Structural Dynamics Lunchtime Series - #1.2 The Modal Filter with Applications

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Introduction

- Basics of the Modal Filter
- Root Extraction from Experimental FRFs using Analytical Mode Shapes
- Basic SWAT Force Reconstruction
- Modal Filter in SMAC Modal Parameter Identification
- Constraining Boundary Conditions Analytically with the Modal Filter on Experimental Models

Basics of Modal Filter

- Define the modal filter for a single mode as a column vector $\overline{\Psi}$

$$\overline{\Psi}^T \ddot{\overline{\mathbf{x}}} = \ddot{q}$$

Row vector * acceleration column vector = SDOF response

$$\overline{\mathbf{x}} = \mathbf{\Phi}\overline{\mathbf{q}}$$

$$\mathbf{\Phi}^T \mathbf{M} \mathbf{\Phi} \ddot{\overline{\mathbf{q}}}_{\overline{\Psi}} = \mathbf{I}\ddot{\overline{\mathbf{q}}}$$

Orthogonality of mode shapes with mass or stiffness matrix

$$\mathbf{\Phi}^T \mathbf{M} \ddot{\overline{\mathbf{x}}} = \mathbf{I}\ddot{\overline{\mathbf{q}}}$$

$$\overline{\mathbf{\Phi}}_r^T \mathbf{M} \ddot{\overline{\mathbf{x}}} = \ddot{q}_r$$

$$\mathbf{\Phi}^+ \ddot{\overline{\mathbf{x}}} = \ddot{\overline{\mathbf{q}}}$$

Most common modal filter

Example system – Wind Turbine Nacelle hanging from crane

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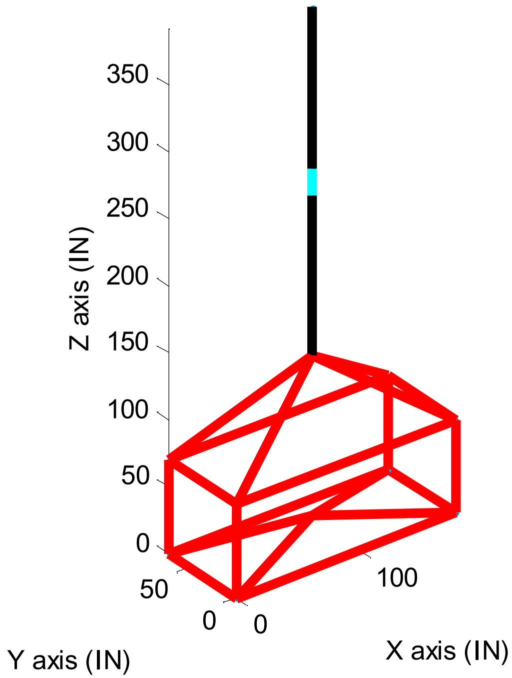
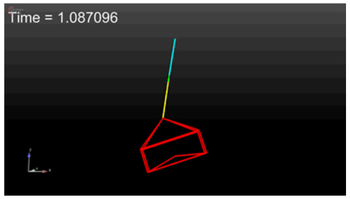
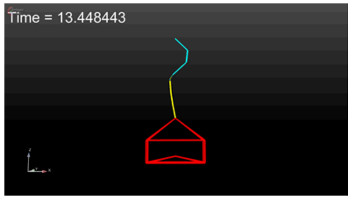
- The nacelle is a rigid body
- The mass and lateral stiffness of the crane wire rope, hoist block, hook and strap are modeled discretely
- Assume measurements are made at
  - triaxes on 4 corners of nacelle
  - Lateral biaxial on block and hook
- With that instrumentation set the mode shape matrix becomes ill-conditioned beyond the crane block mode
- Only modes up to crane block mode could be successfully filtered

