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Development of a Single Input Multiple Output (SIMO) Input Derivation Algorithm for Oscillatory Decaying Shocks

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Abstract

During shaker shock testing of a complex system it may be desirable to match a Shock Response Spectra (SRS) at one location while controlling the test at a different location. Further, it may be desirable to match SRS at multiple locations. This paper describes an algorithm for deriving an optimum shaker shock input such that a weighted combination of the responses for multiple locations is matched with respect to the field data measured at those locations. This work assumes the shock environment is characterized by a SRS. Since the SRS is a nonlinear transformation of the underlying acceleration waveform, the optimization process will be based on the decayed sine synthesis algorithms developed by David Smallwood.

I. Introduction

In traditional shaker shock testing, the shaker input is computed to match the requirement at the input to the system with no attempt being made to match the response at other locations of interest. If the boundary conditions and/or forcing functions differ between the test lab and the field environment the responses produced during the shaker test may be very different from the field measurements, and even if an attempt were made to match internal responses then it was based on trial and error and therefore required significant effort and rarely produced optimal results. The algorithm presented in this paper describes a process for generating an optimal "single input" that produces a weighted best fit for "multiple outputs" using an automated function (i.e., a SIMO model). The reader should understand that that with only one input a perfect match for multiple locations is typically not possible.

II. Smallwood's Method to Compute Shaker Input for a Single Output

This method considers the shaker input a sum of decaying sinusoids as shown in equation 1.

$$A(t) = \sum_{i=1}^{N} A_i e^{-\lambda_i w_i t} \sin(w_i t + \varphi_i) \quad (1)$$

where

- A = acceleration amplitude of sinusoid
- w = frequency of sinusoid
- φ = delay of sinusoid
- λ = decay rate of sinusoid
- t = time
- N = number of sine tones

The objective of Smallwood's algorithm [1] is to produce a set a decayed sinusoids that will produce an SRS that will suitably match the reference SRS. The reference SRS location is typically at the input to the test article on the shaker head. The test is controlled at this point.

To start the analysis, the user must enter an estimate of the decayed sine parameters (amplitudes, frequencies, decay rates, amplitude polarities, and delays) to be used throughout the optimization. Both Smallwood's original routine

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and the SIMO method described in this paper optimize only the amplitude and leave all other decayed sine parameters as originally defined by the user. Furthermore, both methods use Smallwood's convergence algorithm for deriving the optimal sine tone amplitude for each frequency. Smallwood's convergence algorithm starts with the lowest frequency sine tone, iterating on the amplitude of that sine tone until the amplitude producing the best match for the desired SRS at that particular frequency is achieved. The amplitudes for each sequentially higher frequency tone is then iterated upon until the SRS associated with the sum of the current and all previously defined sine tones matches the desired SRS for the frequency of interest. The algorithm steps from low frequency to high frequency because the low frequencies can strongly affect the higher frequencies while the high frequencies only weakly affect the low frequencies.

Once a guess of the initial sine tone parameters is made, the algorithm first calculates the SRS of the initial guess input and calculates the error with respect to the reference SRS at the lowest sine tone (see equation 2).

$$Error = \frac{SRS(f) - SRS_{ref}(f)}{SRS_{ref}(f)} \tag{2}$$

Then, in an effort to determine how sensitive the SRS is to a change in the decayed sine amplitude, the amplitude is perturbed and a new SRS is computed. Then the sensitivity is computed by equation 3.

$$Sensitivity = \frac{A(f) - A_{perturb}(f)}{SRS(f) - SRS_{perturb}(f)} \tag{3}$$

The sensitivity is used to compute the new amplitude in equation 4.

$$New\ Amplitude = 0.8 * sensitivity * \left(SRS_{reference} - SRS_{pre-perturb}\right) + |A_{current}| \tag{4}$$

With the new decayed sine amplitude, the SRS can be recomputed and the error at the first decayed sine tone can be recomputed. If the error is more than a defined tolerance then the amplitude is iterated on. If the error is less than the defined tolerance then the algorithm steps to iterate on the amplitude of the next decayed sine tone.

Once a full sweep is done the compensating pulse is calculated. The compensating pulse is necessary to ensure that the acceleration, velocity, and displacement begin and end at 0. However, due to the interaction between sine tones, the estimates for the lower frequency tones are no longer optimized. Therefore, it is common practice to re-iterate on the entire set of tones again, starting with the lowest frequency tone. This re-iteration process is typically done 1-2 times before the optimal solution is achieved.

For the test cases in section IV, the parameters are as follows: Six sine tones per octave, the decay was set such that all the sine tones decay at the same rate, and no delay was set for any sine tones. Also the SRS was calculated with 3% critical damping ratio.

