Can-Crush Model and Simulations for Verifying Uncertainty Quantification Method for Sparse Stress-Strain Curve Data

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Abstract

This work examines the variability of predicted responses when multiple stress-strain curves (reflecting variability from replicate material tests) are propagated through a transient dynamics finite element model of a ductile steel can being slowly crushed. An elastic-plastic constitutive model is employed in the large-deformation simulations. The present work assigns the same material to all the can parts: lids, walls, and weld. Time histories of 18 response quantities of interest (including displacements, stresses, strains, and calculated measures of material damage) at several locations on the can and various points in time are monitored in the simulations. Each response quantity's behavior varies according to the particular stress-strain curves used for the materials in the model. We estimate response variability due to variability of the input material curves. When only a few stress-strain curves are available from material testing, response variance will usually be significantly underestimated. This is undesirable for many engineering purposes. This paper describes the can-crush model and simulations used to evaluate a simple classical statistical method, Tolerance Intervals (TIs), for effectively compensating for sparse stress-strain curve data in the can-crush problem. Using the simulation results presented here, the accuracy and reliability of the TI method are being evaluated on the highly nonlinear input-to-output response mappings and non-standard response distributions in the can-crush UQ problem.

Introduction

This work examines the variability of predicted responses when multiple stress-strain curves (reflecting variability from replicate material tests) are propagated through an explicit transient dynamics finite element model of a cylindrical steel can being crushed. The can has welded lids and rests in a V-shaped support structure. The ductile steel can is crushed at a relatively slow rate with a displacement-controlled ram plate. A multi-linear elastic plastic (MLEP) constitutive plasticity model is employed in the large-deformation simulations. Over 70 response quantities of interest (e.g. displacement, Von Mises stress, and equivalent-plastic-strain and tearing-parameter material damage indicators) at several locations on the can lids, welds, and walls and various points in time are monitored.

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Each response quantity's time behavior varies according to the particular stress-strain curves used for the materials in the model. The stress-strain curves originate from material tensile tests at two temperatures, 200C and 400C. Typically, replicate tests are performed and each yields a slightly but non-trivially different stress-strain curve. This variability reflects test-to-test random variations in the experiments, involving suspected stochastic material variability and also suspected variability of test conditions and measurement errors. Because the effects of random variability of test conditions and measurement errors are not presently deconvolvable from the effects of stochastic material variability in the tests, all the stress-strain curve variability is presently carried forward and interpreted as material variability.

The test curves of measured engineering stress and strain are converted via an iterative fitting procedure into strain-rate-independent Cauchy stress, log strain curves for use in the multi-linear elastic plastic (MLEP) constitutive model [Wellman, 2012]. Necking of the cylinder test specimens is accounted for in the fitting procedure. Of interest is the sensitivity of the response quantities to various steps in the fitting procedure (e.g. whether smoothing of the raw measured stress-strain curves is performed, and if so, what type of smoothing is used). Of further interest is the variability of predicted response from the multiple replicate stress-strain curves.

For the latter purpose it is important to note that each stress-strain curve comprises a ***discrete*** random function that has no readily identifiable parametric relationship to other stress-strain curves from the tests. Furthermore, a stress-strain curve bounded by two other stress-strain curves at that same temperature will not necessarily result in predicted model output results that are bounded by the model responses using the two bounding curves. This is because aggregate structural response depends on the history or path of a given stress-strain curve. Another difficulty in the current uncertainty quantification (UQ) problem is that the number of material variability tests (number of replicate stress-strain curves) is relatively small. When only a few samples of a random variable or function are available, these will usually significantly misrepresent the randomness properties of the source of variability that was sampled. The variability properties of the source (full population of random values or functions) generally cannot be accurately constructed from just a few samples of the population. Thus, substantial epistemic "sampling uncertainty" exists in addition to the aleatory uncertainty due to stochastic variability in the source population.

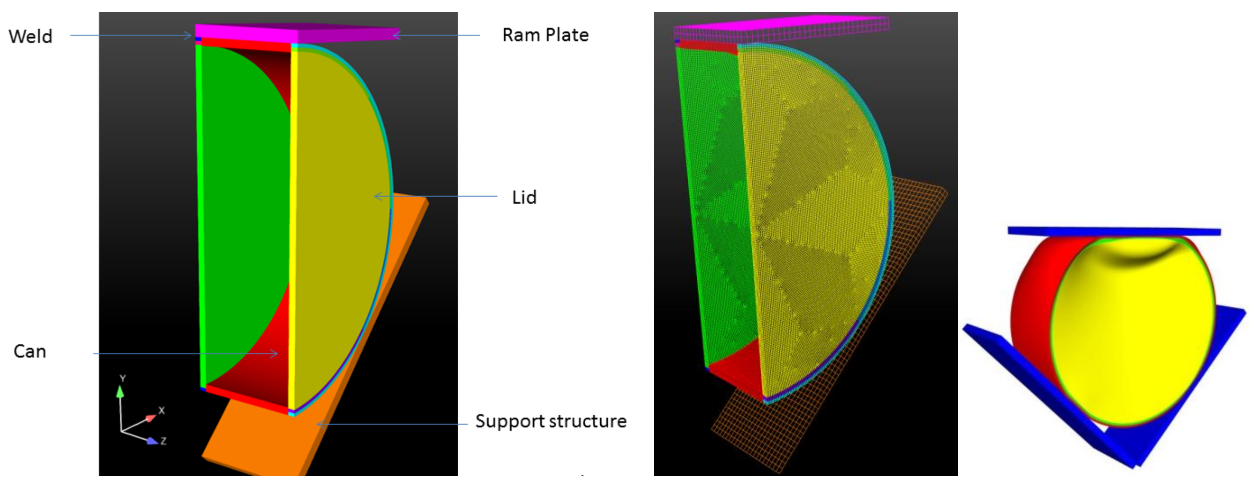
The likely error that accompanies sparse sampling has a bias toward underestimating the true full-population variance. This unconservative bias is undesirable for many engineering purposes. If a structure or pressure-vessel model were perfect in every other way, use of the model with sparse samples of the random-data inputs would likely underestimate the (strength or displacement) response variance of the real system. In design and risk analysis one would normally want to avoid such underestimation.

Appropriate UQ techniques for working from just a few samples from the full population of discrete random processes or functions is an active area of research. This paper describes the model and simulations used to evaluate a classical statistical method, Tolerance Intervals ([Hahn, G.J., and Meeker, 1991]; [Montgomery and Runger, 1994]), for dealing with sparse data. The approach treats the stress-strain curves as discrete random functions with no readily identifiable parametric relationship between them. Yet the approach recognizes that the stress-strain curves issue from the same characteristic population of discrete random functions. Furthermore, the approach mitigates chances of underestimating the full-population variability from relatively few data samples.

The present paper describes the model, output quantities monitored, and supporting simulations for evaluation of the TI UQ method. The TI method and a strategy for evaluating its performance are briefly summarized in this paper, with presentation of some interim results, but details of the method and its accuracy and reliability performance are currently under preparation in [Romero et al., 2017] and are not presented in this paper. The present work assigns the same material to all the can parts: lids, walls, and weld. Future work will investigate multi-material solid mechanics problems where only a few stress-strain curves exist for each material.