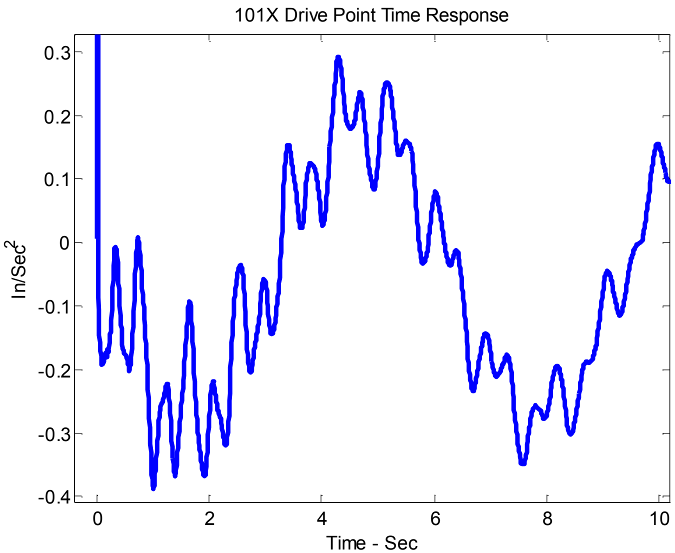




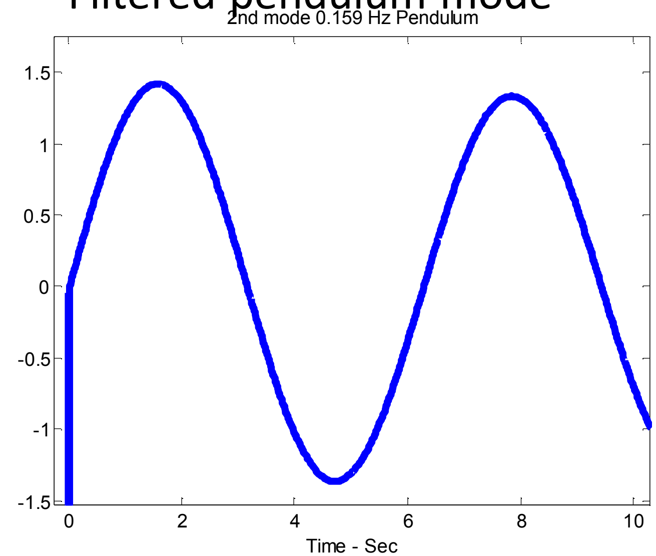


Modal Filter Nacelle results from impact

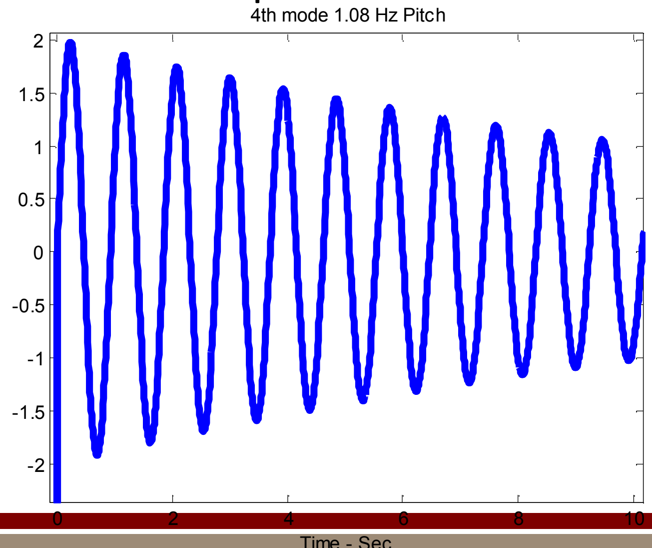
Drive point time history



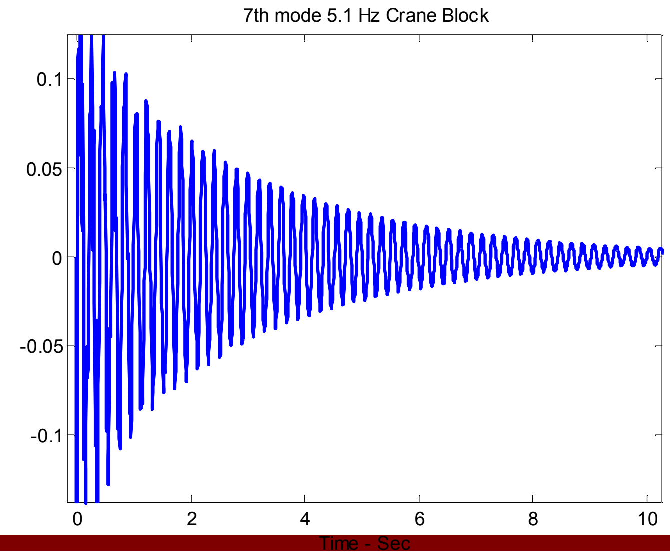