III. Method to Compute Shaker Input for Multiple Outputs

The method to compute the shaker input when accounting for multiple output locations is very similar to the single output location explained in the previous section. The major difference is that transfer functions and the decayed sine input are convolved to determine the response at the multiple output locations. The frequency domain convolution equation is shown in equation 5.

$$X_o(\omega) = H_{xq}(\omega) X_I(\omega) \tag{5}$$

where

$X_o(\omega)$ = Fourier transform of response at output location
$H_{xq}(\omega)$ = frequency response function
$X_I(\omega)$ = Fourier transform of decayed sinusoid input excitation

The SRS error at the sine tone frequency is computed with equation 6. Each individual response location error is weighted based on the user input weighing function.

$$Weighted\ Error = \frac{\sum \frac{\mu_i \left(SRS_i(f) - SRS_{ref_i}(f)\right)}{SRS_{ref_i}(f)}}{\sum \mu_i} \tag{6}$$

The sensitivity of the SRS to the decayed sine amplitude is computed using equation 6. Similar to the error calculation, the sensitivity incorporates the multiple output locations and user input weighting function.

$$Sensitivity = \frac{A(f) - A_{perturb}(f)}{\left(\frac{\sum \mu_i \left(SRS_i(f) - SRS_{perturb_i}(f)\right)}{\sum \mu_i}\right)} \tag{7}$$

The new decayed sine amplitude is computed using equation 8.

$$New\ Amplitude = c * Sensitivity * \left(\frac{\sum \mu_i \left(SRS_i(f) - SRS_{ref_i}(f)\right)}{\sum \mu_i}\right) + A(f) \tag{8}$$

Similar to Smallwood's method, once the amplitude for a decayed sine tone has been computed, the SRS is computed using the sum of that tone and all previously computed tones and the weighted error for that frequency can be recomputed. If the weighted error is more than a defined tolerance then the amplitude of the sine tone is iterated on. If the weighted error is less than the defined tolerance then the algorithm advances to the next sine tone and start iterating. Once a full sweep is done the compensating pulse is calculated. The entire set of sine tones is re-iterated upon as many times as the user requests.

IV. Test Cases

In order to demonstrate how the process works, a multi-degree of freedom (MDOF) spring/mass/damper system, referred to as the "toy model", was considered. The toy model is shown in figure 1. The model was excited using an experimentally measured oscillatory decaying waveform for the purpose of defining the "field environment" for three of the internal masses (M1, M2, and M3).

Two test cases are presented**.** Both the Smallwood model and the SIMO model will be used so as to demonstrate the differences. For both cases the Smallwood model simply produces an acceleration waveform whose SRS matches the input SRS, while for the SIMO model field responses are used as 'target' SRS along with the transfer functions and then the algorithm attempts to compute an input that produces optimal matches to the target SRS. For both models the responses were computed for the target masses using the inputs and the transfer functions.

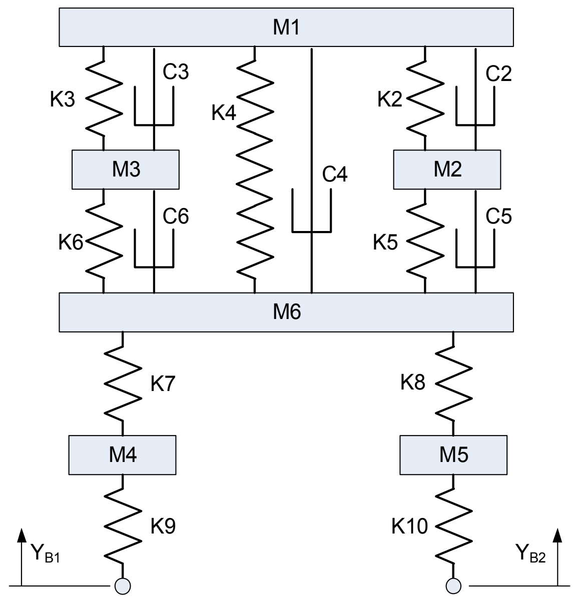

Figure 1: Babuska 'Toy Model'

V. Test Case 1: Single Input, Equally Weighted Outputs, Coupled System

This test case excites the toy model with the experimental input shown in figure 2. In this case both input locations in the toy model, Y_{B1} and Y_{B2}, are coupled together and the points of interest (M1, M2, and M3) have equal weightings in the SIMO approach. It is anticipated that the only error will be associated with the fact that the experimental input is not a simple sum of decayed sines.

