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Application of Line Search Methods to Return Mapping Algorithms for Isotropic and Anisotropic Plasticity Models

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Motivation

- Ductile failure in large deformation plasticity
  - Mechanics of failure
  - Numerical methods
  - Boundary value problem
  - Load paths

- Improve models for plastic deformation
  - Yield functions

Yield Function

- The general form we will use for the yield function is as follows

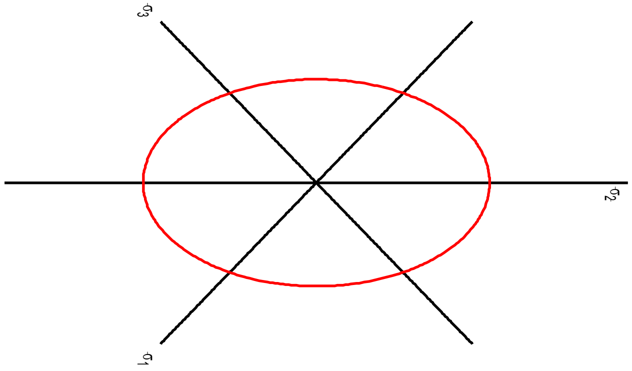
$$f = \phi\left(\boldsymbol{\sigma}\right) - \bar{\sigma}\left(\bar{\varepsilon}^{p}\right) = 0$$

  - This defines a surface in stress space – the yield surface
  - Assume associated flow

Isotropic Plasticity Models

von Mises – 1 parameter

$$\phi = \sqrt{\frac{3}{2}\mathbf{s} : \mathbf{s}} \qquad \mathbf{s} = \boldsymbol{\sigma} - \frac{1}{3}\left(\mathrm{tr}\boldsymbol{\sigma}\right)\,\mathbf{I}$$



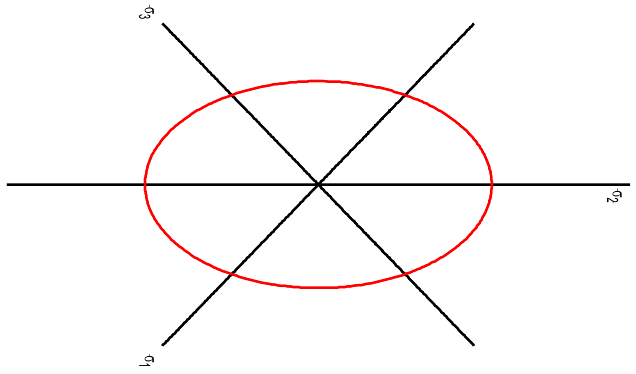
Isotropic Plasticity Models

normal

$$\frac{\partial \phi}{\partial \boldsymbol{\sigma}} = \frac{3}{2\phi}\mathbf{s}$$

curvature

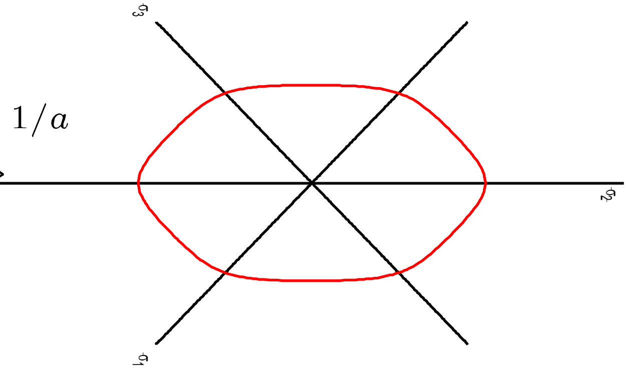
$$\frac{\partial^2 \phi}{\partial \boldsymbol{\sigma} \partial \boldsymbol{\sigma}} = \frac{3}{2\phi}\left(\mathbf{I\!I}' - \frac{2}{3}\frac{\partial \phi}{\partial \boldsymbol{\sigma}} \otimes \frac{\partial \phi}{\partial \boldsymbol{\sigma}}\right)$$



Isotropic Plasticity Models

Hosford – 2 parameters

$$\phi = \left\{ \frac{1}{2} \left[|\sigma_1 - \sigma_2|^a + |\sigma_2 - \sigma_3|^a + |\sigma_3 - \sigma_1|^a \right] \right\}^{1/a}$$



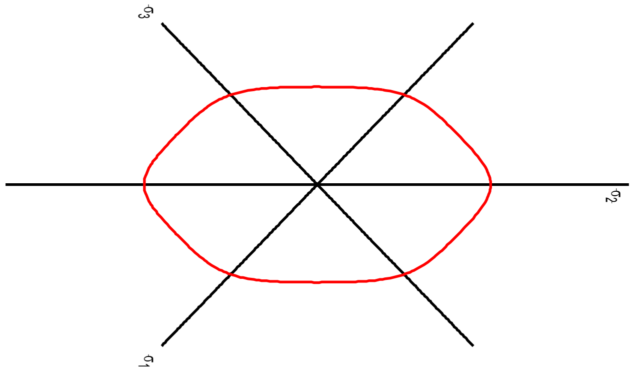
$$\boldsymbol{\sigma} = \sum_{i=1}^{3} \sigma_i \hat{\mathbf{e}}_i \otimes \hat{\mathbf{e}}_i$$

$$\bar{\sigma}_i = \frac{\sigma_i}{\sigma_{\mathrm{vm}}} \quad \rightarrow \quad \phi = \sigma_{\mathrm{vm}} \left\{ \frac{1}{2} \left[|\bar{\sigma}_1 - \bar{\sigma}_2|^a + |\bar{\sigma}_2 - \bar{\sigma}_3|^a + |\bar{\sigma}_3 - \bar{\sigma}_1|^a \right] \right\}^{1/a}$$

Isotropic Plasticity Models

normal

$$\frac{\partial \phi}{\partial \boldsymbol{\sigma}} = \sum_{i=1}^{3} \frac{\partial \phi}{\partial \sigma_i} \hat{\mathbf{e}}_i \otimes \hat{\mathbf{e}}_i$$



$$\frac{\partial \phi}{\partial \sigma_1} = \frac{\phi}{2\phi^a} \left[\frac{|\sigma_1 - \sigma_2|^a}{\sigma_1 - \sigma_2} - \frac{|\sigma_3 - \sigma_1|^a}{\sigma_3 - \sigma_1} \right]$$

$$\hat{\sigma}_i = \frac{\sigma_i}{\phi} \quad \rightarrow \quad \frac{\partial \phi}{\partial \sigma_1} = \frac{1}{2} \Big[(\hat{\sigma}_1 - \hat{\sigma}_2) \, |\hat{\sigma}_1 - \hat{\sigma}_2|^{a-2} - (\hat{\sigma}_3 - \hat{\sigma}_1) \, |\hat{\sigma}_3 - \hat{\sigma}_1|^{a-2} \Big]$$

Isotropic Plasticity Models

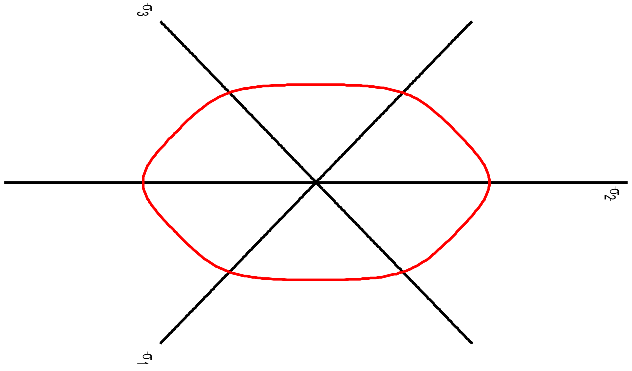
curvature

$$\frac{\partial^2 \phi}{\partial \boldsymbol{\sigma} \partial \boldsymbol{\sigma}} = A_{ijkl} \hat{\mathbf{e}}_i \otimes \hat{\mathbf{e}}_j \otimes \hat{\mathbf{e}}_k \otimes \hat{\mathbf{e}}_l$$

$$A_{1111} = \frac{\partial^2 \phi}{\partial \sigma_1 \partial \sigma_1}$$

$$A_{1122} = \frac{\partial^2 \phi}{\partial \sigma_1 \partial \sigma_2}$$

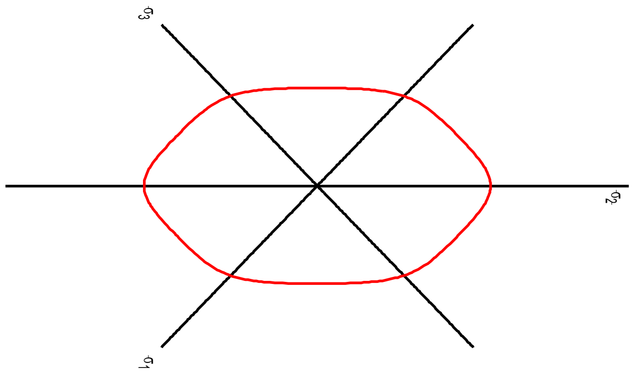
$$A_{1212} = \frac{\partial \phi / \partial \sigma_1 - \partial \phi / \partial \sigma_2}{2 \left(\sigma_1 - \sigma_2 \right)}$$



Isotropic Plasticity Models

curvature

$$A_{1212} = \lim_{\sigma_1 \to \sigma_2} \frac{\partial\phi/\partial\sigma_1 - \partial\phi/\partial\sigma_2}{2\left(\sigma_1 - \sigma_2\right)} = \frac{1}{2}\left(\frac{\partial^2\phi}{\partial\sigma_1\partial\sigma_1} - \frac{\partial^2\phi}{\partial\sigma_1\partial\sigma_2}\right)$$

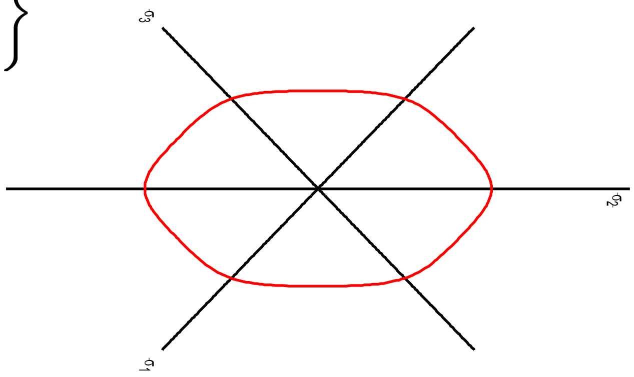


Isotropic Plasticity Models

curvature

$$\frac{\partial^2\phi}{\partial\sigma_1\partial\sigma_1} = \frac{a-1}{\phi}\left\{\frac{1}{2}\left[|\hat{\sigma}_1 - \hat{\sigma}_2|^{a-2} + |\hat{\sigma}_3 - \hat{\sigma}_1|^{a-2}\right] - \frac{\partial\phi}{\partial\sigma_1}\frac{\partial\phi}{\partial\sigma_1}\right\}$$

$$\frac{\partial^2\phi}{\partial\sigma_1\partial\sigma_2} = \frac{a-1}{\phi}\left\{-\frac{1}{2}|\hat{\sigma}_1 - \hat{\sigma}_2|^{a-2} - \frac{\partial\phi}{\partial\sigma_1}\frac{\partial\phi}{\partial\sigma_1}\right\}$$

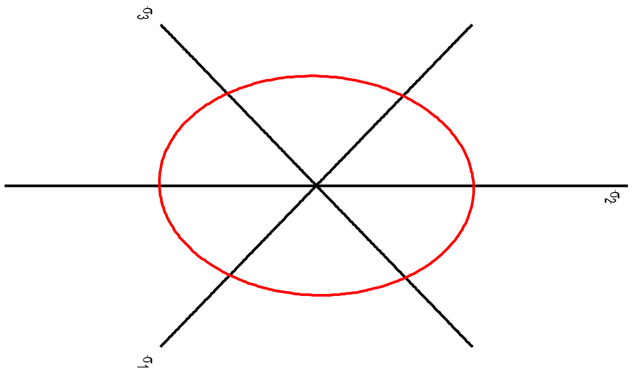


Orthotropic Plasticity Models

Hill – 7 parameters

$$\phi^2(\boldsymbol{\sigma}) = F(\hat{\sigma}_{22} - \hat{\sigma}_{33})^2 + G(\hat{\sigma}_{33} - \hat{\sigma}_{11})^2 + H(\hat{\sigma}_{11} - \hat{\sigma}_{22})^2$$

$$+ 2L\hat{\sigma}_{23}^2 + 2M\hat{\sigma}_{31}^2 + 2N\hat{\sigma}_{12}^2$$



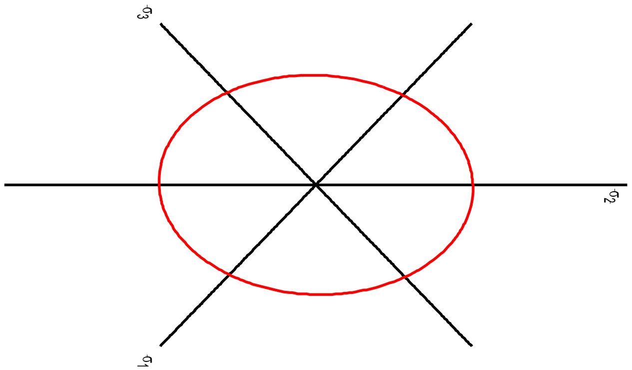
$$\phi = \sqrt{\frac{3}{2}\boldsymbol{\sigma} : \mathbf{P} : \boldsymbol{\sigma}}$$

depends on material orientation

Orthotropic Plasticity Models

normal

$$\frac{\partial \phi}{\partial \boldsymbol{\sigma}} = \frac{3}{2\phi}\mathbf{P} : \boldsymbol{\sigma}$$



curvature

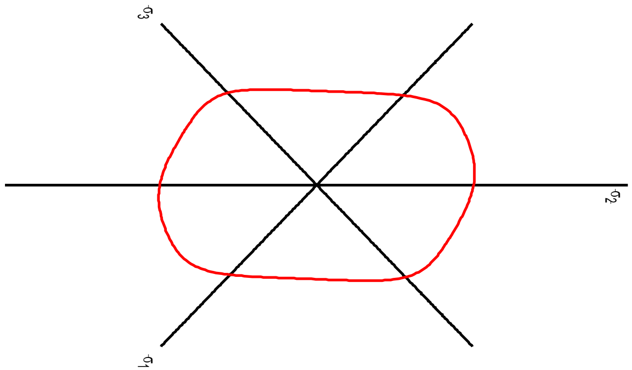
$$\frac{\partial^2 \phi}{\partial \boldsymbol{\sigma} \partial \boldsymbol{\sigma}} = \frac{3}{2\phi}\left(\mathbf{P} - \frac{2}{3}\frac{\partial \phi}{\partial \boldsymbol{\sigma}} \otimes \frac{\partial \phi}{\partial \boldsymbol{\sigma}}\right)$$

Orthotropic Plasticity Models

Barlat (Yld2004-18p) – 20 parameters *

$$\mathbf{s}' = \mathbf{L}' : \boldsymbol{\sigma} \quad ; \quad \mathbf{s}'' = \mathbf{L}'' : \boldsymbol{\sigma}$$

$$\phi(\boldsymbol{\sigma}) = \left\{ \frac{1}{4} \Big[|s'_1 - s''_1|^a + |s'_1 - s''_2|^a + |s'_1 - s''_3|^a + |s'_2 - s''_1|^a + |s'_2 - s''_2|^a + |s'_2 - s''_3|^a + |s'_3 - s''_1|^a + |s'_3 - s''_2|^a + |s'_3 - s''_3|^a \Big] \right\}^{1/a}$$



* Barlat et. al., "Linear transformation based anisotropic yield functions", IJP, v. 21, 2005.

Orthotropic Plasticity Models

$$\mathbf{s}' = \underbrace{\mathbf{C}' : \mathbb{I}'}_{\mathbf{L}'} : \boldsymbol{\sigma} = \mathbf{C}' : \mathbf{s}$$

$$\begin{Bmatrix} s'_{11} \\ s'_{22} \\ s'_{33} \\ s'_{12} \\ s'_{23} \\ s'_{31} \end{Bmatrix} = \begin{bmatrix} 0 & -c'_{12} & -c'_{13} & 0 & 0 & 0 \\ -c'_{21} & 0 & -c'_{23} & 0 & 0 & 0 \\ -c'_{31} & -c'_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c'_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c'_{66} \end{bmatrix} \begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ s_{12} \\ s_{23} \\ s_{31} \end{Bmatrix}$$

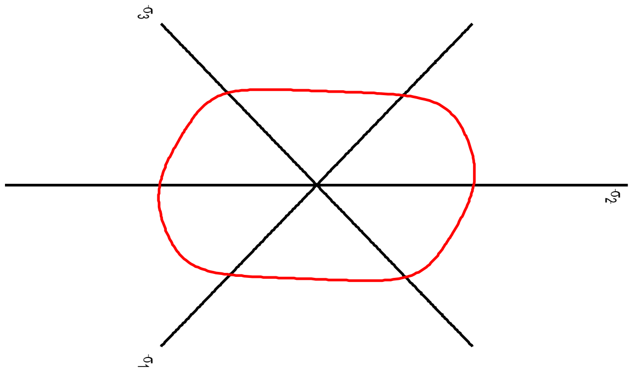
Orthotropic Plasticity Models

normal

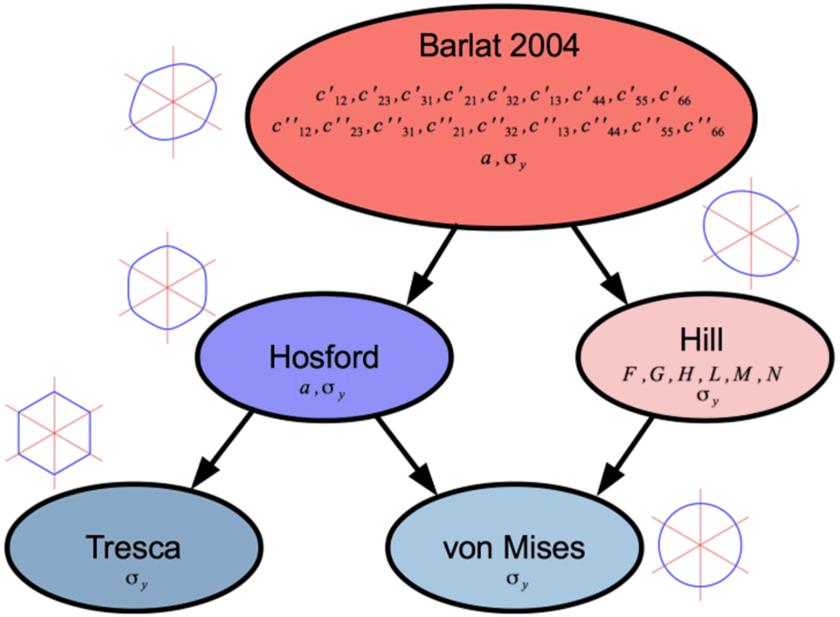
$$\frac{\partial \phi}{\partial \boldsymbol{\sigma}} = \frac{\partial \phi}{\partial \mathbf{s}'} : \frac{\partial \mathbf{s}'}{\partial \boldsymbol{\sigma}} + \frac{\partial \phi}{\partial \mathbf{s}''} : \frac{\partial \mathbf{s}''}{\partial \boldsymbol{\sigma}}$$

$$= \frac{\partial \phi}{\partial \mathbf{s}'} : \mathbf{L}' + \frac{\partial \phi}{\partial \mathbf{s}''} : \mathbf{L}''$$

curvature



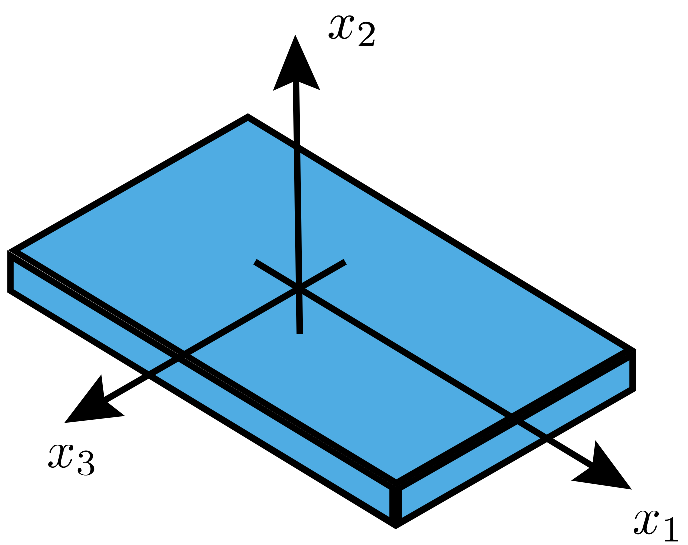
Model Hierarchy



- Models are related to each other
- Other models to consider
  - Karafillis-Boyce
  - Cazacu

Fitting the Barlat Model

- Uniaxial tension test
  - Rolling direction
  - Transverse direction
  - Every 15º between
- Hydraulic bulge test
- Disk compression test
- Crystal plasticity simulations



- Where possible, look at flow directions

Barlat Model

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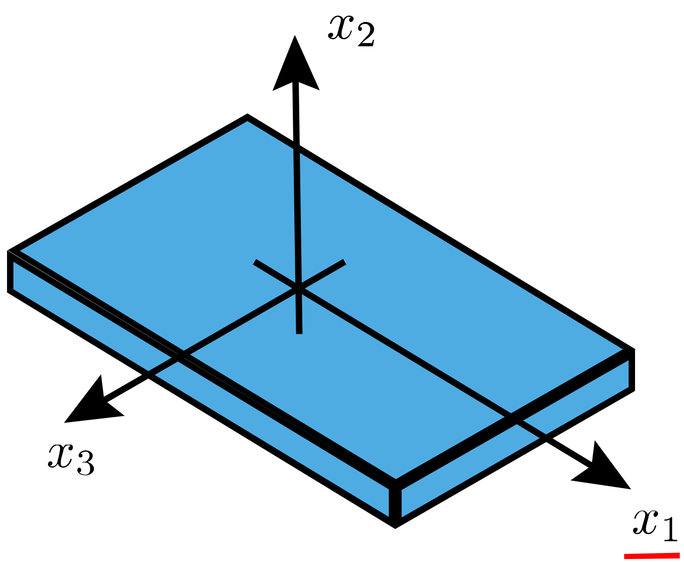
2090-T3 Al *

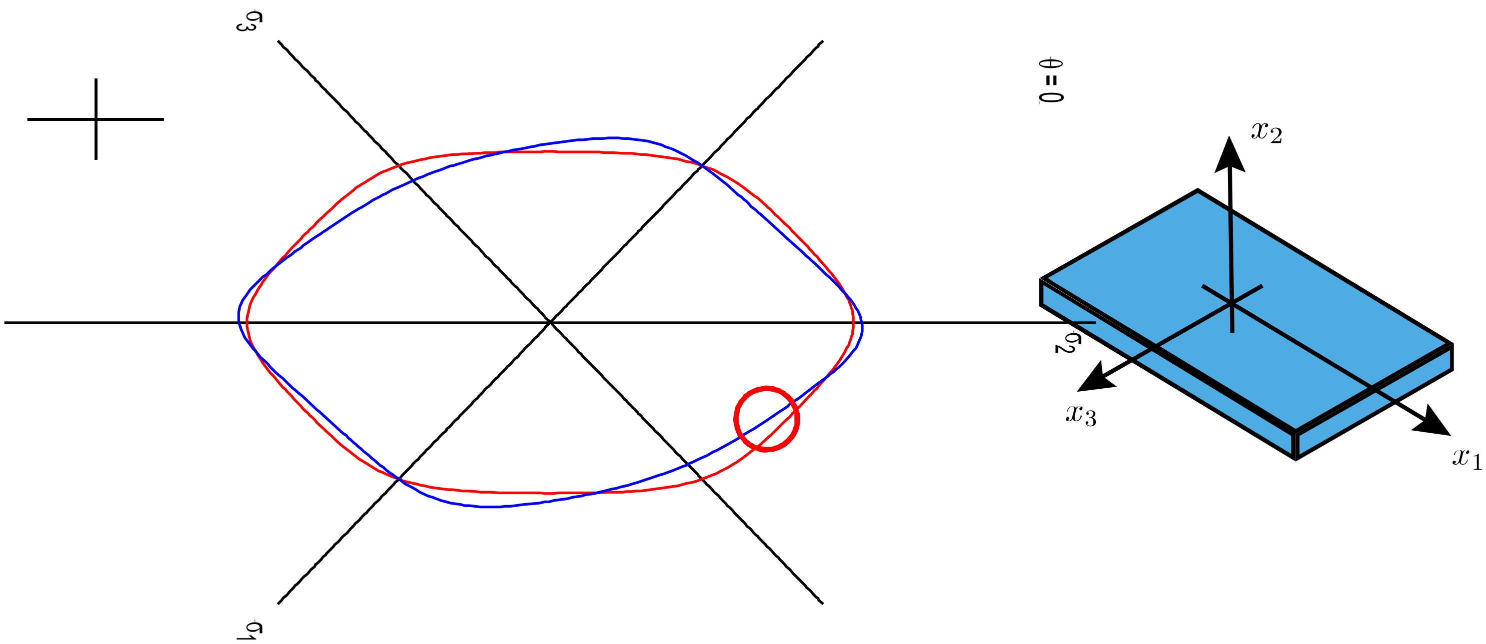
$$a = 8$$

$$c'_{12} = -0.069888 \quad ; \quad c''_{12} = 0.981171$$
$$c'_{13} = 0.936408 \quad ; \quad c''_{13} = 0.476741$$
$$c'_{21} = 0.079143 \quad ; \quad c''_{21} = 0.575316$$
$$c'_{23} = 1.003060 \quad ; \quad c''_{23} = 0.866827$$
$$c'_{31} = 0.524741 \quad ; \quad c''_{31} = 1.145010$$
$$c'_{32} = 1.363180 \quad ; \quad c''_{32} = -0.079294$$
$$c'_{44} = 1.023770 \quad ; \quad c''_{44} = 1.051660$$
$$c'_{55} = 1.069060 \quad ; \quad c''_{55} = 1.147100$$
$$c'_{66} = 0.954322 \quad ; \quad c''_{66} = 1.404620$$

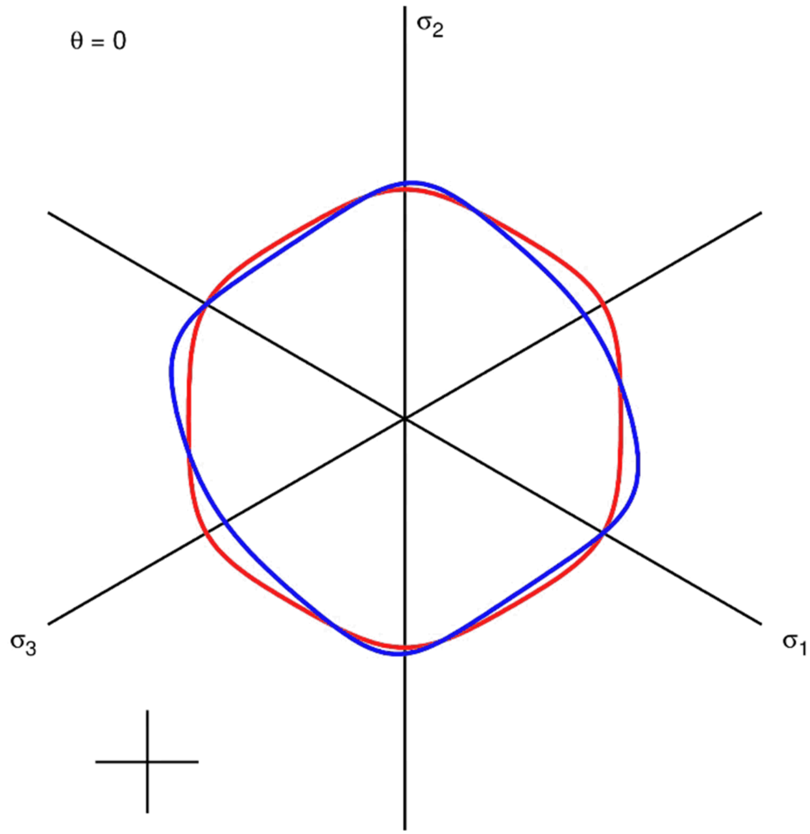


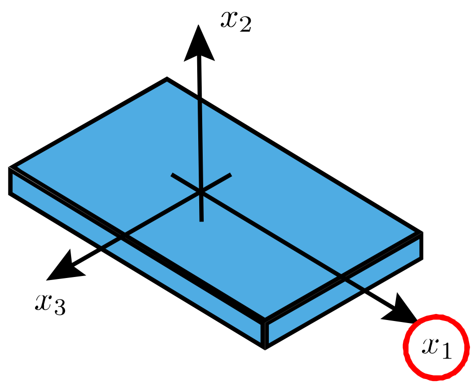
* Barlat et. al., "Linear transformation based anisotropic yield functions", IJP, v. 21, 2005.

Hosford and Barlat

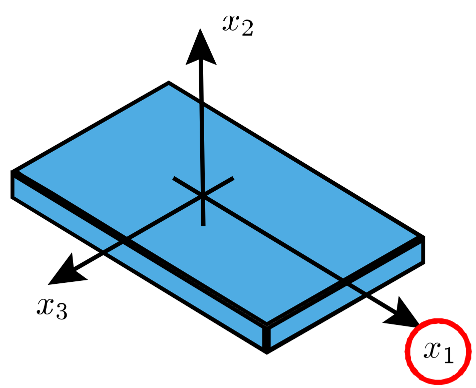


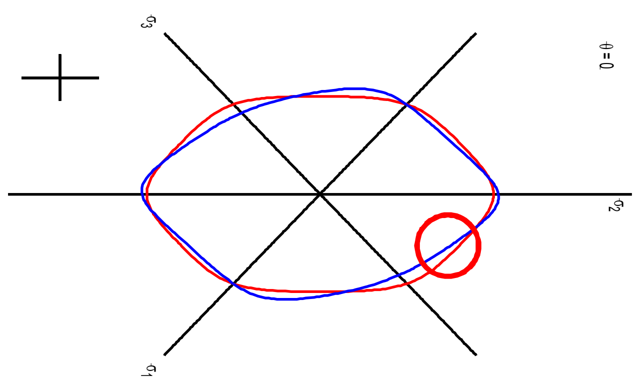
Hosford and Barlat Model

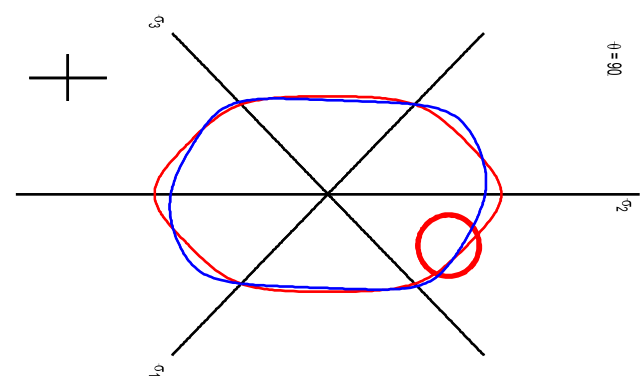




Hosford and Barlat Model

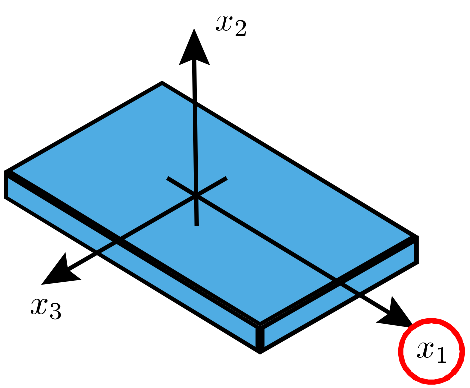


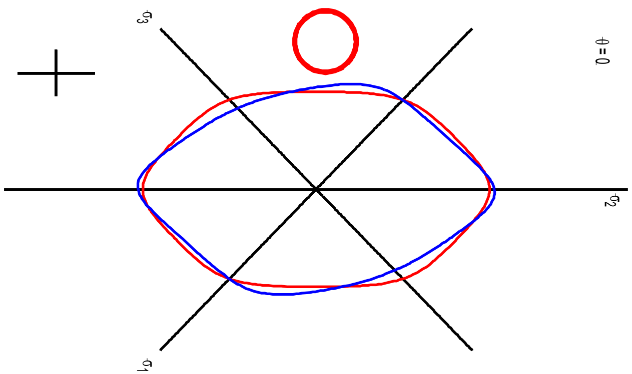


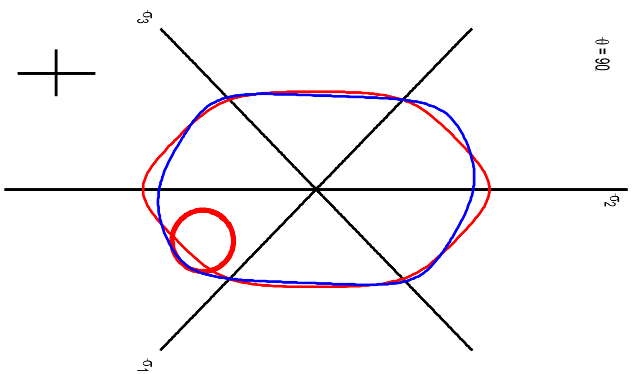


Hosford and Barlat Model

Intersection of yield surface on σ_2 axis moves to σ_3 axis

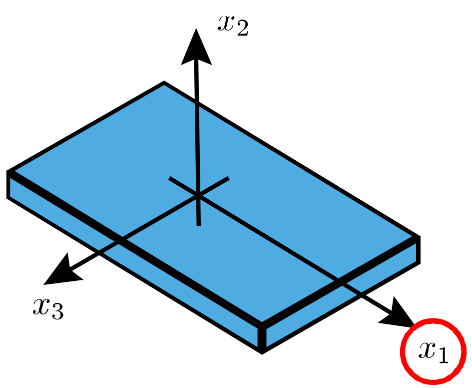


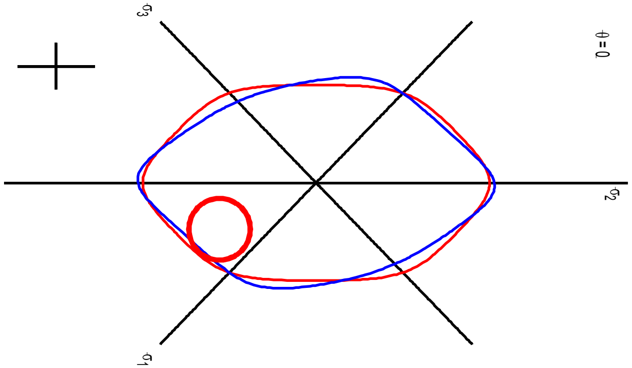


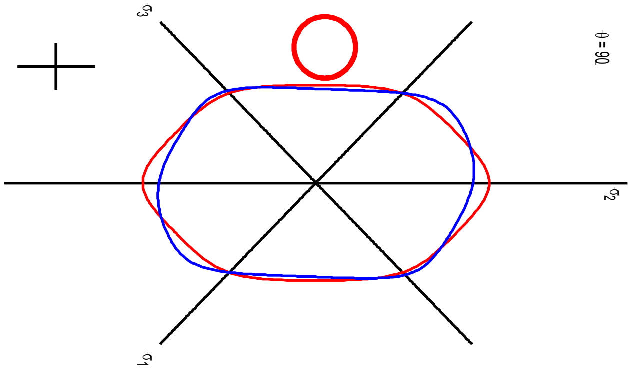


Hosford and Barlat Model

Intersection of yield surface on σ_3 axis moves to σ_2 axis



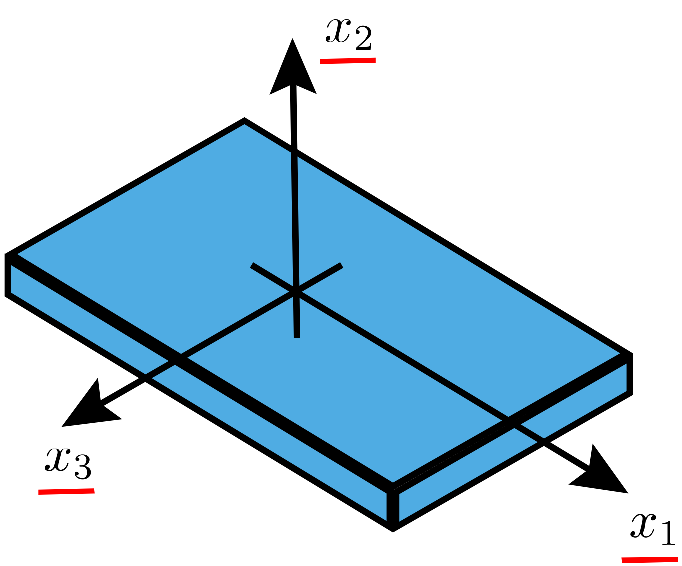




Hill Model

2090-T3 Al *

$$F = 0.690940 \quad , \quad L = 2.85422$$
$$G = 0.206643 \quad ; \quad M = 3.67927$$
$$H = 0.790646 \quad ; \quad N = 2.19515$$