Finite element model description

An explicit transient dynamics finite element model was created to simulate slow crush of a ductile metal can type structure as depicted in Figure 1. Shown in ½ plane symmetry geometry, the model contains a ram plate that crushes the can into a wedged support structure. Lids are welded to the can top and bottom as shown.



(a) (b) (c)

Figure 1 – Half-symmetry finite element model of can crush configuration showing: (a) geometry and components; (b) finite element mesh; (c) example simulation results of can deformation.

All components of the can are contiguously meshed. The ram plate and support structure are modeled with a contact algorithm with the Sandia solid mechanics and dynamics analysis code

[Adagio]. Table 1 lists the components of the can crush structure along with mesh specifications of the model. Figure 2 depicts an experimental setup used for model validation studies.

Table 1 – Can crush configuration and model specifications

Component	Material	Dimensions	No. Elements	No. thru thickness
Ram Plate	Rigid	1" wide x 0.1" tk	1200	2
Wedge	Rigid	1" wide x 0.1" tk	2400	2
Lid	304L SSTL	3" dia x .062" tk	117264	4
Can	304L SSTL	3" dia x 1" wide x .067" tk	74400	4
Weld	304L SSTL	3" dia. x .03" wide x .062" tk	4800	2x4

(a) (b)

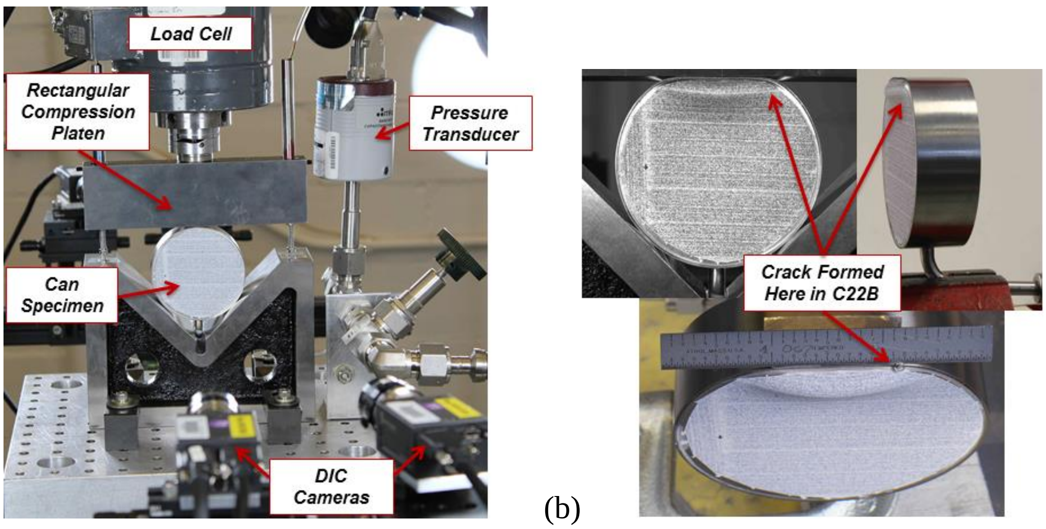


Figure 2 –Can crush test setup (a), with post-test can crush damage (b). Figure from Kramer et al. (2015).

Material characterization and modeling

Material characterization tensile tests to failure at elevated temperatures of 200C and 400C were performed (Antoun et al., 2016) with 304L stainless steel cylindrical rod specimens of 3 in. length and 0.25 in. diameter. The same billet of material is used to machine out the can crush components as well as for rod samples used for tensile testing and constitutive model material fits. In this series of testing, the rod material samples are taken axially from a billet that the can is made from. The samples are tested using a strain rate of 0.001/s at 200C and 400C with several repeats as depicted in Figure 3. Engineering stress vs engineering strain tensile test results with repeats are presented for 304L stainless steel at room temperatures through 800C. Some ratcheting is noted at 600C for these tests. Therefore these stress strain curves were smoothed by running-window averaging before fitting. Yield and ultimate stress decrease with temperature for all of these tests, while strain to failure (ductility) increases with temperature in the range from room temperature to roughly 600C, then increases with temperature in the range from 600 to 800C.

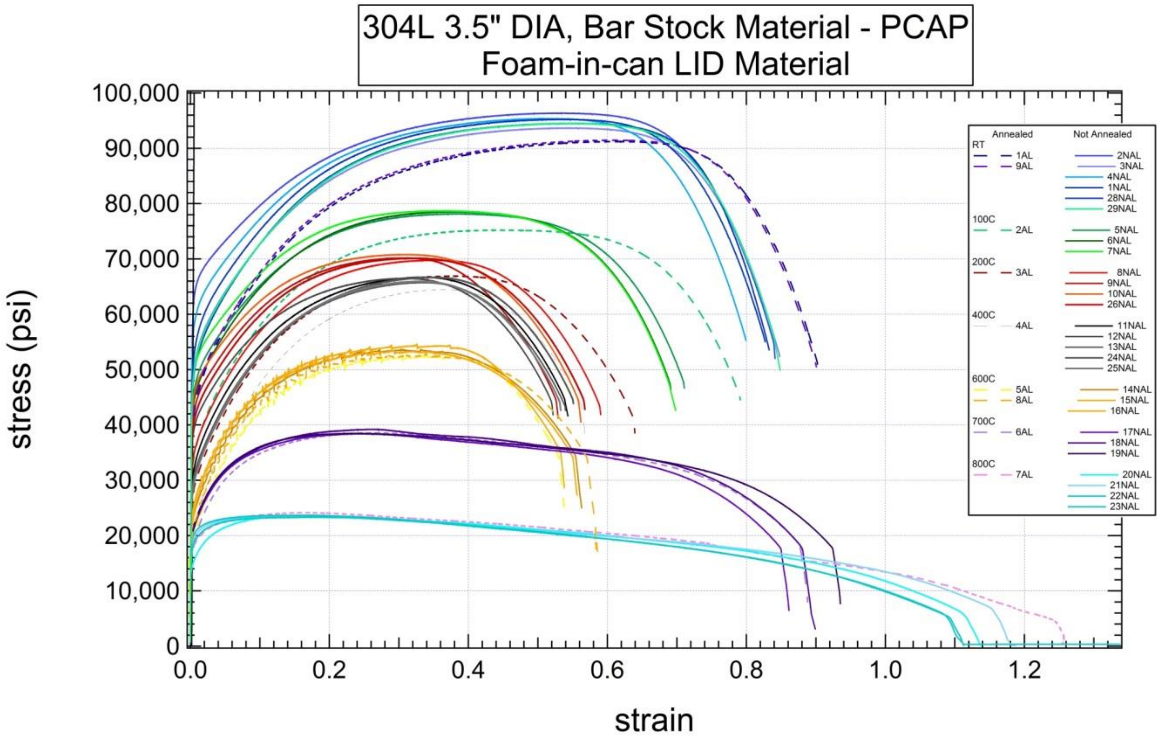


Figure 3 – Engineering stress vs engineering strain tensile test results with repeats for 304L stainless steel at room temperatures through 800C.