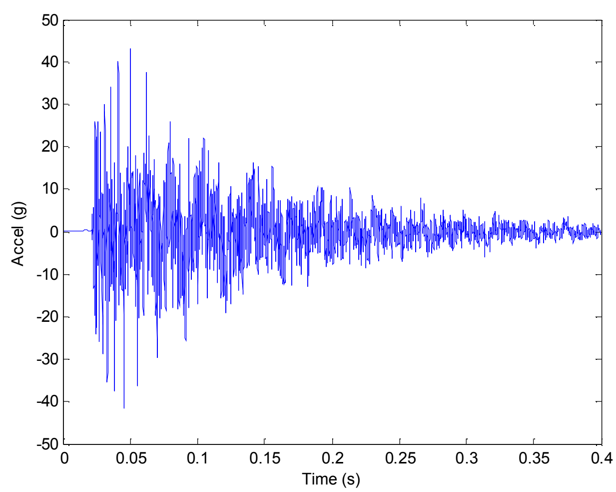

Figure 2: 'Experimental' Input

Figure 3 shows the SRS and acceleration waveforms for the experimental input along with the Smallwood and SIMO models. The Smallwood model matches the input SRS extremely well, but the corresponding acceleration waveform is significantly lower in amplitude. The SIMO model matches the input SRS until about 1300 Hz at which point it trends high (the reasons for this trend are not totally understood). The SIMO model's input acceleration is closer in amplitude to the experimental waveform.

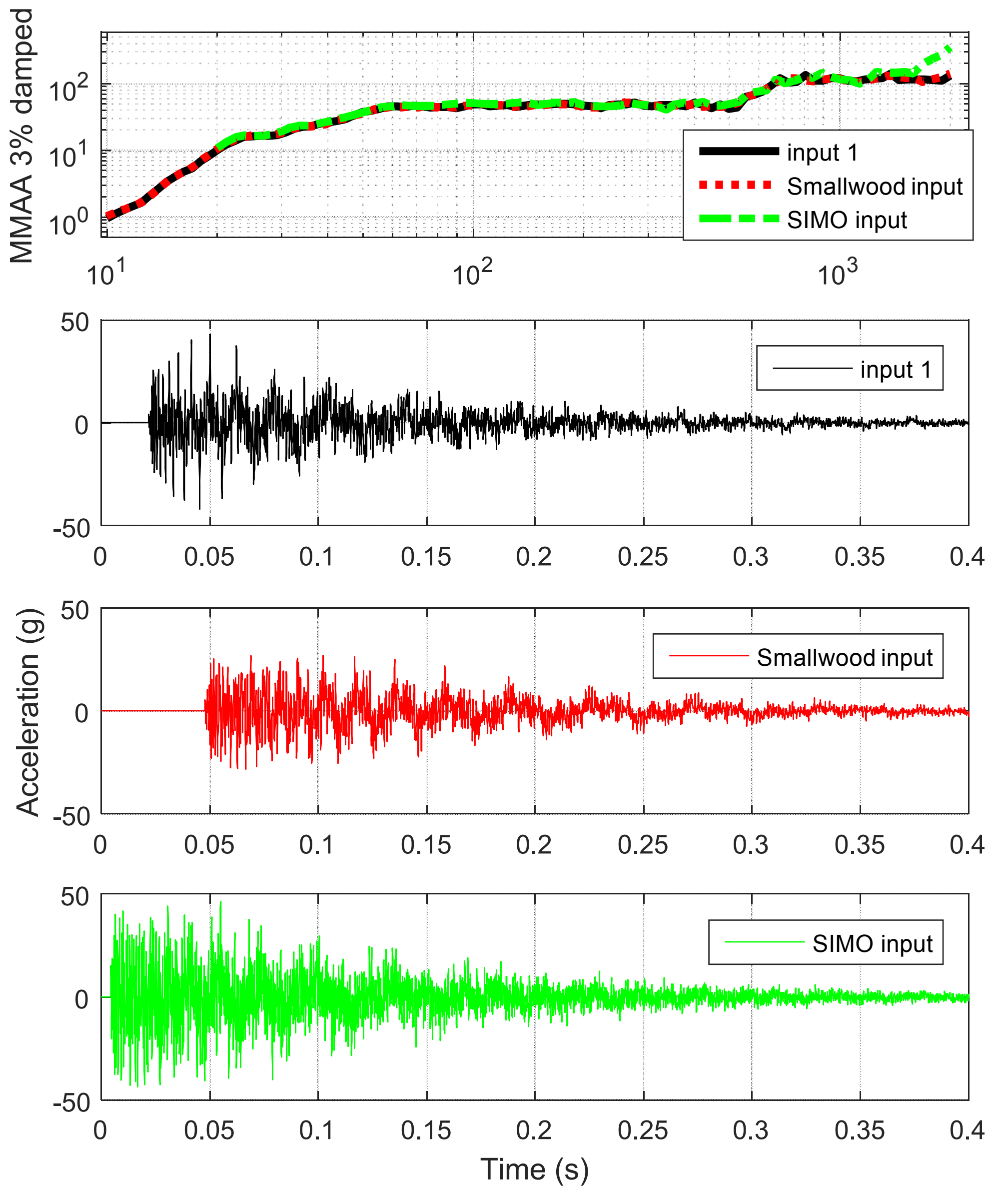

Figure 3: Comparison of input SRS and Acceleration Waveforms for Case 1

Figure 4 shows comparisons of the SRS for the target masses. The top three plots in Figure 5 present the dB ratios of the model results divided by the target responses for the Smallwood and SIMO models respectively. The bottom plot presents the arithmetic average of the dB errors for the three masses for the SIMO and Smallwood models respectively. The Smallwood model tends to under predict the responses for frequencies above 800 Hz. The SIMO model responses are considerably closer to the target responses.