Filtered pendulum mode



Filtered pitch mode



Filtered crane block mode

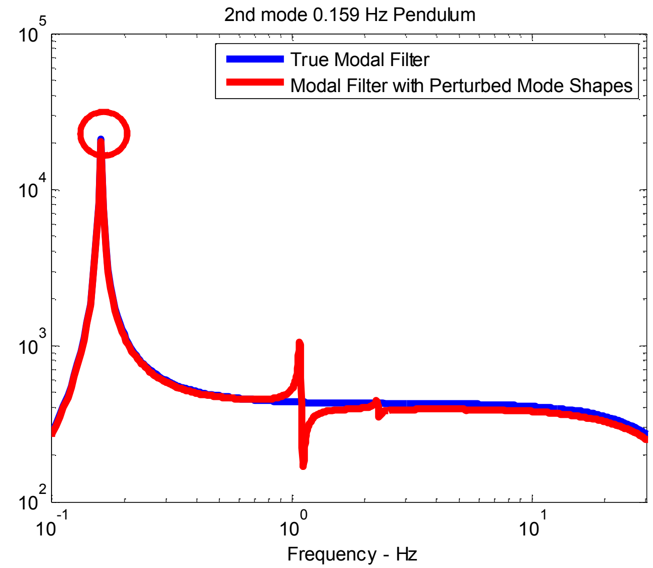


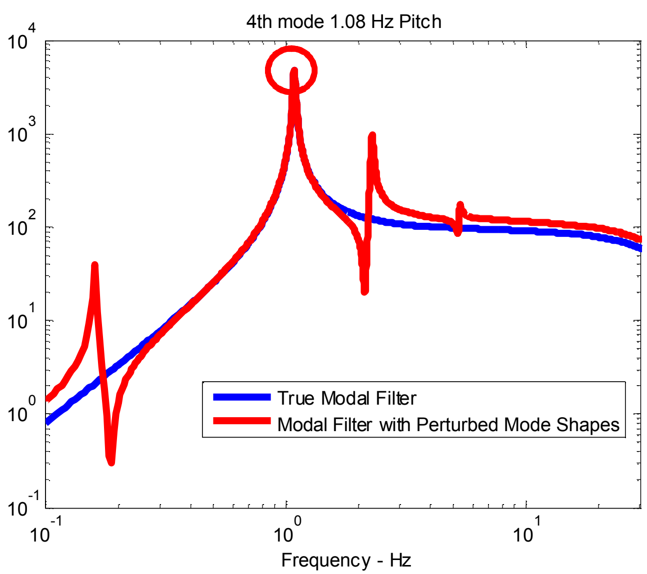
Extracting roots using modal filter from FE model mode shapes