Table 2 lists some temperature dependent mechanical properties used to characterize this material at 200C and 400C, for these low strain-rate tensile tests. Values for Young's Modulus and Poisson's ratio were taken from literature sources [ASM, 1995] and [BSSA, 2014] respectively. Yield stresses were taken from test data at 0.2% strain offset. A specialized finite element fitting procedure [Wellman, 2012] was used to convert the engineering stress-strain test results to Cauchy stress, log strain (or "true stress", "true strain") by modeling the experiments, including tensile material necking through failure as described next.

Table 2 - Temperature dependent mechanical properties used to characterize the material at 200C and 400C.

Low rate	Modulus (MSI)	Yield stress (ksi)	Poisson ratio
200C	30.7	24.6	0.28
400C	25.4	21.1	0.29

To characterize the material for use in the MLEP constitutive model, a model of the test specimens is created as shown in Figure 4. The modeled rod is pulled into tension by fixing one end and applying a displacement controlled deflection to the other end. This deflection produces stress and strain in the modeled specimen and it begins to deform uniaxially through yield. At yield, a slope-based estimate of a projected true stress and true strain is calculated. Adagio then runs a simulation using this definition (dashed partial red curve) to check how well the assumption fits (green curve) to the original test data (black curve). If the initial guess produced a bad fit outside a specified tolerance, a new estimate is made and the process is repeated until a full true stress, true strain curve is produced. As the true stress, true strain response gets computed, a tearing parameter measure of material damage is also computed and stored. At the point of failure when the test specimen separates in the necked zone, a critical tearing parameter is computed and defined.

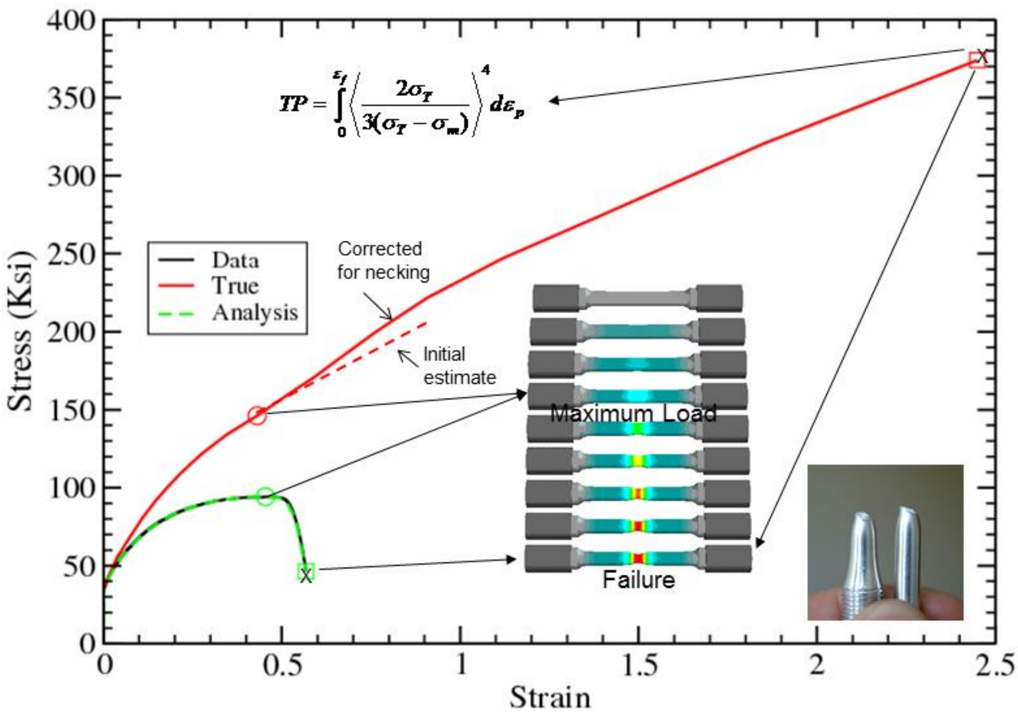


Figure 4 – Schematic showing the process of computing and checking true stress, true strain from test data and through FEA simulation. Lower curves are measured engineering stress, engineering strain. The upper curves are true stress; true strain fits to measured data.

In Figures 5 and 6, 304L stress strain curves are shown at temperatures of 200C and 400C, respectively. The upper curves are true stress, true strain fits to the measured lower curve data. In this case, low strain rate (0.001/s) fits are shown with repeats.

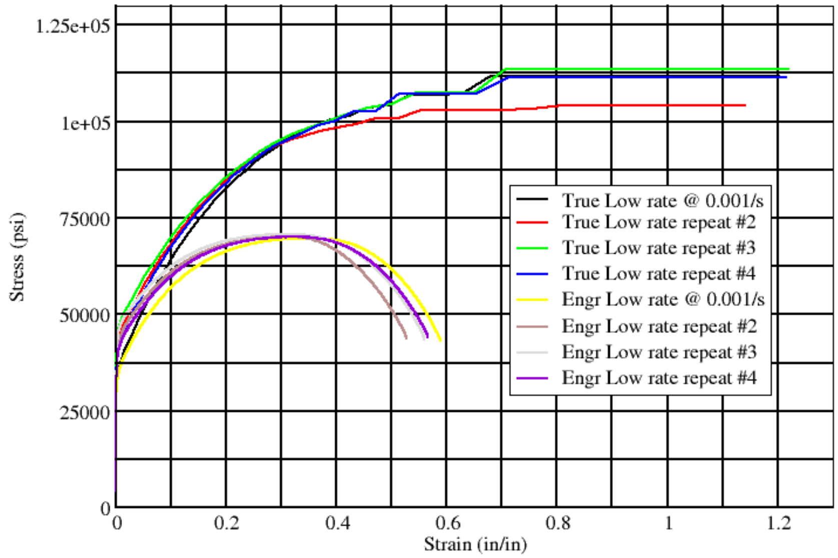


Figure 5 – 304L room (200 C) temperature true stress, true strain material fits for low (0.001/s) strain rate tests. Lower curves are measured engineering stress, engineering strain. The upper curves are true stress; true strain fits to the measured data.