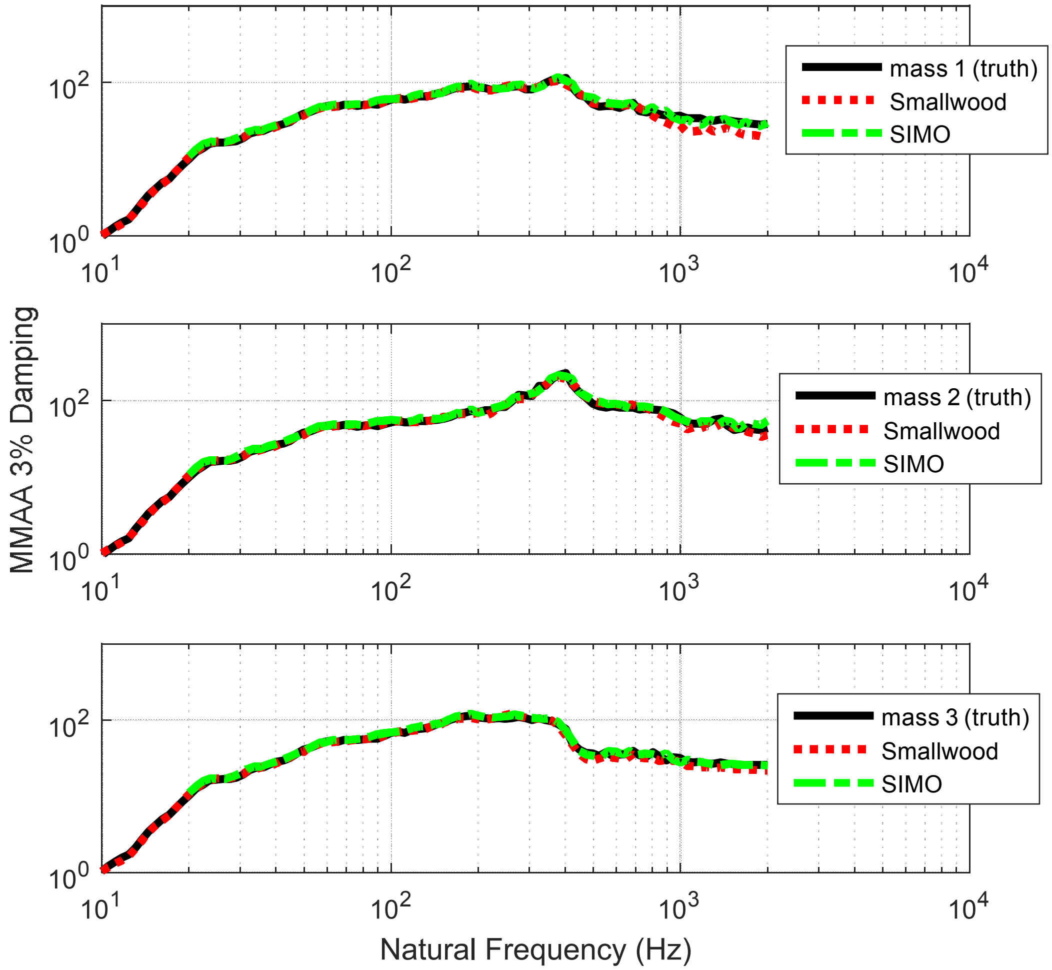

Figure 4: Comparison of Responses for Case 1

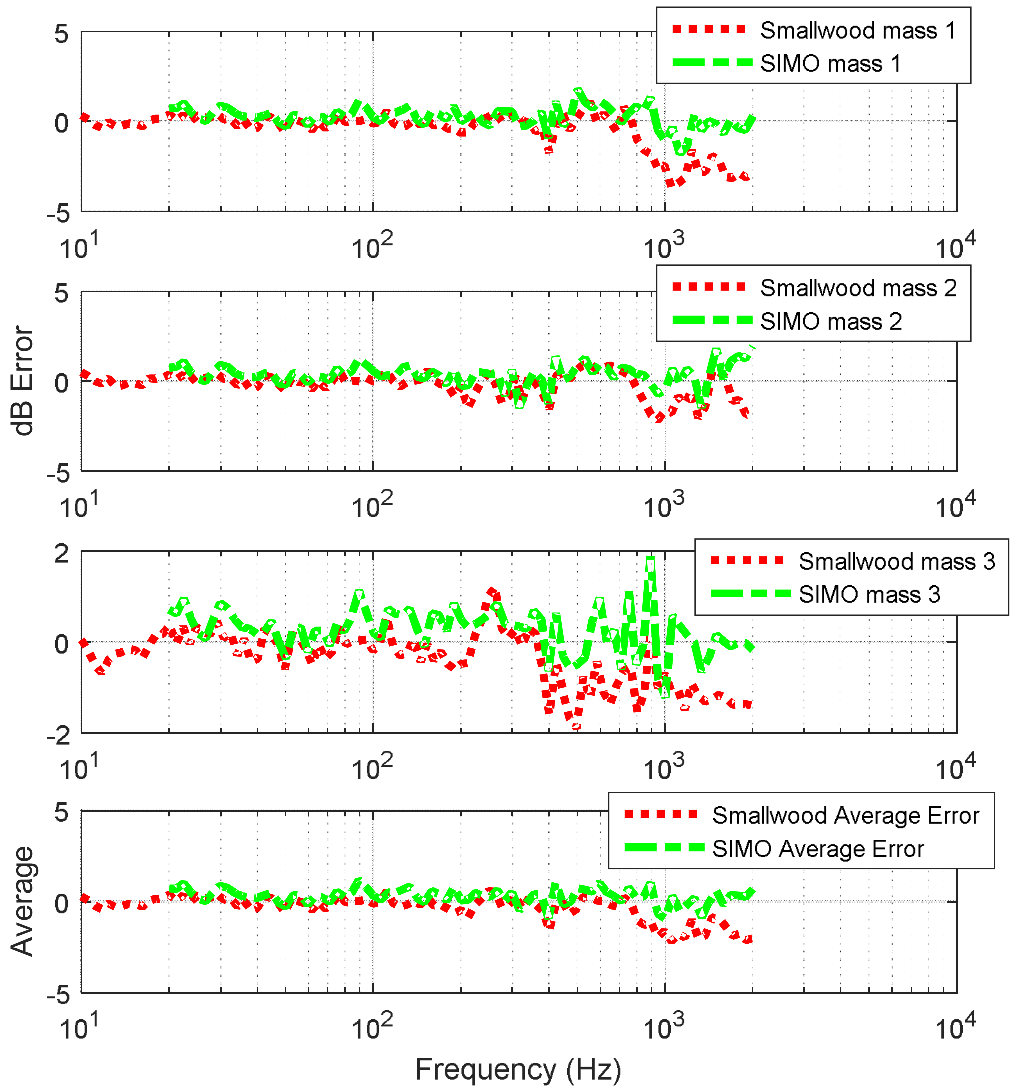

Figure 5: Response dB Error for Case 1

VI. Test Case 2: Decoupled System

It is often the case that the test spec or target SRS are obtained from a field test that has different boundary conditions and independent excitation levels for the various input points where as the shaker test must constrain the input points and apply a single input. The second test case will replicate this scenario by using different inputs for YB1 and YB2 during the "field" event (Figure 6) while forcing those points to act in unison during the laboratory test (Figure 7). Input 1 (Y_{B1}) is the same input used in the last test case (Figure 2). The second input (Y_{B2}) is shown in figure 8.

As would mostly likely be done in an actual scenario, the envelope of the SRS for both inputs was used as the target inputs for the Smallwood model.