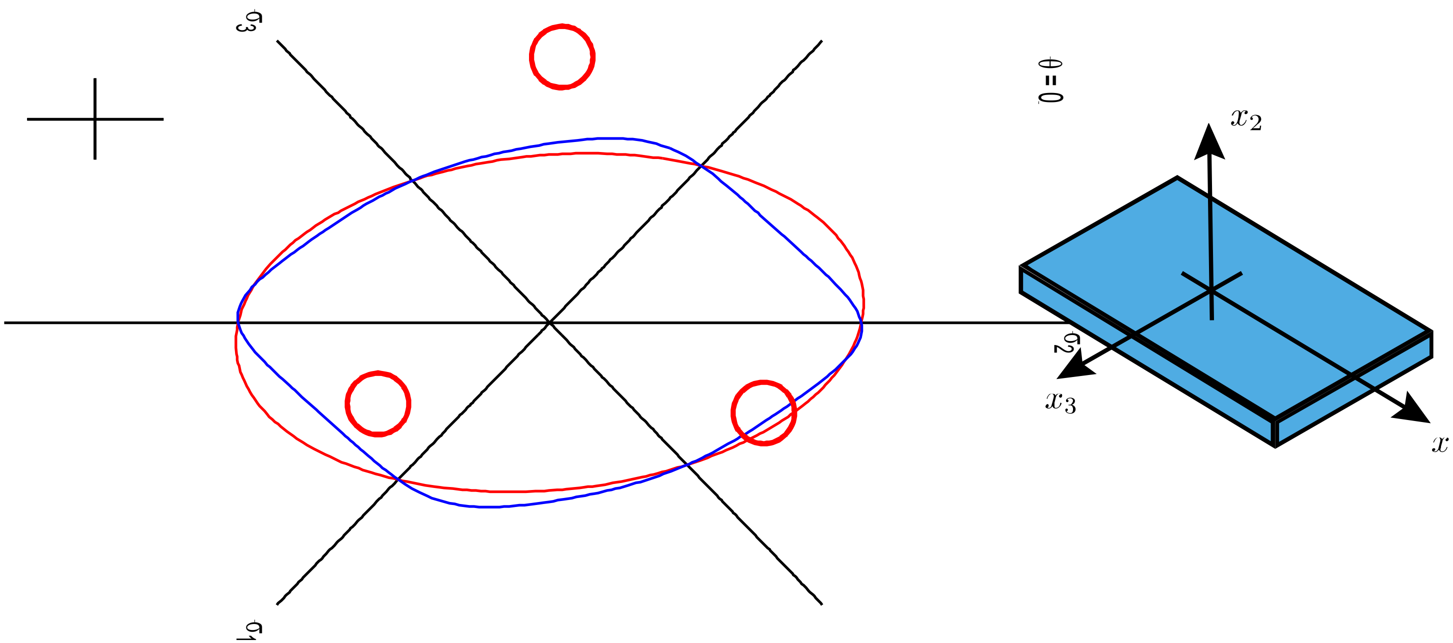


$$\phi^2(\boldsymbol{\sigma}) = F(\hat{\sigma}_{22} - \hat{\sigma}_{33})^2 + G(\hat{\sigma}_{33} - \hat{\sigma}_{11})^2 + H(\hat{\sigma}_{11} - \hat{\sigma}_{22})^2$$

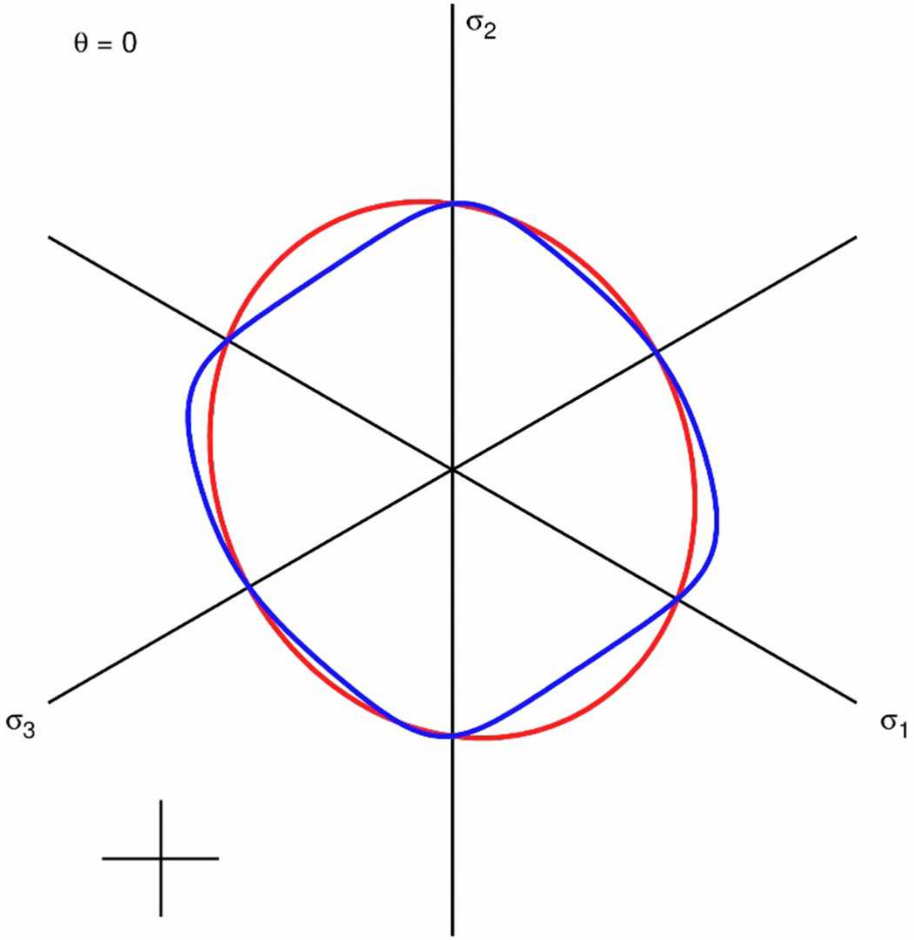
$$+ 2L\hat{\sigma}_{23}^2 + 2M\hat{\sigma}_{31}^2 + 2N\hat{\sigma}_{12}^2$$

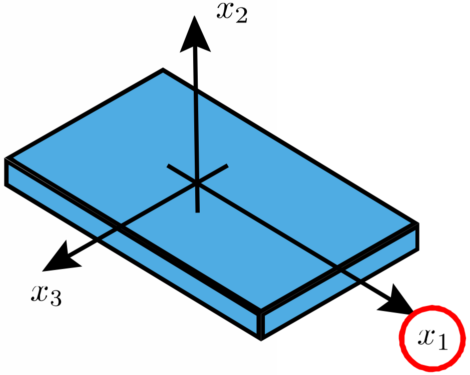
* Barlat et. al., "Linear transformation based anisotropic yield functions", IJP, v. 21, 2005.

Hill and Barlat

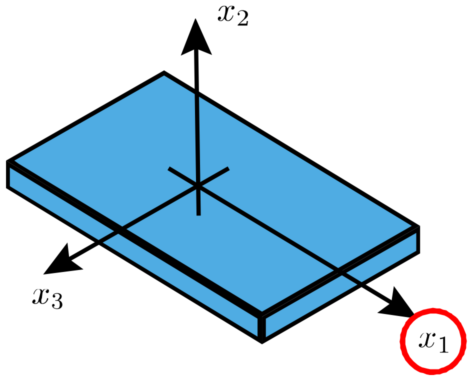


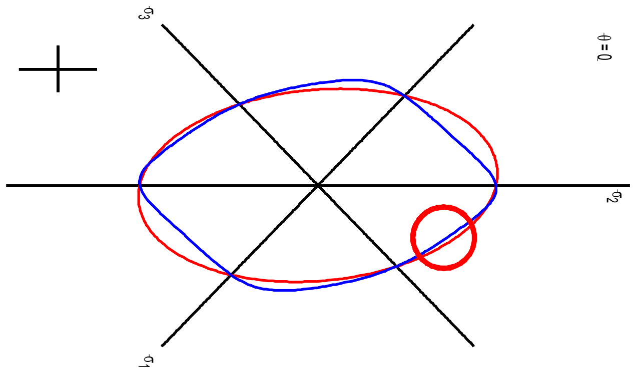
Hill and Barlat Model

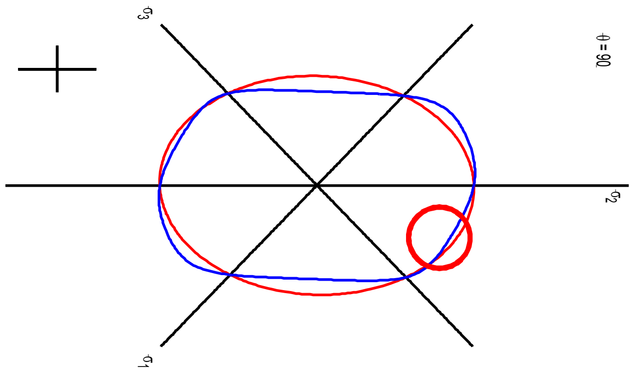




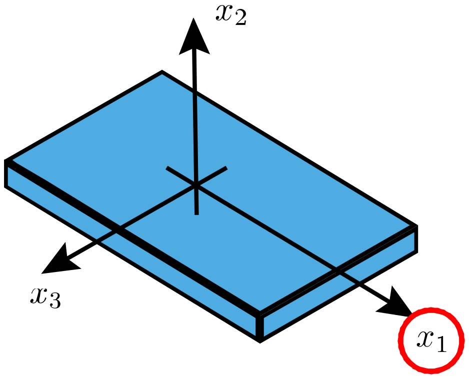
Hill and Barlat Model

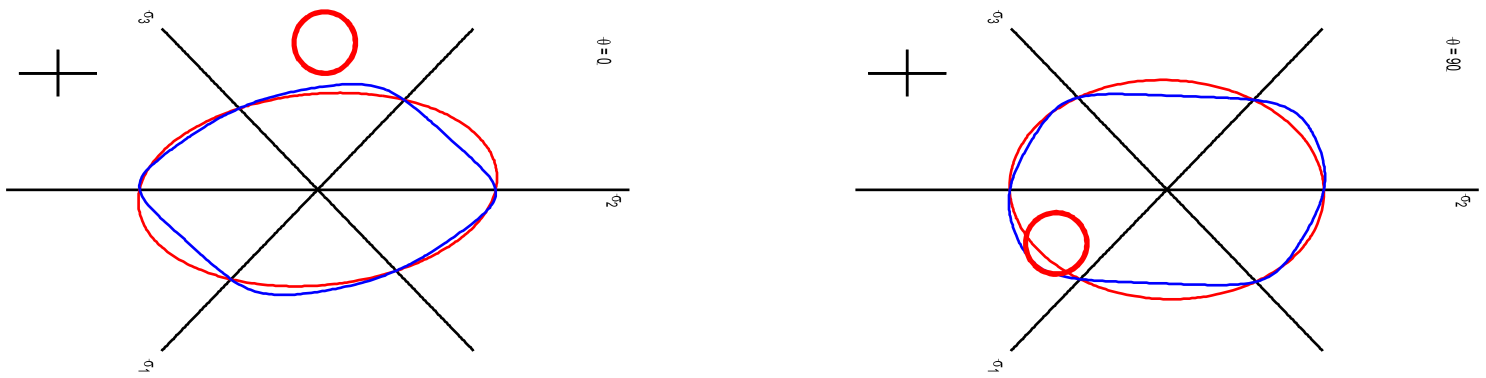




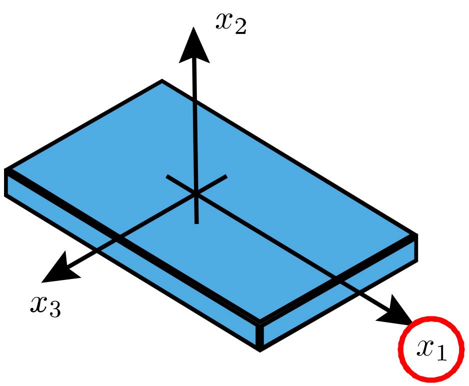


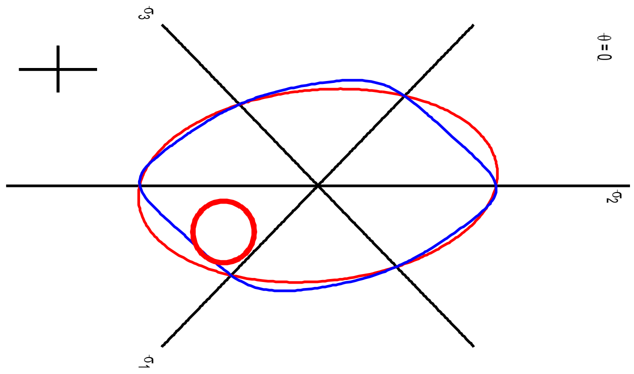
Hill and Barlat Model

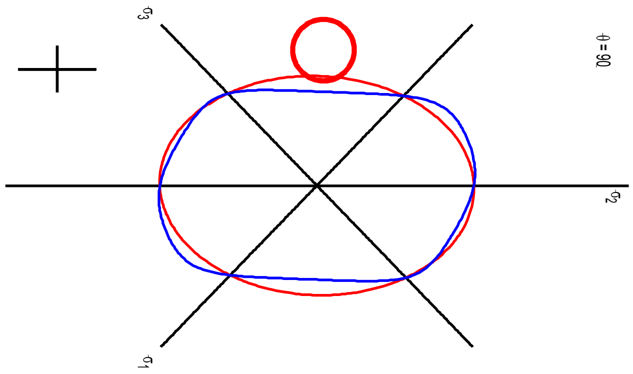




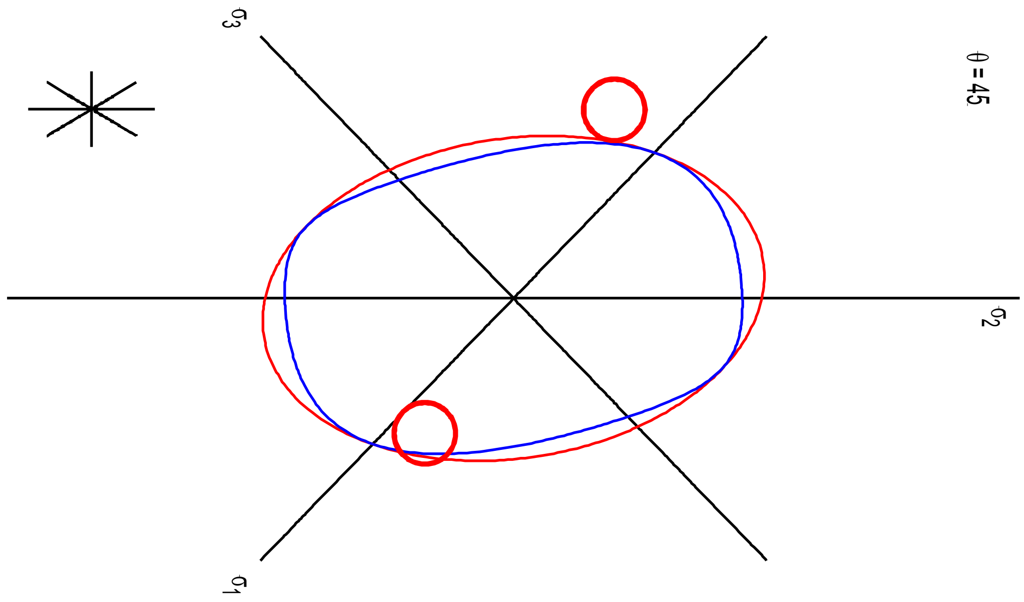
Hill and Barlat Model







Hill and Barlat Model



Rate Formulation

Rate form of the model

$$\dot{\boldsymbol{\sigma}} = \mathbb{C} : \dot{\boldsymbol{\varepsilon}}^e$$

Additive decomposition of strain rate

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p$$

Associated flow

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\gamma} \frac{\partial \phi}{\partial \boldsymbol{\sigma}}$$

$$\boxed{\dot{\boldsymbol{\sigma}} = \mathbb{C} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\gamma} \frac{\partial \phi}{\partial \boldsymbol{\sigma}} \right)}$$

Return Mapping Algorithms

Trial stress

$$\boldsymbol{\sigma}^{\mathrm{tr}} = \boldsymbol{\sigma}^{n} + \mathbb{C} : \Delta\boldsymbol{\varepsilon}$$

Yield function

$$f = \phi\left(\boldsymbol{\sigma}\right) - \bar{\sigma}\left(\Delta\gamma\right)$$

Plastic strain increment

$$\mathbf{R} = -\Delta\boldsymbol{\varepsilon}^{p} + \Delta\gamma\frac{\partial\phi}{\partial\boldsymbol{\sigma}}$$

$$\Delta\boldsymbol{\varepsilon}^{p} = \mathbb{C}^{-1} : \left(\boldsymbol{\sigma} - \boldsymbol{\sigma}^{tr}\right)$$

Equations we want to solve

$$f = 0$$

$$\mathbf{R} = \mathbf{0}$$

Unknowns

$$\Delta\gamma \ , \ \boldsymbol{\sigma}$$

Iterative Algorithm

Create iterative solution for unknowns

$$\Delta\gamma^{(k+1)} = \Delta\gamma^{(k)} + \Delta\left(\Delta\gamma\right)$$

$$\boldsymbol{\sigma}^{(k+1)} = \boldsymbol{\sigma}^{(k)} + \Delta\boldsymbol{\sigma}$$

Two algorithm to solve for increment in unknowns

- Newton
- Line search based on Newton

Newton Algorithm

$$\Delta\left(\Delta\gamma\right) = \frac{f^{(k)} - \mathbf{R}^{(k)} : \mathscr{L}^{(k)} : \dfrac{\partial\phi^{(k)}}{\partial\boldsymbol{\sigma}}}{\dfrac{\partial\phi^{(k)}}{\partial\boldsymbol{\sigma}} : \mathscr{L}^{(k)} : \dfrac{\partial\phi^{(k)}}{\partial\boldsymbol{\sigma}} + H'_{(k)}}$$

$$\Delta\boldsymbol{\sigma} = -\mathscr{L}^{(k)} : \left(\mathbf{R}^{(k)} + \Delta\left(\Delta\gamma\right)\frac{\partial\phi^{(k)}}{\partial\boldsymbol{\sigma}}\right)$$

Slope of hardening curve

$$H' = \frac{d\bar{\sigma}}{d\Delta\gamma}$$

Hessian

$$\mathscr{L}^{-1} = \mathbb{C}^{-1} + \Delta\gamma\,\frac{\partial^2\phi}{\partial\boldsymbol{\sigma}\partial\boldsymbol{\sigma}}$$

von Mises

$$\Delta\left(\Delta\gamma\right) = \frac{f^{(k)}}{3\mu + H'_{(k)}} \qquad \Delta\boldsymbol{\sigma} = -\Delta\left(\Delta\gamma\right)\frac{3}{2\phi}\mathscr{L}^{(k)} : \mathbf{s}$$

Line Search Algorithm

$$\Delta\gamma^{(k+1)} = \Delta\gamma^{(k)} + \alpha\,\Delta\left(\Delta\gamma\right)$$

$$\boldsymbol{\sigma}^{(k+1)} = \boldsymbol{\sigma}^{(k)} + \alpha\,\Delta\boldsymbol{\sigma}$$

search direction

search magnitude

$$\alpha \in [0, 1]$$

Search direction comes from Newton algorithm

Need to determine "best" value for α

Line Search Algorithm *

Residual

$$\mathbf{r} = \left(\frac{f}{2\mu} \,,\, \mathbf{R} \right)$$

Merit function based on residual

$$\psi = \frac{1}{2} \mathbf{r} \cdot \mathbf{r}$$

…as a function of α

$$\psi(\alpha) = \frac{1}{2} \mathbf{r}(\alpha) \cdot \mathbf{r}(\alpha)$$

We want

$$\psi(\alpha) < \psi(0)$$

If we get this, then the solution is improving

* Perez-Foguet and Armero, "On the formulation of closest point projection algorithms in elastoplasticity – part II: globally convergent schemes", IJNME, v. 53, 2002.

Line Search Algorithm

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check

$$\psi(1) < \psi(0)$$

Yes

$$\alpha = 1$$

use Newton

No

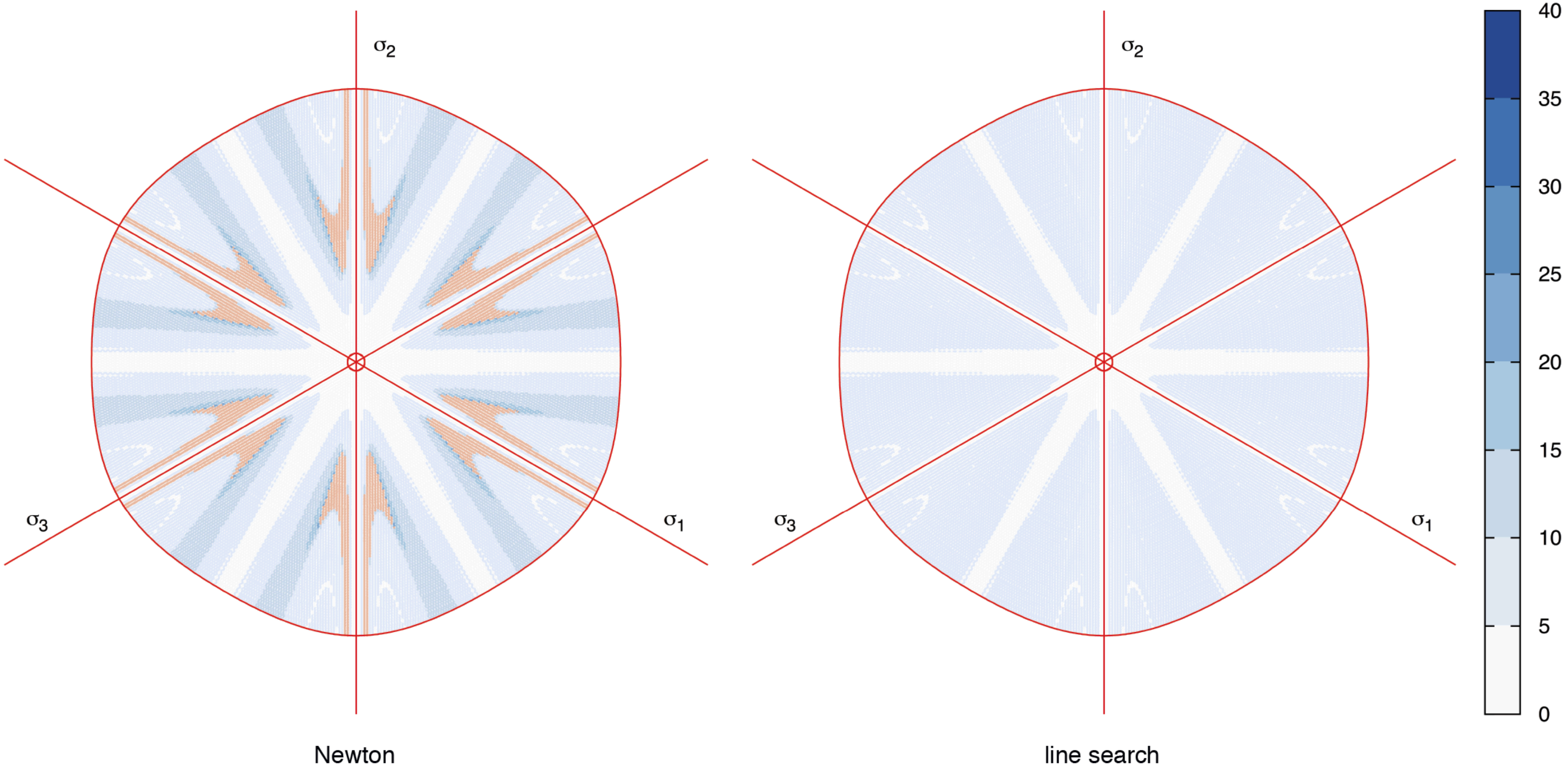
$$\hat{\psi} = \psi_0 + \alpha\psi_1 + \alpha^2\psi_2$$

$$\hat{\psi} = \left(1 - 2\alpha + \alpha^2\right)\psi(0) + \alpha^2\psi(1)$$

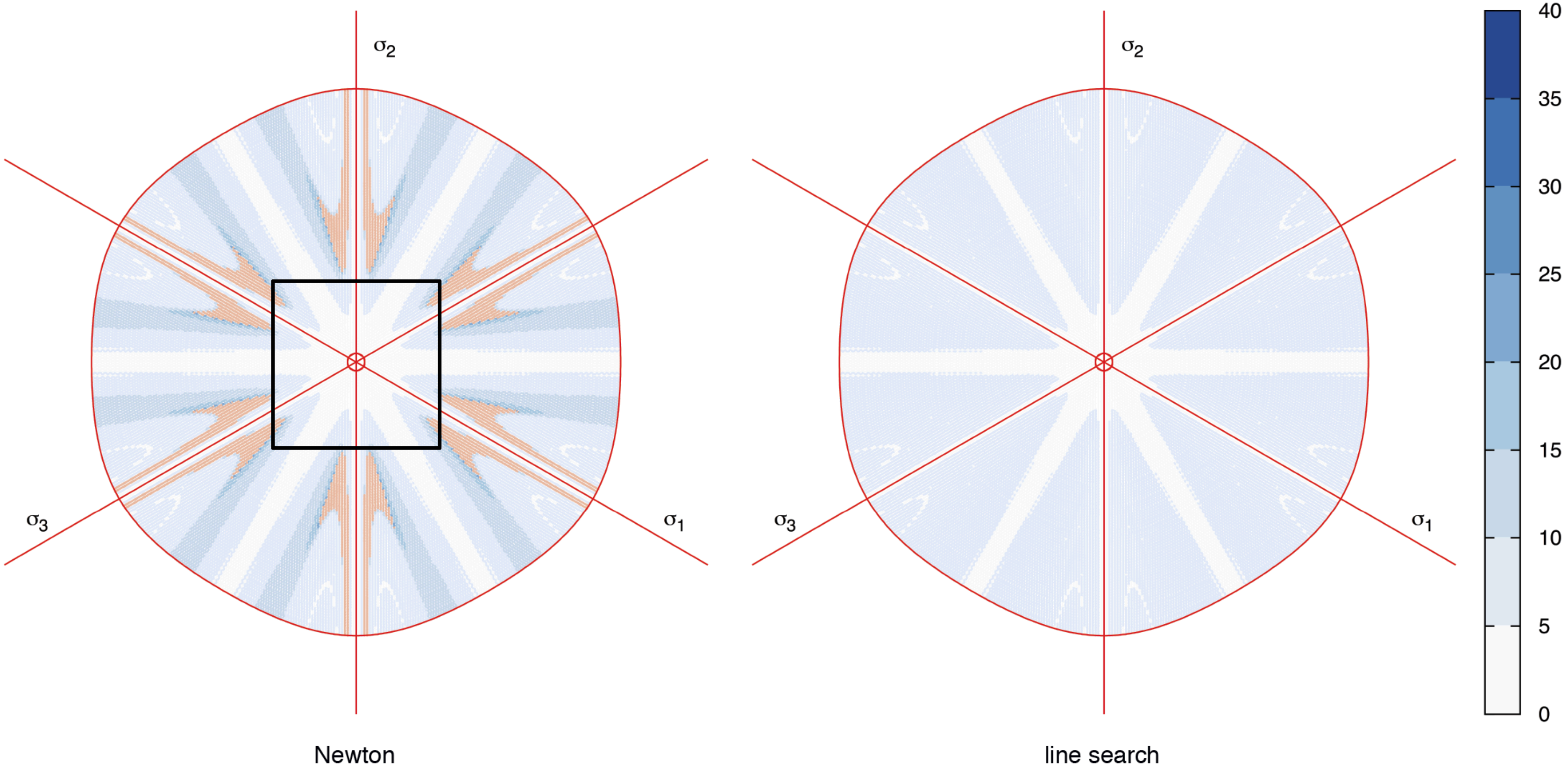
$$\alpha = \frac{\psi(0)}{\psi(1) + \psi(0)}$$

Hosford Model

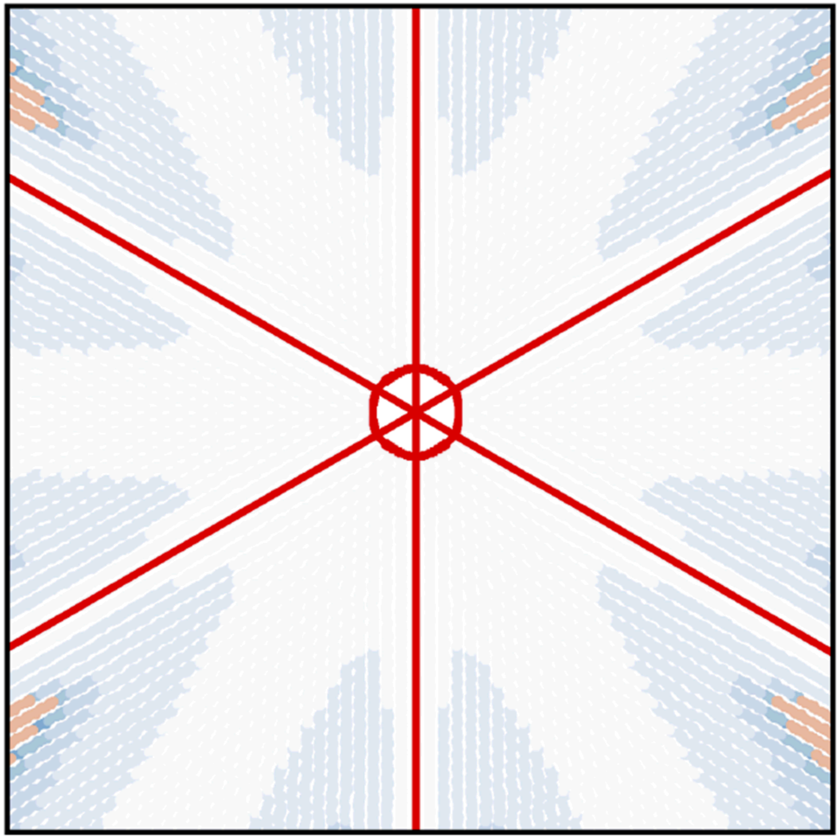
Hosford (a=6)



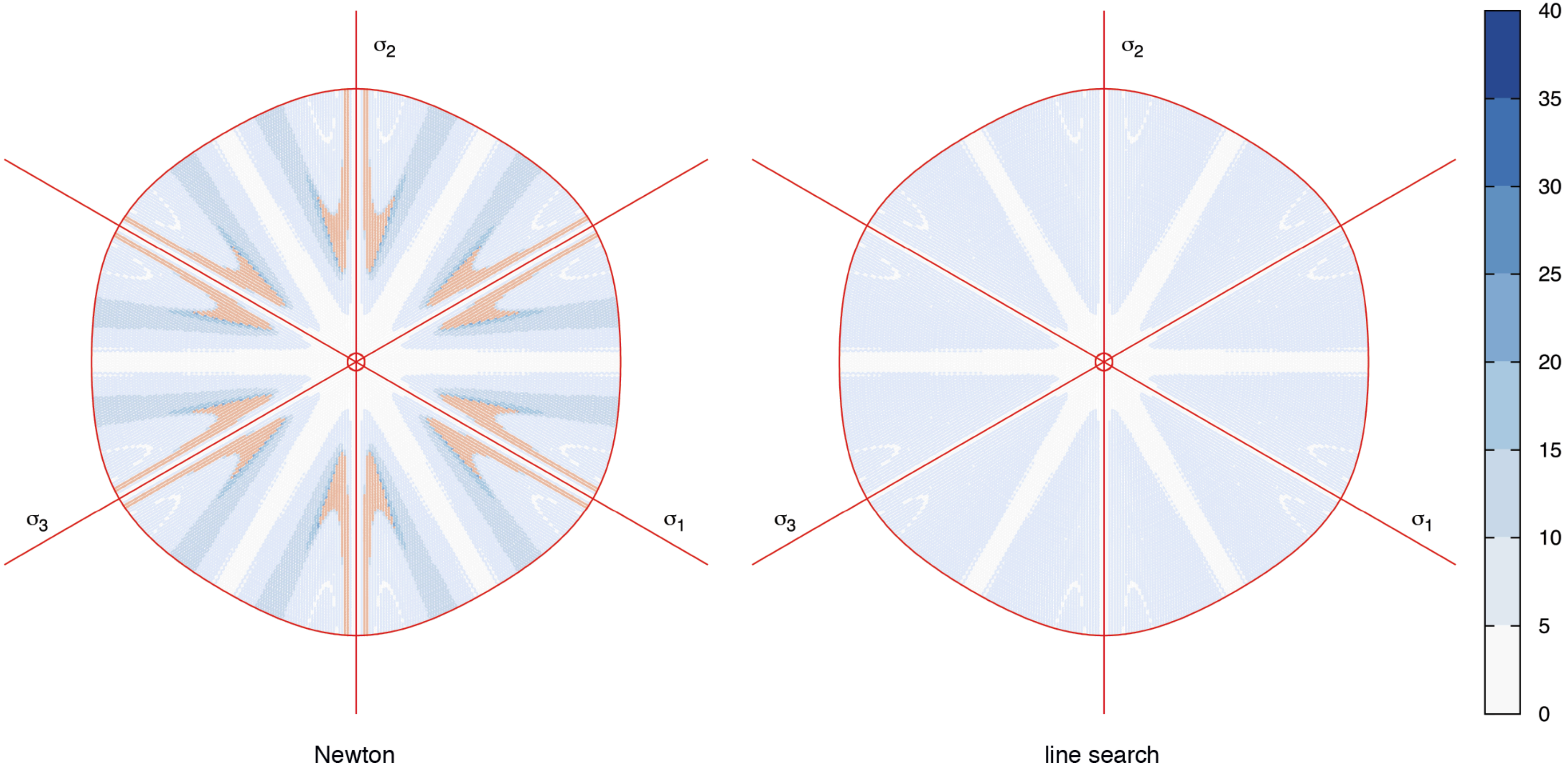
Hosford (a=6)