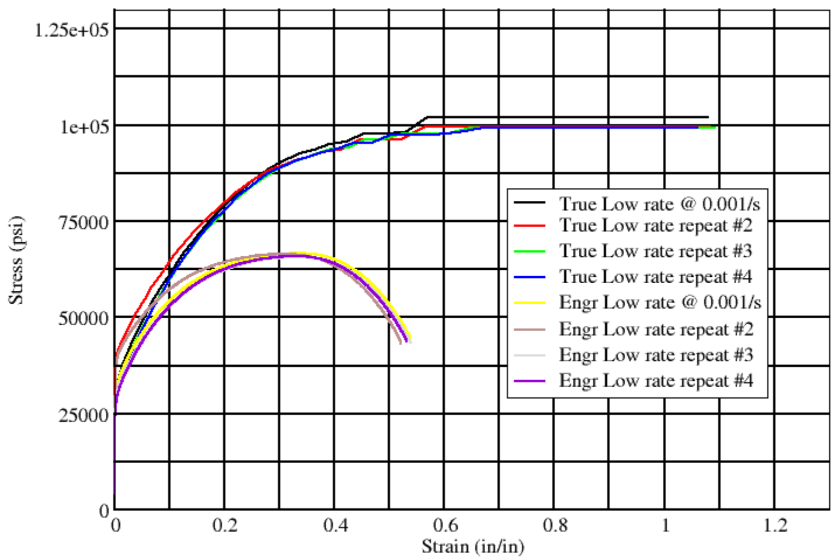


Figure 6 – 304L (400 C) temperature true stress, true strain material fits for low (0.001/s) strain rate tests. Lower curves are measured engineering stress, engineering strain. The upper curves are true stress; true strain fits to the measured data.

Finite element simulations and response to ductile crush

The present work assigns the same material to all the can parts: lids, walls, and weld. An explicit transient-dynamics finite element modeling approach was used to perform ductile can crush simulations. A ram plate is used to crush the can into a wedged support structure, Figure 3. As the ram loads the can in a displacement control mode, the ram and lid/can displacements are computed and stored, Figure 7. The averaged ram reaction load is also computed as shown in Figure 7.

Figures 8 through 12 plot responses for several output variables at the positions shown. These outputs include displacements, Von Mises stress, and several computed measures of material damage: equivalent plastic strain (EQPS), tensile equivalent plastic strain (TEQPS) and tearing parameter (TP). For many of these outputs, a location-varying global maximum, and several fixed-location element or nodal maximums of the weld, lid and can are output, as shown in the figures and explained in their captions. Substantial time and effort was invested to select these output quantities and optimize the spatial locations for their output to ensure significant and diverse response signals were obtained.

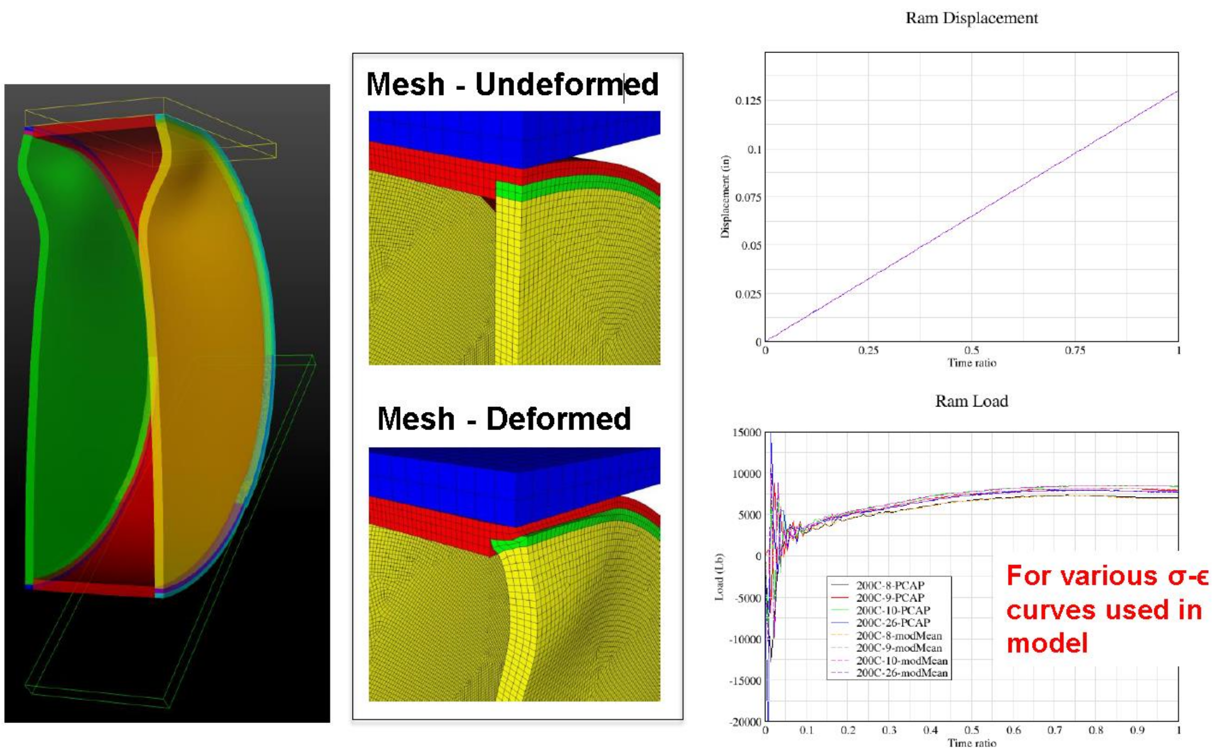


Figure 7 –Vertical ram displacement and the averaged reaction load to the ram from can vs time fraction for various stress strain curves tried in the model.

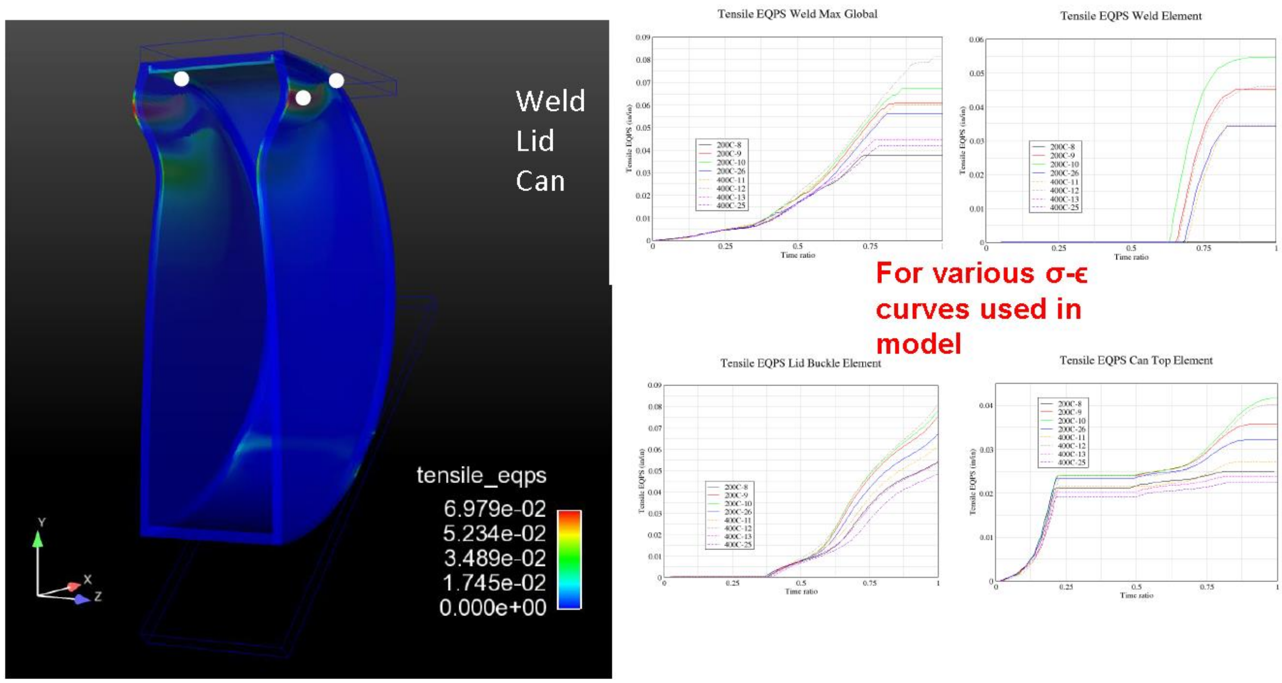


Figure 8 – Tensile EQPS values tracked for weld, lid and can at the locations shown, as well as the global maximum in the weld material, for various stress strain curves tried in the model.