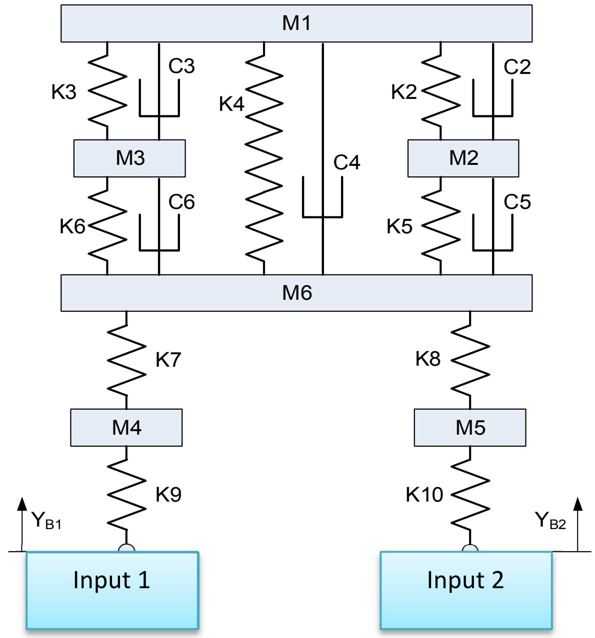

Figure 6: Toy model used to compute the target SRS

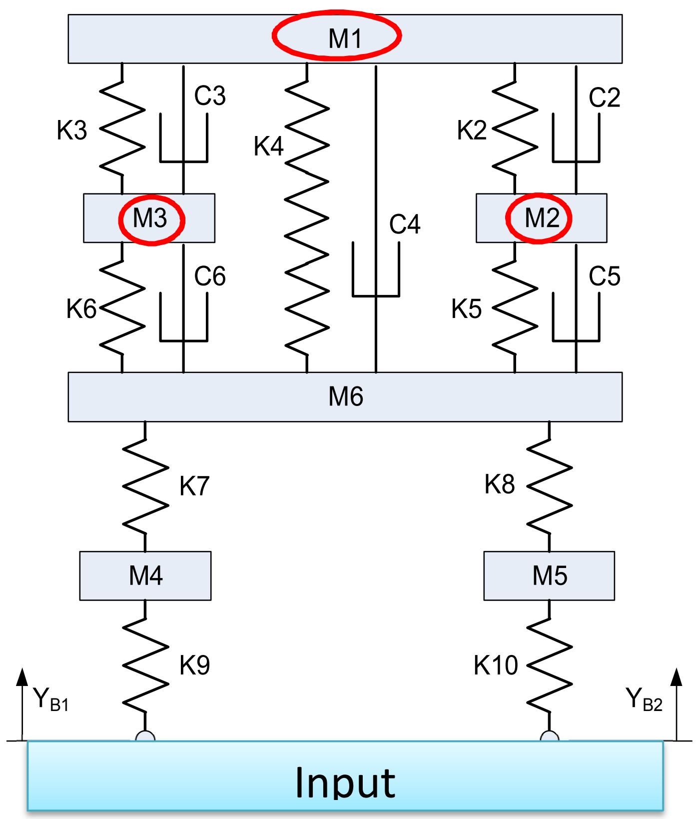

Figure 7: Toy model used to compute optimized input

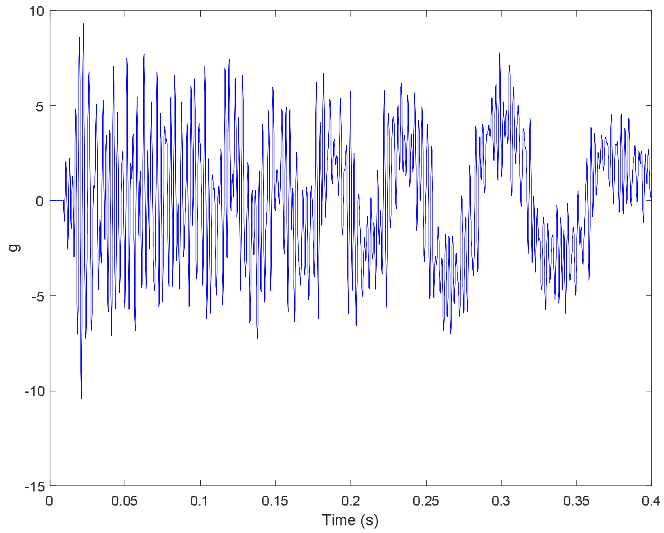

Figure 8: Input 2

Figure 9 shows the SRS and acceleration waveforms for the experimental input along with the Smallwood and SIMO models. As one would expect given how the test case was defined, the Smallwood input SRS matches the envelope of the input 1 and input 2 SRS. Also, as one would expect, the SRS for the SIMO input lies in between the two experimental input SRS.