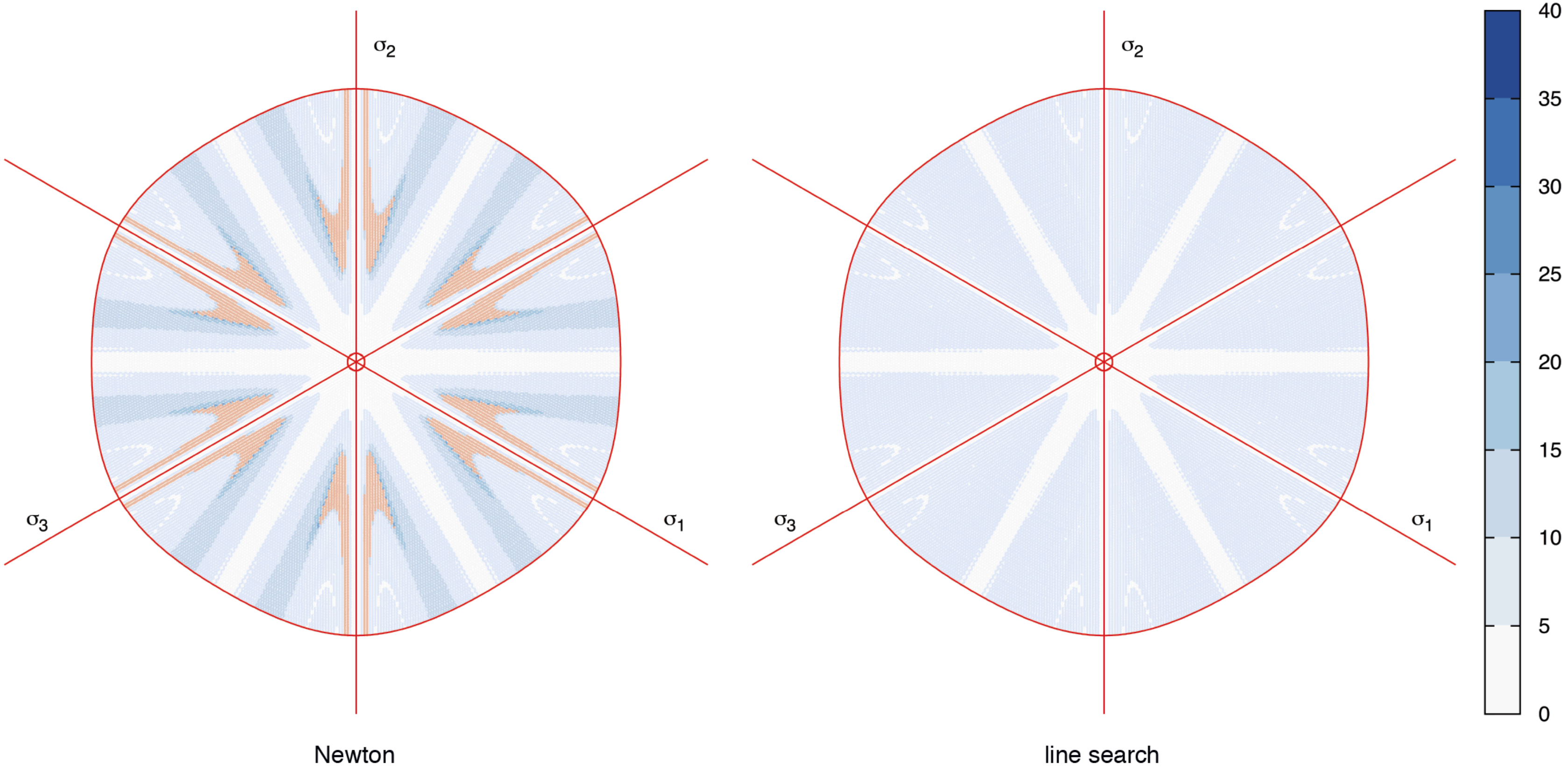
Hosford (a=6)



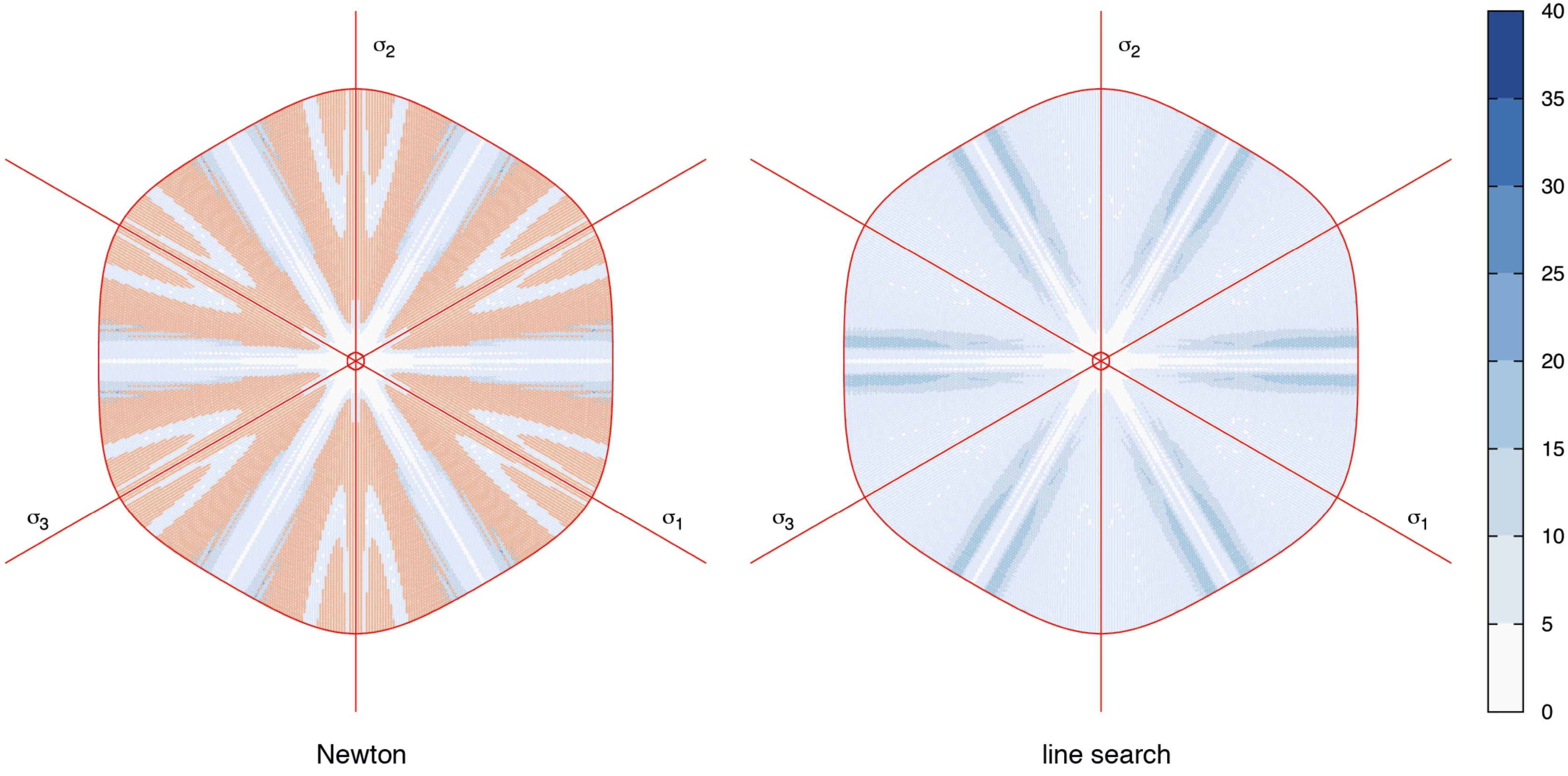
Hosford (a=6)



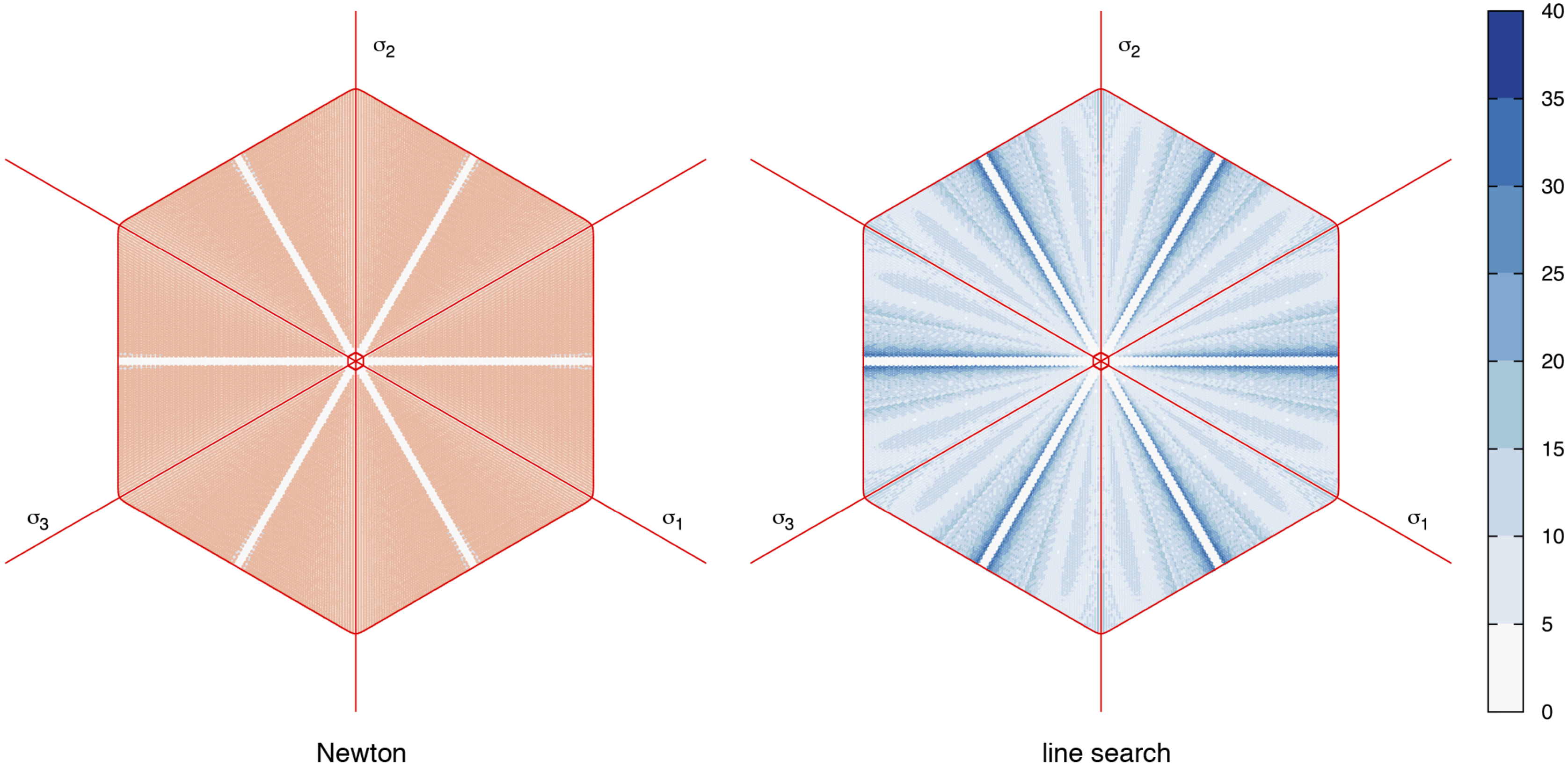
Hosford (a=6)



Hosford (a=8)



Hosford (a=100)



Visualization

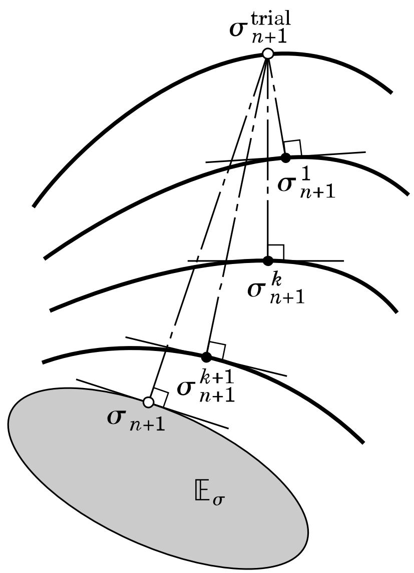
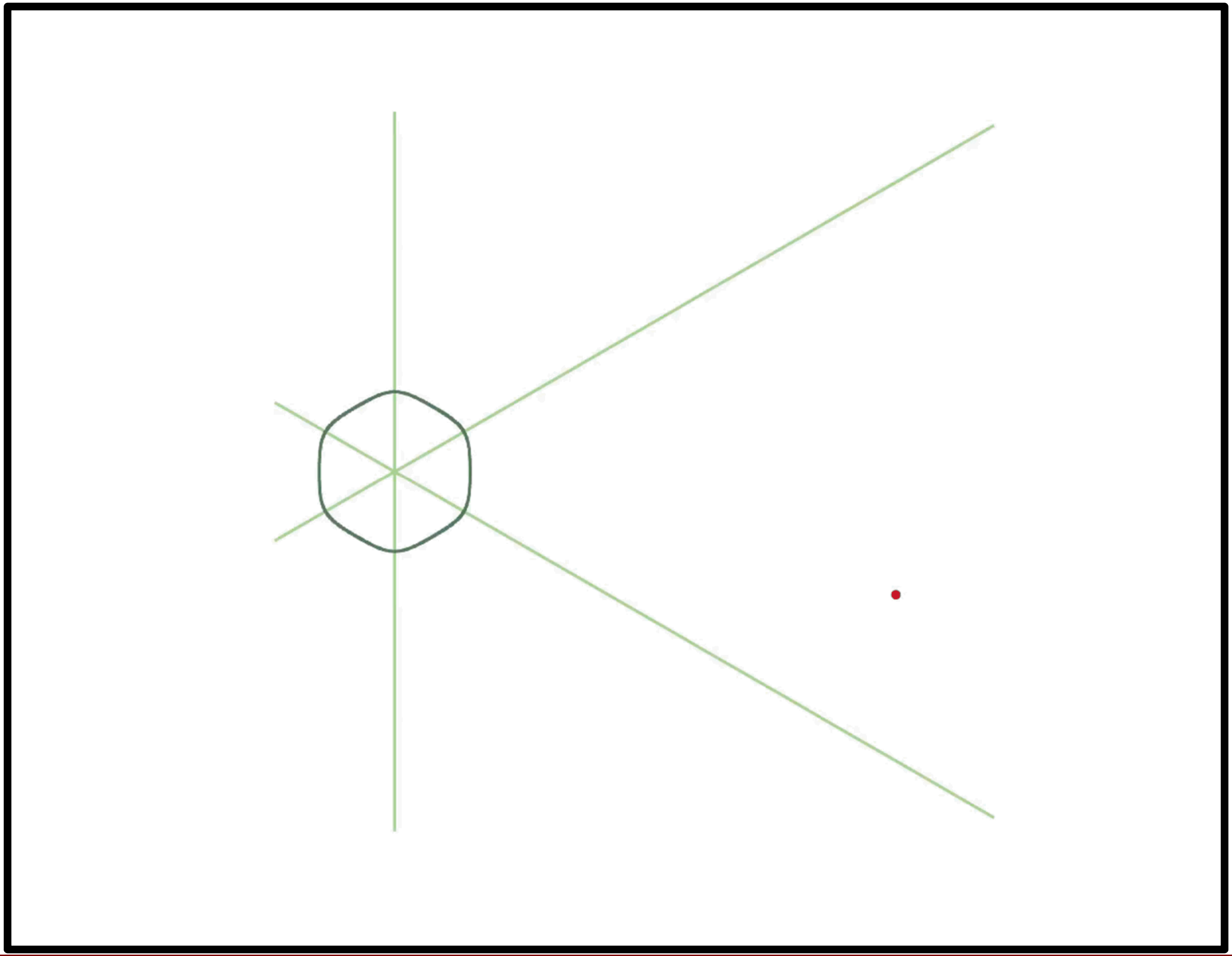


FIGURE 3-10. A geometric interpretation of the closest point projection algorithm in stress space. At each iterate $(\bullet)^{(k)}$, the constraint is linearized to find the intersection (cut) with $f = 0$. The next iterate $(\bullet)^{(k+1)}$, located on level set $f_{n+1}^{(k+1)} > 0$, is the closest point of that level set to the previous iterate $(\bullet)^{(k)}$ in the metric defined by the elasticities **C**.

Simo and Hughes, Computational Inelasticity, 1998.

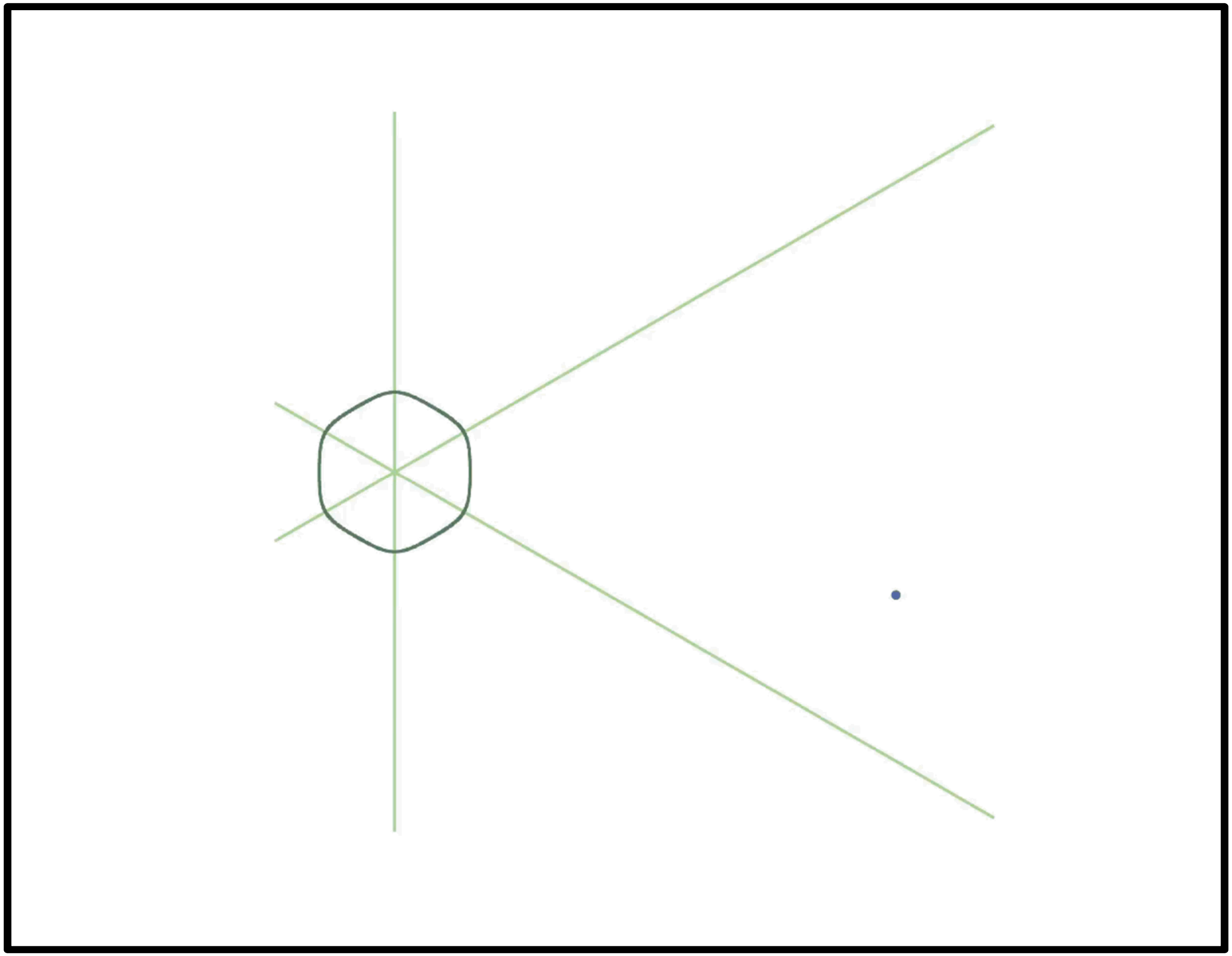
Hosford (a=8)

Newton algorithm



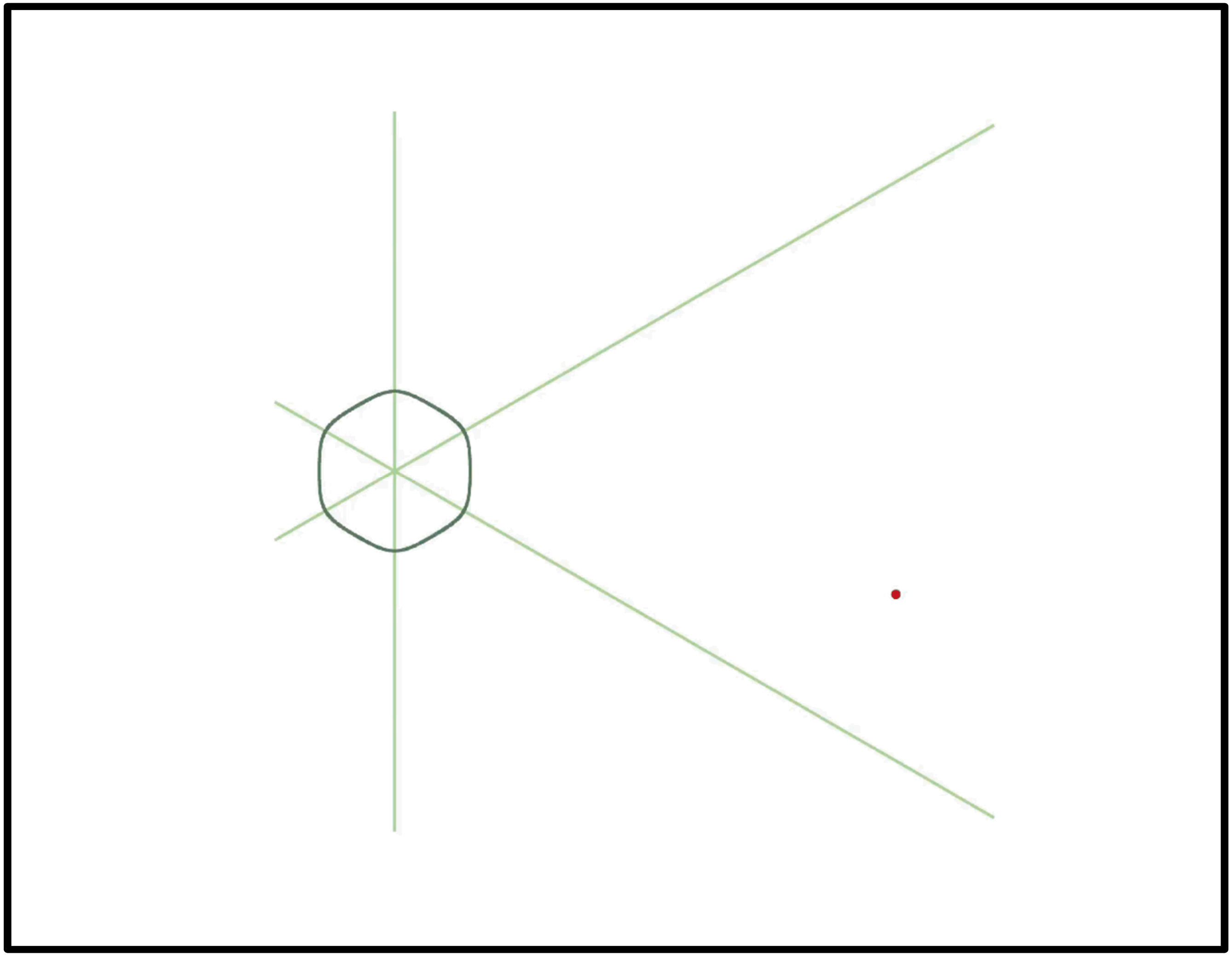
Hosford (a=8)

line search algorithm

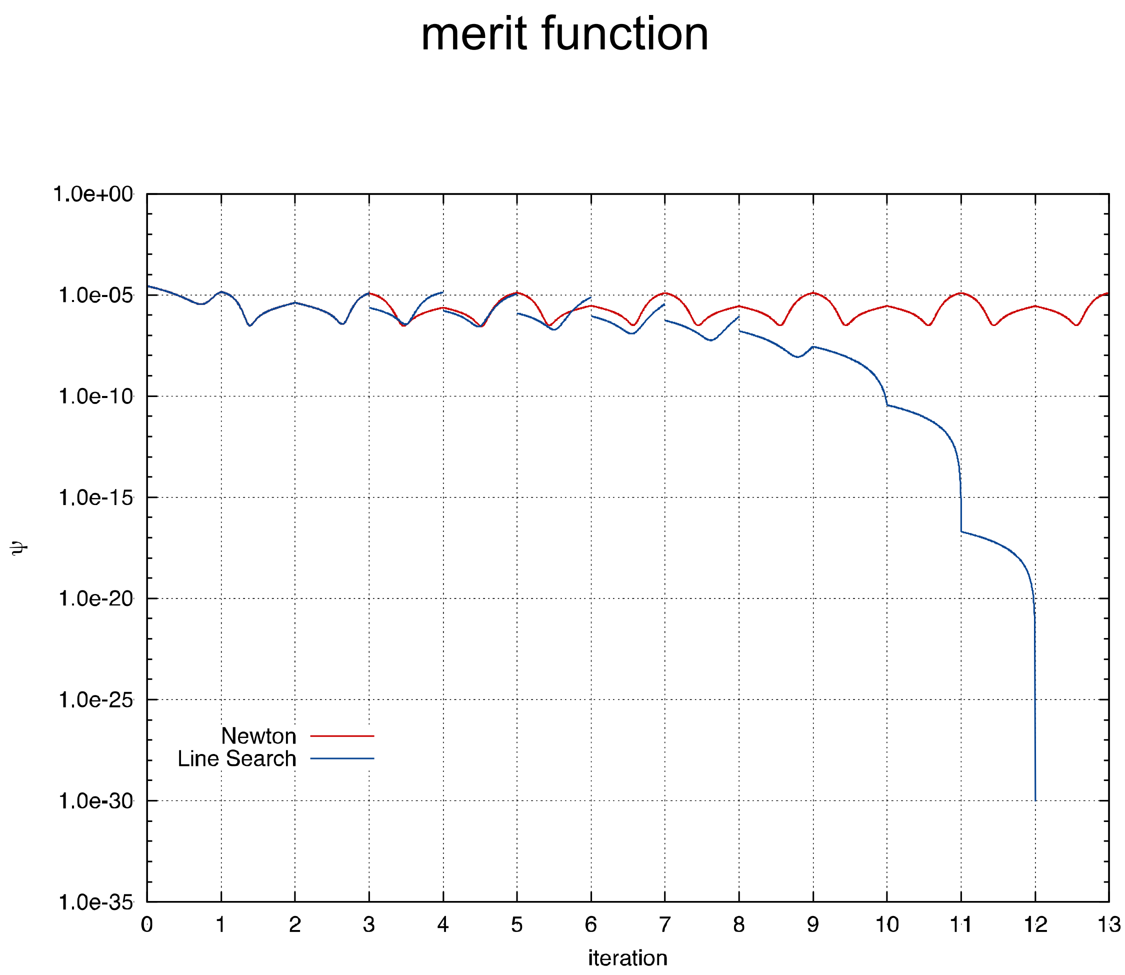


Hosford (a=8)

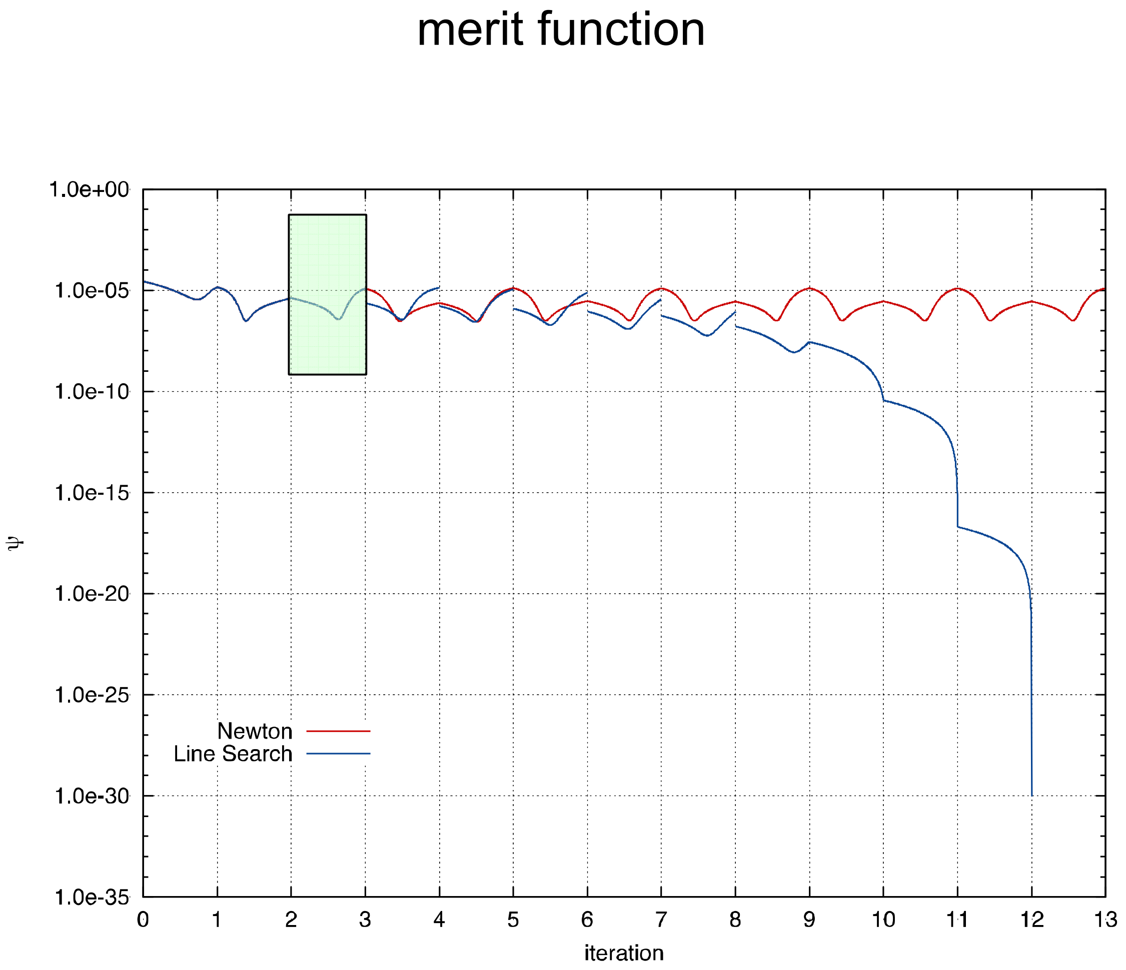
Newton and line search algorithms



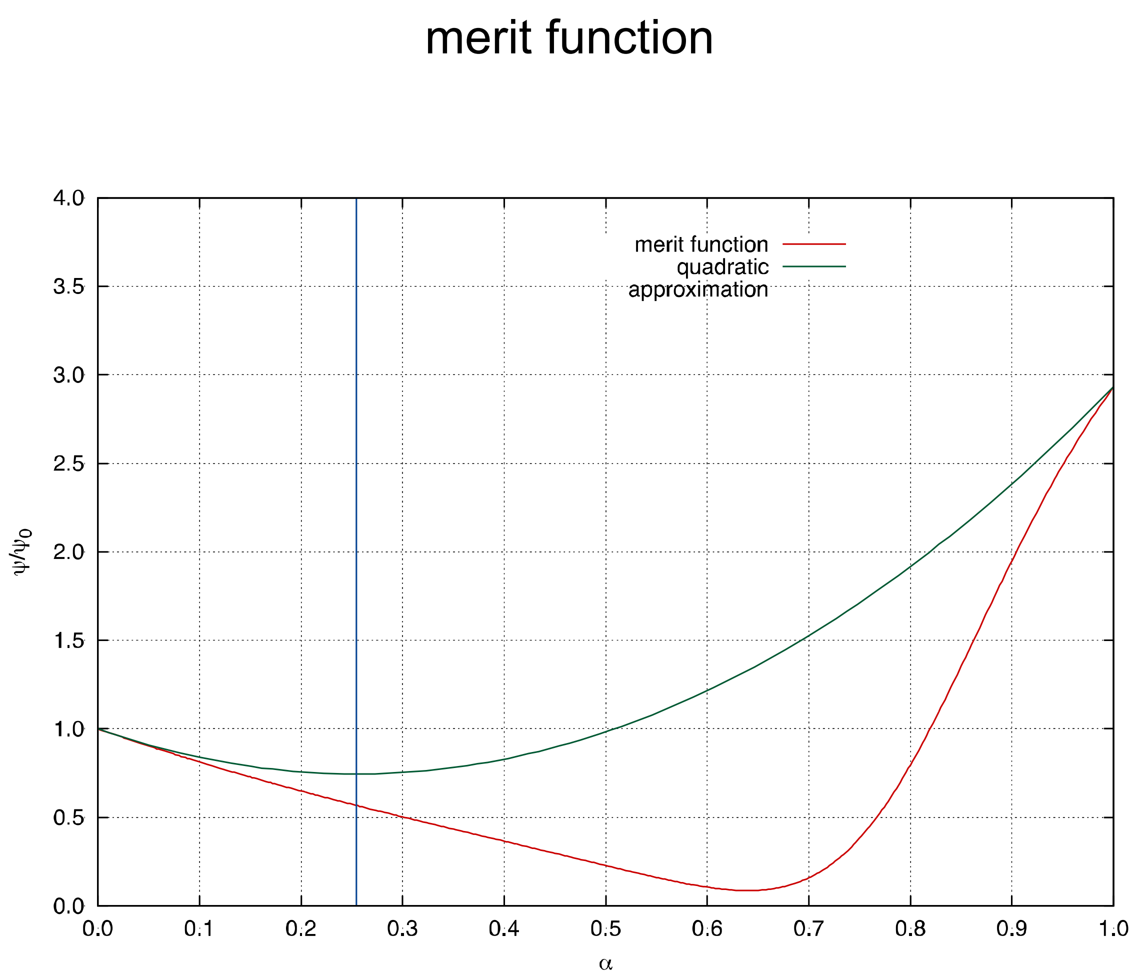
Hosford (a=8)


merit function
1.0e+00
1.0e-05
1.0e-10
1.0e-15
1.0e-20
1.0e-25
1.0e-30
1.0e-35
ψ
0
1
2
3
4
5
6
7
8
9
10
11
12
13
iteration
Newton
Line Search