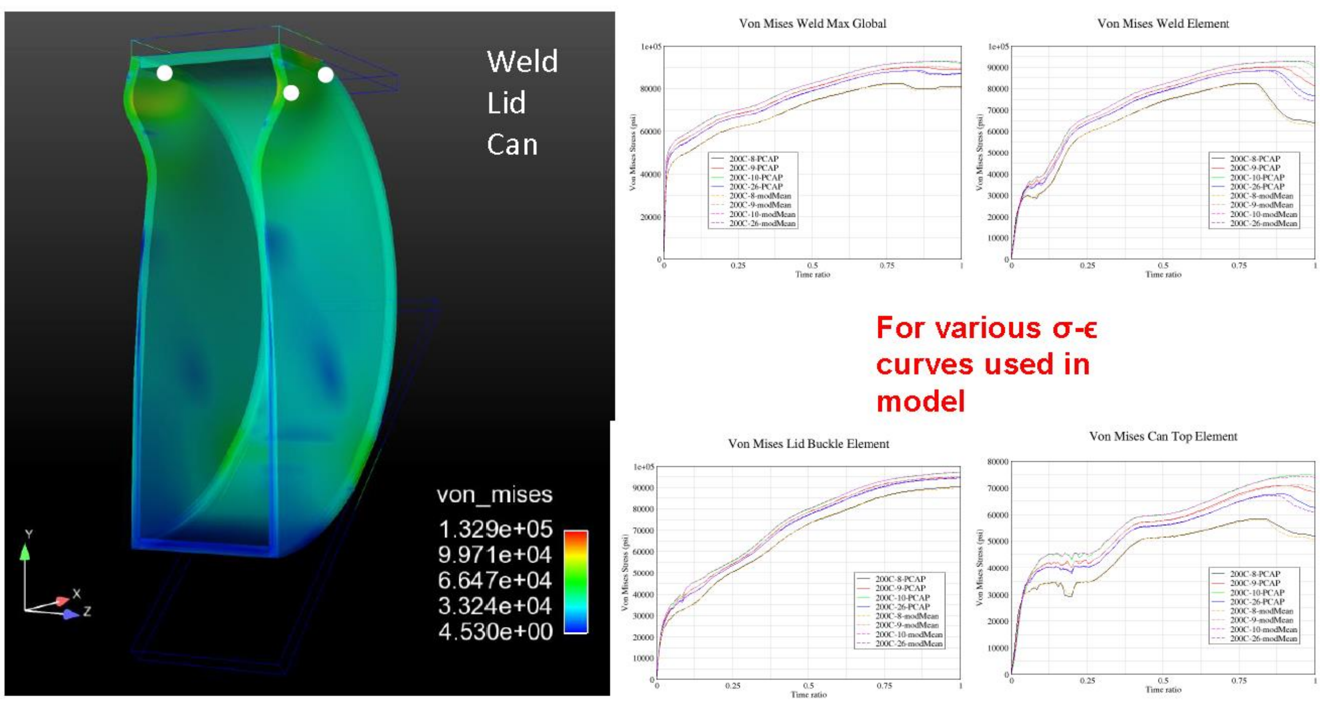


Figure 9 – Von Mises stress values tracked for the weld, lid and can at the locations shown, as well as the maximum in the weld material, for various stress strain curves tried in the model.

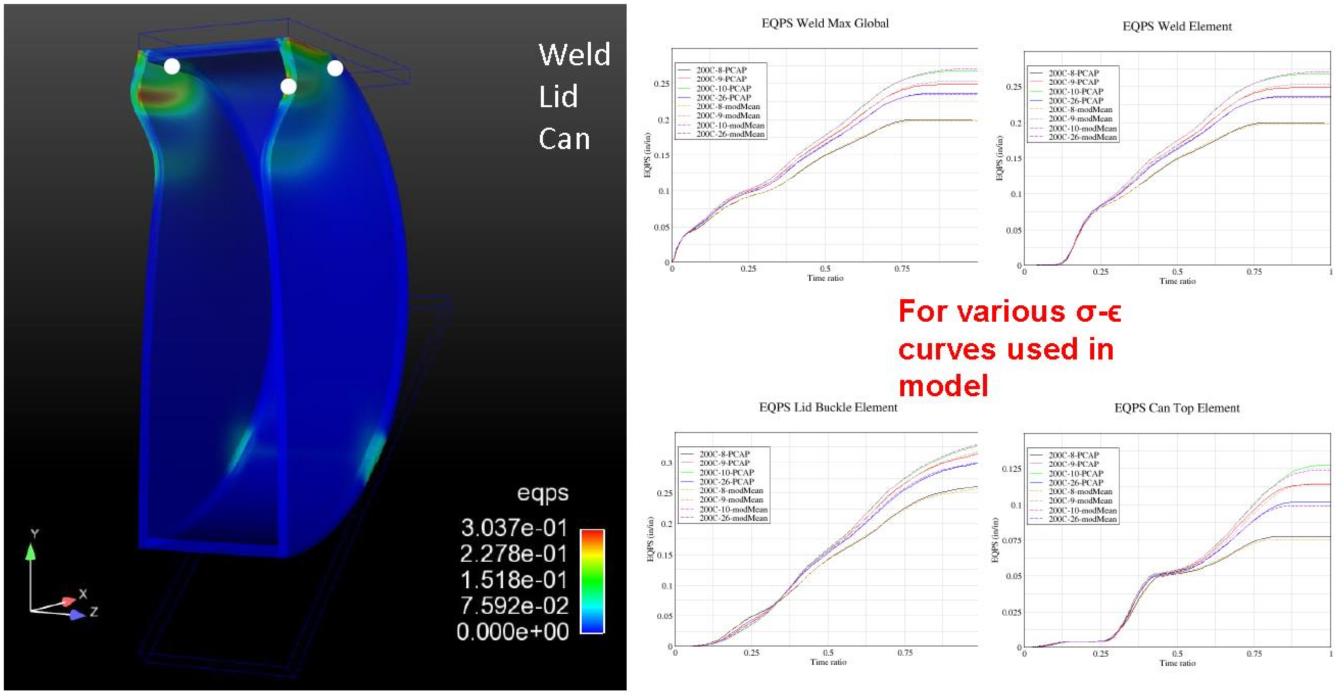


Figure 10 –EQPS values tracked for the weld, lid and can at the locations shown, as well as the global maximum in the weld material, for various stress strain curves tried in the model.