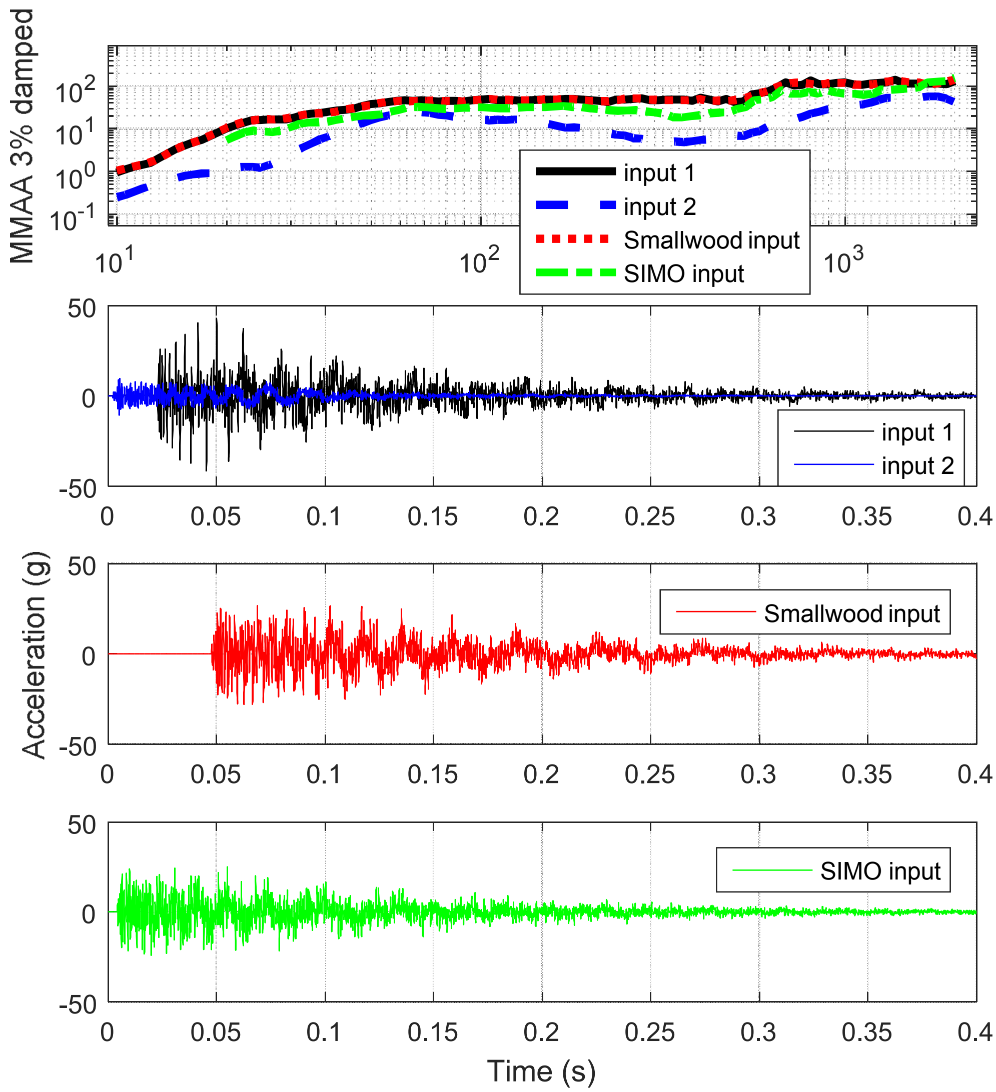

Figure 9: Comparison of input SRS and Acceleration Waveforms for Case 2

Figure 10 shows how well the predicted SRS match the target SRS. There is not as good agreement as in the first test case particularly with mass 1 and 3. This is expected with the more difficult (realistic) scenario of this test case. The dB errors are shown in Figure 11. The SIMO model produced generally better results on average.