Hosford (a=8)


merit function
1.0e+00
1.0e-05
1.0e-10
1.0e-15
1.0e-20
1.0e-25
1.0e-30
1.0e-35
ψ
0
1
2
3
4
5
6
7
8
9
10
11
12
13
iteration
Newton
Line Search

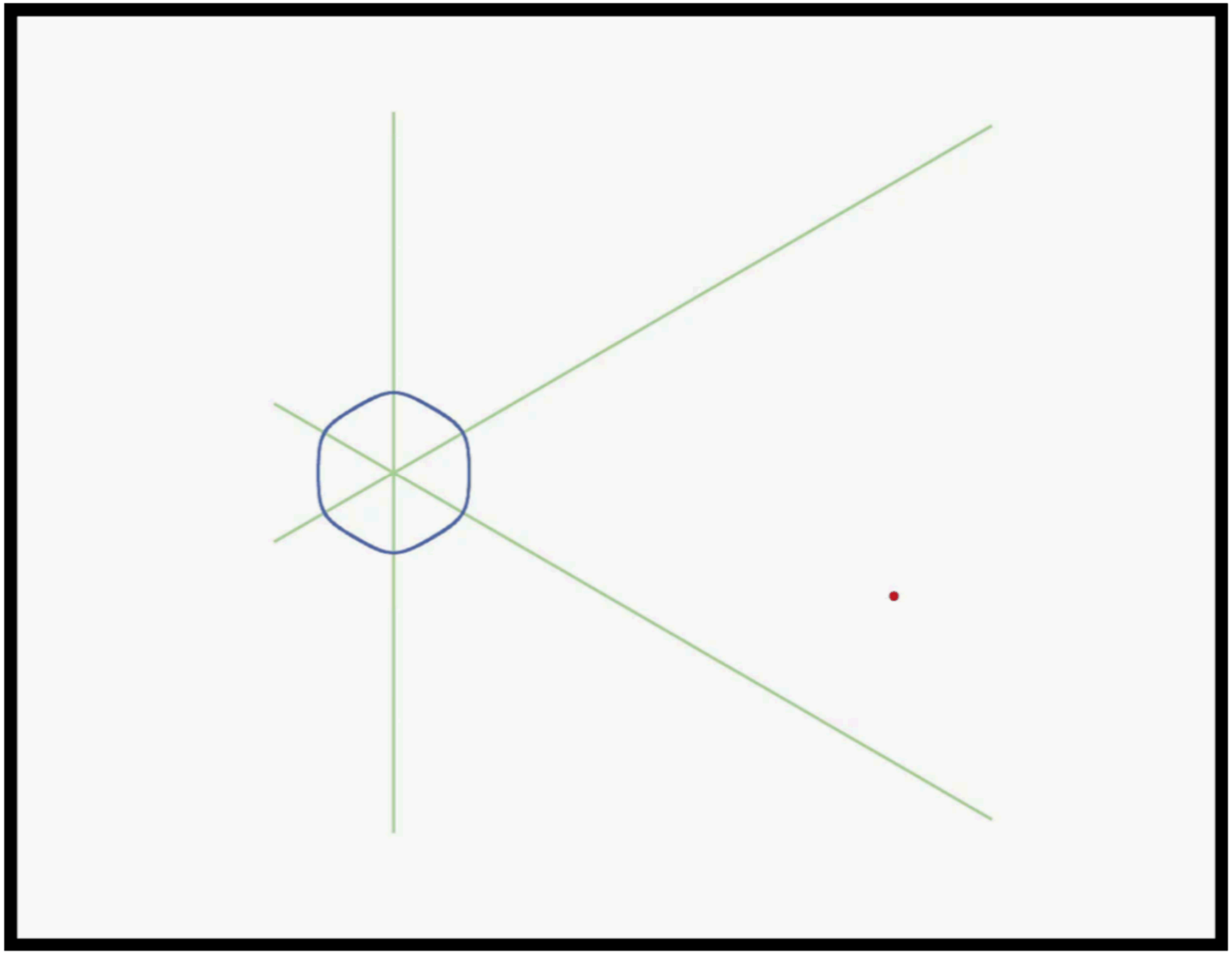
Hosford (a=8)



merit function
merit function
quadratic approximation
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
ψ/ψ0
α

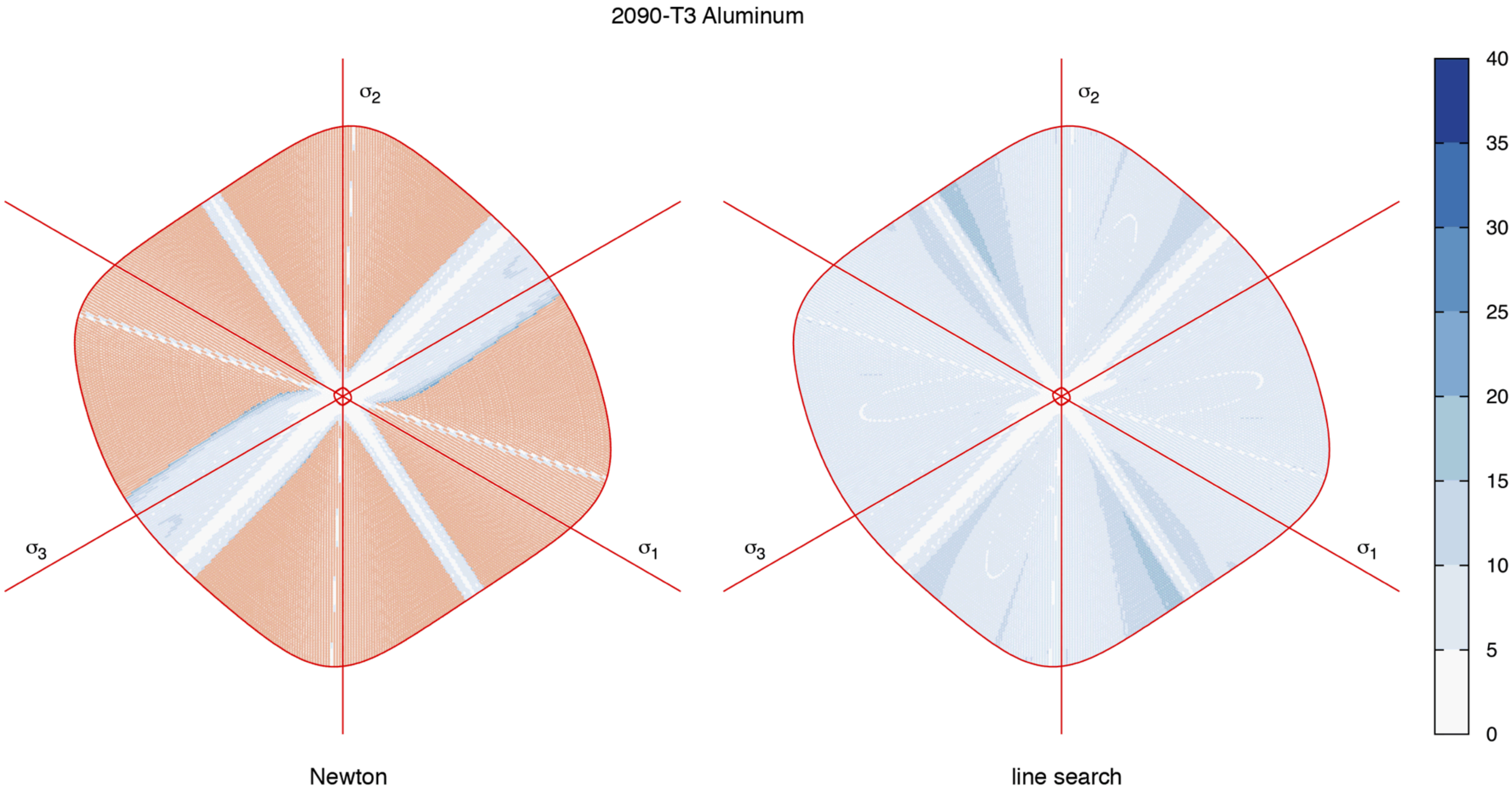
Hosford (a=8)

return map with hardening



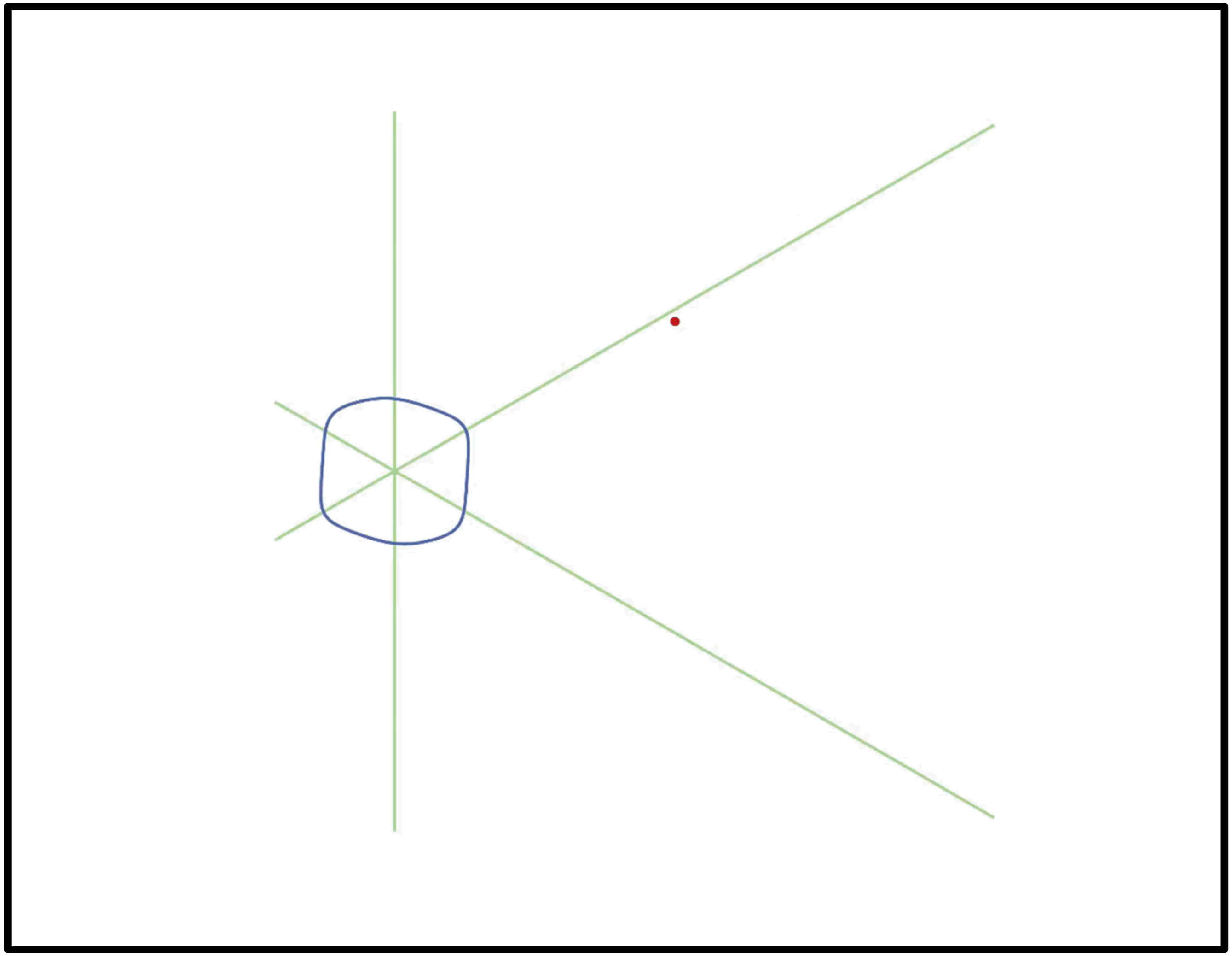
Barlat Model (Yld2004-18p)

Barlat Model



Aligned with Material Axes

line search algorithm



Orthotropic Plasticity Models

$$\begin{Bmatrix} s'_{11} \\ s'_{22} \\ s'_{33} \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0 & -c'_{12} & -c'_{13} & 0 & 0 & 0 \\ -c'_{21} & 0 & -c'_{23} & 0 & 0 & 0 \\ -c'_{31} & -c'_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c'_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c'_{66} \end{bmatrix} \begin{Bmatrix} s_{11} \\ s_{22} \\ s_{33} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

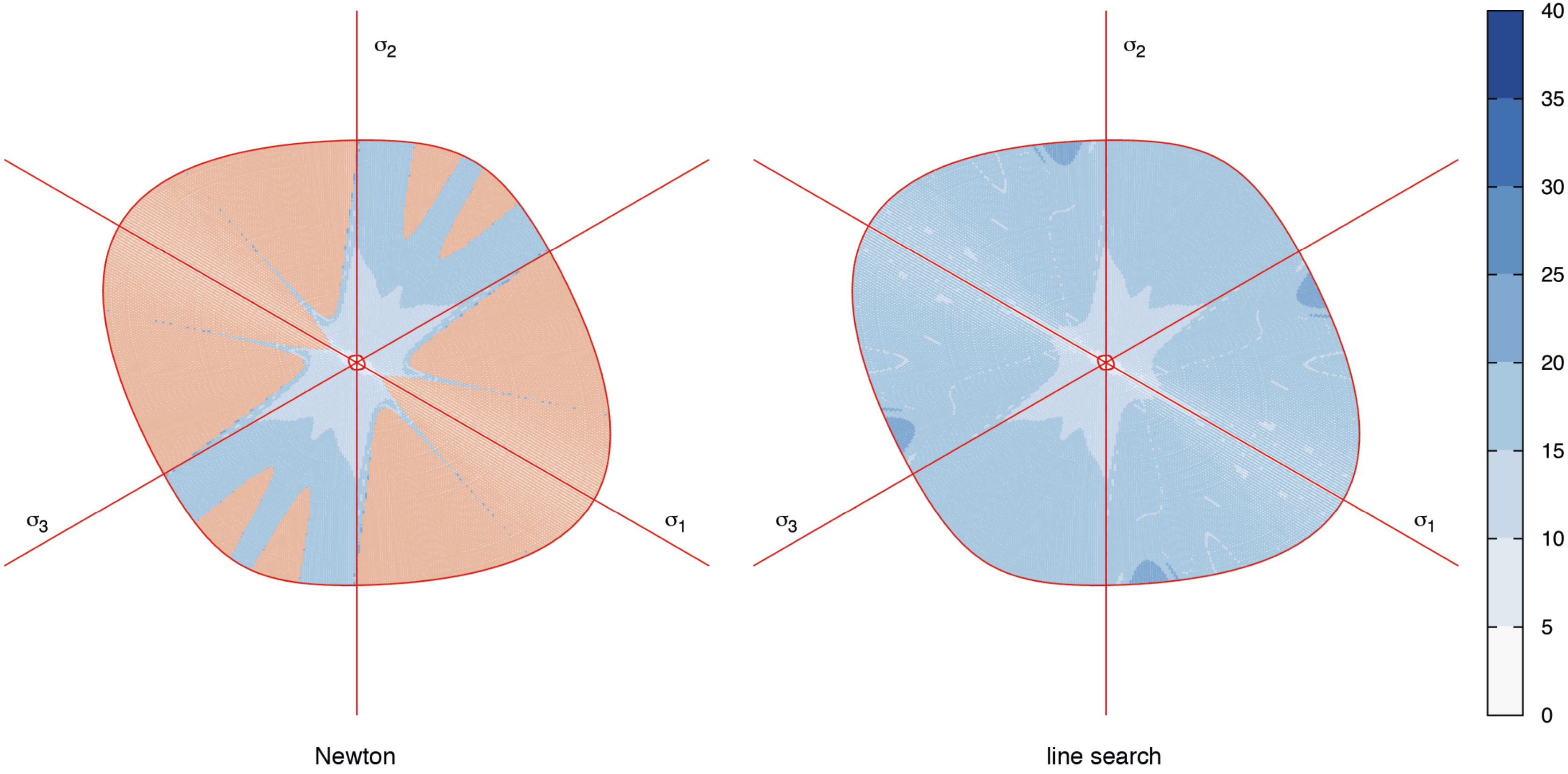
← Components in material coordinate system

$$s'_1 = s'_{11} \quad ; \quad s'_2 = s'_{22} \quad ; \quad s'_3 = s'_{33}$$

If trial stress is aligned with material coordinate system, then the solution stays aligned with the material coordinate system