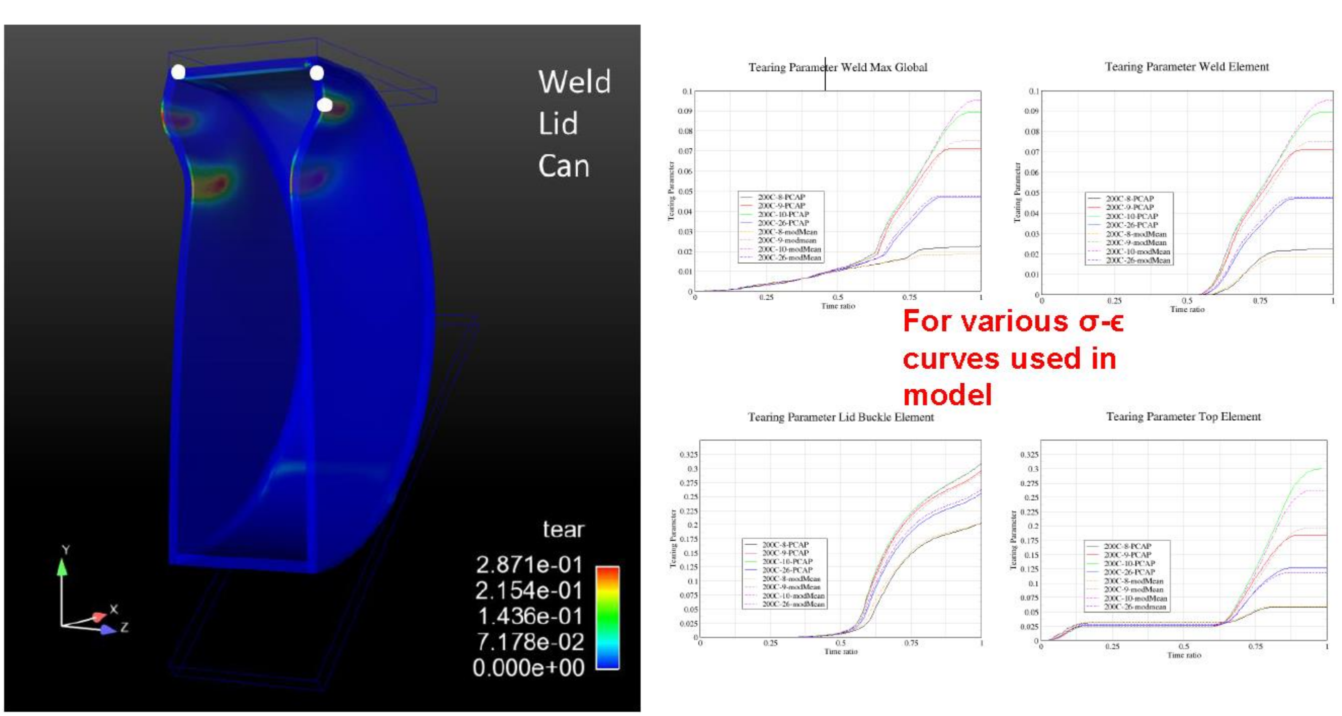


Figure 11 –Tearing parameter values tracked for the weld, lid and can at the locations shown, as well as the global maximum in the weld material, for various stress strain curves tried in the model.

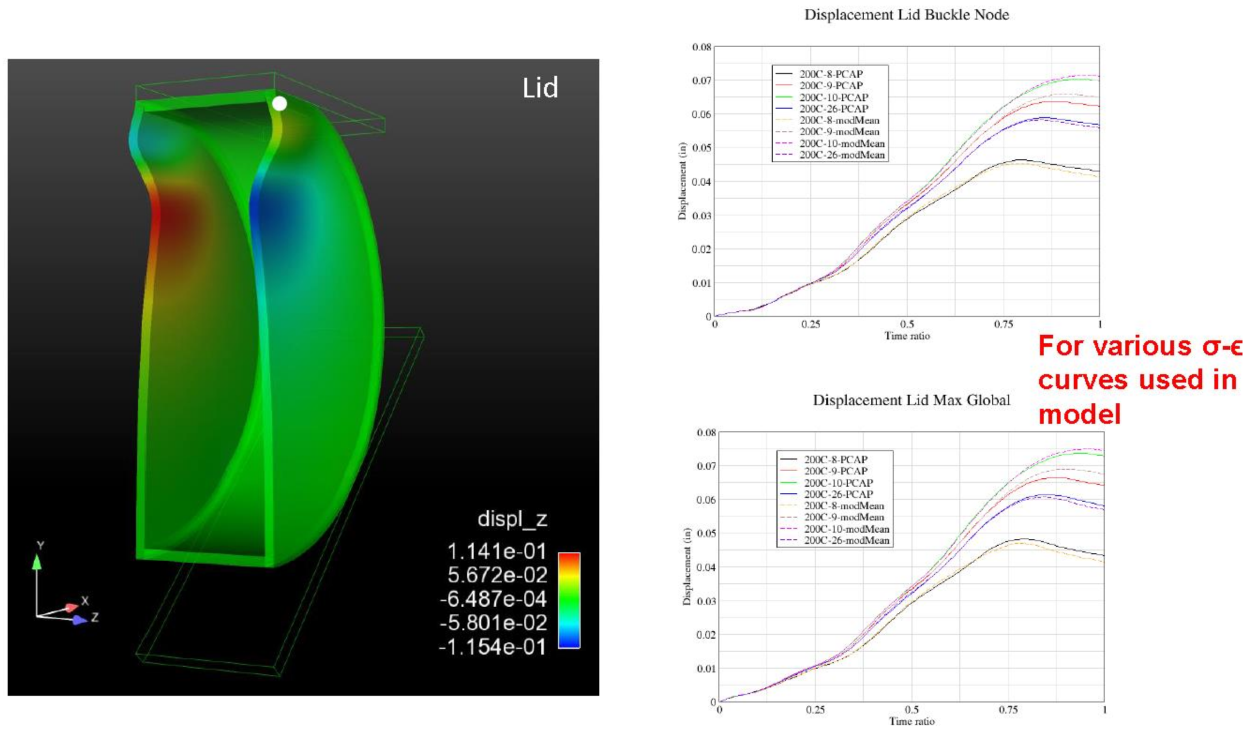


Figure 12 –Buckle displacement values tracked for the lid at the location shown, as well as and global maximum in the lid, for various stress strain curves tried in the model.

UQ Method Performance Testing Methodology and Interim Results

The Tolerance Interval (TI) method for modeling the variability of predicted response from model runs with multiple replicate stress-strain curves is briefly summarized here, along with a strategy for assessing TI method performance,. Some intermediate results are presented. Details of application of the TI UQ method to the can-crush problem and in-depth evaluation of the method's accuracy and reliability are currently under preparation in [Romero et al., 2017] and are not presented in this paper.