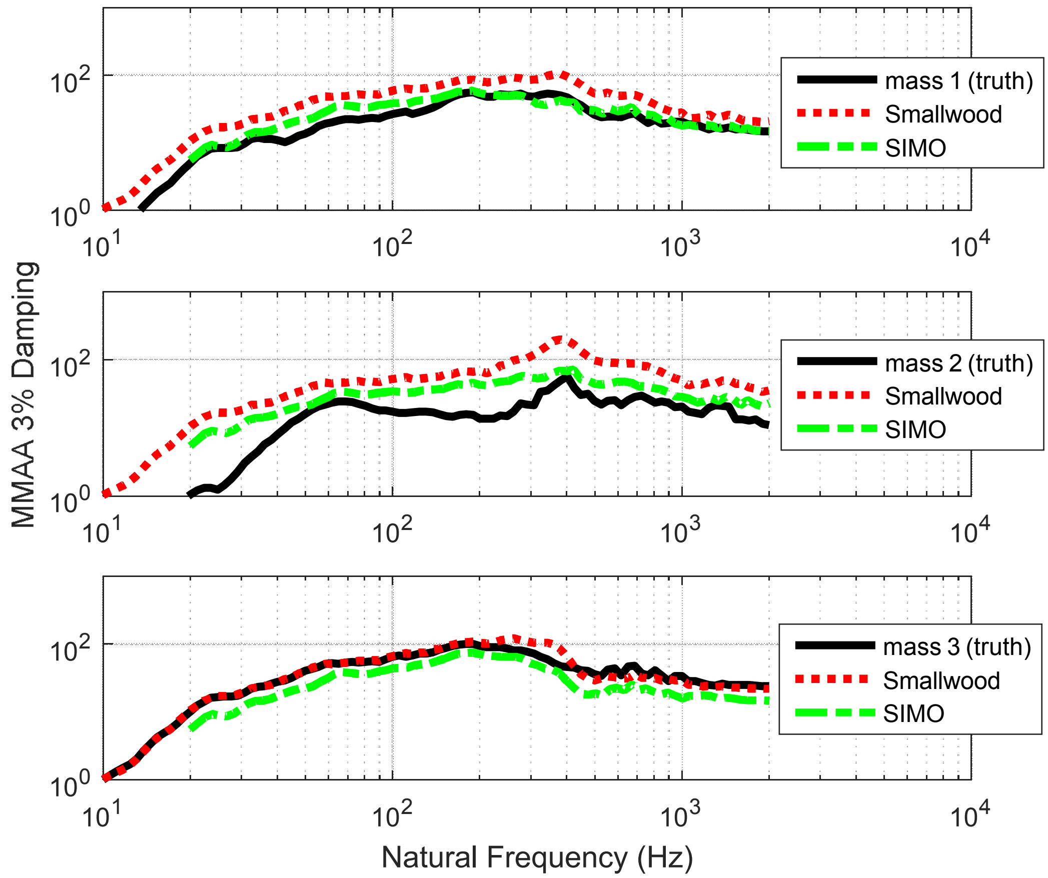

Figure 10: Comparison of Responses for Case 2

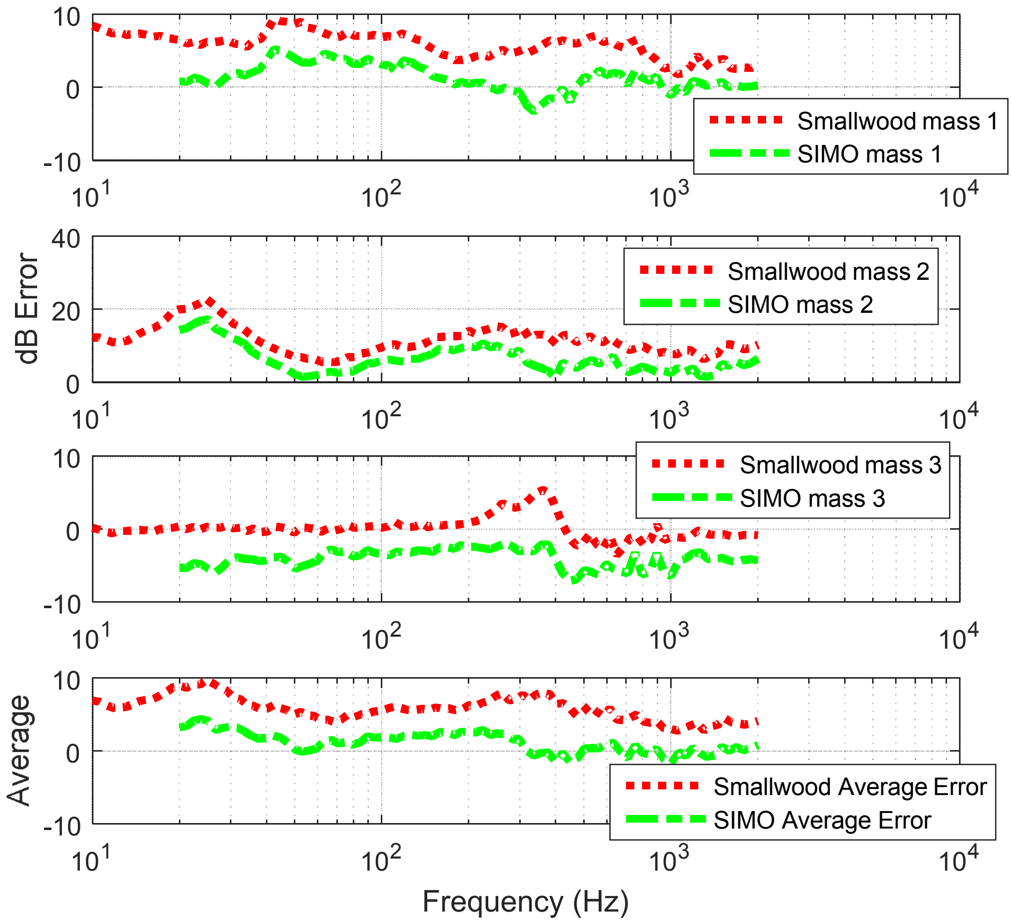

Figure 11: Response dB Error for Case 2

V. Conclusions

This paper described a method to compute a shaker shock input such that a weighted combination of the responses for multiple locations is matched with respect to the field data measured at those locations. The simplified test cases presented in this paper each took a few minutes to run, which is orders of magnitude faster than a manual iteration approach.

Test case 1 demonstrated that the SIMO model is both stable and accurate for a scenario for which the 'field test' has a single input location and the same boundary conditions as the 'laboratory test'. Test case 2 presented a more realistic scenario with multiple 'field' inputs and boundary conditions that are not possible in the laboratory. Under these more difficult conditions, the SIMO model created an input that produced a better average error over all frequencies than the traditional approach using the Smallwood model. It is therefore our recommendation that whenever data are available for multiple response locations the laboratory test should be based on a SIMO model rather than the traditional base input model.

However, there is still work to be done to prove out this method for a wider variety of systems. For example, based on the results from Case 1, it appears that when the system transmissibility is low, the SIMO model tends to produce higher than expected input levels, presumably because it generates slightly more accurate responses.

It may also be useful to have a response limited weighting scheme where the objective is to prevent an over test at one or more locations. This would mean that each response location SRS could be equal to or less than the reference SRS at each frequency.

References

[1] Smallwood DS. An Improved Recursive Formula for Calculating Shock Response Spectra. The Shock and Vibe Bulletin, May 1981