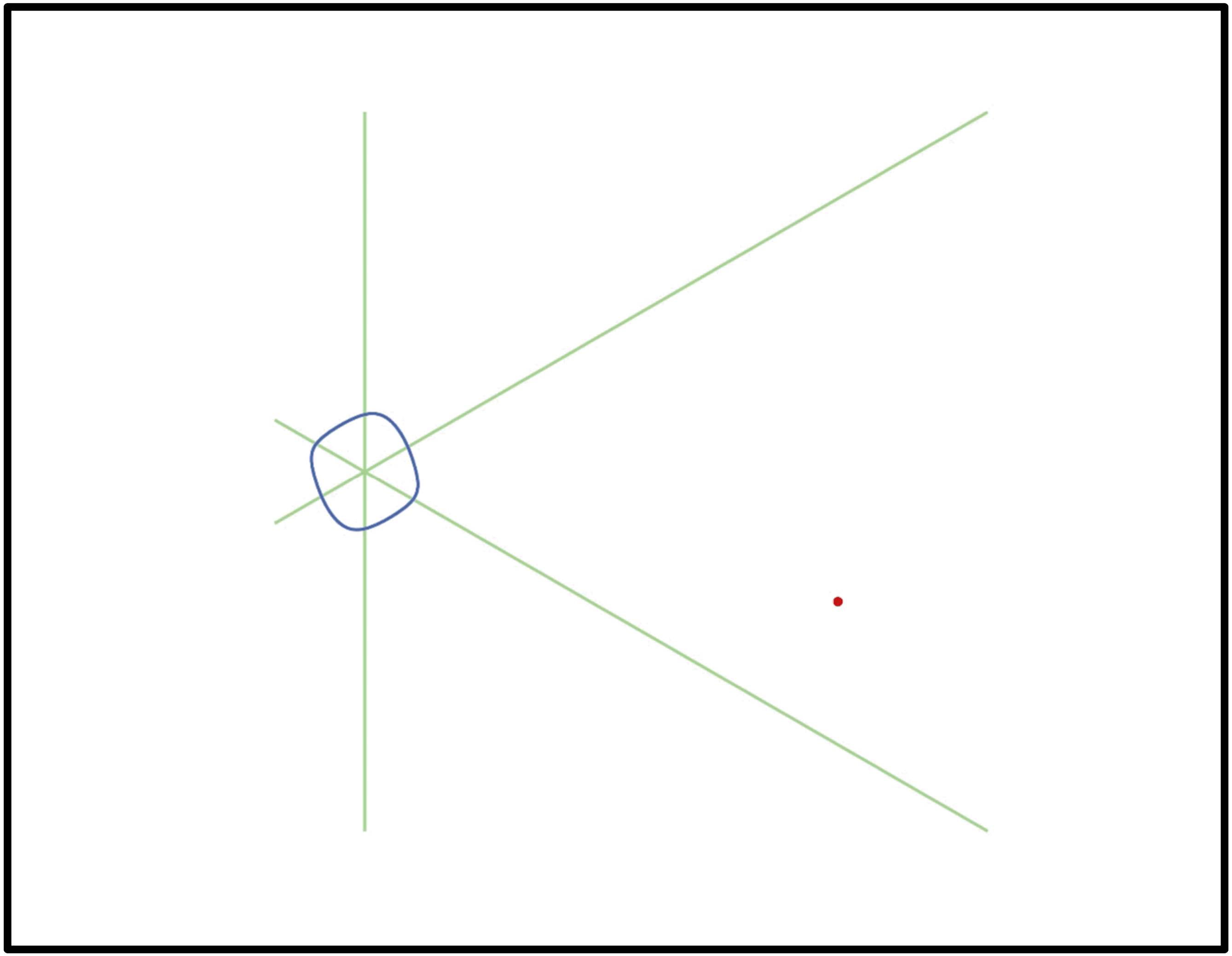
Barlat Model

45 degrees about x_1 axis



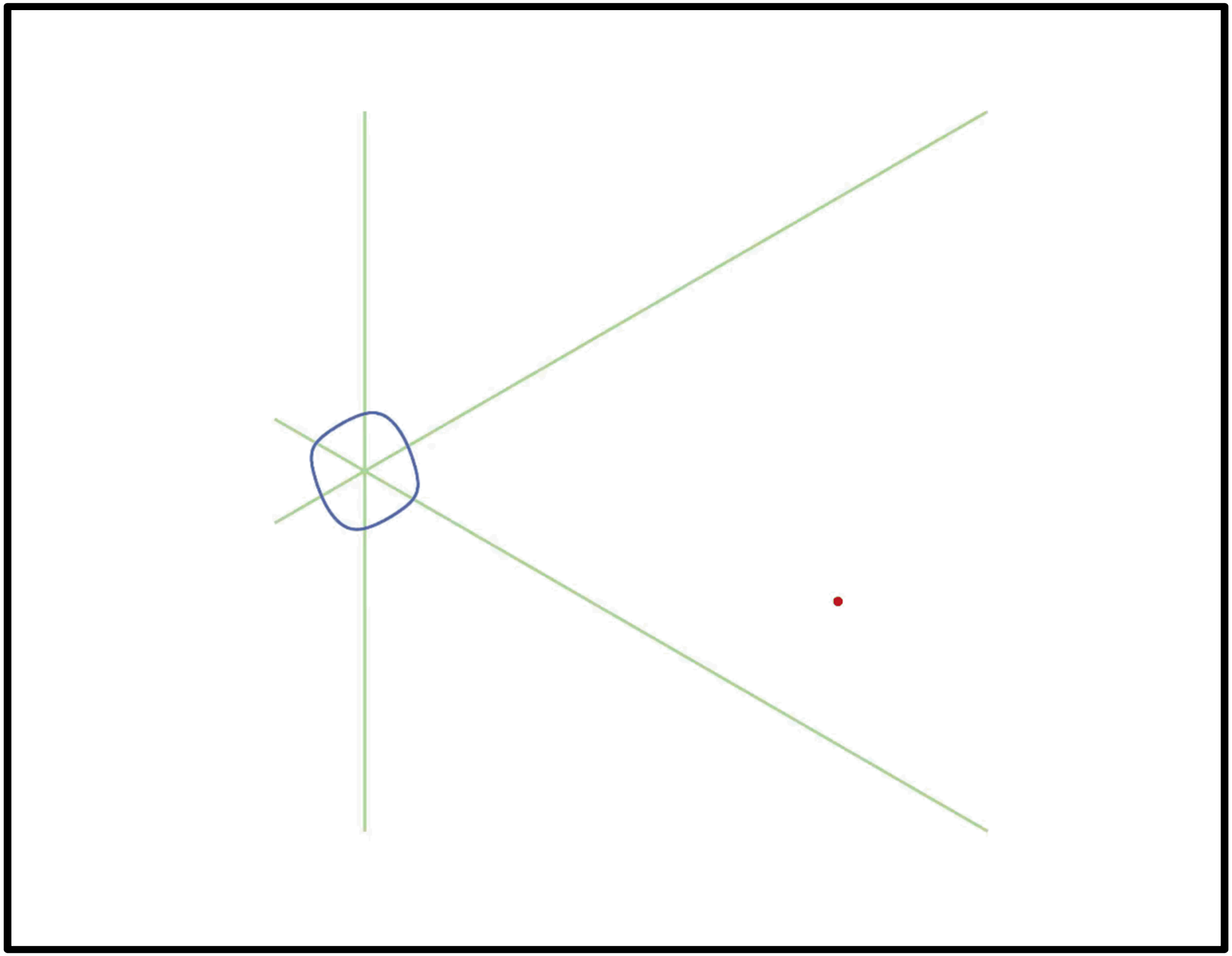
Not Aligned with Material Axes

Newton algorithm

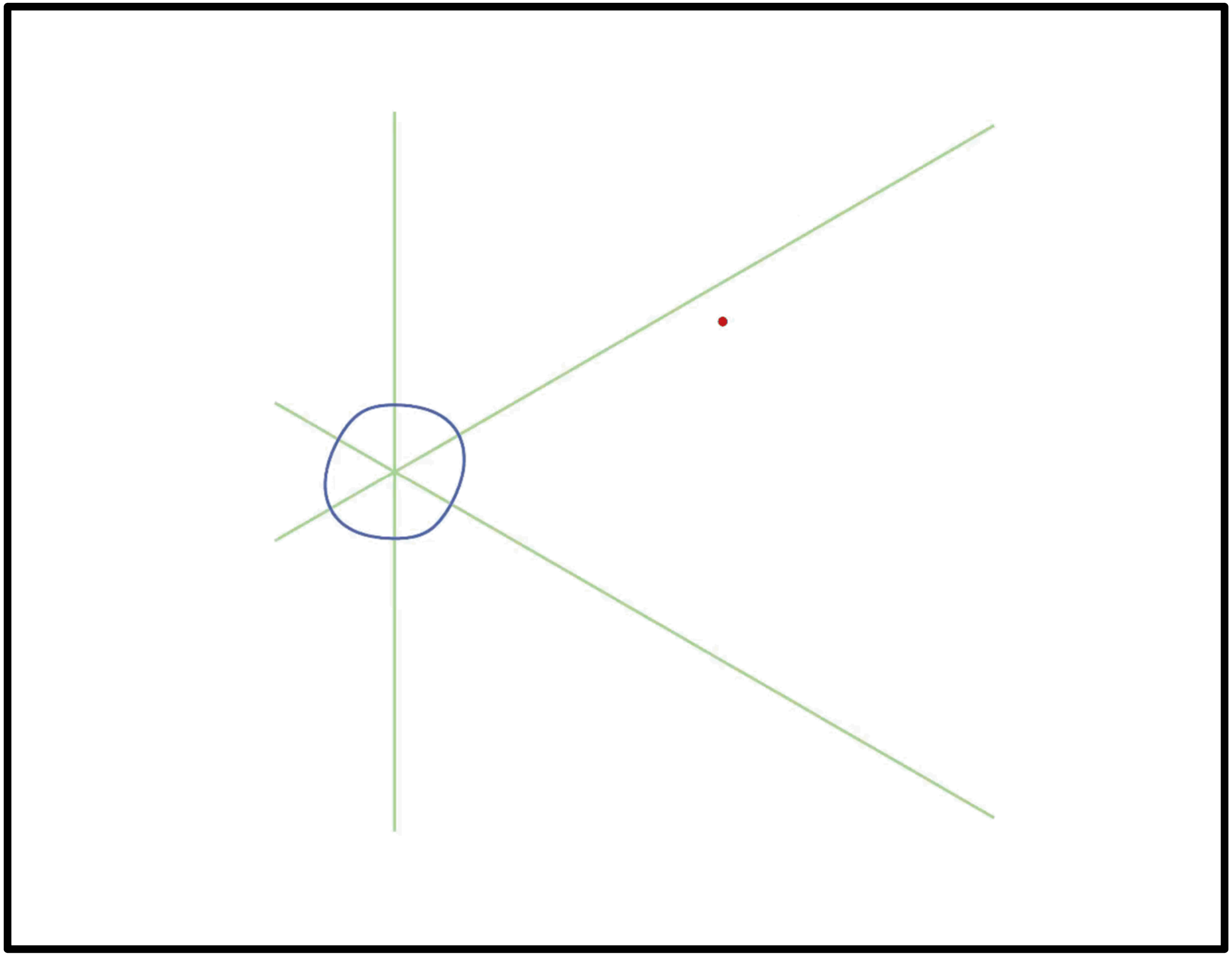


Not Aligned with Material Axes

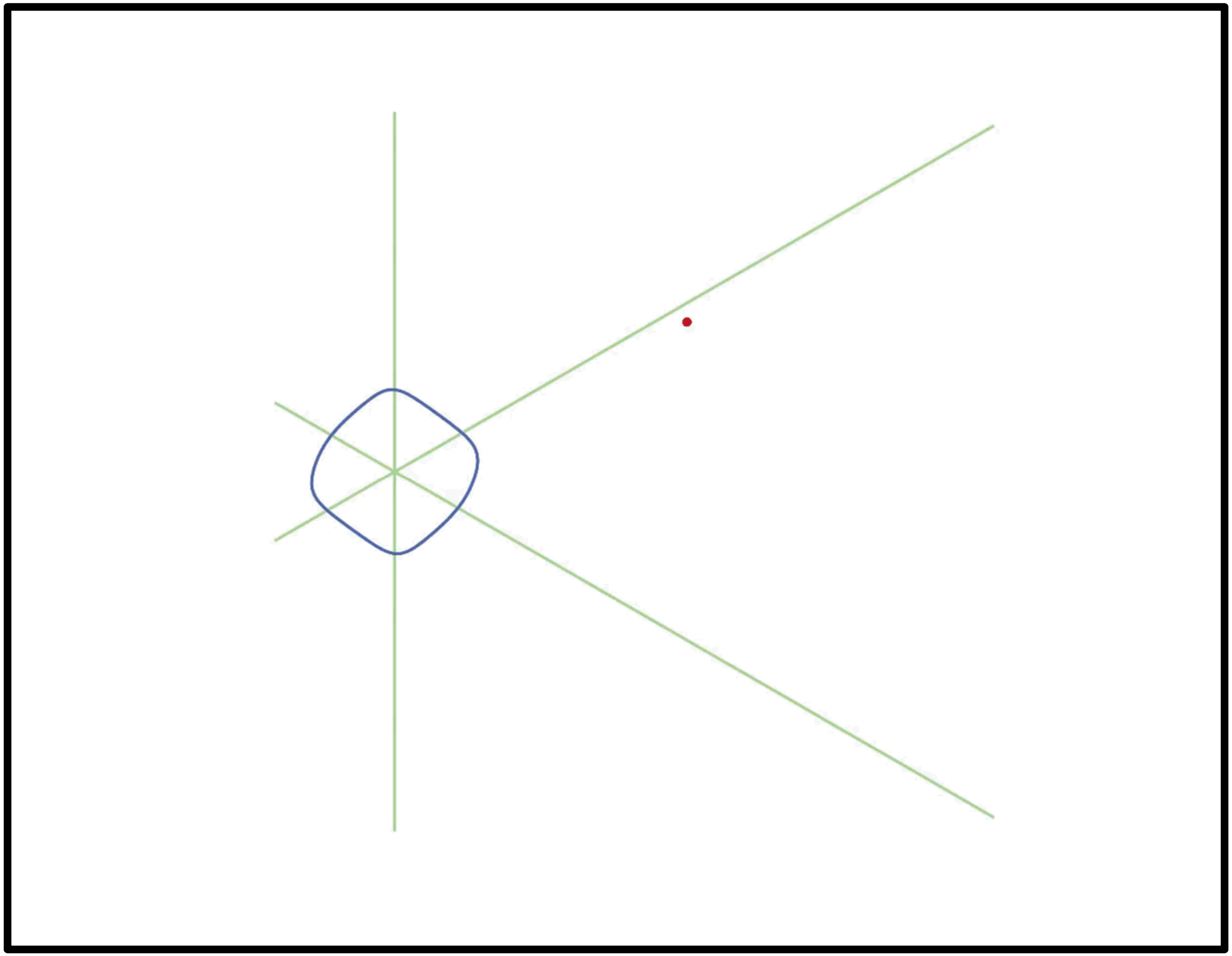
line search algorithm



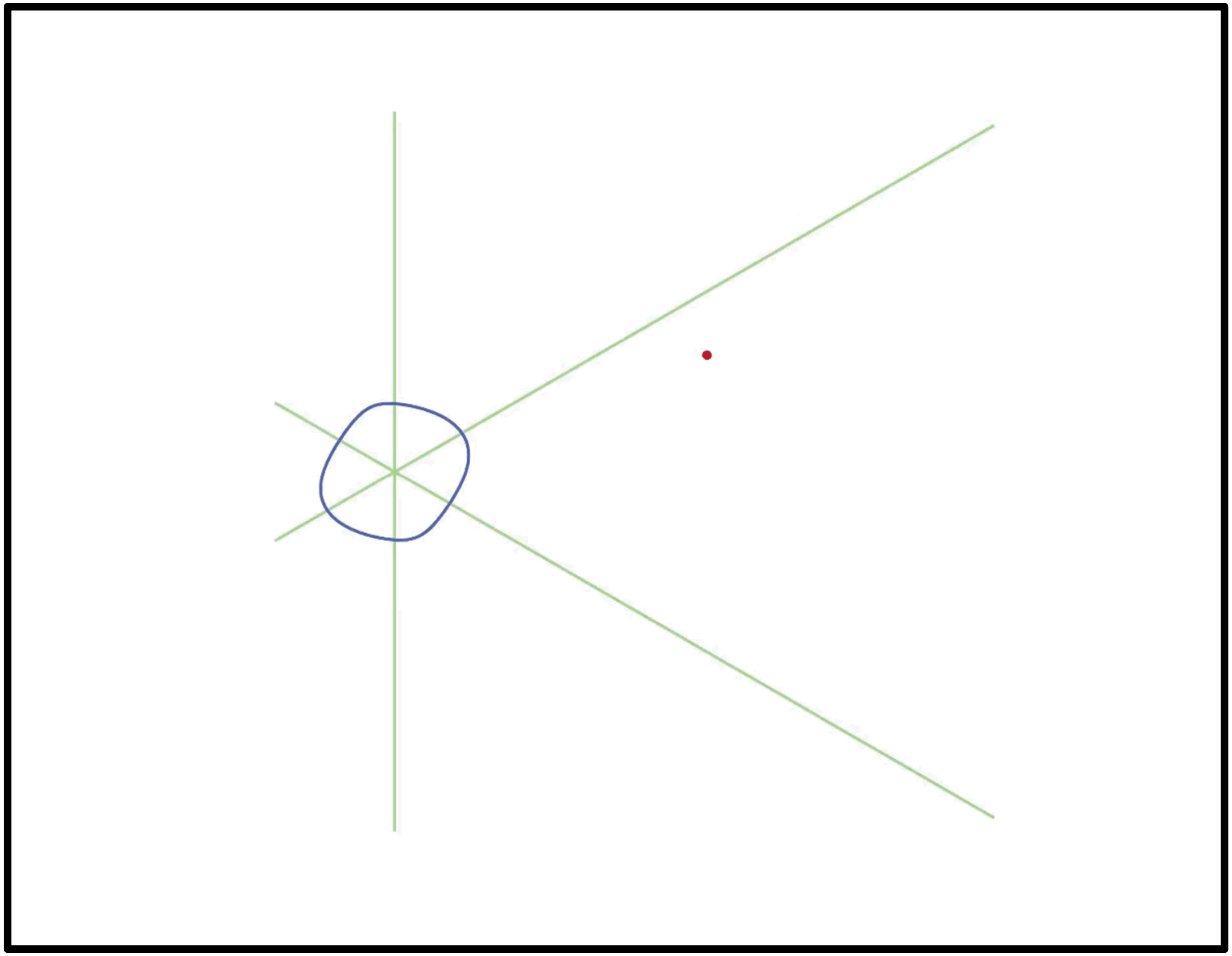
Not Aligned with Material Axes



Not Aligned with Material Axes



Not Aligned with Material Axes



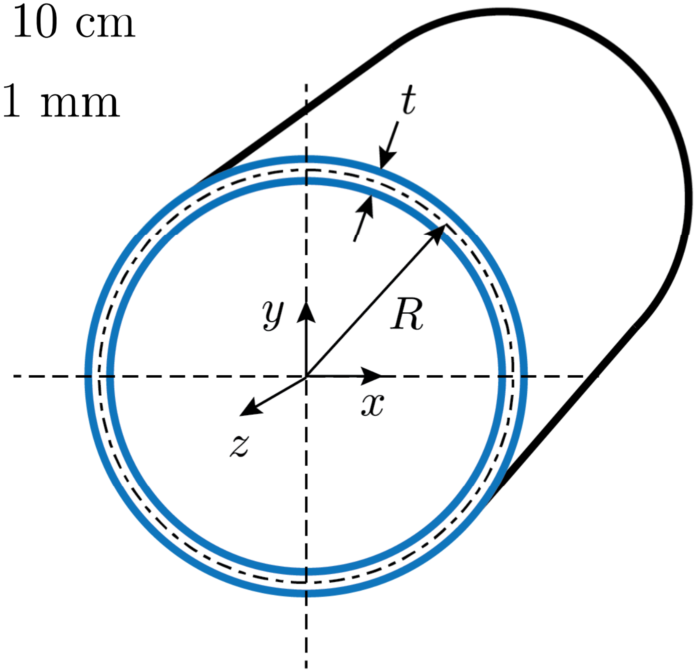
Example Problem

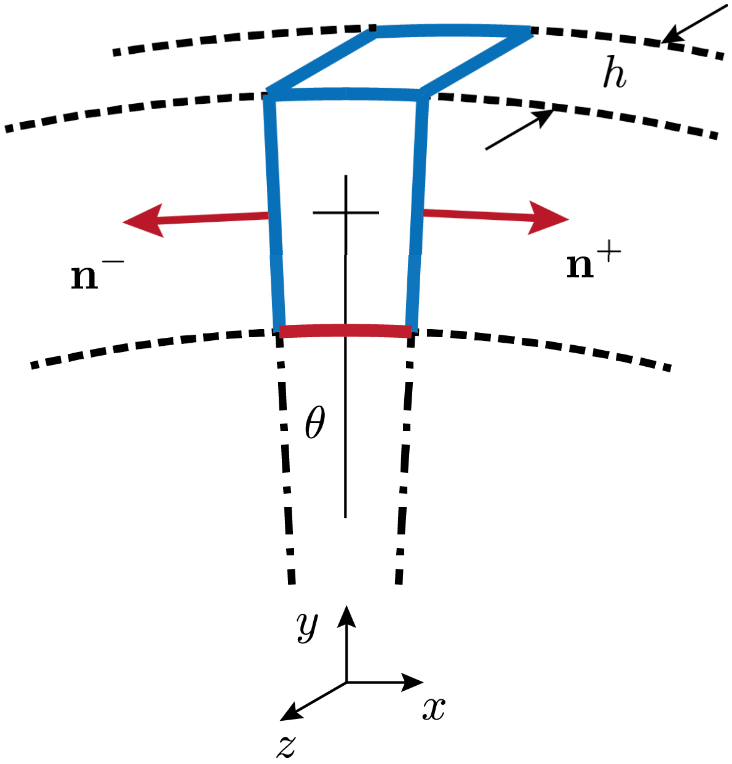
Example Problem

2090-T3 Aluminum

$R = 10$ cm

$h = 1$ mm



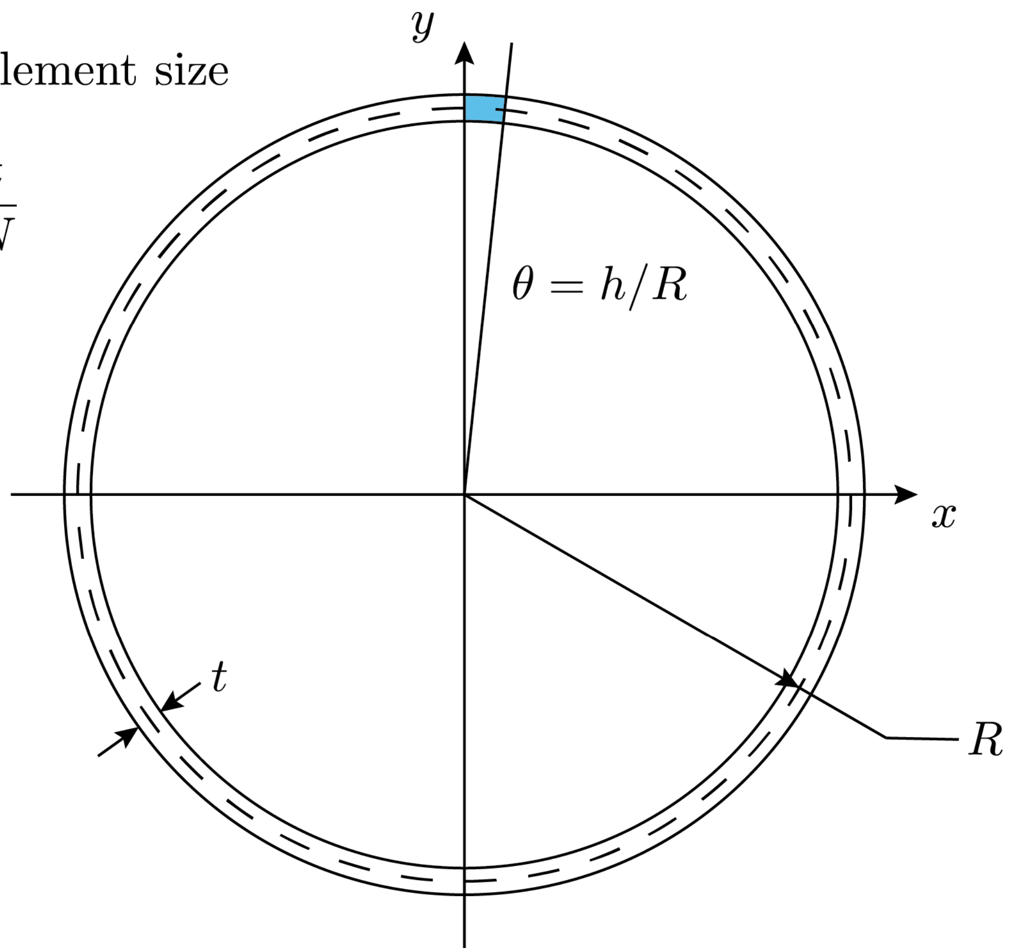


$$\bar{\sigma} = 200\left(1 - \exp(-20\,\bar{\varepsilon}^p)\right)\ \text{MPa} \quad \rightarrow \quad \sigma_y = 200\ \text{MPa}$$