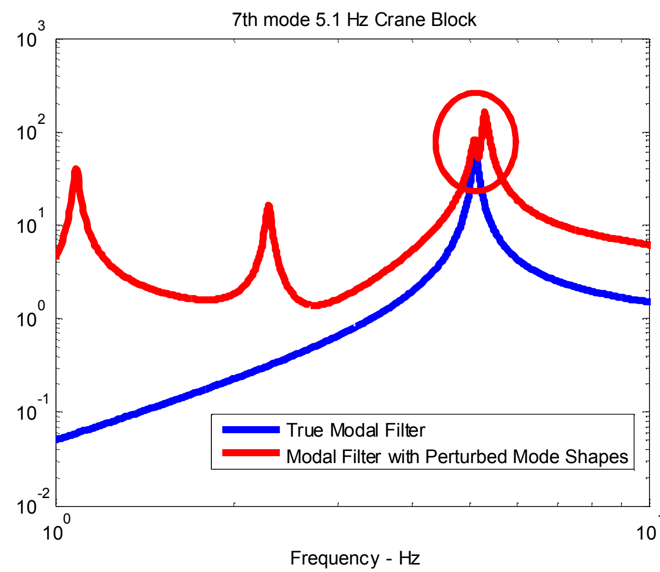
- If FE mode mode shapes are reasonably good, one can use the modal filter developed from those shapes to extract sdof FRF for each mode which can be used for root extraction/correlation purposes

$$\mathbf{\Phi}_{FE\_meas}{}^{+}\ddot{\mathbf{x}}_{meas} = \ddot{\mathbf{q}}$$

$$\mathbf{\Phi}_{FE\_meas}{}^{+}\ddot{\mathbf{H}}_{meas} = \ddot{\mathbf{H}}_{q}$$







Classic SWAT force reconstruction

- SWAT is the Sum of Weighted Accelerations Technique for force reconstruction
- SWAT is applied to a free system with rigid body modes assumed to have a frequency of zero

SWAT Theory of force reconstruction

- SWAT reduces ill-conditioning of typical FRF inverse

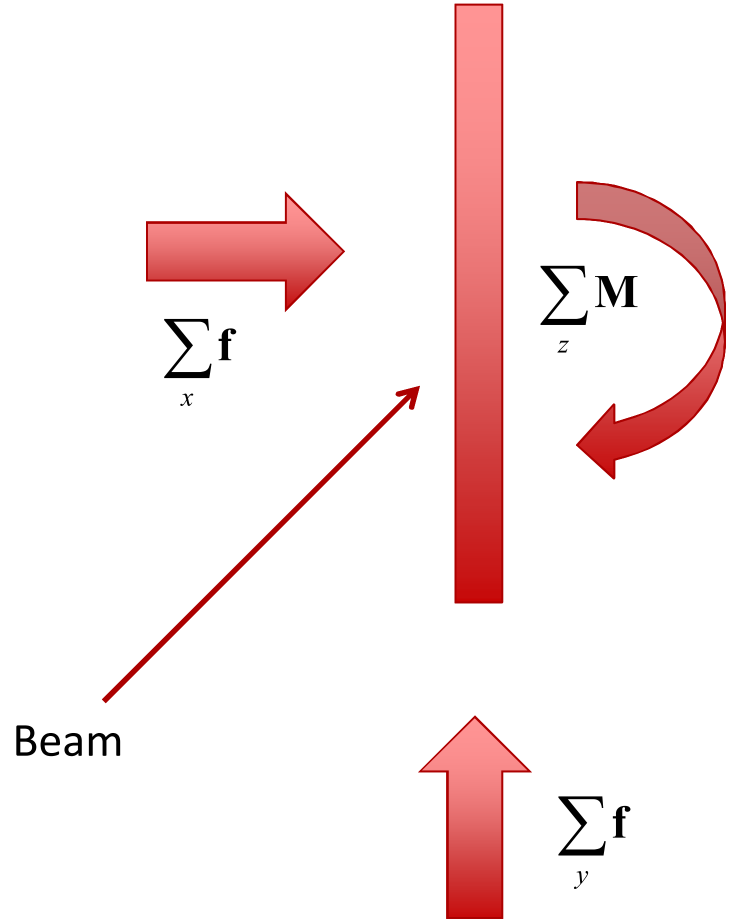
$$\mathbf{M}\boldsymbol{\Phi}\ddot{\bar{\mathbf{q}}} + \mathbf{C}\boldsymbol{\Phi}\dot{\bar{\mathbf{q}}} + \mathbf{K}\boldsymbol{\Phi}\bar{\mathbf{q}} = \bar{\mathbf{f}}$$