Tolerance Intervals (TIs) are a simple approach to compensate for the epistemic sampling uncertainty introduced from non-infinite samples. Investigations by Romero et al. (2013a,b) confirmed that TIs provide reliably conservative estimates of the combined epistemic and aleatory uncertainty associated with very sparse data samples for normal, uniform, and right-triangular random variables, and for distributions resulting from convolving various combinations of these three random-variable types. The TI approach is also much easier to use than the four other sparse-data methods investigated. However, it was also found that TIs can egregiously exaggerate the true variability when very few samples are involved. The approach of Pradlwarter and Schuëller (2008) worked well for sparse samples in many cases, with usually significantly less exaggeration of the true variability when few samples are available, but is somewhat more involved to implement and its performance remains to be broadly tested and characterized because the method is very new. The rather common practice of simply fitting the random data with a normal distribution was found to have substantial risk of under-estimating the true variability of the population being sampled—especially if the sampled distribution is Normal.

TI robustness on several other distribution shapes is established in [Kanwar and Haftka, 2015]. They also found that the TI approach is more reliable than bootstrapping methods for numbers of samples below about 30.

Although the TI UQ method has been recently used in several engineering applications at Sandia National Laboratories (Romero et al., 2014, 2015, 2016), it has not yet been verified on highly non-linear application problems and non-standard distribution shapes because of the computational expense of the application models and because of other time and resource constraints in those projects. Consequently, the can-crush model and UQ problem are being used as a test bed to evaluate the accuracy and reliability of the TI method on the highly nonlinear input-to-output response mappings and non-standard response distributions that arise in the can-crush solid mechanics UQ problem.

Figure 13 outlines a strategy of computational experiments and assessment methodology for characterizing the accuracy and reliability of the TI sparse-data UQ method. The process steps in Figure 13 are explained in detail in [Romero et al., 2017].

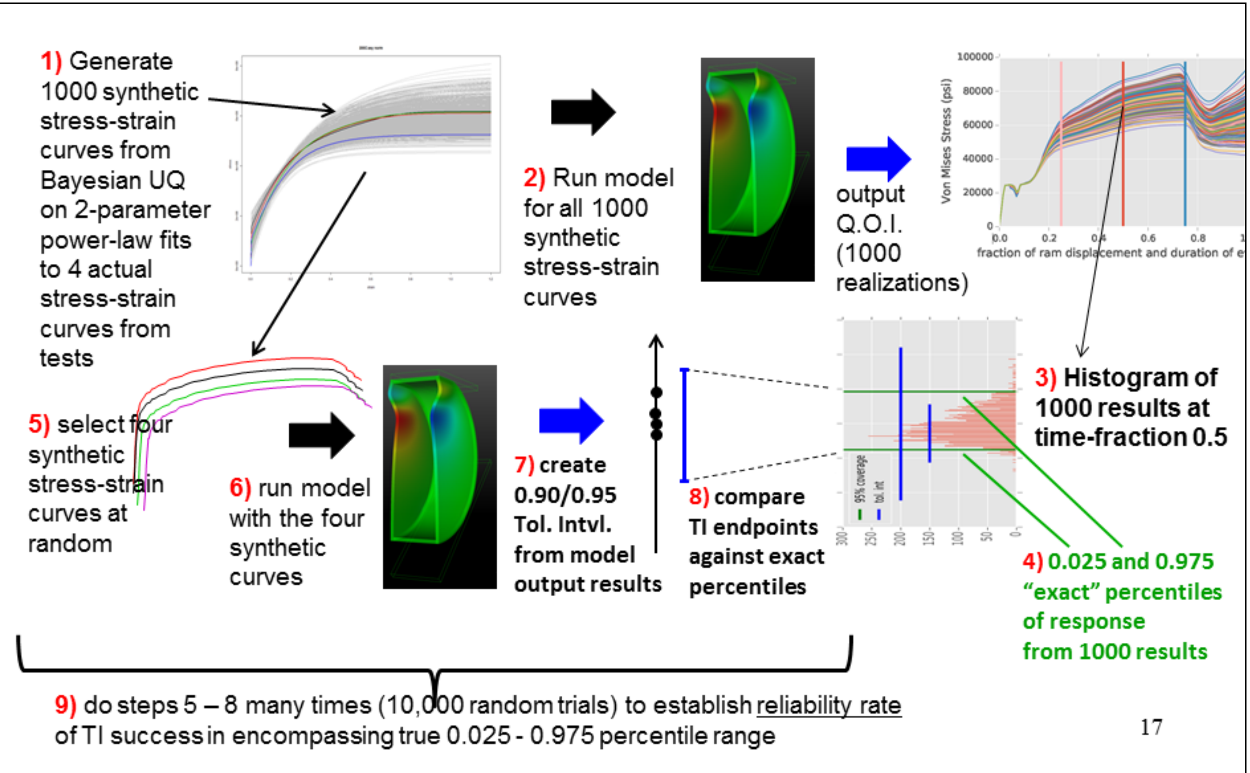


Figure 13 –Computational experiments and assessment strategy for TI method performance.

Figure 14 shows output results of 1000 can-crush simulations with the 1000 synthetic stress-stain curves shown in step 1 of Figure 3. 1000 random realizations of transient response are shown for each of 18 output quantities of interest (QOIs) introduced in Figures 7-12. All responses are catalogued in Figure 14 except for tensile EQPS for the weld element from Figure 8 (inadvertently overlooked in the analysis) and ram displacement with time from Figure 7 (an input to the analyses so the same in each simulation). The transient responses are seen to be highly diverse and nonlinear in time, with substantial spreads or variance of the response populations, especially at later time fractions. Some response quantities retain their bottom-to-top ordering relatively well over time, while others exhibit substantial crossing with time. As shown in Step 3 in Figure 13, the distributions (histograms) of response can differ significantly from the standard distribution shapes Normal, uniform, and right-triangular that TI performance has already been reasonably affirmed for (Romero et al., 2013a,b).