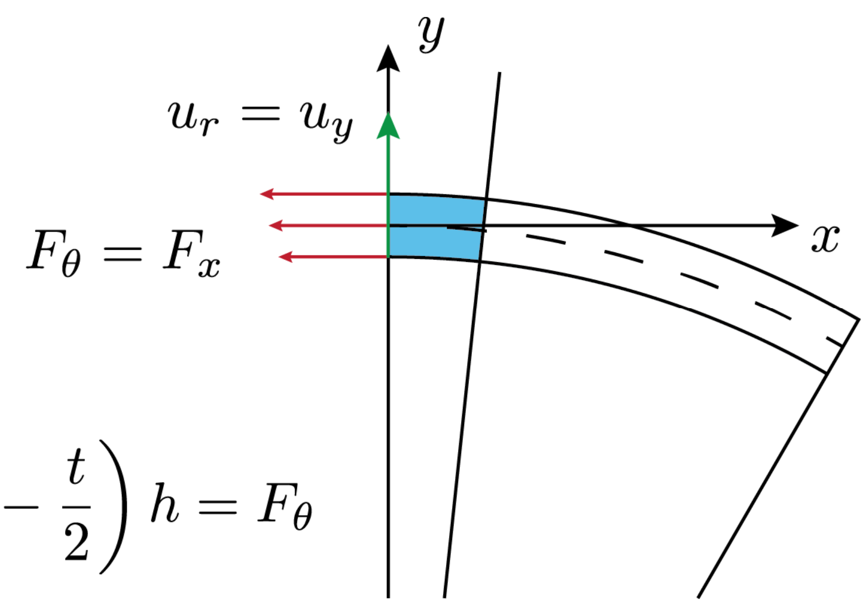
Example Problem

$h -$ element size

$$h = \frac{t}{N}$$



Example Problem

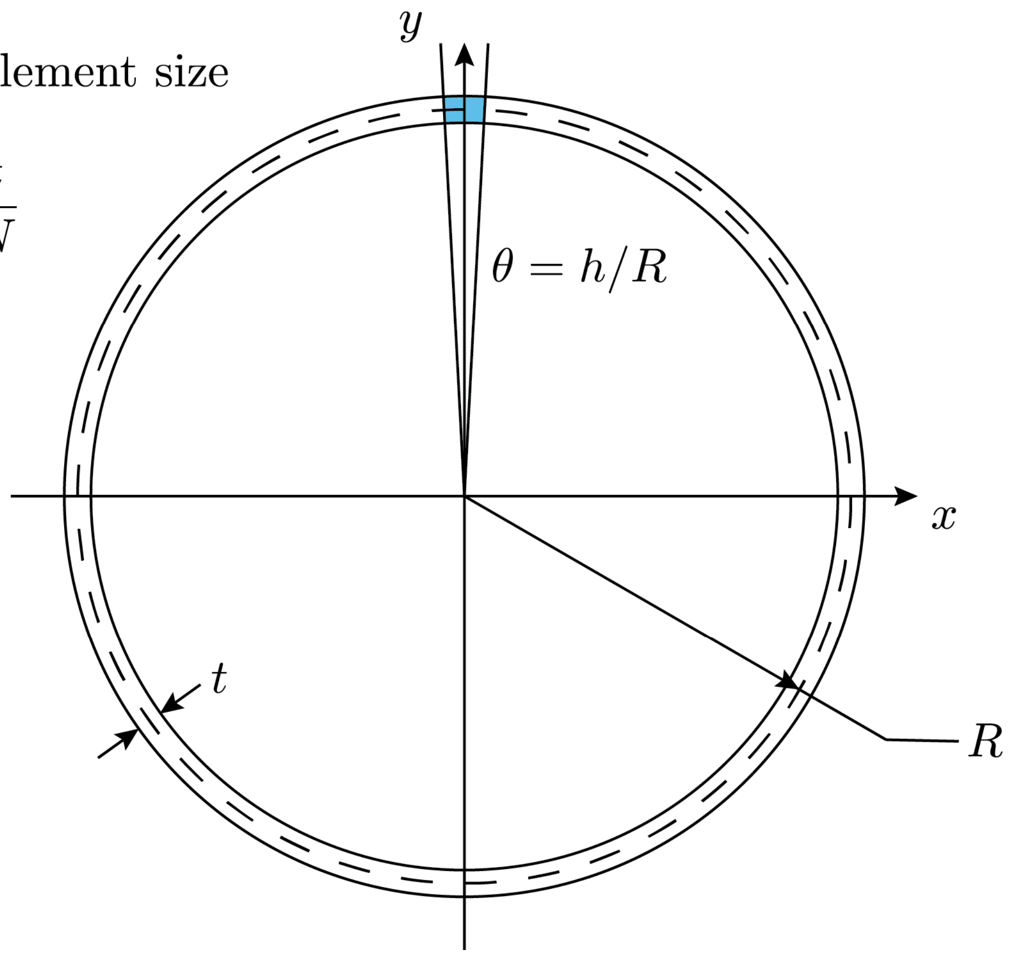


$$p\left(R + u_r - \frac{t}{2}\right)h = F_\theta$$

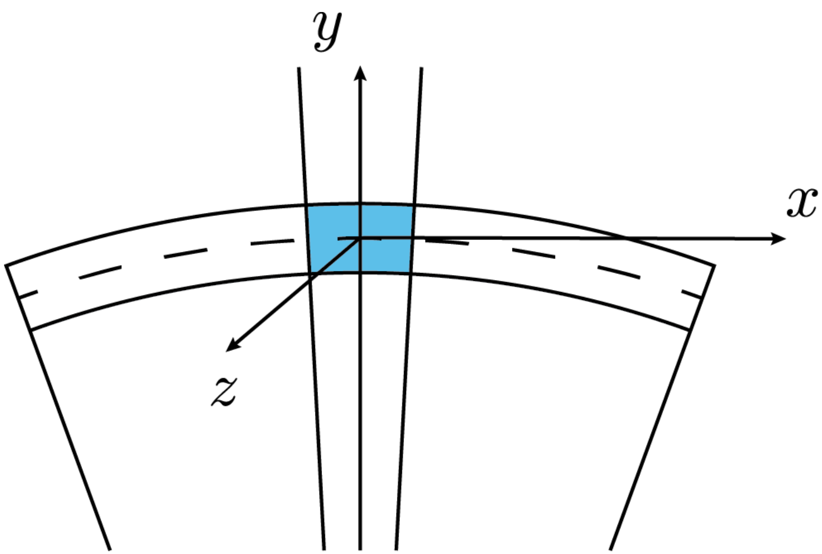
Example Problem

$h -$ element size

$$h = \frac{t}{N}$$



Example Problem

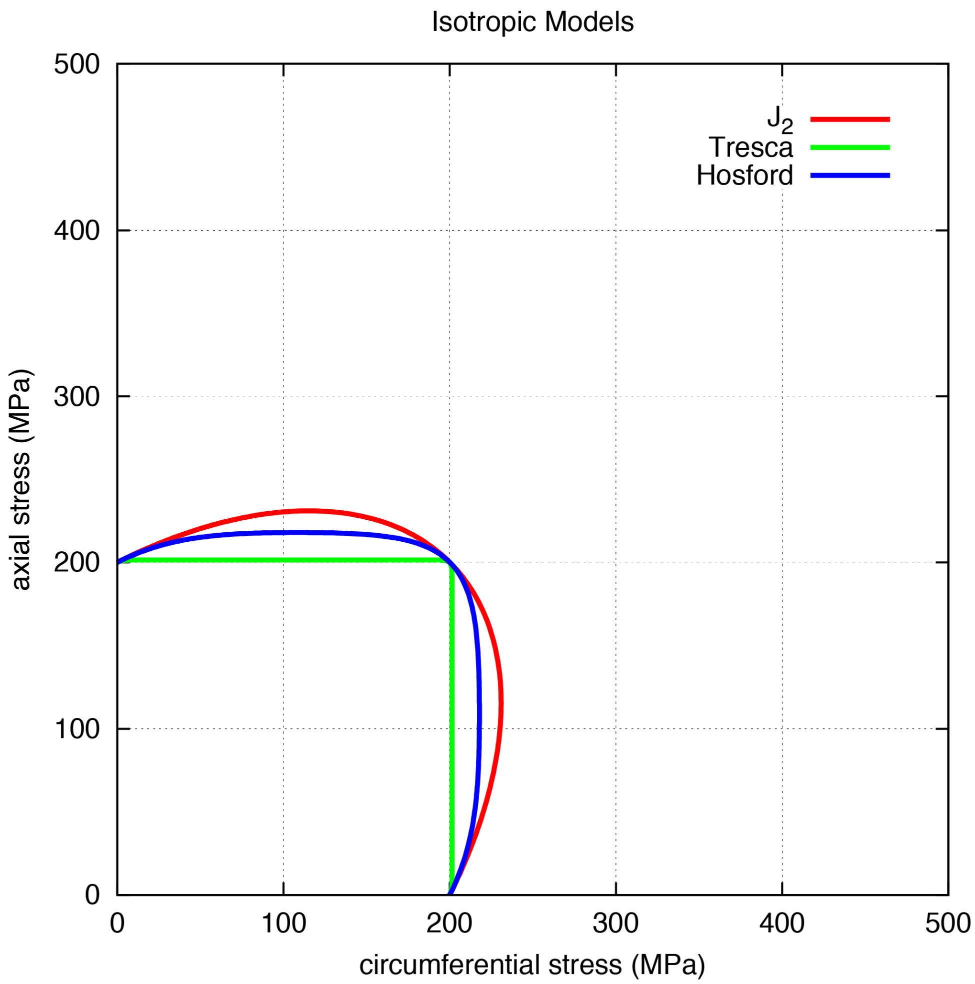


$$\sigma_{rr} = \sigma_{yy}$$

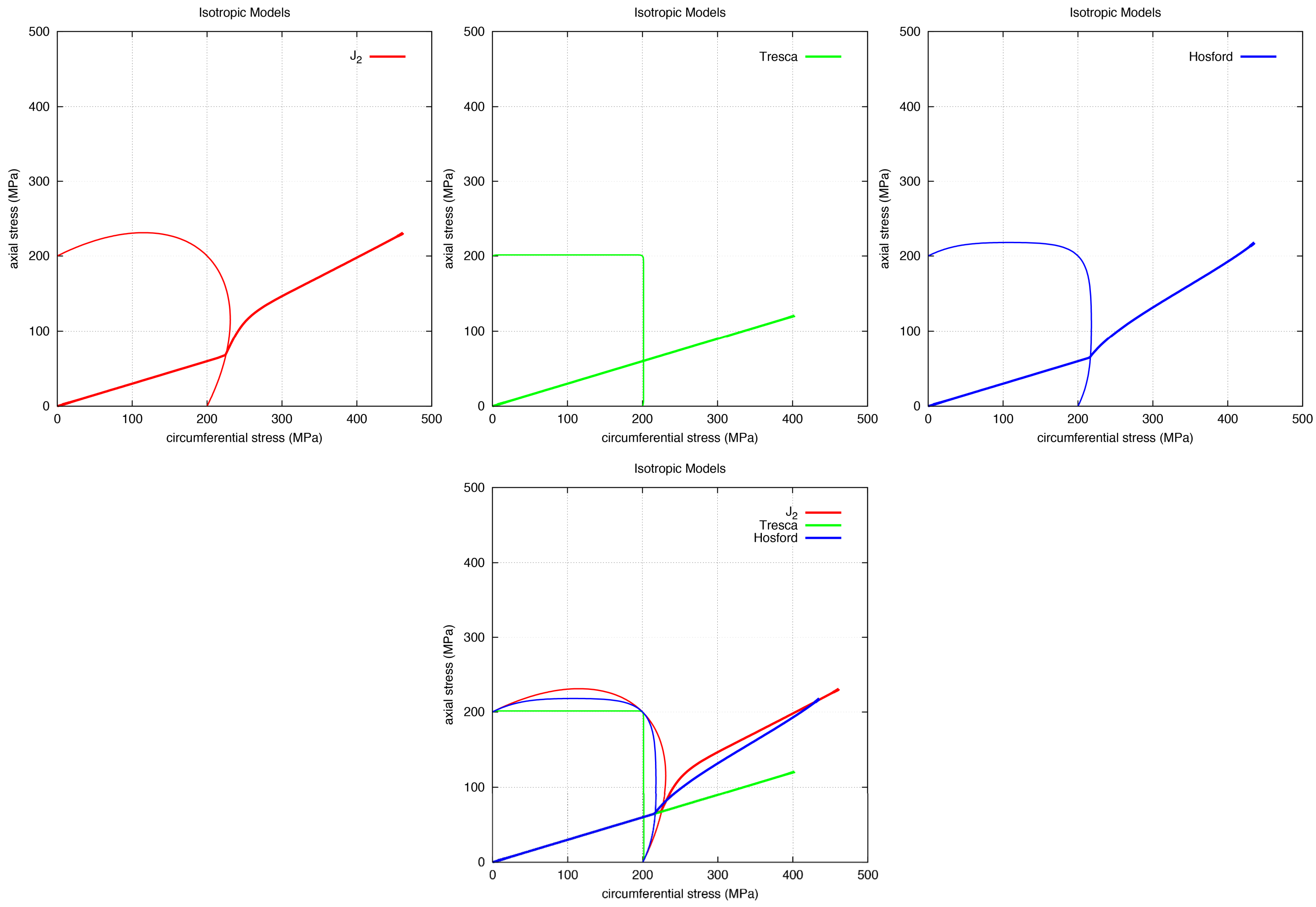
$$\sigma_{\theta\theta} = \sigma_{xx}$$

$$\sigma_{zz} = \sigma_{zz}$$

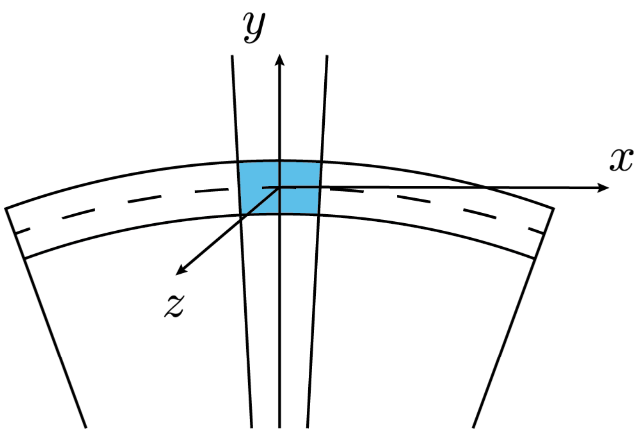
Yield Surface – Isotropic Models



Stress Paths – Isotropic Models



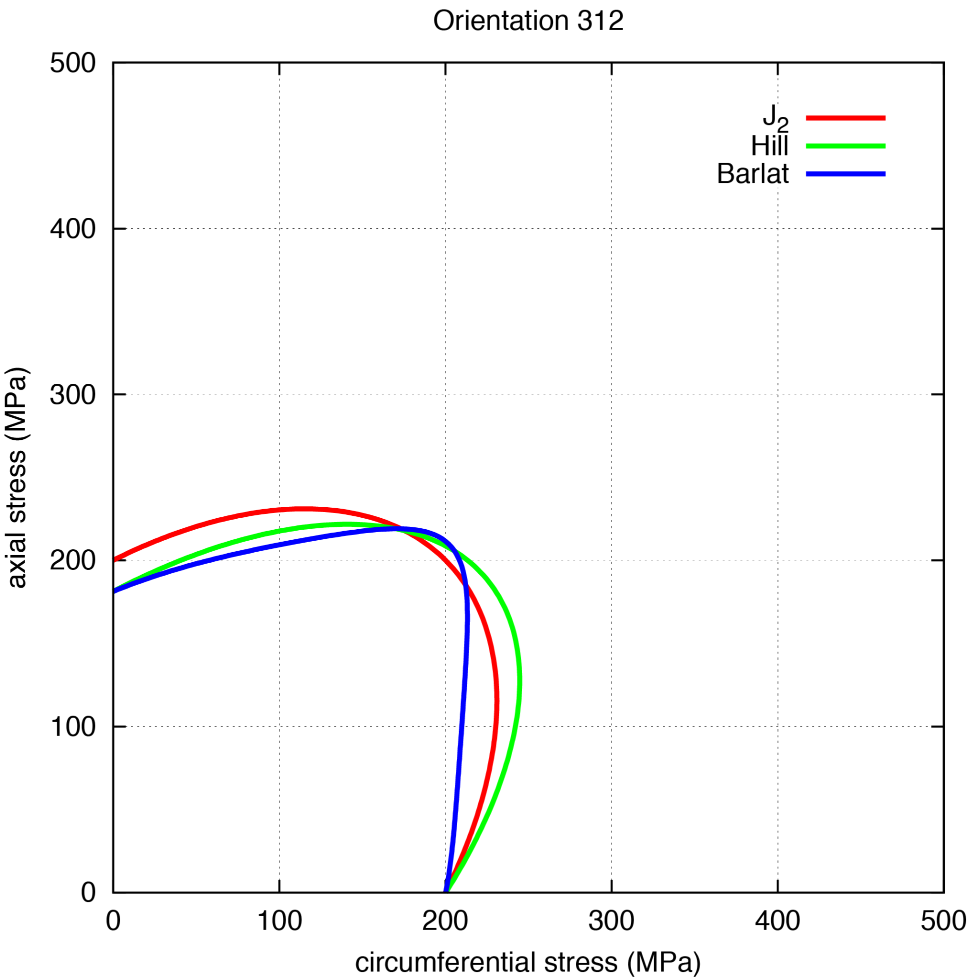
Yield Surface – Orthotropic Models



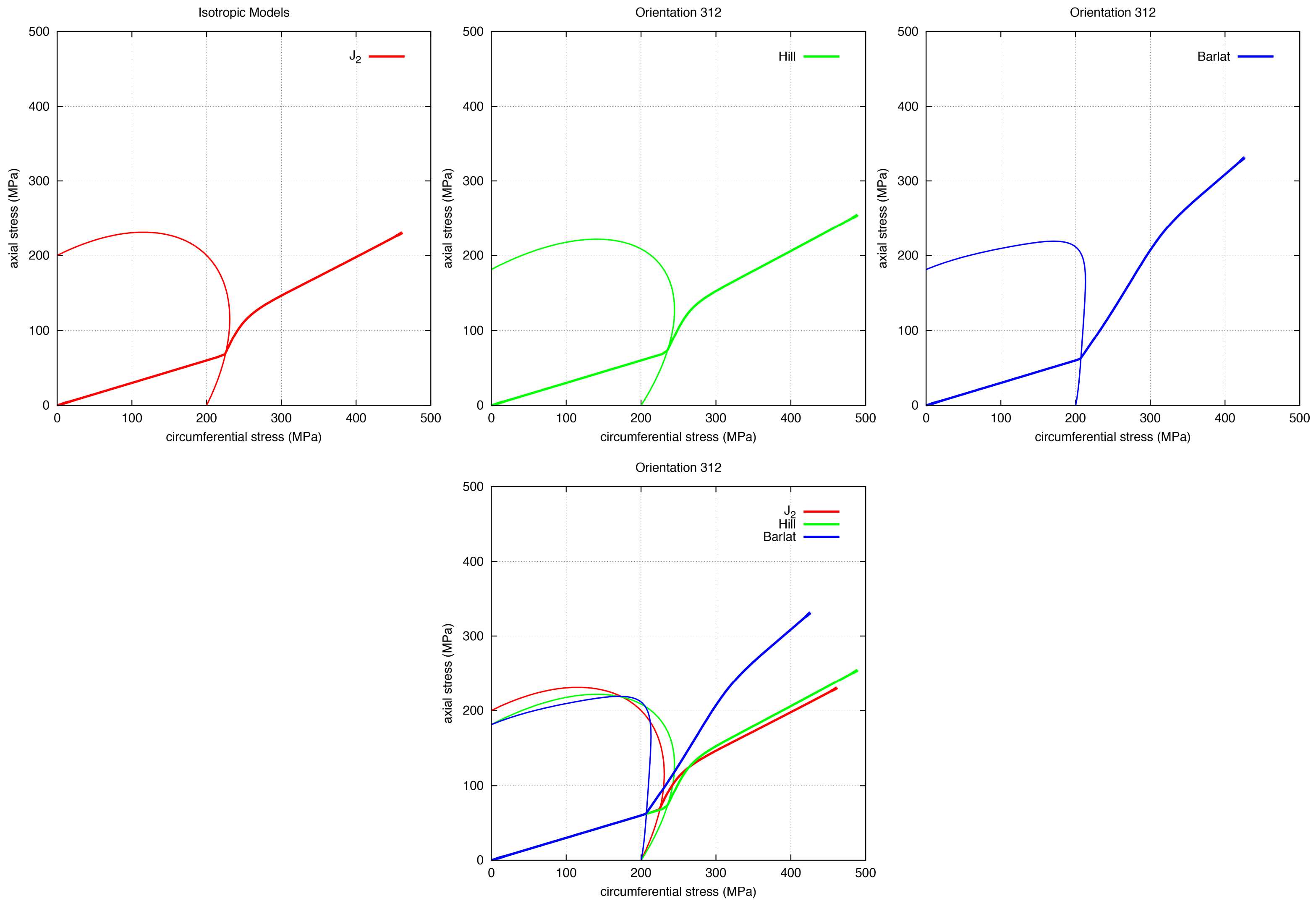
$$\sigma_{rr} = \sigma_{33}$$

$$\sigma_{\theta\theta} = \sigma_{11}$$

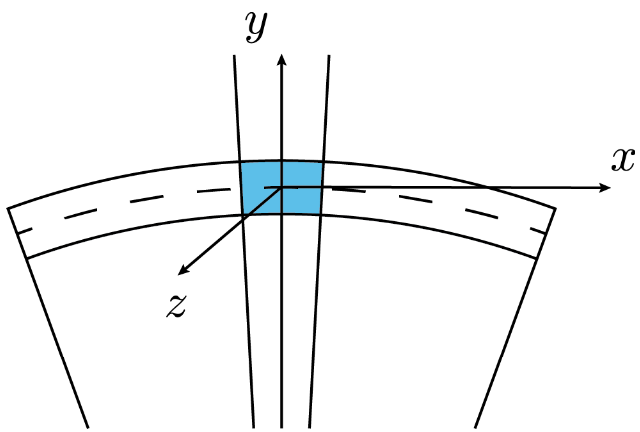
$$\sigma_{zz} = \sigma_{22}$$



Stress Paths – Orthotropic Models



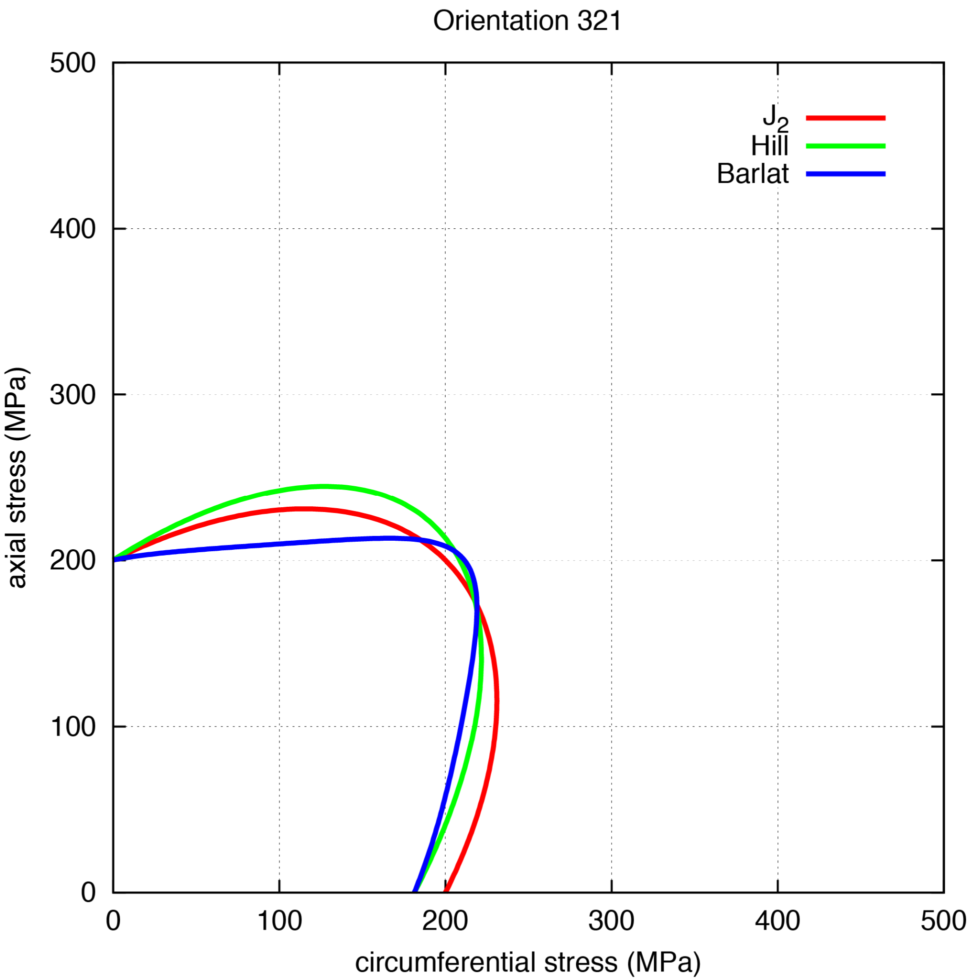
Yield Surface – Orthotropic Models



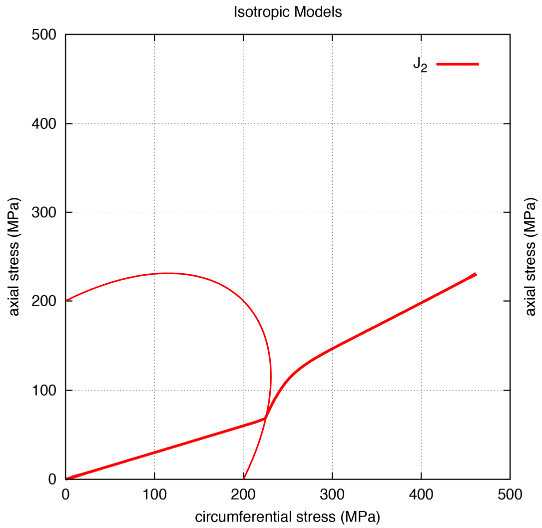
$$\sigma_{rr} = \sigma_{33}$$

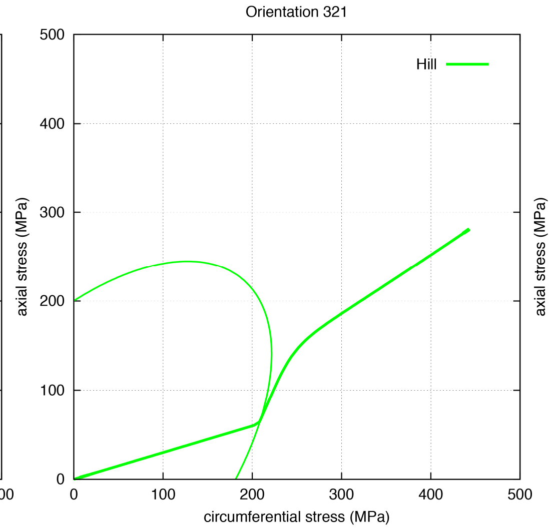
$$\sigma_{\theta\theta} = \sigma_{22}$$

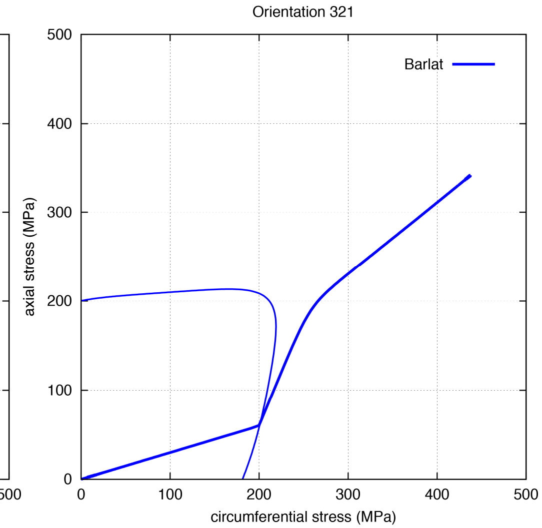
$$\sigma_{zz} = \sigma_{11}$$

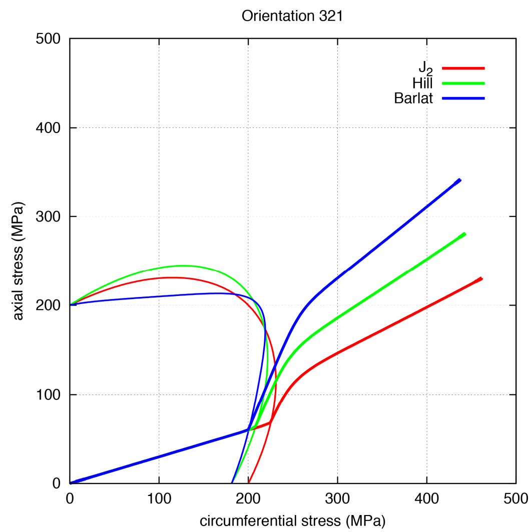


Stress Paths – Orthotropic Models

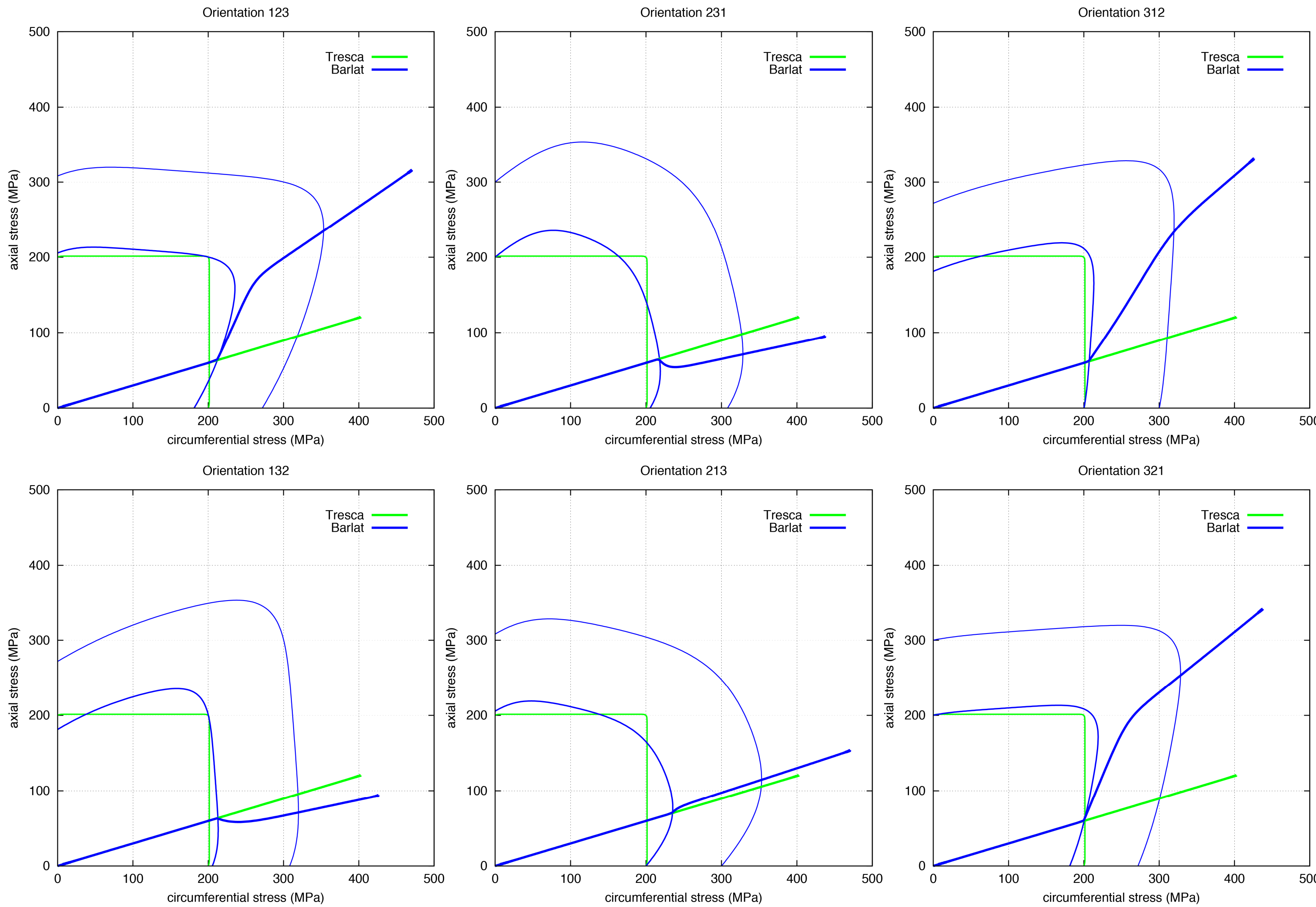








Stress Paths – Yield Surface Normal



Yield Surface – Orthotropic Models

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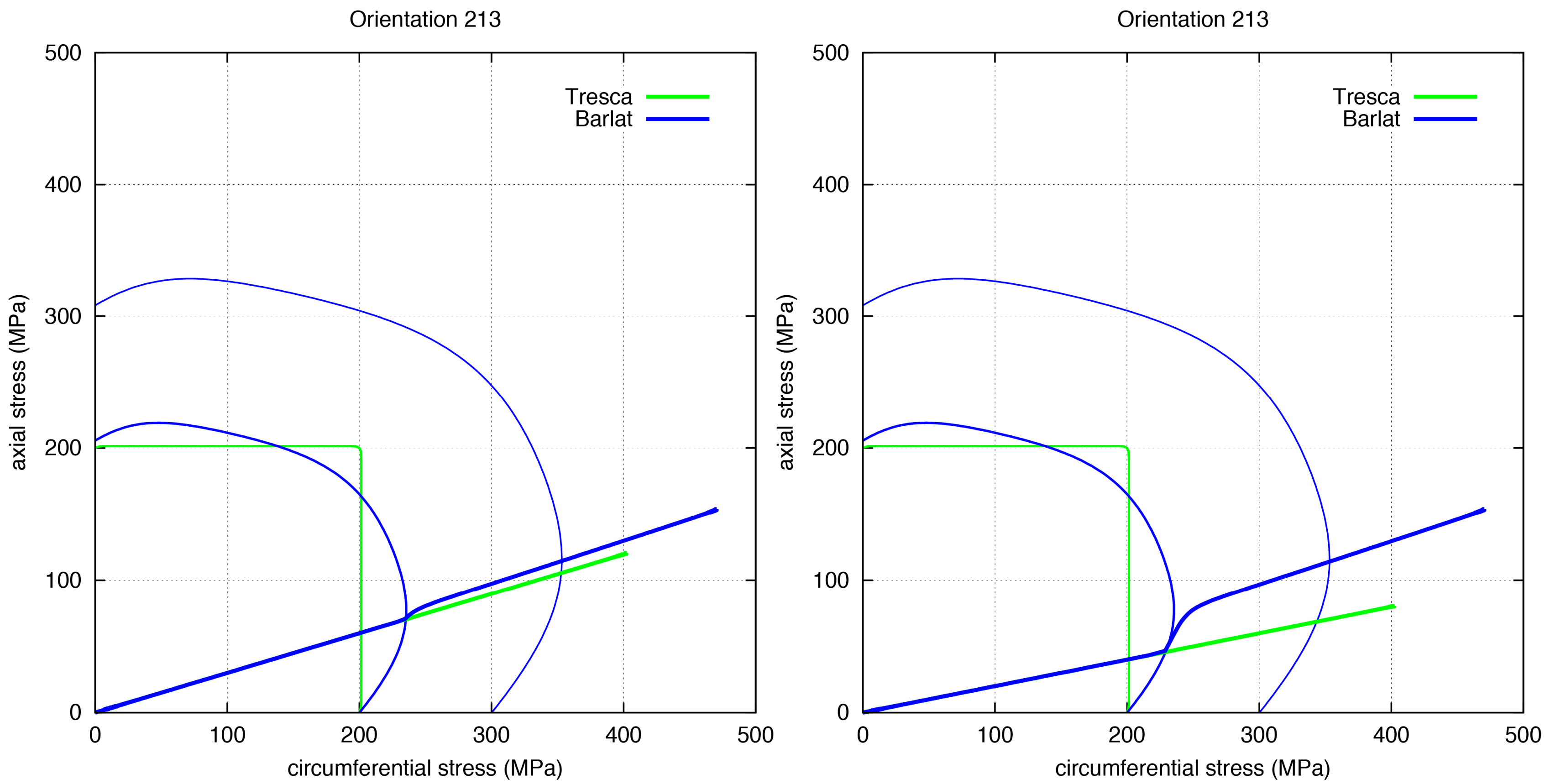
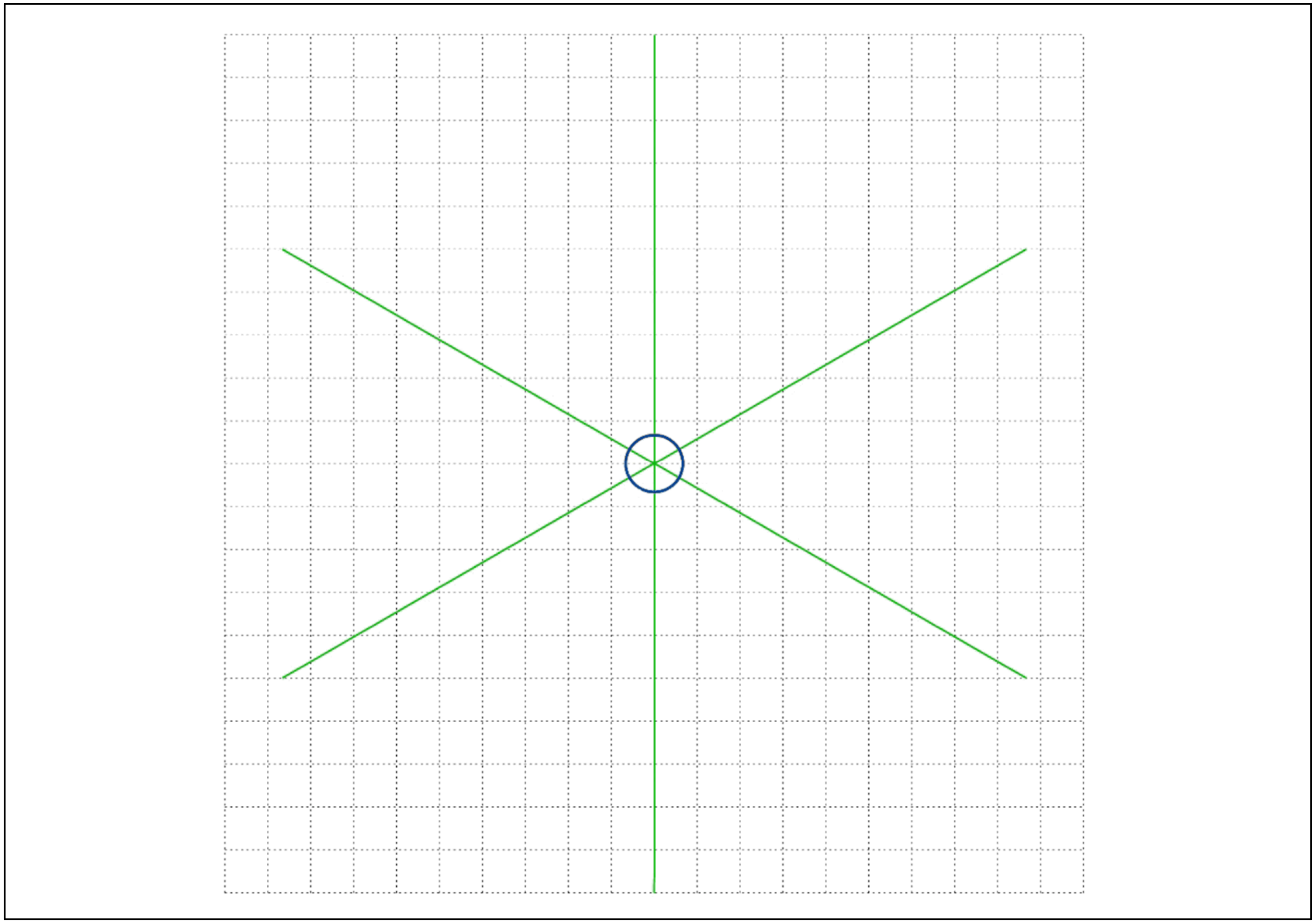
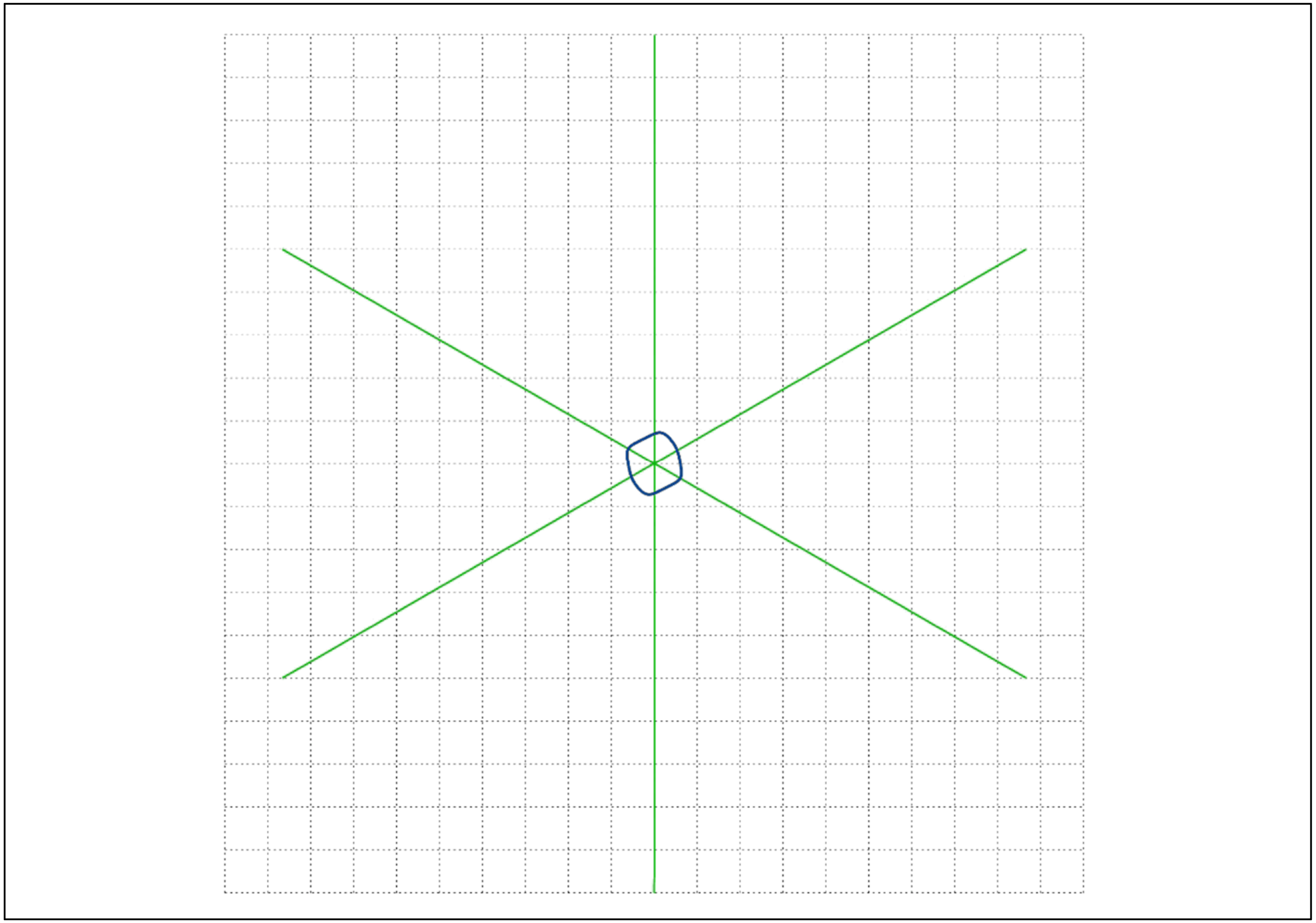


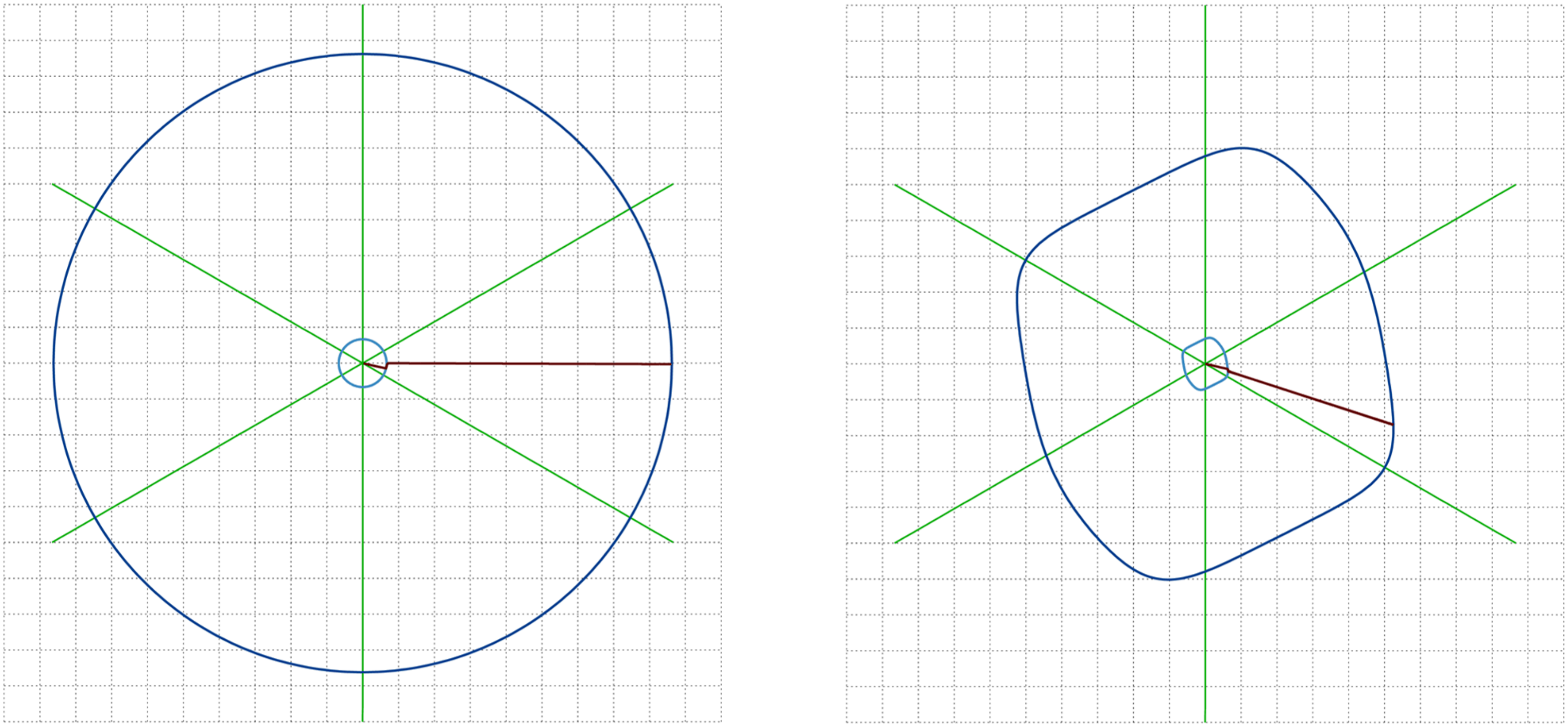
Stress Path – J2 Model



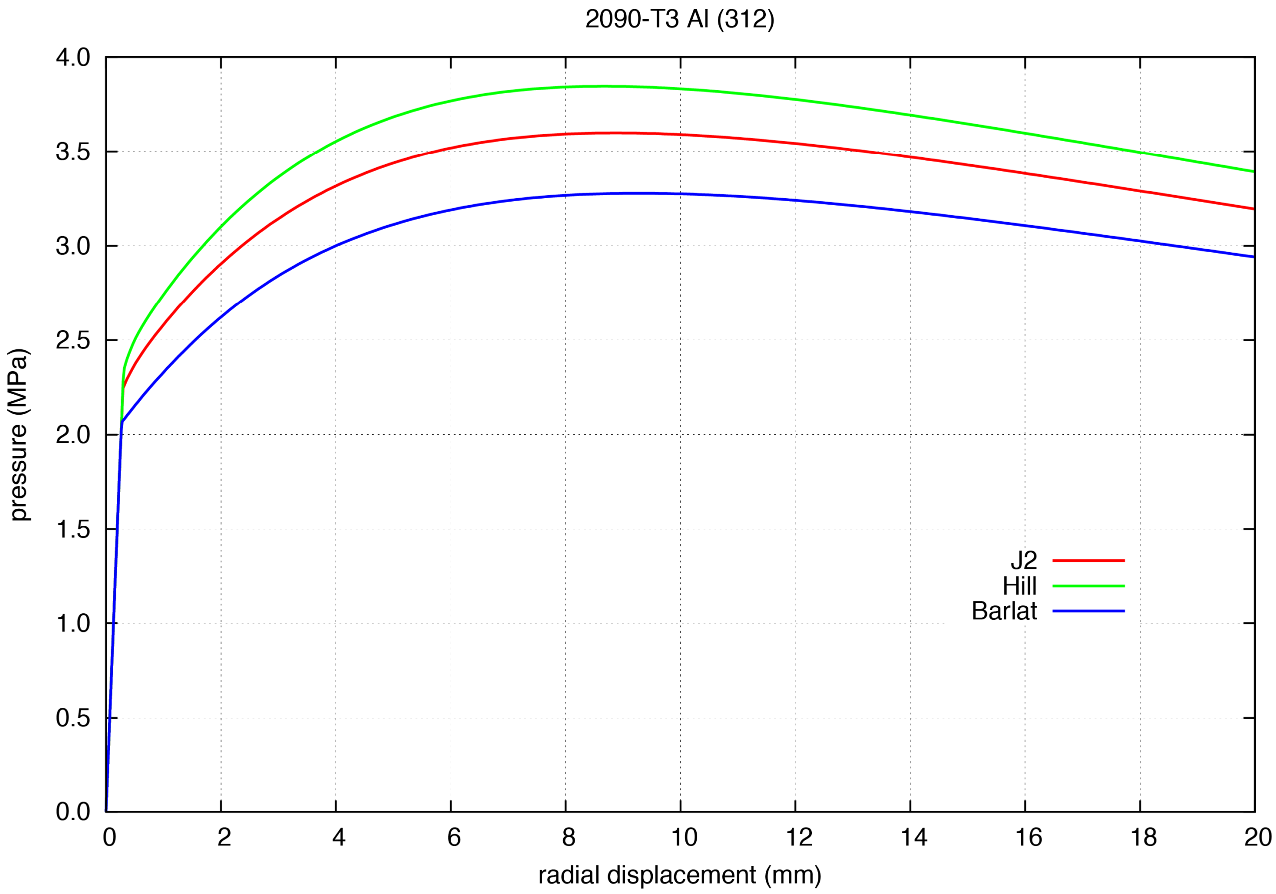
Stress Path – Barlat Model



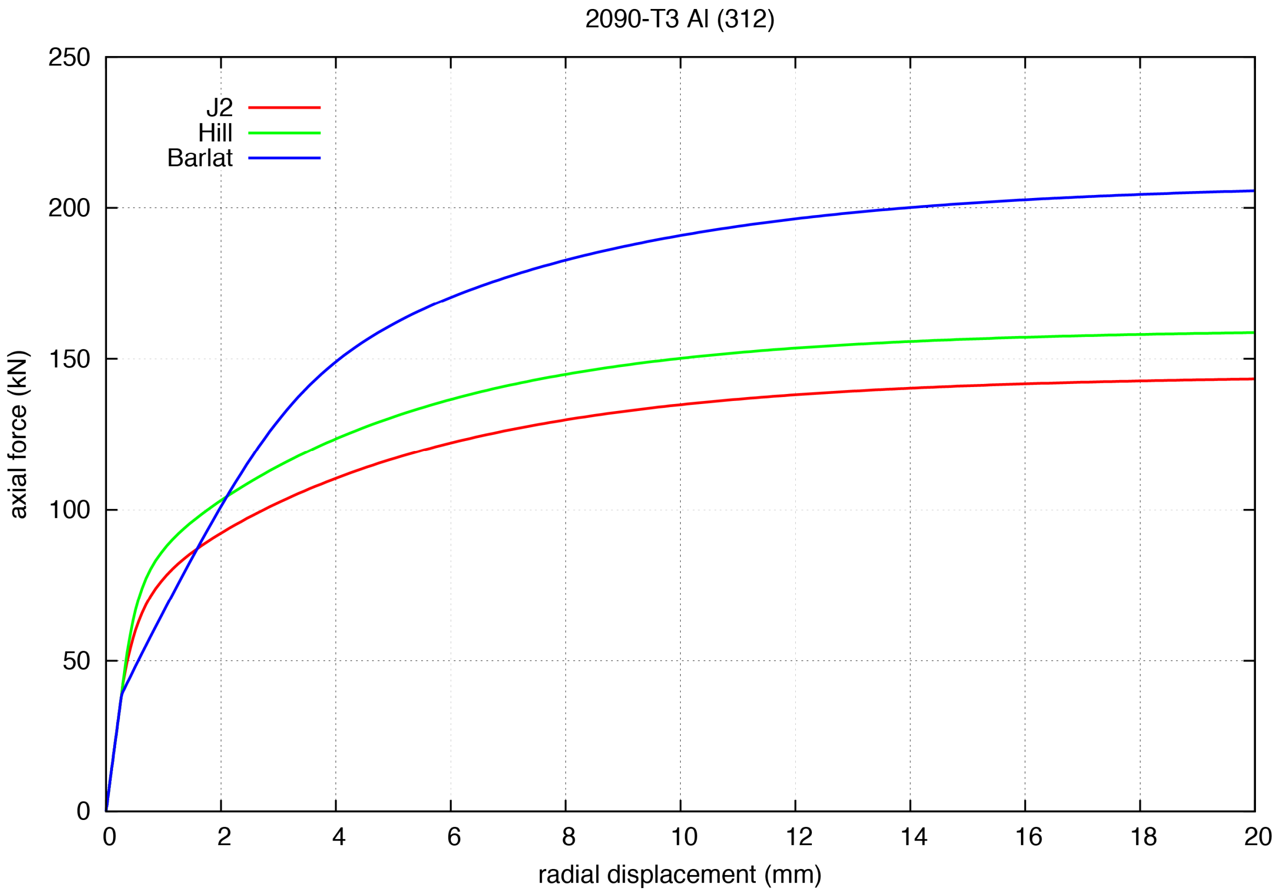
Stress Path – Comparison



Pressure

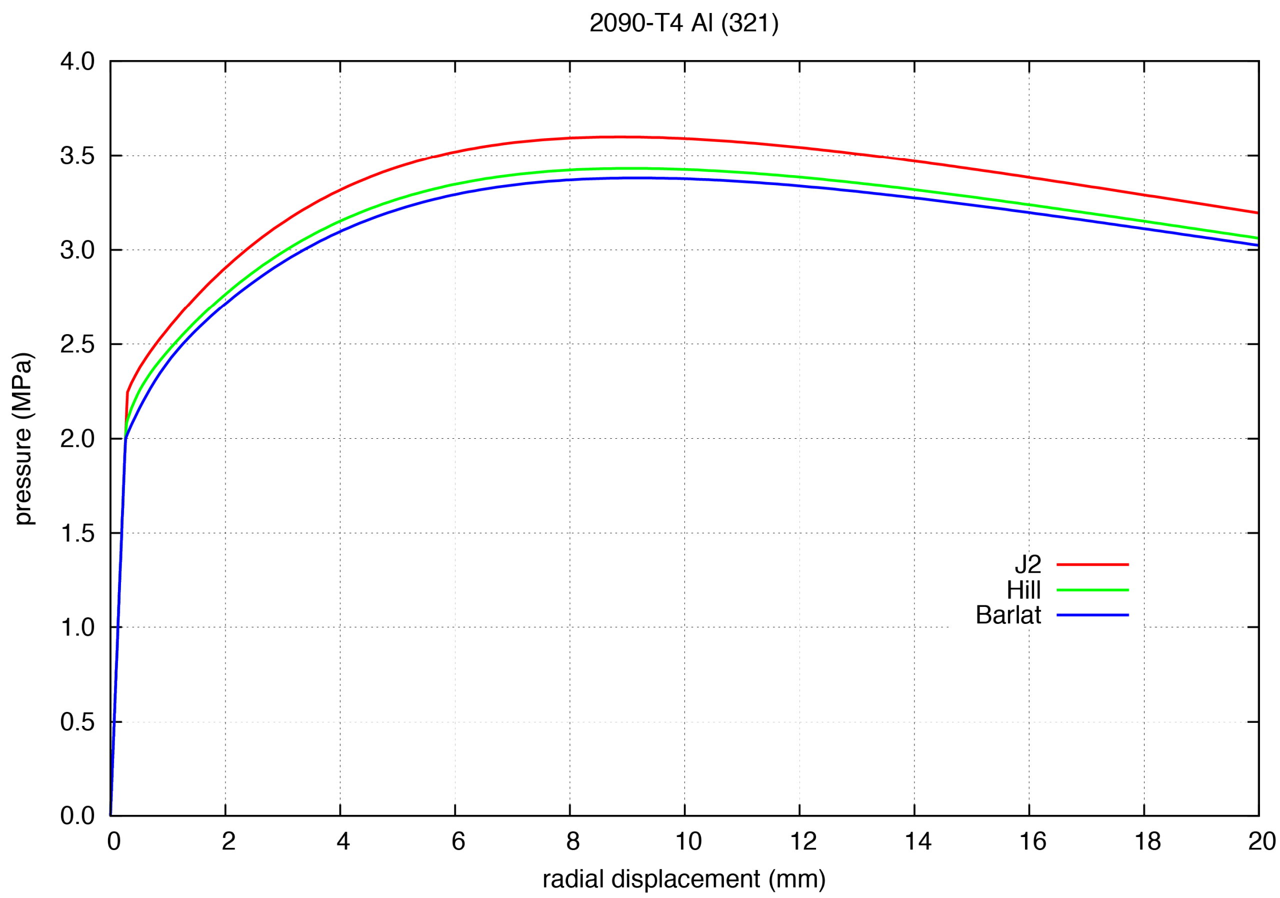

2090-T3 Al (312)
pressure (MPa)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
12
14
16
18
20
radial displacement (mm)
J2
Hill
Barlat

Axial Load

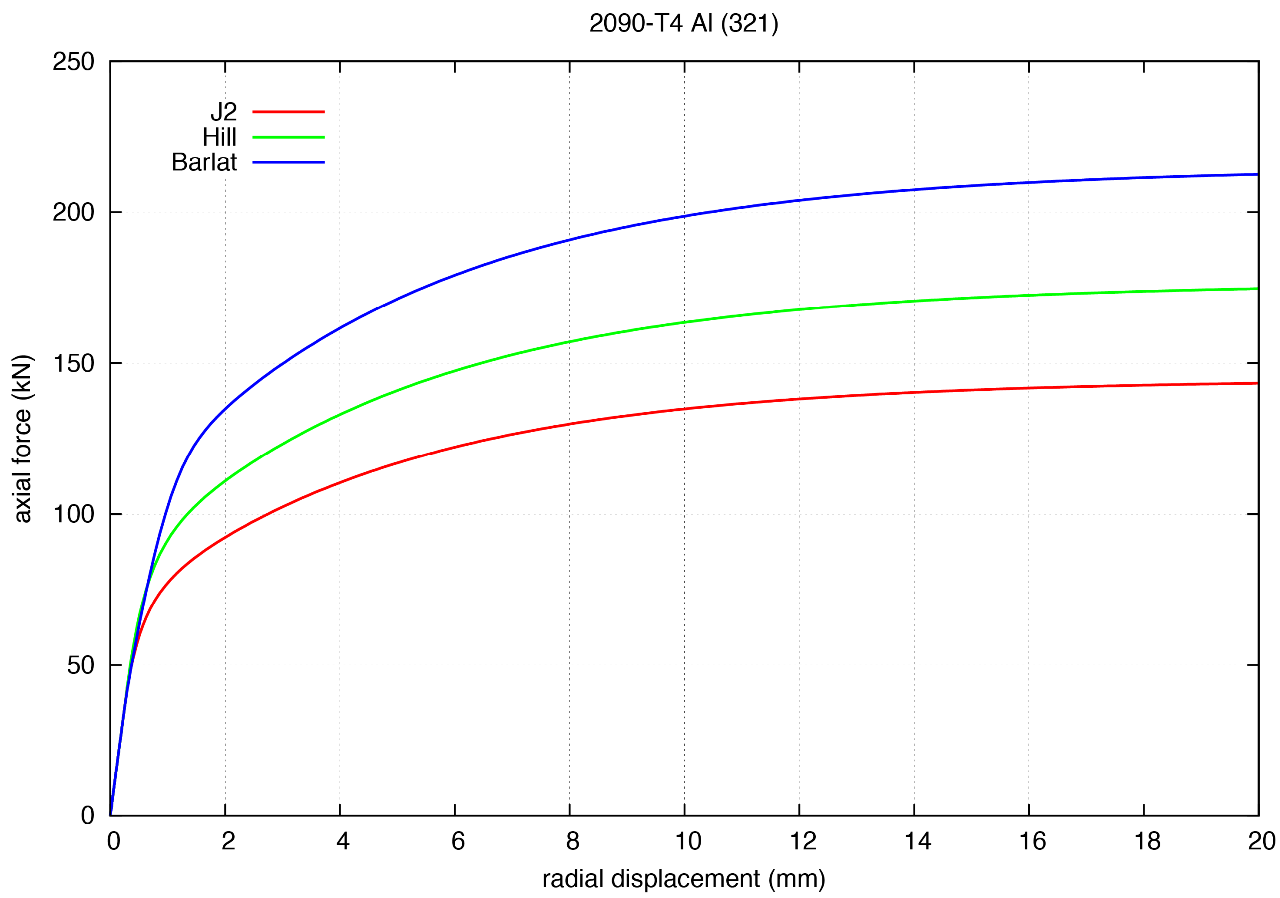


2090-T3 Al (312)
J2
Hill
Barlat
axial force (kN)
radial displacement (mm)
0
50
100
150
200
250
0
2
4
6
8
10
12
14
16
18
20

Pressure

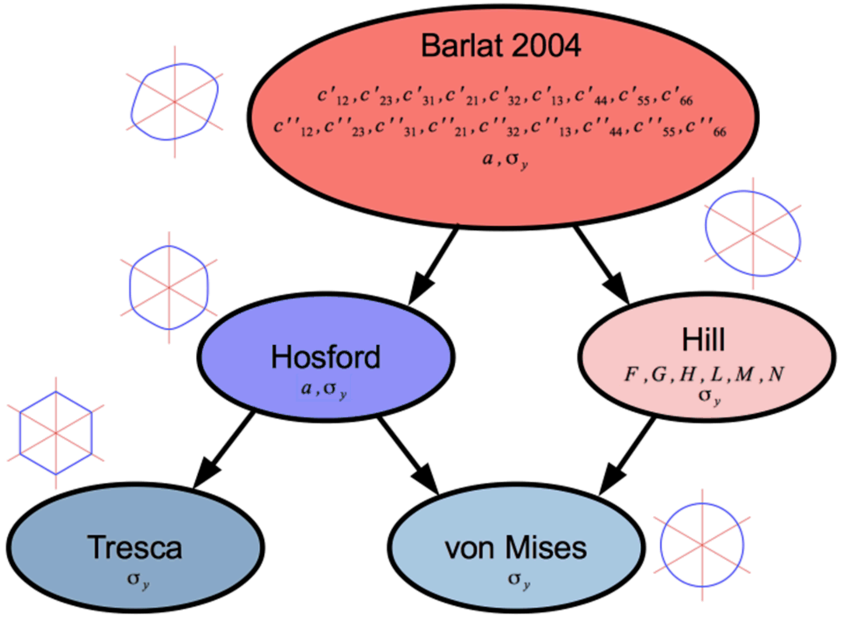

2090-T4 Al (321)
pressure (MPa)
radial displacement (mm)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
12
14
16
18
20
J2
Hill
Barlat

Axial Load



2090-T4 Al (321)
J2
Hill
Barlat
axial force (kN)
radial displacement (mm)
0
50
100
150
200
250
0
2
4
6
8
10
12
14
16
18
20

Uncertainty Quantification



- Can we use the Barlat model to same something **quantitative** about model form uncertainty?
- How does the choice of model affect our analysis?