$$\boldsymbol{\Phi}_R^T\mathbf{M}\boldsymbol{\Phi}\ddot{\bar{\mathbf{q}}} + \boldsymbol{\Phi}_R^T\mathbf{C}\boldsymbol{\Phi}\dot{\bar{\mathbf{q}}} + \boldsymbol{\Phi}_R^T\mathbf{K}\boldsymbol{\Phi}\bar{\mathbf{q}} = \boldsymbol{\Phi}_R^T\bar{\mathbf{f}}$$

$$\begin{bmatrix} \backslash\, \mathbf{m}_{r\backslash} & \mathbf{0} \end{bmatrix} \begin{matrix} \ddot{\mathbf{q}}_R \\ \ddot{\mathbf{q}}_E \end{matrix} + \begin{bmatrix} 0 & 0 \end{bmatrix}\dot{\bar{\mathbf{q}}} + \begin{bmatrix} 0 & 0 \end{bmatrix}\bar{\mathbf{q}} = \begin{Bmatrix} \sum_x \mathbf{f} \\ \sum_y \mathbf{f} \\ \sum_z \mathbf{M} \end{Bmatrix}$$

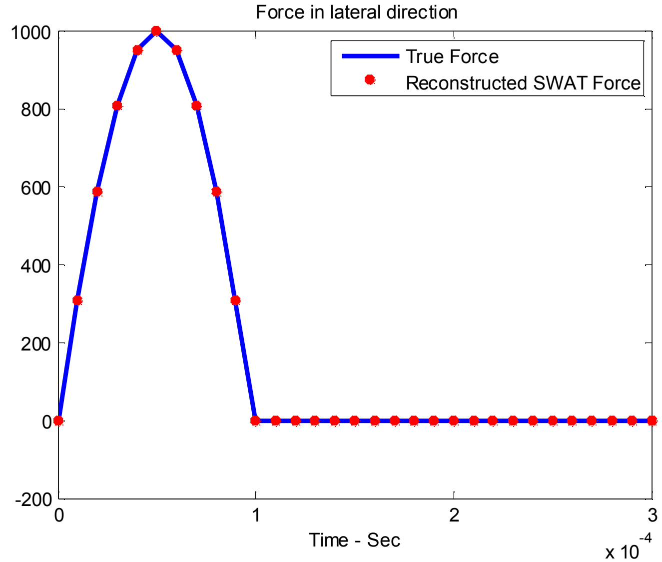
$$\boldsymbol{\Phi}_R^T\mathbf{M}\ddot{\mathbf{x}}_{measured} = \begin{Bmatrix} \sum_x \mathbf{f} \\ \sum_y \mathbf{f} \\ \sum_z \mathbf{M} \end{Bmatrix}$$

$$\boldsymbol{\Phi}^+_{RigidParti\ tion}\ddot{\mathbf{x}}_{measured} = \begin{Bmatrix} \sum_x \mathbf{f} \\ \sum_y \mathbf{f} \\ \sum_z \mathbf{M} \end{Bmatrix}$$