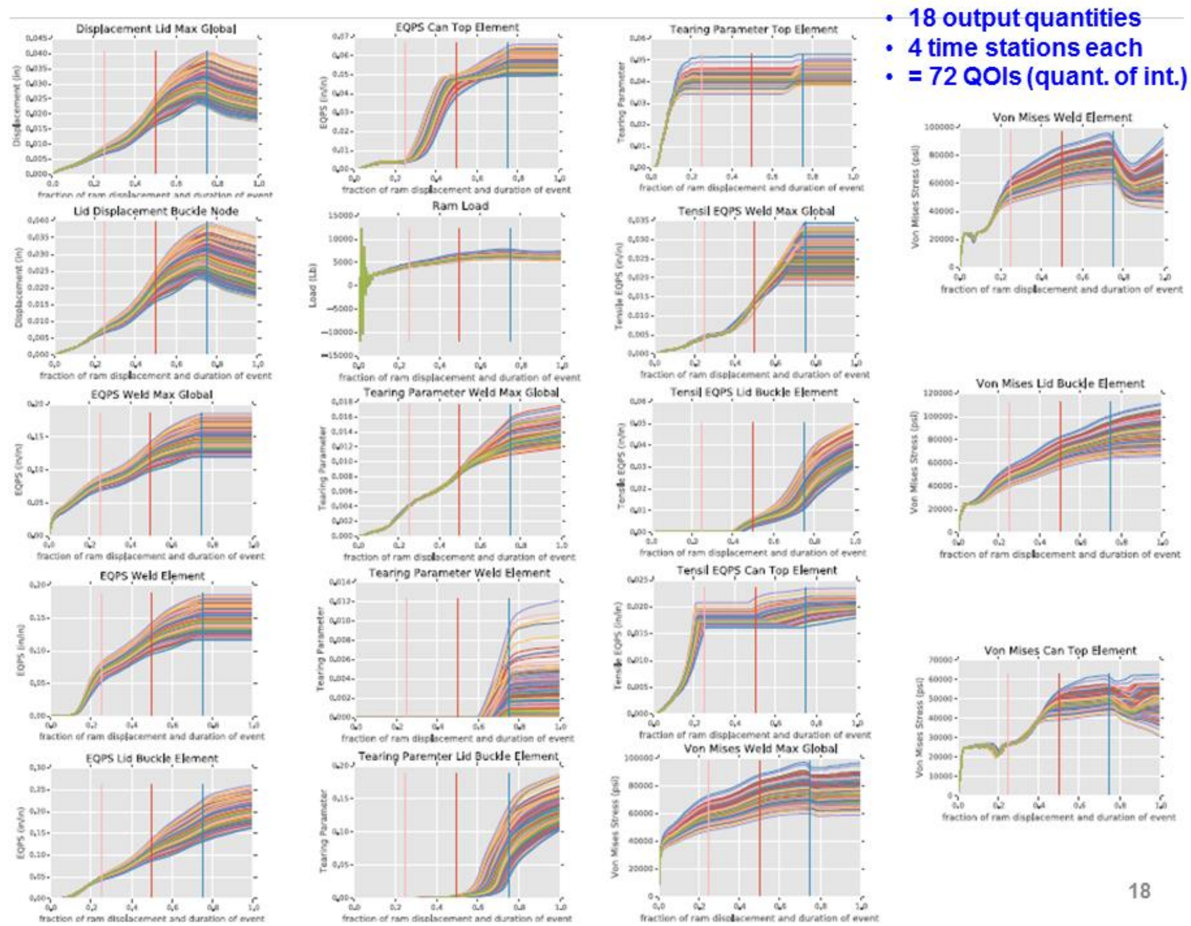


Figure 14 –1000 random realizations of transient response for each of 18 output quantities.

Conclusions

The can-crush solid mechanics UQ problem appears to be an appropriate and challenging test bed to evaluate the accuracy and reliability of sparse-data UQ methods on the highly diverse and nonlinear input-to-output response mappings and non-standard response distributions that arise in the can-crush problem. Details of application of the TI UQ method to this problem, and evaluation of the method's performance, are currently under preparation [Romero et al., 2017]. Future work will investigate multi-material solid mechanics problems where the can lids, walls, and weld are each a different steel material and only a few stress-strain curves are available for each material.

References

Adagio, Sierra/Solid Mechanics and Structural Dynamics, 4.30 Users guide, Computational Solid Mechanics and Structural Dynamics Department, Engineering Sciences Center, SAND Report, printed September 18, 2013.

Antoun, B.R., Sartor, G.B., Briner, C.F., Dempsey, J.F., Duong, H., "Mechanical Characterization of 304L VAR Stainless Steel as a Function of Temperature and Orientation", Sandia National Laboratories report in preparation.

ASM Specialty handbook, Stainless Steels, printed January 1, 1995.

Bhachu, K.S., R.T. Haftka, N.H. Kim, "Using Bootstrap to Assess Sampling Uncertainty in Fatigue Crack Growth Life," 17th AIAA Non-Deterministic Approaches Conference, AIAA SciTech 2015, Jan. 5-9, Kissimee, FL.

British Stainless Steel Association (BSSA), "Elevated temperature physical properties of stainless steels", Online publication, http://www.bssa.org.uk/topics.php?article=139, 2014, BSSA, Park Suite, Forsyth Enterprise Centre, Bramall Lane, Sheffield S2 4SU.

Hahn, G.J., and Meeker, W.Q., *Statistical Intervals—A Guide for Practitioners*, Wiley & Sons, 1991.

Kramer, S., J. Gearhart, J. Laing, T. Bosijevac, D. Pendley, A. Jackson, "Laser Weld Can Crush Study Update," Sandia National Laboratories presentation, March 2015.

Montgomery, D.C., and Runger, G.C., *Applied Statistics and Probability for Engineers*, Wiley & Sons, 1994.

Pradlwarter, H.J., and G.I. Schuëller, "The use of kernel densities and confidence intervals to cope with insufficient data in validation experiments," *Computer Methods in Applied Mechanics and Engineering*. Vol. 197, Issues 29-32, May 2008, pp. 2550-2560.

Romero, V., J. Mullins, L. Swiler, A. Urbina, "A Comparison of Methods for Representing and Aggregating Experimental Uncertainties involving Sparse Data—More Results," *Soc. Automot. Engrs. Int. J. of Materials and Manufacturing*, 6(3):2013, doi:10.4271/2013-01-0946.

Romero, V., L. Swiler, A. Urbina, J. Mullins, "A Comparison of Methods for Representing Sparsely Sampled Random Quantities," Sandia National Laboratories report SAND2013-4561, September 2013.

Romero, V., J.F. Dempsey, B. Antoun, "UQ and V&V Techniques applied to Experiments and Simulations of Heated Pipes Pressurized to Failure," Sandia National Laboratories report SAND2014-3985, May 2014.

Romero, V., A. Black, N. Breivik, G. Orient, J. Suo-Anttila, B. Antoun, A. Dodd, "Advanced UQ and V&V Procedures applied to Thermal-Mechanical Response and Weld Failure in Heated Pressurizing Canisters," Sandia National Laboratories document SAND2015-3005C presented at Soc. Auto. Engrs. 2015 World Congress, April 21-23, 2015, Detroit, MI.

Romero, V., J.F. Dempsey, B. Schroeder, J. Lewis, N. Breivik, G. Orient, B. Antoun, J. Winokur, M. Glickman, "Evaluation of a Simple UQ Approach to Compensate for Sparse Stress-Strain Curve Data in Solid Mechanics Applications," Sandia National Laboratories document SAND2016-5361 A submitted to 19th Non-Deterministic Approaches Conference, AIAA SciTech 2017, Jan. 9-13, Grapevine, TX.

Romero, V., B. Rutherford, J. Castro, "Model Calibration, Extrapolative Prediction, and Validation with Segregated Aleatory and Epistemic Uncertainties—Demonstration on Prototype Silicon Device," Sandia National Laboratories report in preparation (2016).

Wellman, G. W., "A Simple Approach to Modeling Ductile Failure," Sandia National Laboratories report SAND2012-1343 printed June 2012.