Conclusions

- **isotropic/anisotropic yield descriptions**
- **robust integration algorithm**
  - capability can be used for other yield surfaces
- can we fit the models?
- can we extend to viscoplastic models?
- can we get quantitative model form error and UQ?
- can we model anisotropic hardening and failure?

Extra Slides

Barlat Model

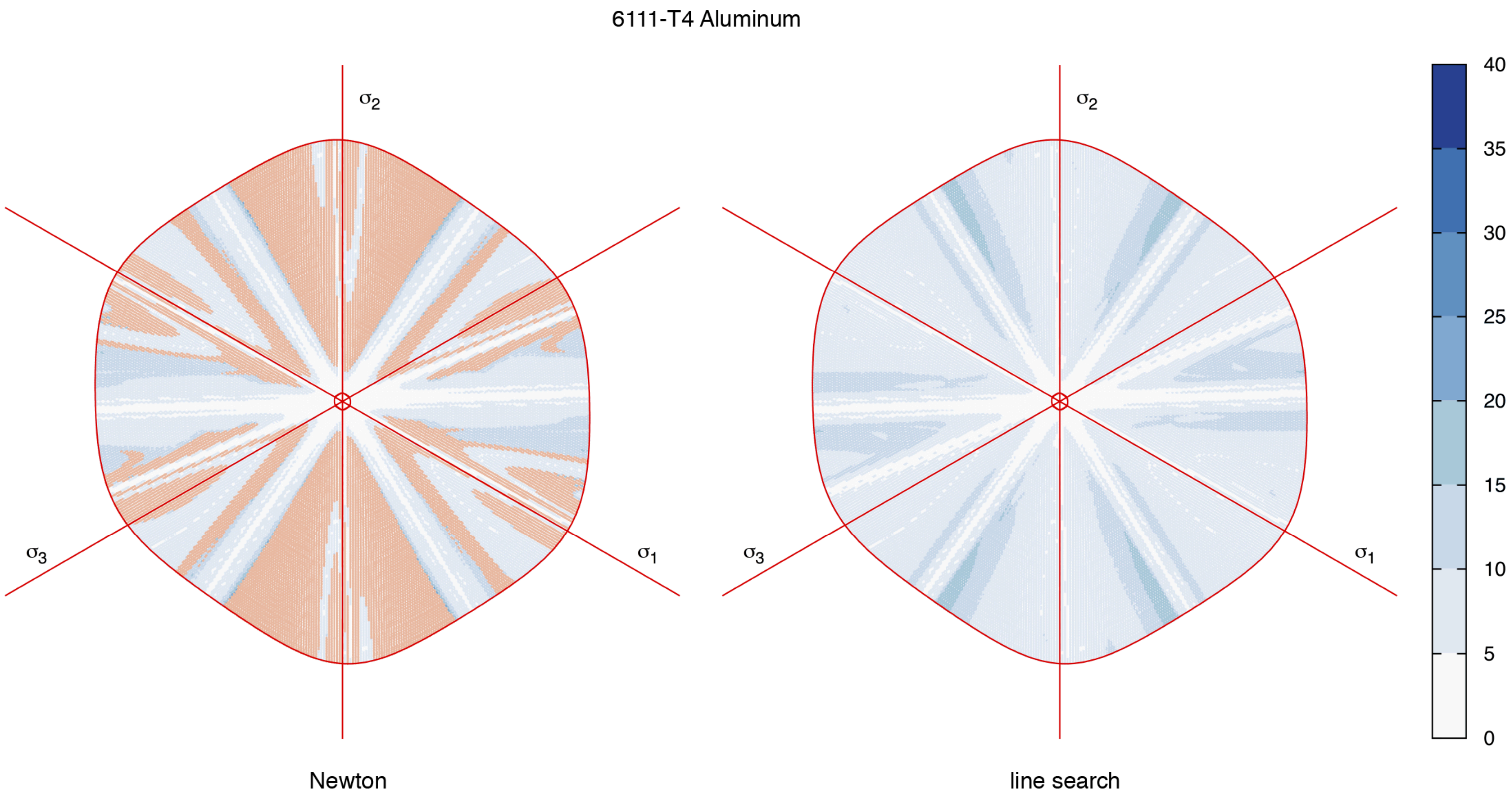
6111-T4 Al *

$$a = 8$$

$$
\begin{aligned}
c'_{12} &= 1.241024 \quad ; \quad c''_{12} = 0.775366 \\
c'_{13} &= 1.078271 \quad ; \quad c''_{13} = 0.922743 \\
c'_{21} &= 1.216463 \quad ; \quad c''_{21} = 0.765487 \\
c'_{23} &= 1.223867 \quad ; \quad c''_{23} = 0.793356 \\
c'_{31} &= 1.093105 \quad ; \quad c''_{31} = 0.918689 \\
c'_{32} &= 0.889161 \quad ; \quad c''_{32} = 1.027625 \\
c'_{44} &= 0.501909 \quad ; \quad c''_{44} = 1.115833 \\
c'_{55} &= 0.557173 \quad ; \quad c''_{55} = 1.112273 \\
c'_{66} &= 1.349094 \quad ; \quad c''_{66} = 0.589787
\end{aligned}
$$

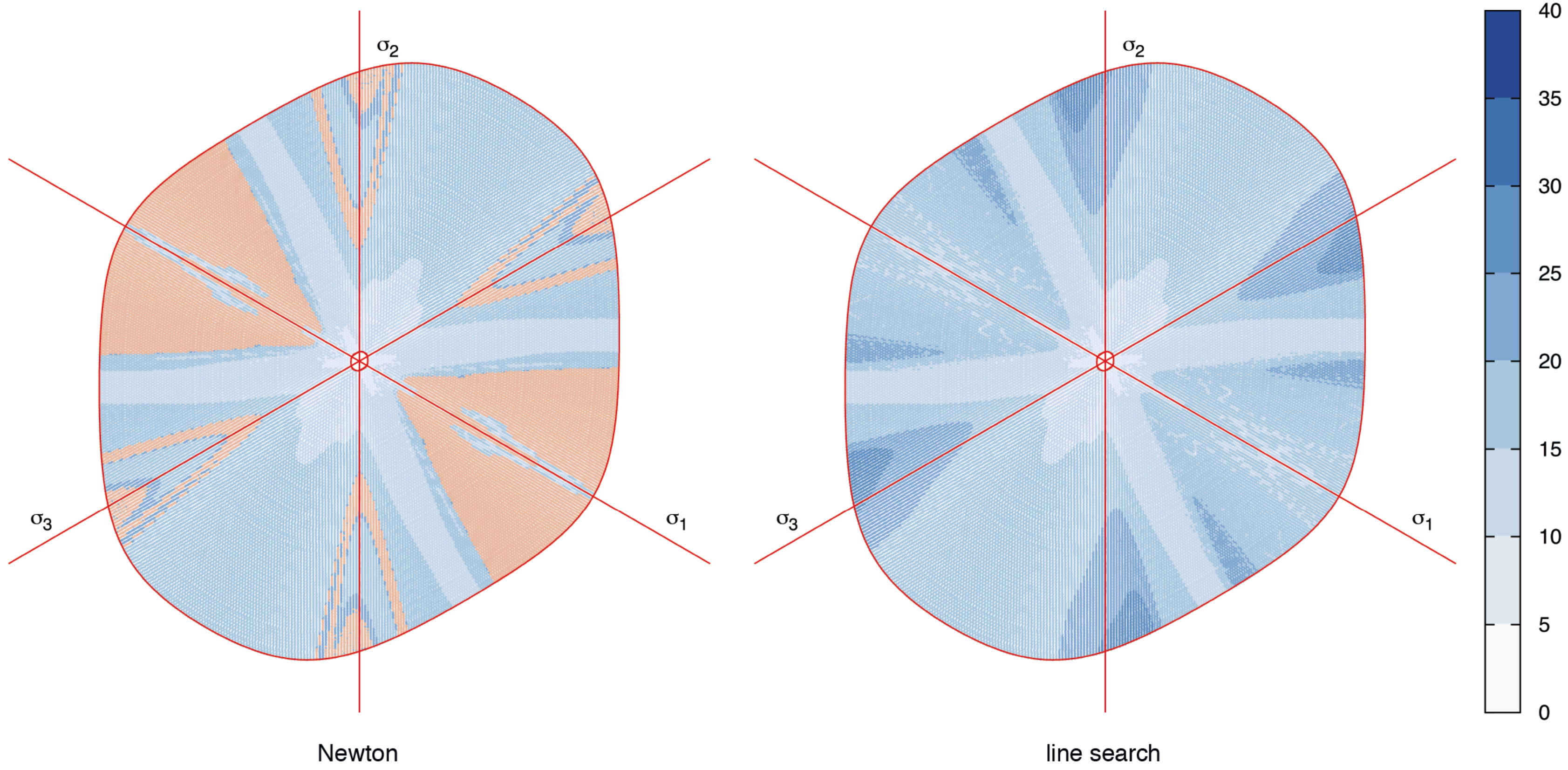
* Barlat et. al., “Linear transformation based anisotropic yield functions”, IJP, v. 21, 2005.

Barlat Model

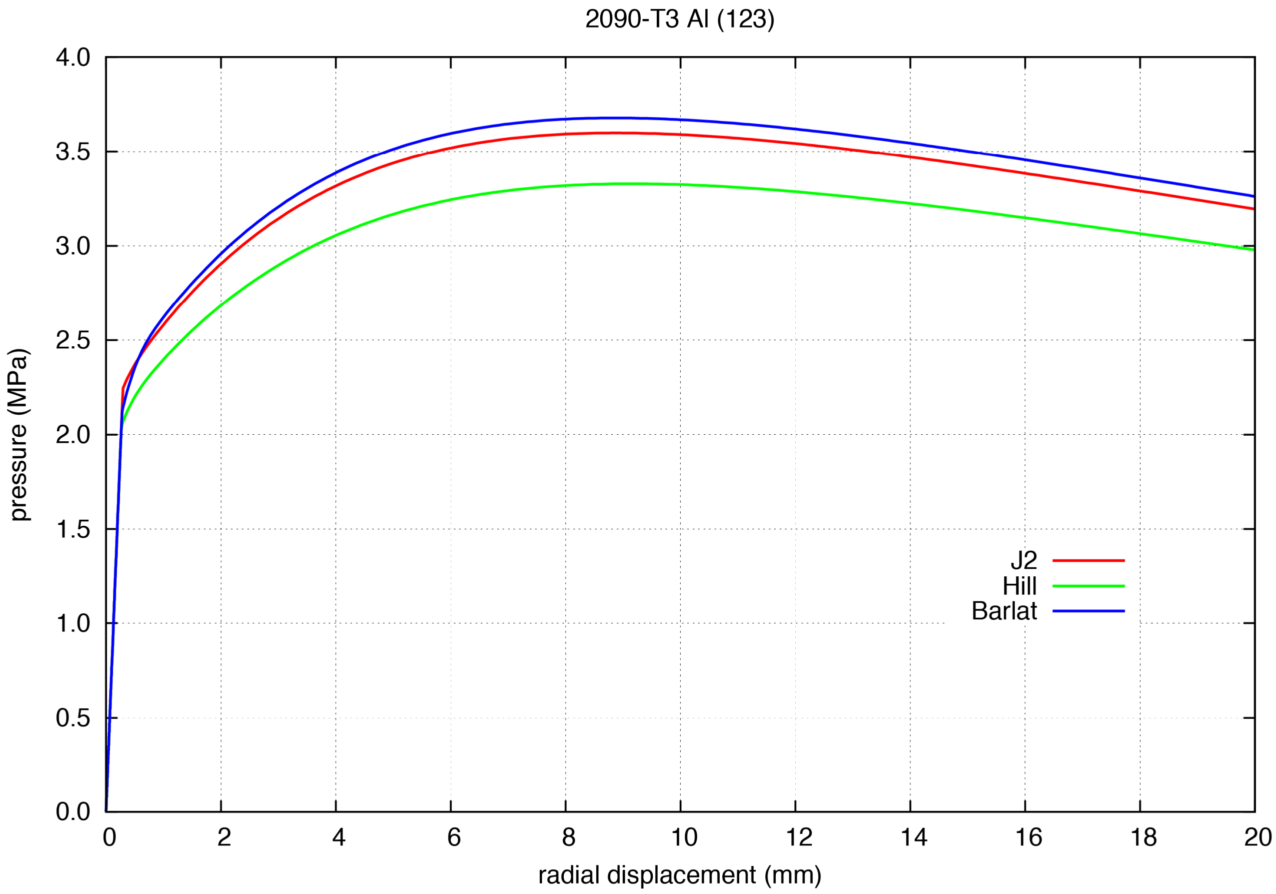


Barlat Model

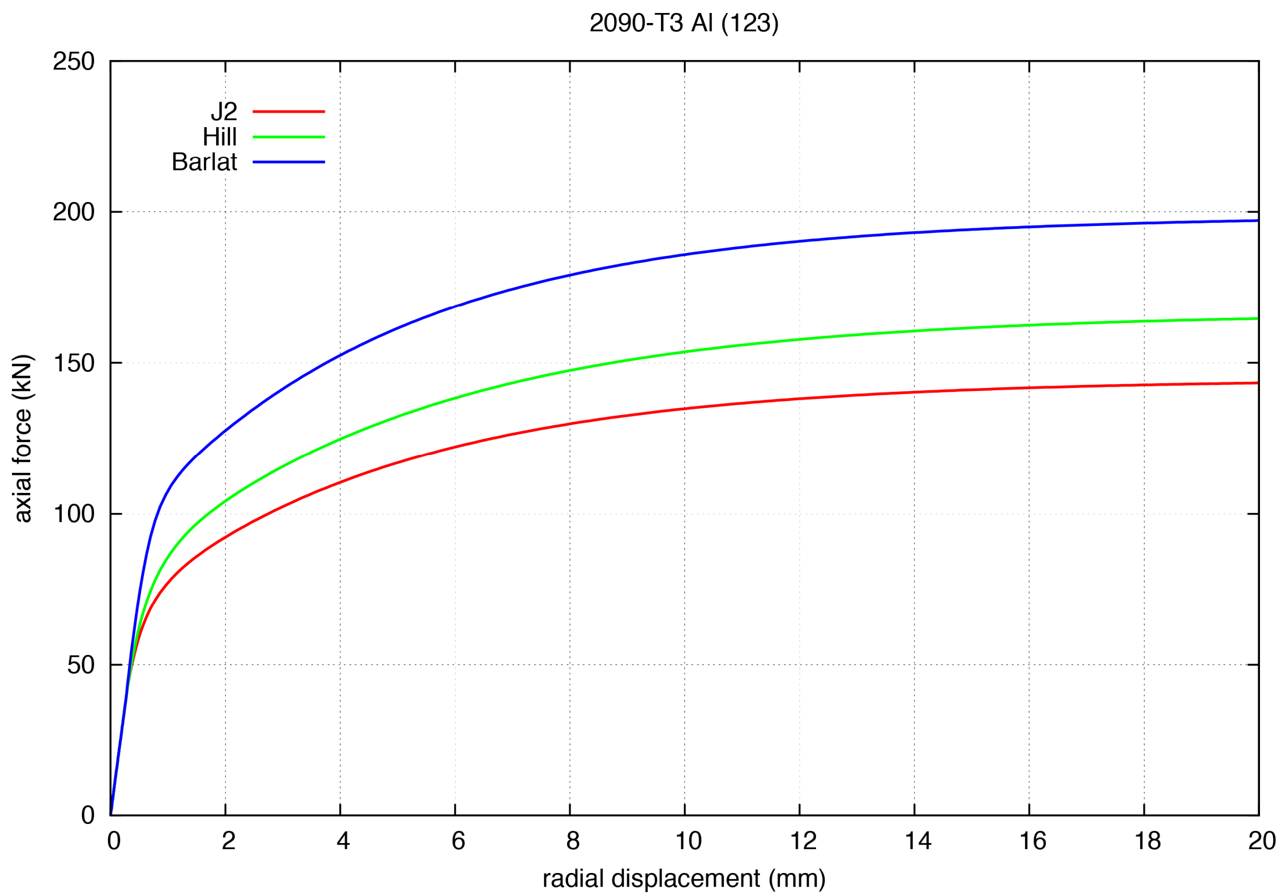
45 degrees about x_1 axis



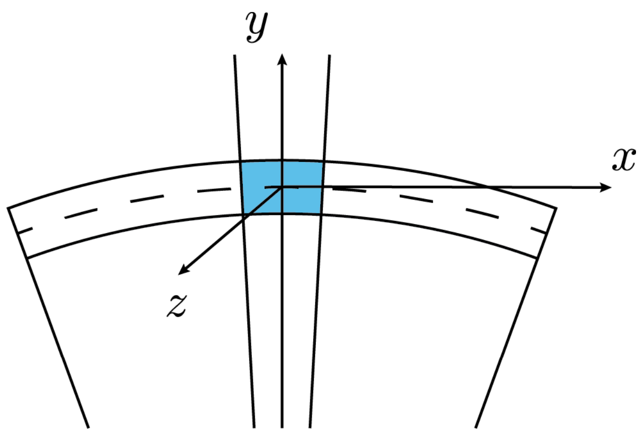
Pressure


2090-T3 Al (123)
pressure (MPa)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
12
14
16
18
20
radial displacement (mm)
J2
Hill
Barlat

Axial Load


2090-T3 Al (123)
J2
Hill
Barlat
axial force (kN)
radial displacement (mm)
0
50
100
150
200
250
0
2
4
6
8
10
12
14
16
18
20

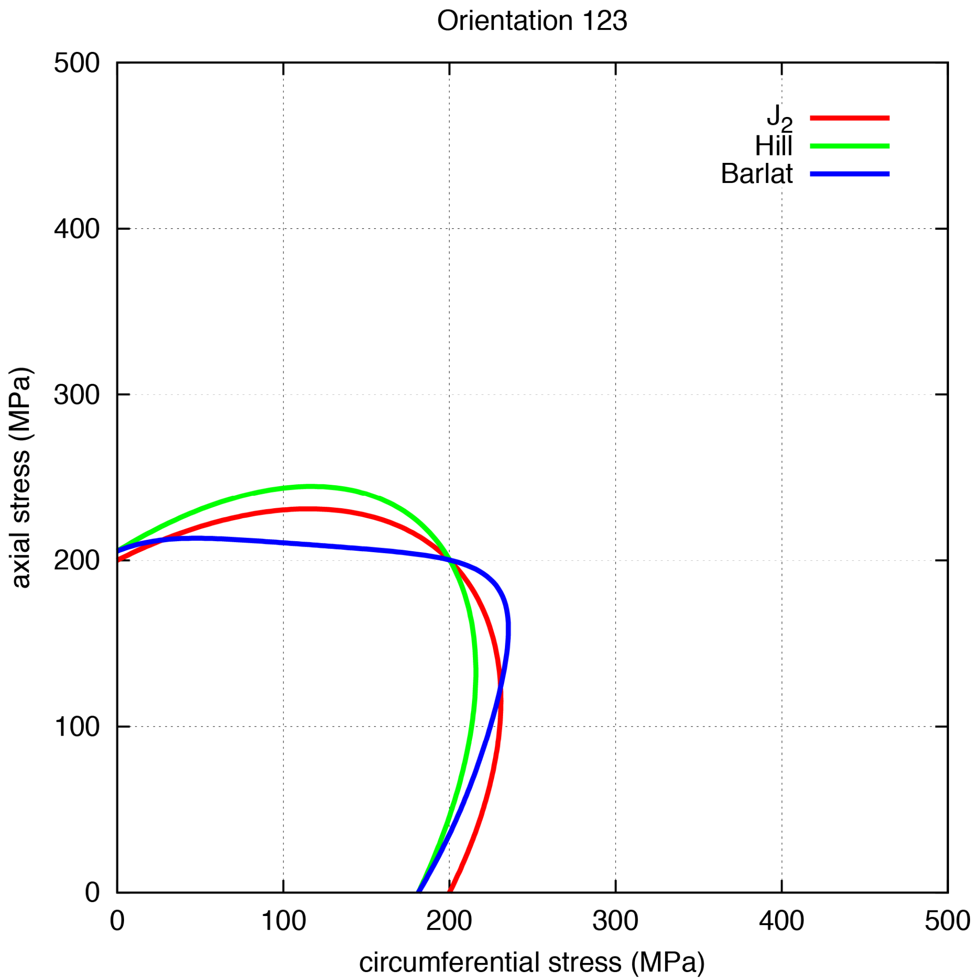
Yield Surface – Orthotropic Models



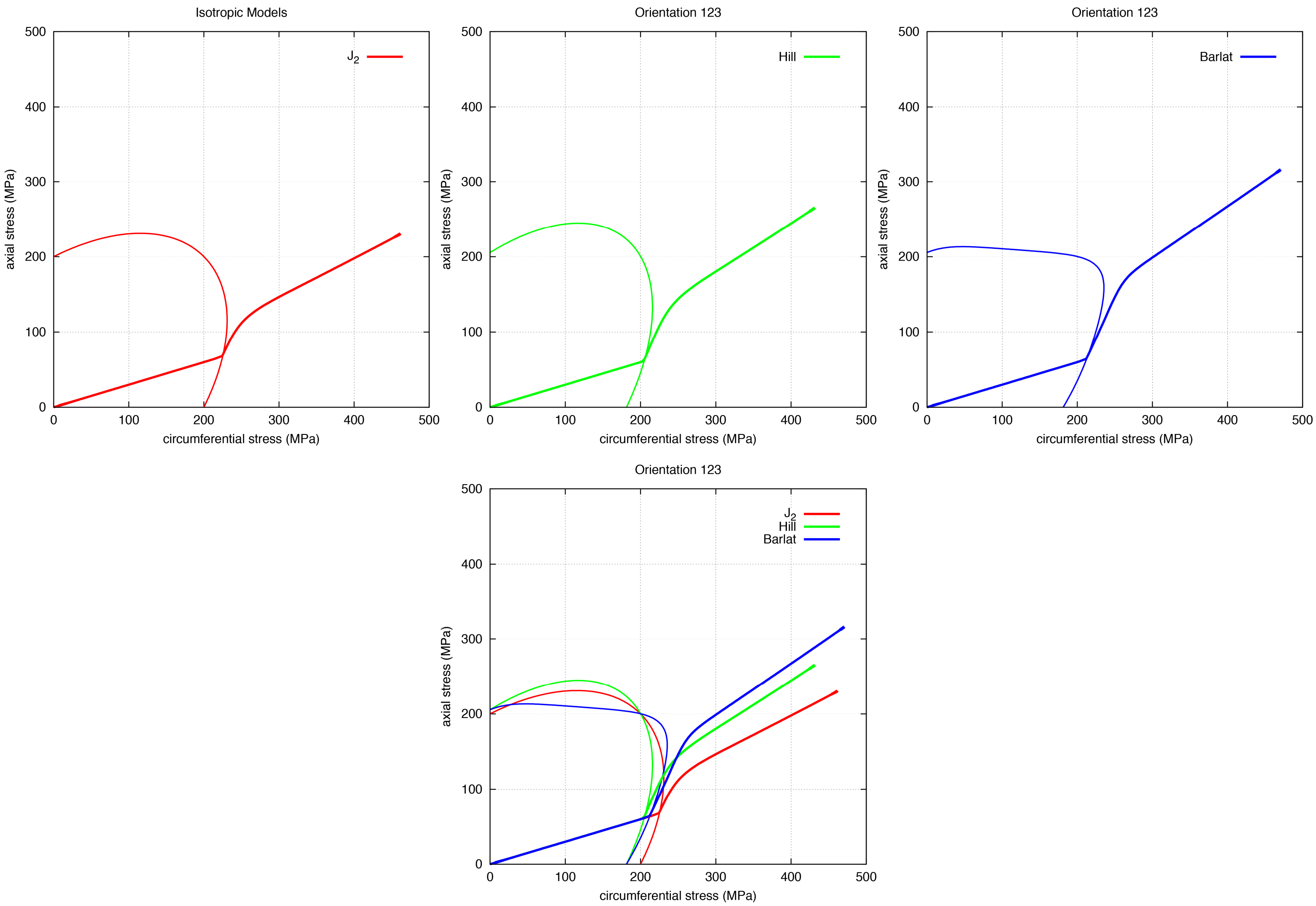
$$\sigma_{rr} = \sigma_{11}$$

$$\sigma_{\theta\theta} = \sigma_{22}$$

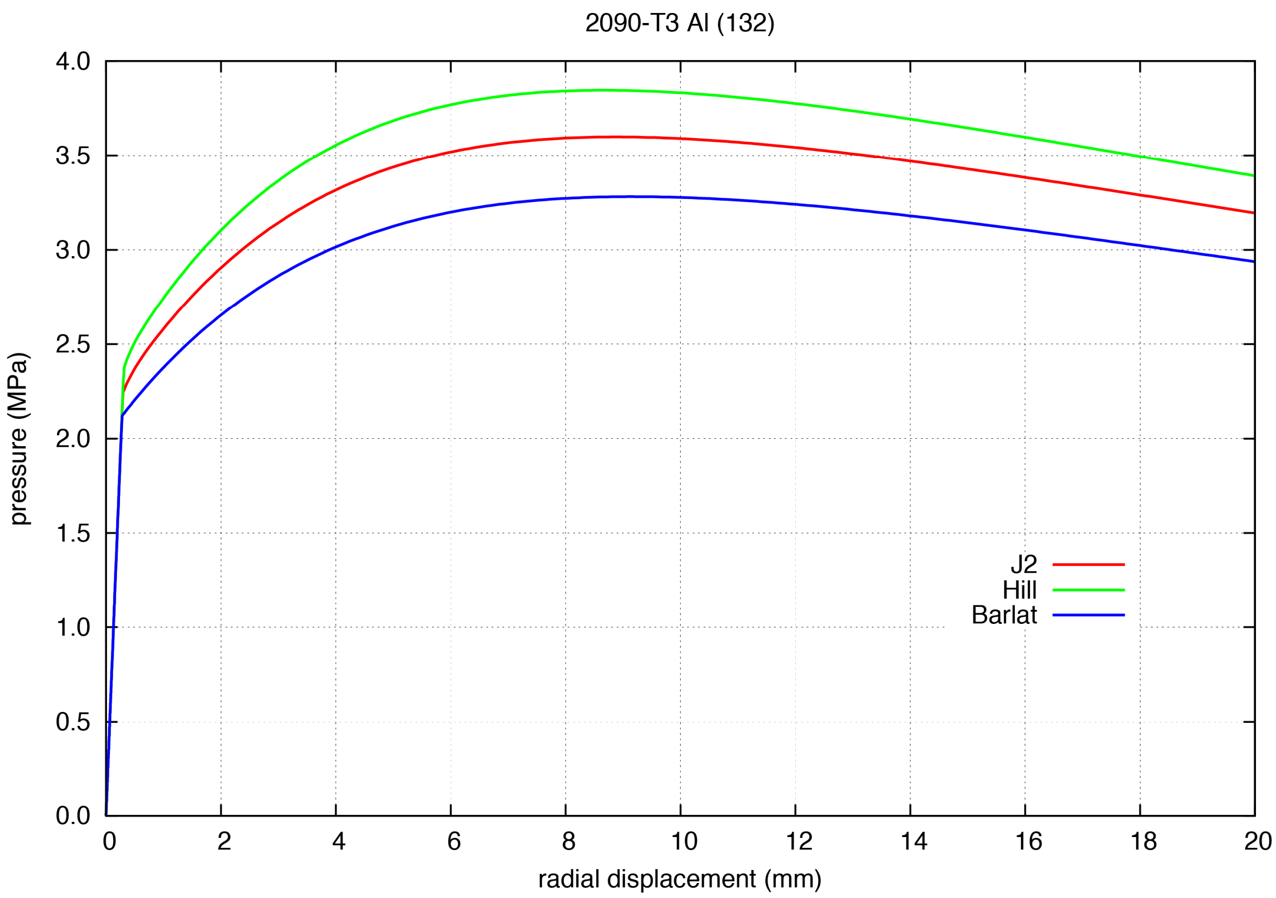
$$\sigma_{zz} = \sigma_{33}$$



Stress Paths – Orthotropic Models

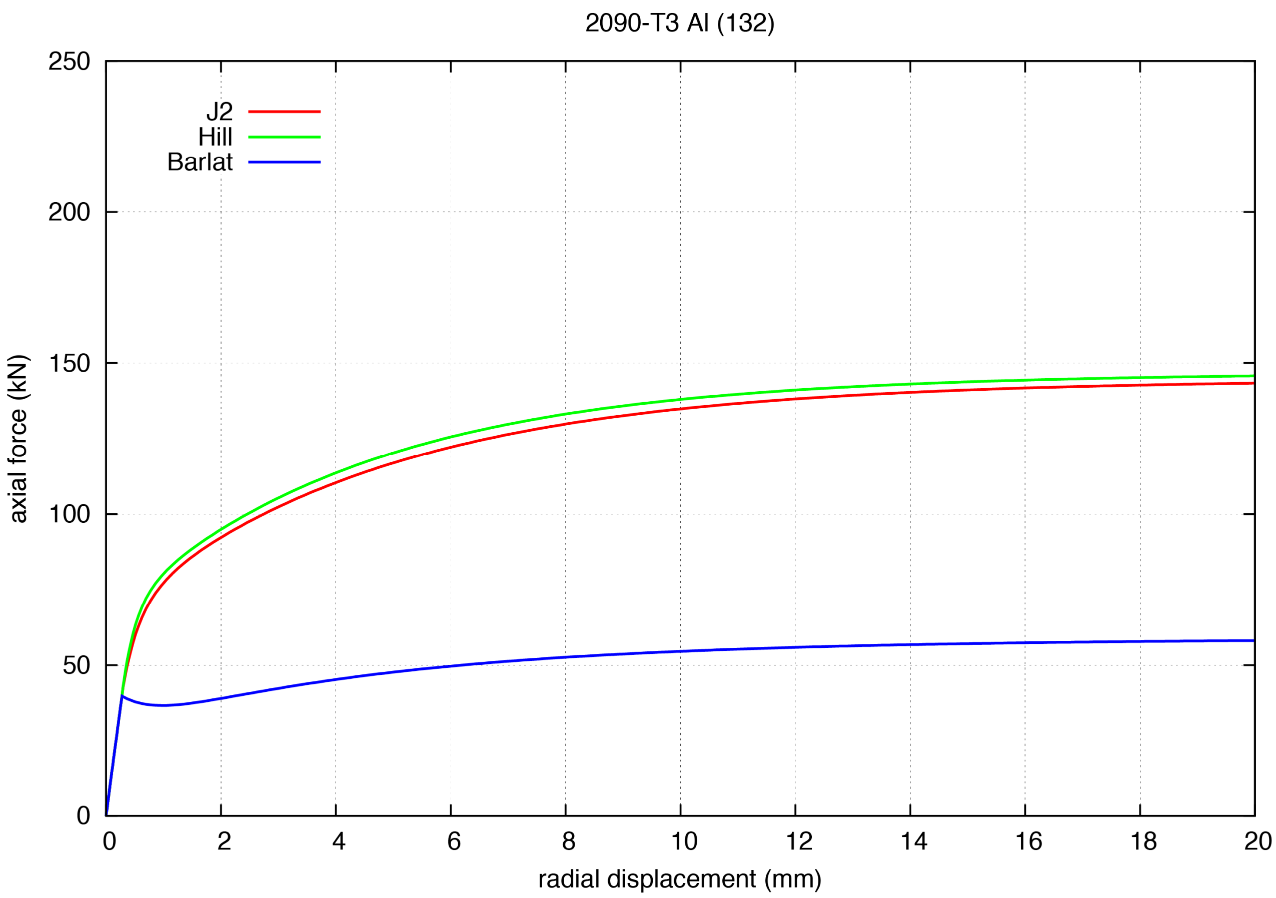


Pressure



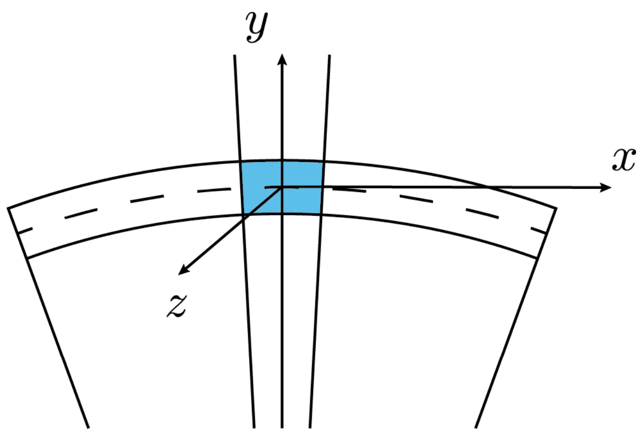
2090-T3 Al (132)
pressure (MPa)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
12
14
16
18
20
radial displacement (mm)
J2
Hill
Barlat