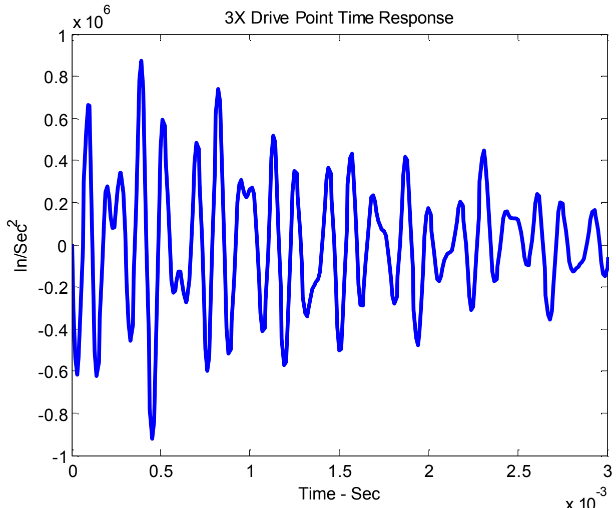


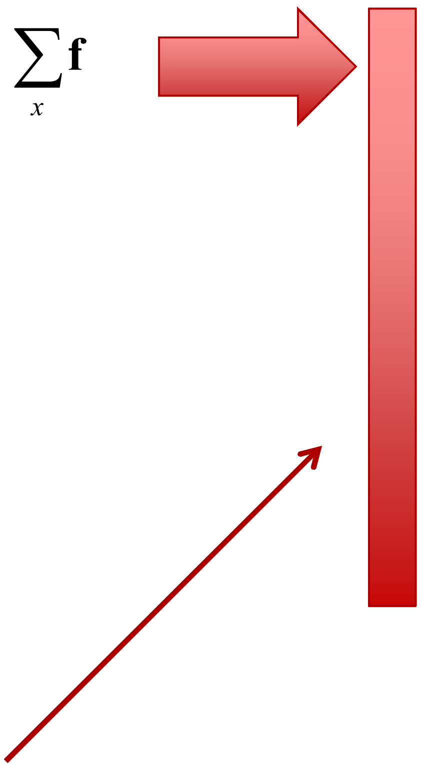
SWAT results force reconstruction

- Plots of reconstructed lateral force vs node 3 applied force (8 mode beam model)



- Drive point accelerometer response for 3 msec





11 lateral accelerometers

2 axial accelerometers

Modal Filter in SMAC modal algorithm

- SMAC (Synthesize Modes and Correlate) modal analysis package utilizes a modal filter instead of polynomial root finder
- Roots are "found" by optimizing frequency and damping until a "match" is found

$$\overline{\Psi}^T \ddot{\overline{\mathbf{x}}} = \ddot{q}$$

$$\overline{\Psi}^T \overline{\mathbf{H}}(f) = H_q(f)$$

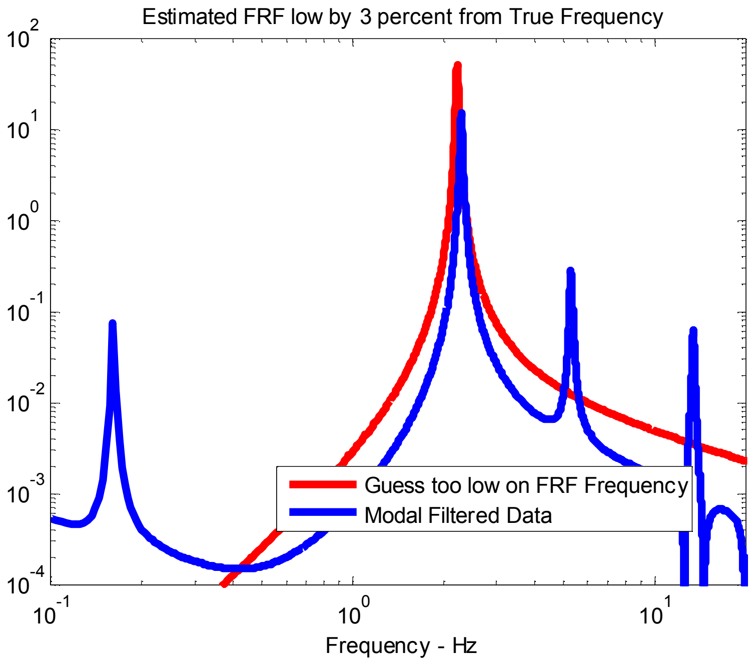
$$\overline{\mathbf{H}}^{.T}(f) * \overline{\Psi} = H_q^{.T}(f)$$

