition

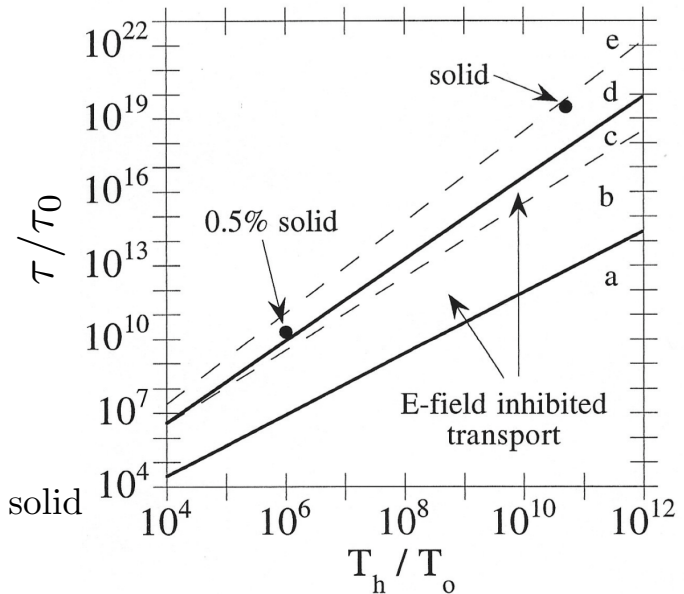
2% absorption of  $3 \times 10^{15} \text{ W/cm}^2$ ,  $\tau = 100 \text{ ps}$ ,  $\lambda = 1 \mu\text{m}$ ,  $T_h = 14 \text{ keV}$



gold foil targets of  
solid and 0.5% solid  
density

$L_n = L_{max}$  for solid

$L_n = 0.25 L_{max}$  for 0.5 % solid



$$L_n \ll \min[(3T_h)^{1/2}t, (Z/2B^2)^{1/2}T_h^2] = L_{max} \text{ if E-field inhibited transport}$$

Bond, Hares, Kilkenny, PRL **45**, 252 (1980).

Beg et al., Phys. Plasmas **4**, 447 (1997). (for  $10^{19} \text{ W/cm}^2$ )

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