Axial Load



2090-T3 Al (132)
J2
Hill
Barlat
axial force (kN)
radial displacement (mm)
0
50
100
150
200
250
0
2
4
6
8
10
12
14
16
18
20

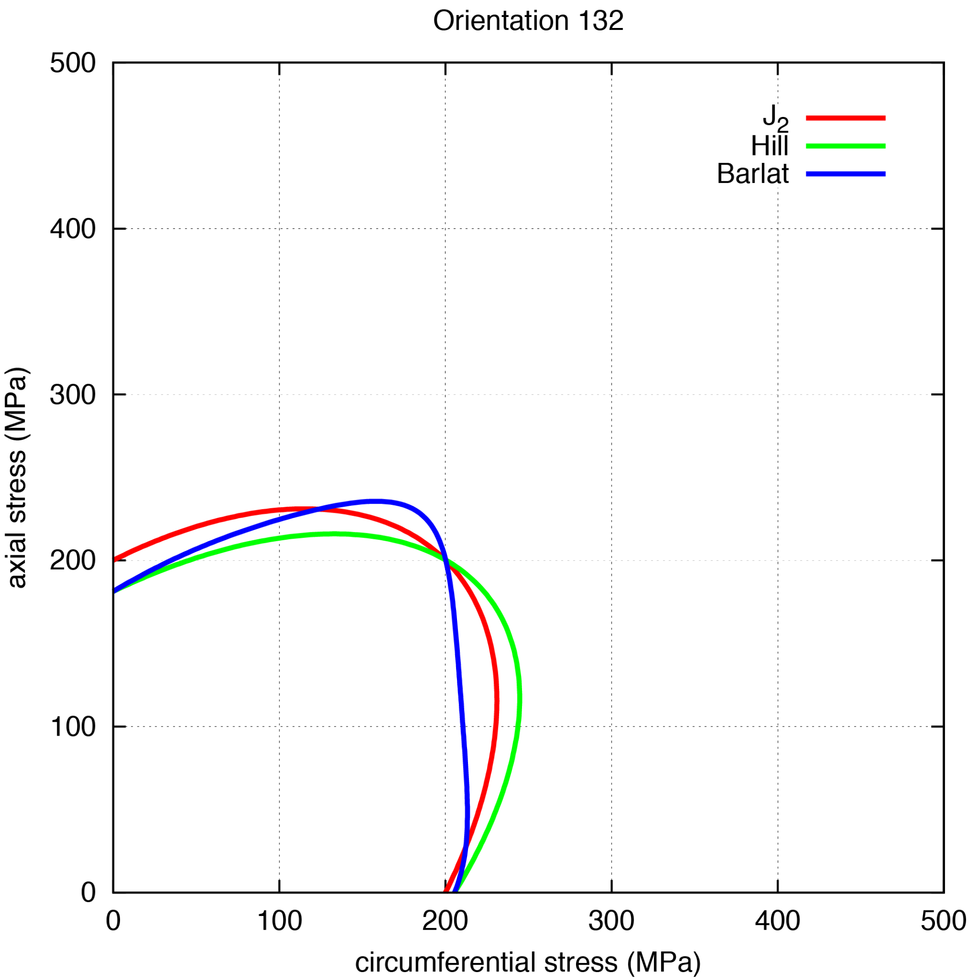
Yield Surface – Orthotropic Models



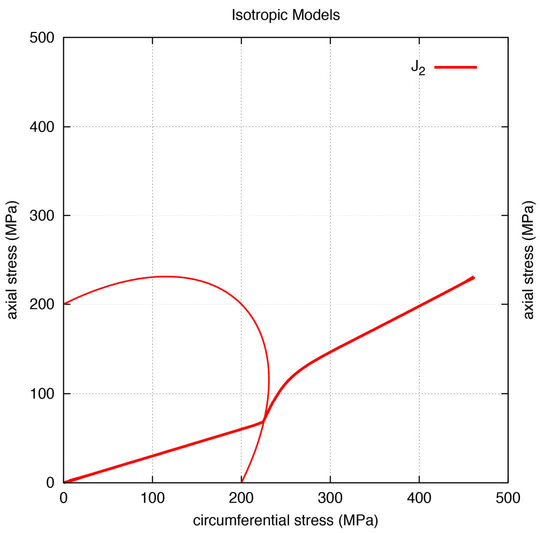
$$\sigma_{rr} = \sigma_{11}$$

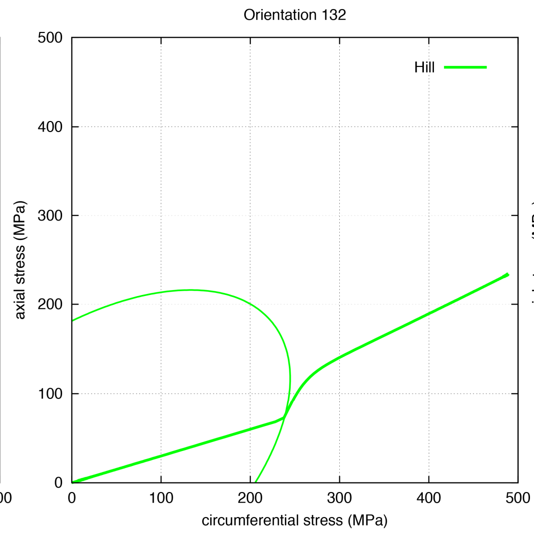
$$\sigma_{\theta\theta} = \sigma_{33}$$

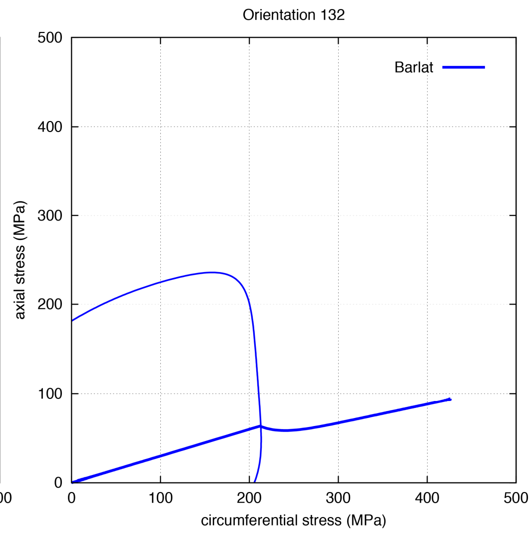
$$\sigma_{zz} = \sigma_{22}$$

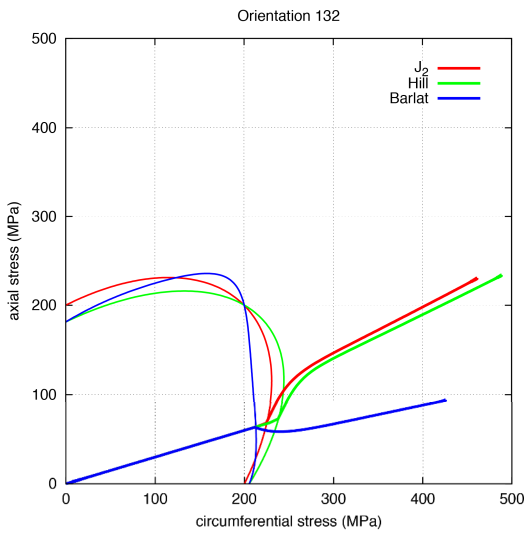


Stress Paths – Orthotropic Models

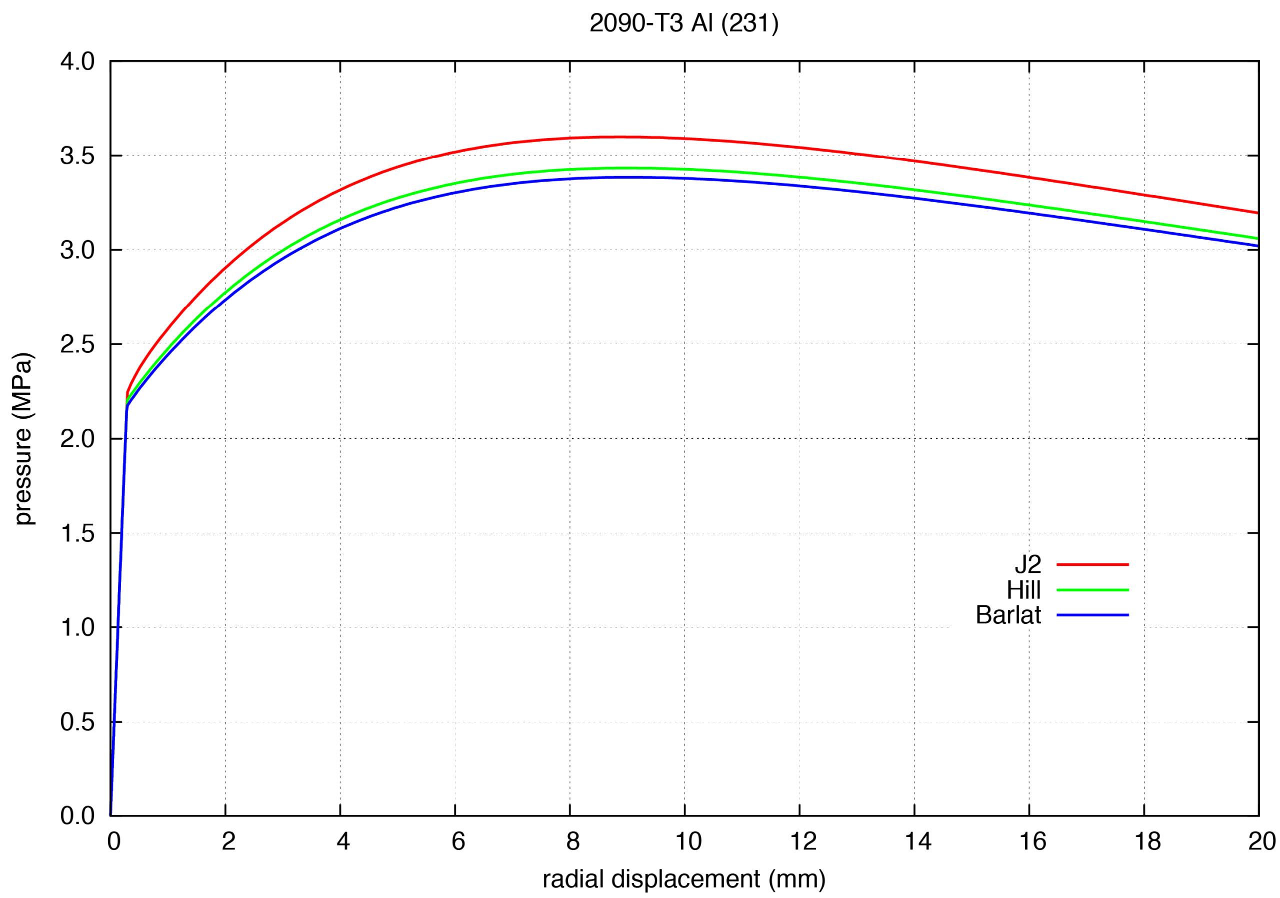






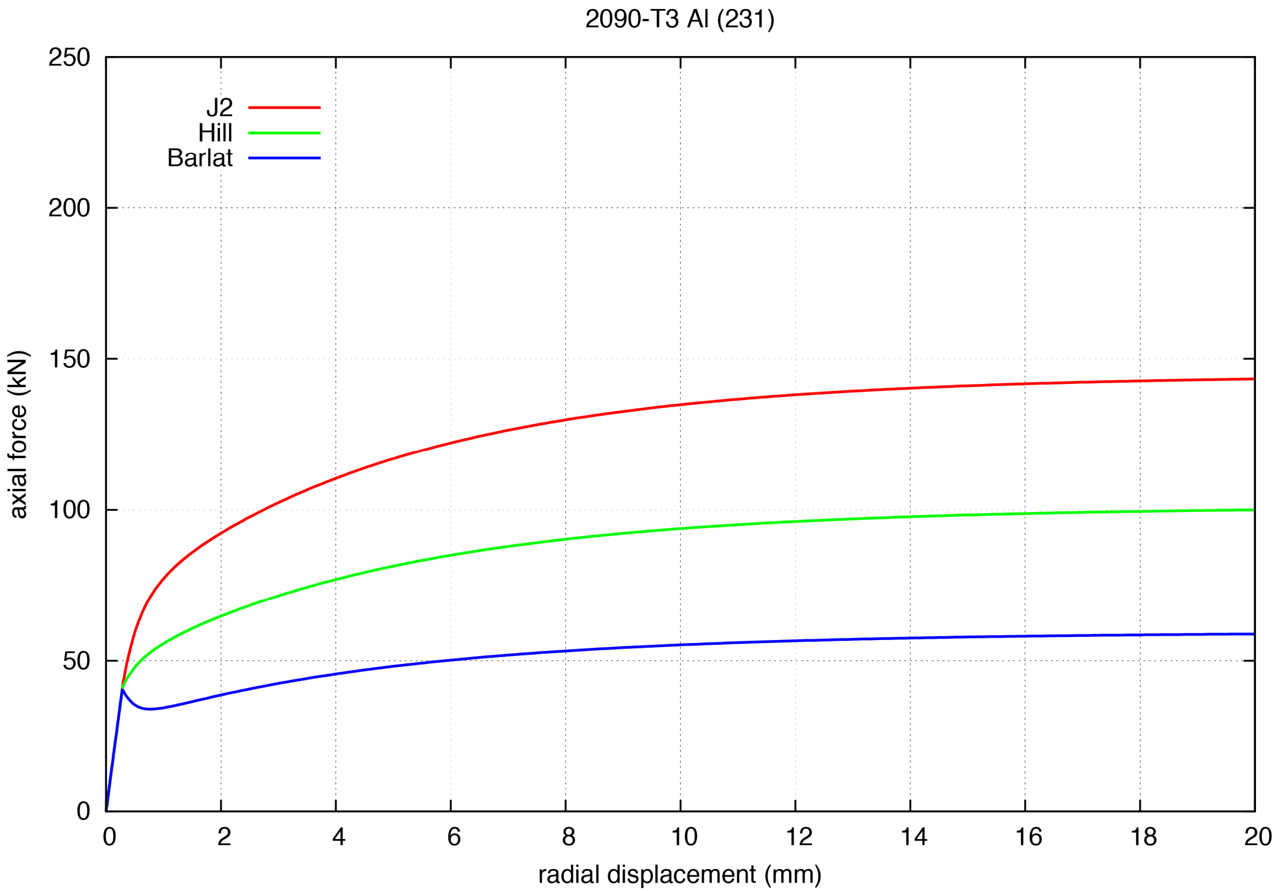


Pressure



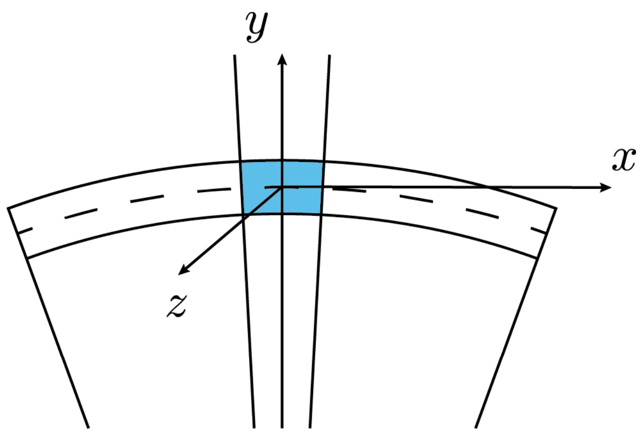
2090-T3 Al (231)
pressure (MPa)
radial displacement (mm)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
12
14
16
18
20
J2
Hill
Barlat

Axial Load



2090-T3 Al (231)
J2
Hill
Barlat
axial force (kN)
radial displacement (mm)
0
50
100
150
200
250
0
2
4
6
8
10
12
14
16
18
20

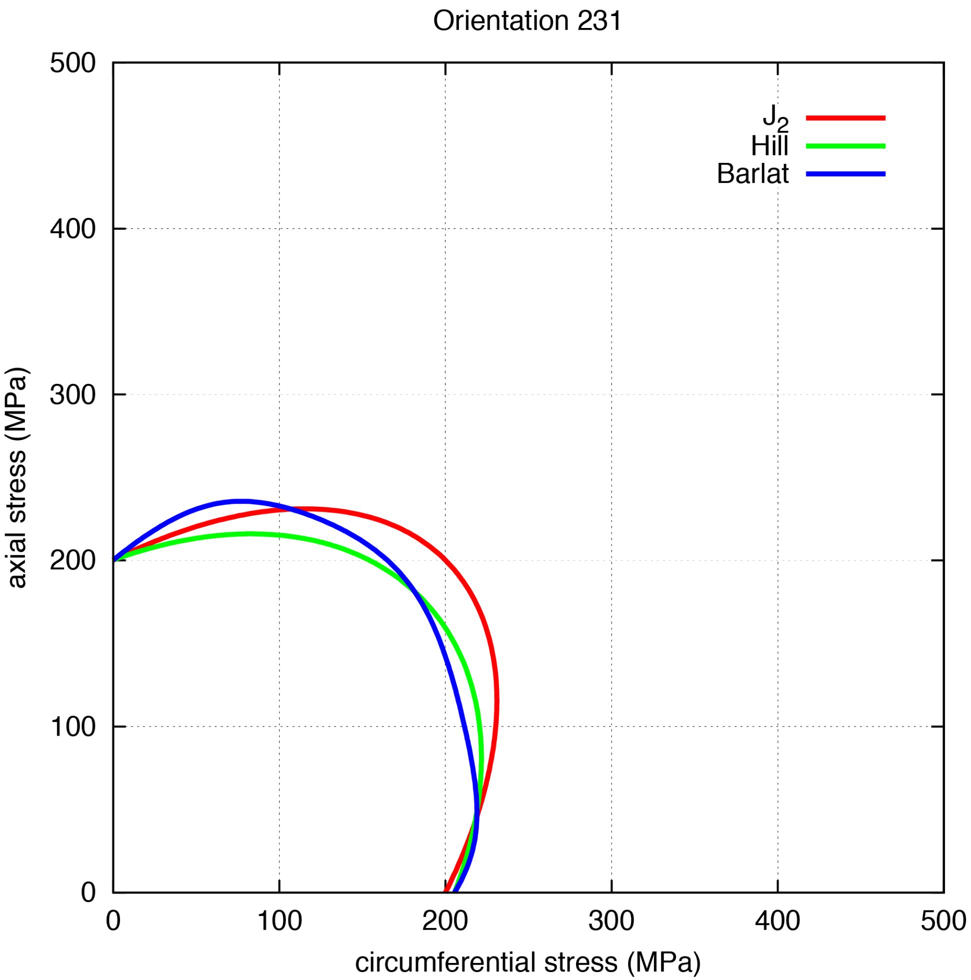
Yield Surface – Orthotropic Models



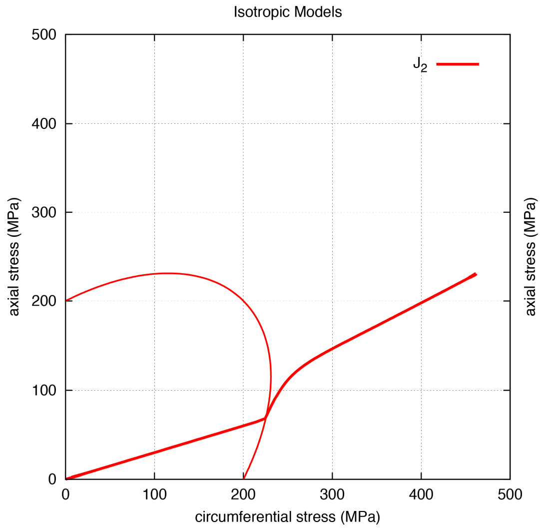
$$\sigma_{rr} = \sigma_{22}$$

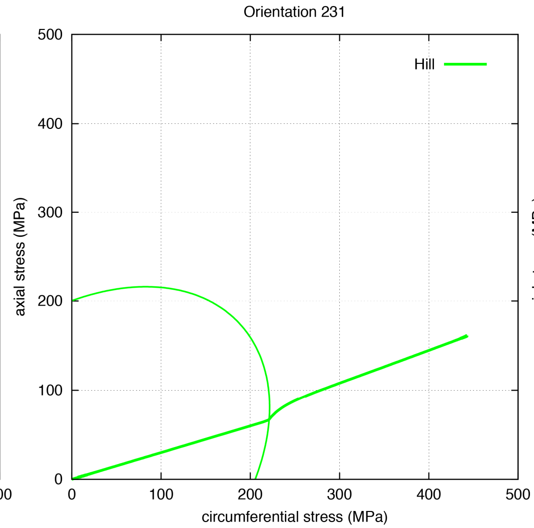
$$\sigma_{\theta\theta} = \sigma_{33}$$

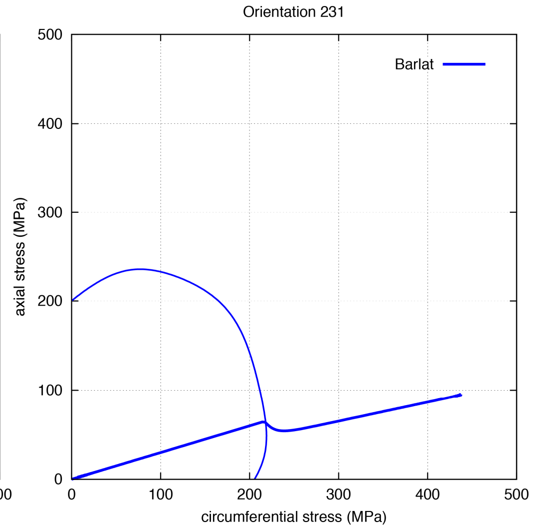
$$\sigma_{zz} = \sigma_{11}$$

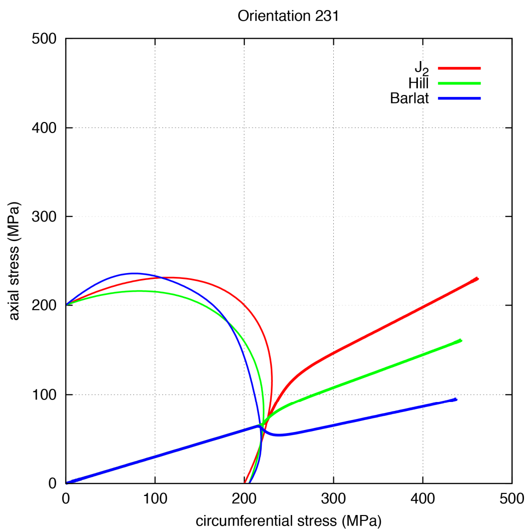


Stress Paths – Orthotropic Models

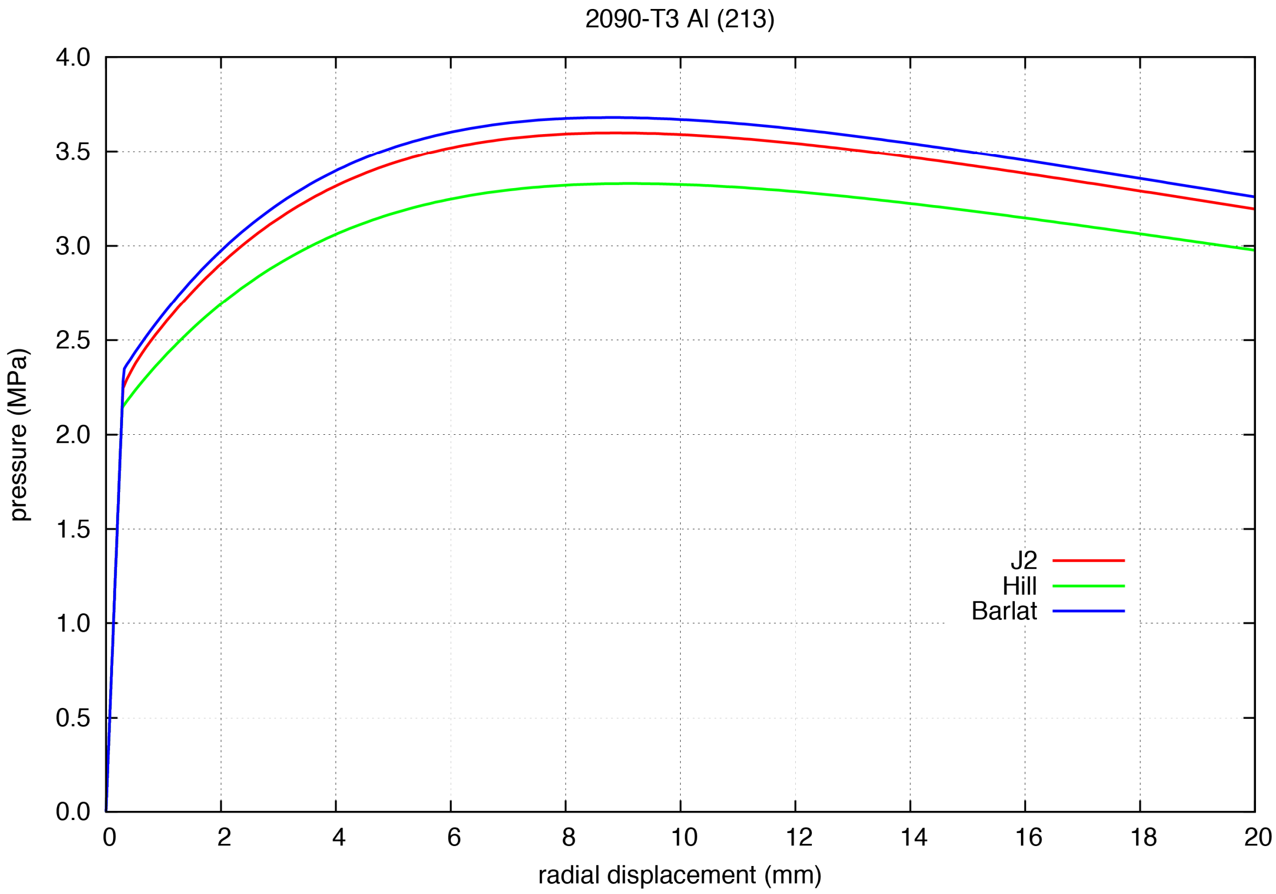






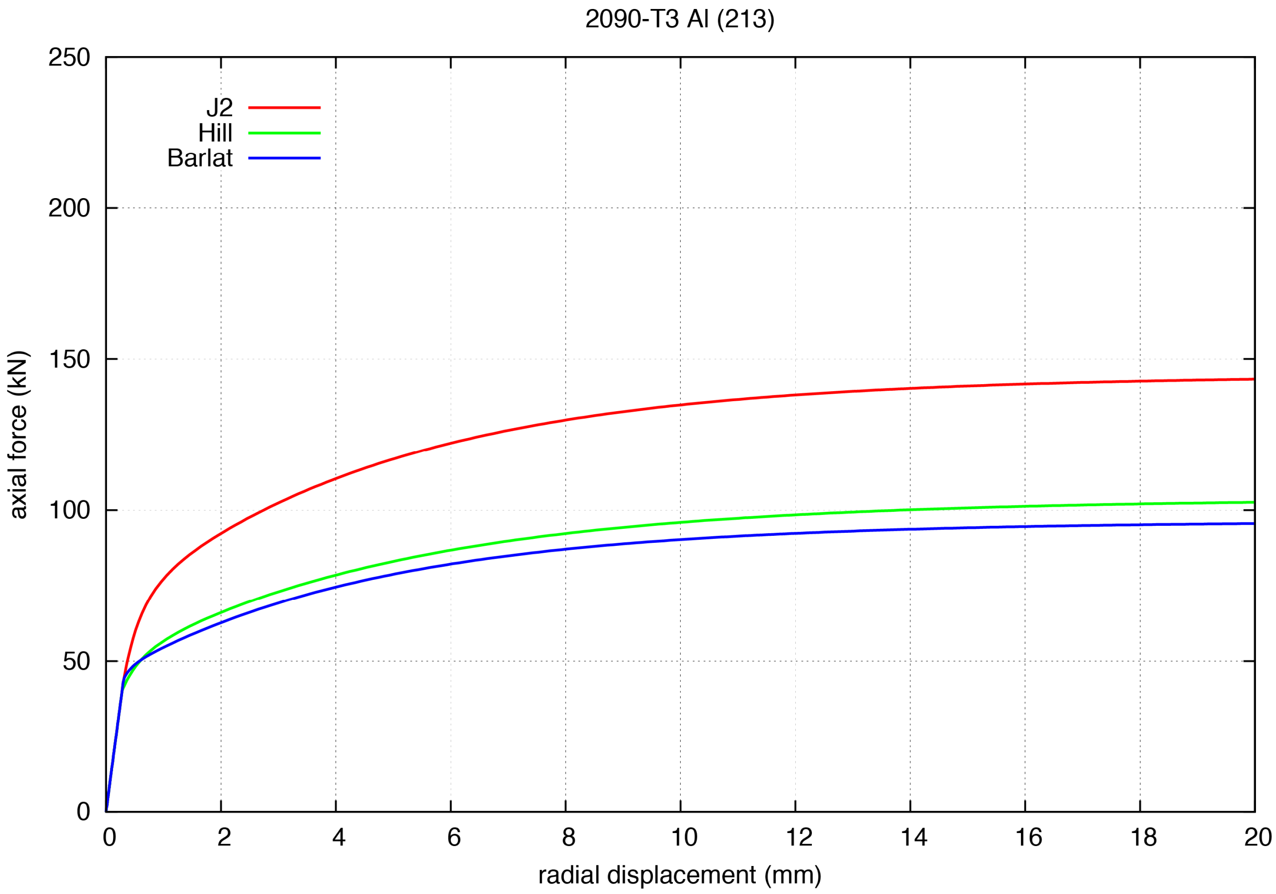


Pressure



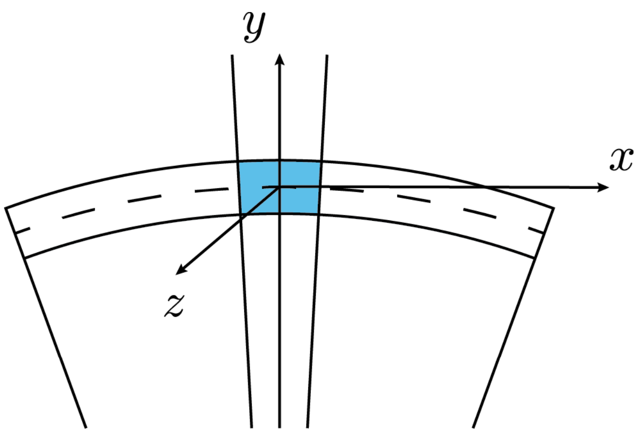
2090-T3 Al (213)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
pressure (MPa)
0
2
4
6
8
10
12
14
16
18
20
radial displacement (mm)
J2
Hill
Barlat

Axial Load



2090-T3 Al (213)
J2
Hill
Barlat
axial force (kN)
radial displacement (mm)
0
50
100
150
200
250
0
2
4
6
8
10
12
14
16
18
20

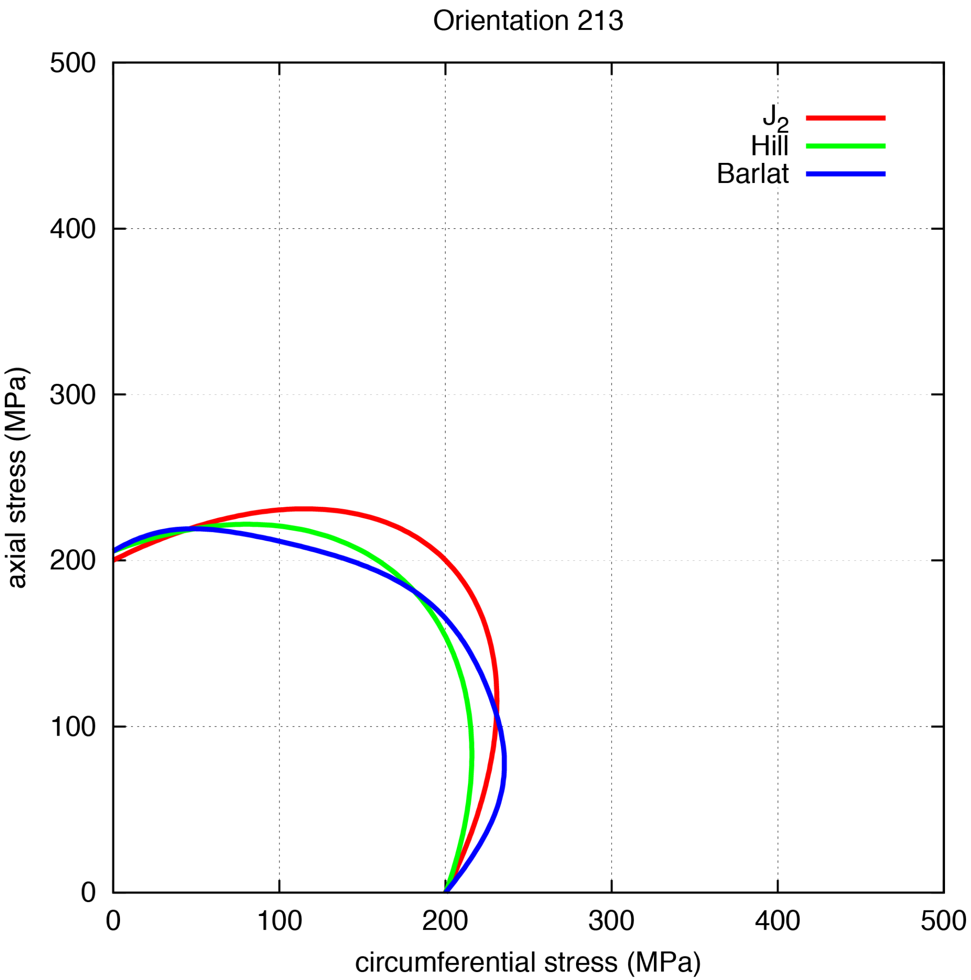
Yield Surface – Orthotropic Models



$$\sigma_{rr} = \sigma_{22}$$

$$\sigma_{\theta\theta} = \sigma_{11}$$

$$\sigma_{zz} = \sigma_{33}$$



Stress Paths – Orthotropic Models