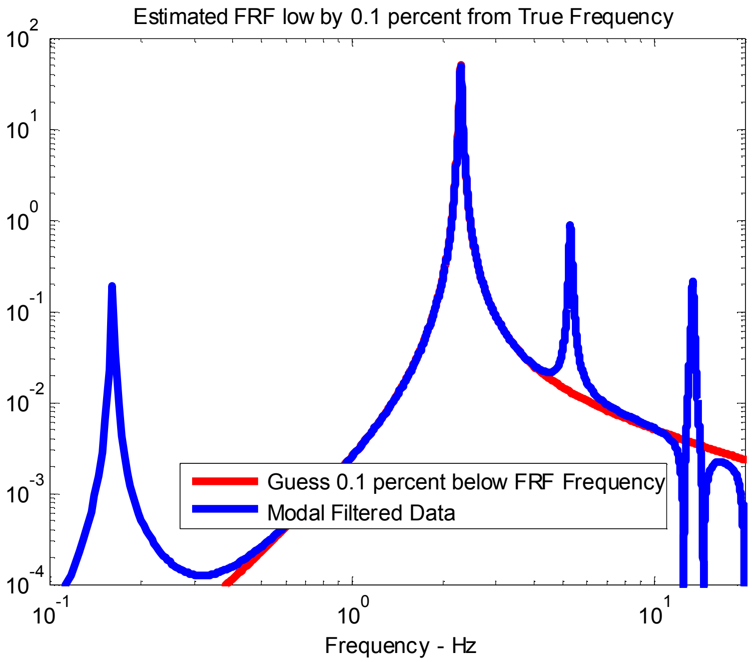
$$\overline{\Psi} = [\overline{\mathbf{H}}^{.T}]^{+} \overline{H}_q^{.T}(f)$$

- Make guesses at frequency and damping of analytical H_q and find $\overline{\Psi}$
- Reconstruct H_q from $\overline{\Psi}$ and measured H and see when you get a match

Modal Filter in SMAC modal algorithm Example of bad match and good match

- First plot shows match when guessed frequency is off by 3%
- 2nd plot shows match when guessed frequency is off by 0.1%
- Algorithm optimizes on both frequency and damping to get best match within a tolerance





Constraining Experimental Free Modes to Approximate Fixed Boundary Condition

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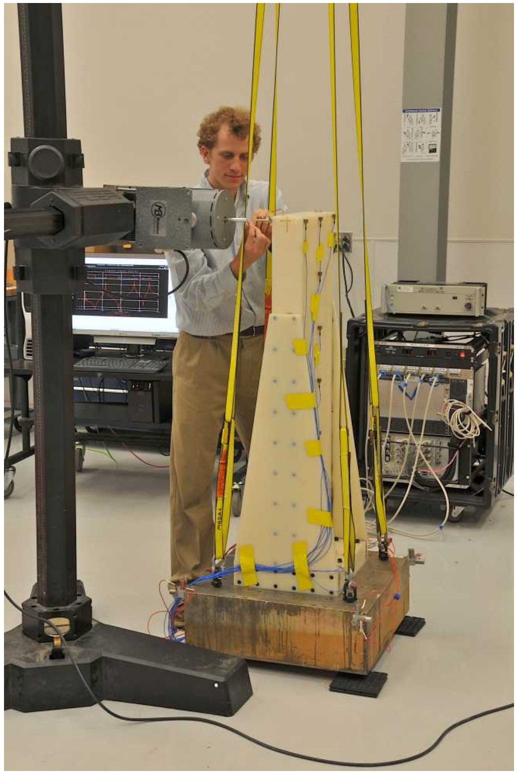
- Given a free modal model (dof q, mode shapes $\mathbf{\Phi}$) of a structure on a fixture, approximate fixed base modes using the free modes of the fixture (dof s, mode shapes $\mathbf{\Gamma}$)

Eqn of Motion
$$\left[-\omega^2\mathbf{I} + j\omega 2\boldsymbol{\zeta}_{\mathbf{m}}\boldsymbol{\omega}_{\mathbf{m}} + \boldsymbol{\omega}_{\mathbf{m}}^{\mathbf{2}}\right]\bar{\mathbf{q}} = 0$$

Constraint
$$\bar{\mathbf{x}}_{base} = 0$$
$$\mathbf{\Phi}_{base}\bar{\mathbf{q}} = \mathbf{\Gamma}_{base}\bar{\mathbf{s}} = 0$$
$$\mathbf{\Gamma}_{base}^{+}\mathbf{\Phi}_{base}\bar{\mathbf{q}} = 0$$
$$\bar{\mathbf{q}} = \mathbf{L}\bar{\boldsymbol{\eta}}$$
$$\mathbf{\Gamma}_{base}^{+}\mathbf{\Phi}_{base}\mathbf{L}\bar{\boldsymbol{\eta}} = 0$$

Transformation
$$\mathbf{L} = null(\mathbf{\Gamma}_{base}^{+}\mathbf{\Phi}_{base})$$

Fixed Base EOM
$$\mathbf{L}^T\left[-\omega^2\mathbf{I} + j\omega 2\boldsymbol{\zeta}_{\mathbf{m}}\boldsymbol{\omega}_{\mathbf{m}} + \boldsymbol{\omega}_{\mathbf{m}}^{\mathbf{2}}\right]\mathbf{L}\bar{\boldsymbol{\eta}} = 0$$



Constraining Experimental Free Modes to Approximate Fixed Boundary Condition

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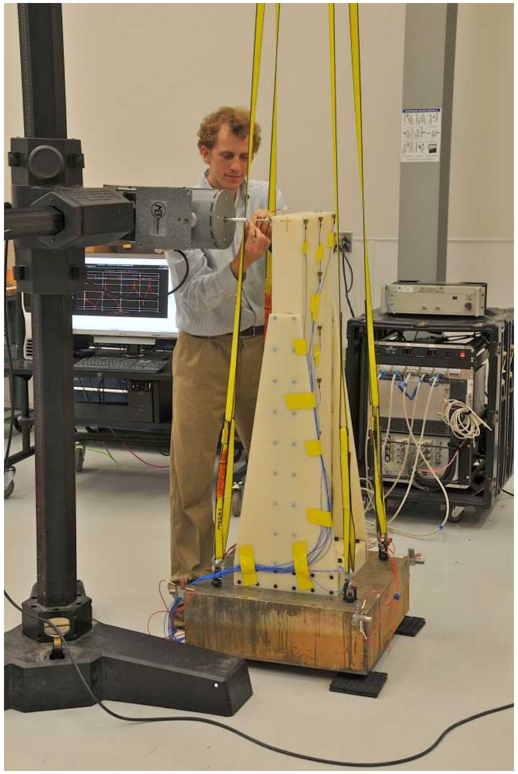
Change in Frequency from Free to Fixed Base

Description	Unconstrained Seismic Mass	Constrained Seismic Mass	Change
	Frequency - Hz	Frequency - Hz	
1st bend soft direction	57.85	38.99	32.60%
1st bend stiff direction	135.18	89.05	34.12%
1st torsion	142	134.31	5.42%
2nd bend soft direction	175.75	163.61	6.91%
2nd bend stiff direction	353.18	338.16	4.25%
2nd torsion	364.91	364.24	0.18%
1st Axial	409.31	392.4	4.13%
3rd bend soft direction	403.9	396.4	1.86%
3rd torsion	598.1	597.6	0.08%
3rd bend stiff	647.3	643.8	0.54%
4th torsion	700.5	700.4	0.01%
4th bend soft	710.4	706.4	0.56%



Conclusions – Modal Filter Enhances Analytical Dynamics Tool Box

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- Modal filter enhances greatly the amount of useful information that can be extracted from experimental response measurements
  - Derive correlation insight from experimental FRFs without modal extraction
  - Derive input forces
  - Extract modal parameters without matrix polynomial root finder
  - Allow manipulation of boundary conditions analytically just like those high falootin' FE modelers (overcome previous experimental error restrictions) – this is a breakthrough in experimental based dynamic modeling

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