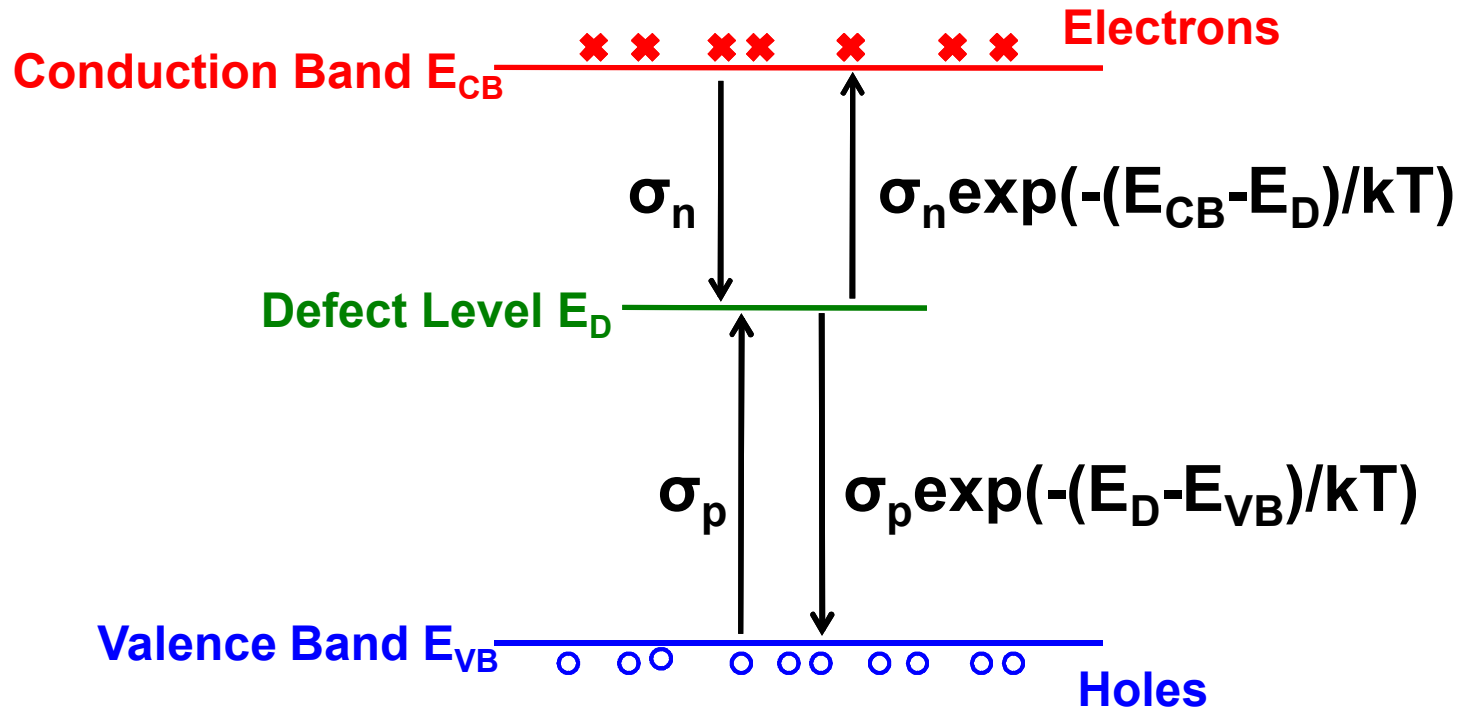


# DFT Calculations for Carrier Capture by Defects in Semiconductors



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# Why Study the EL2 Level in GaAs?

Rare opportunity to validate models of carrier capture

Generally believed to be the 0/+1 level of the As antisite in GaAs

Electron capture cross-sections were measured as a function of temperature by Lang and Logan (1980)

Henry and Lang's Multiphonon Emission Theory (1977) fits the results with 4 parameters

$$E_t = 0.75 \text{ eV}$$

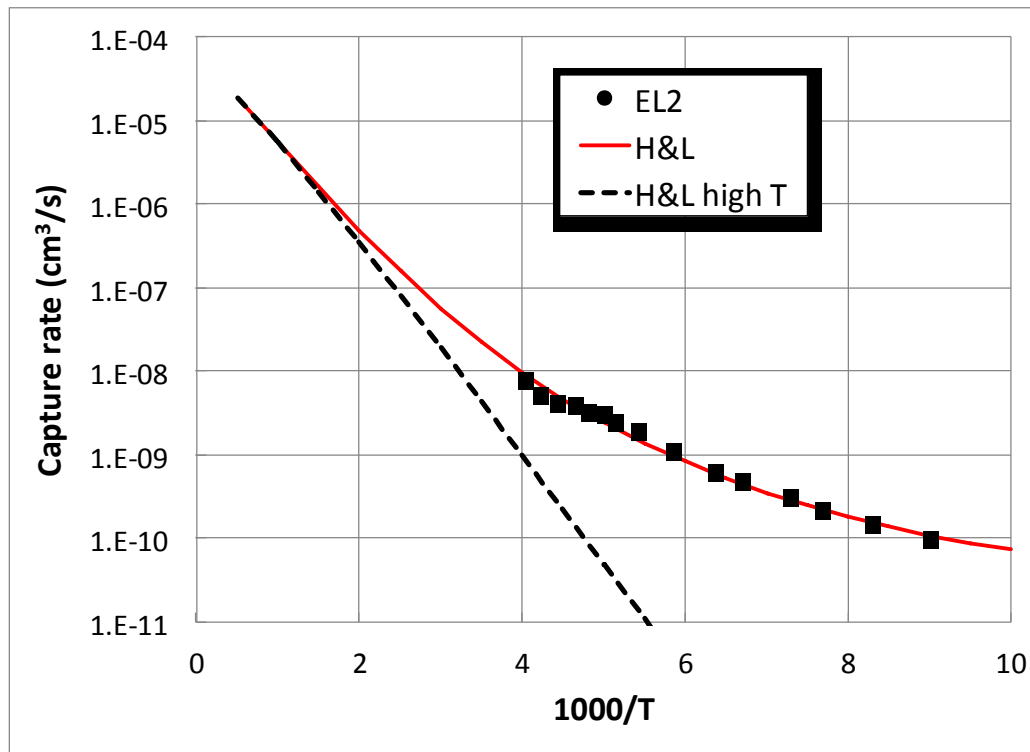
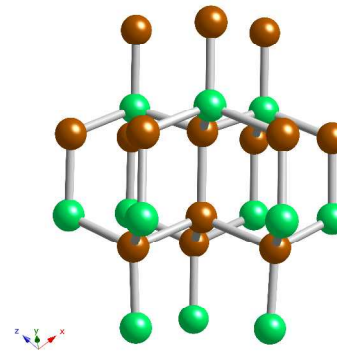
$$E_r = 0.24 \text{ eV}$$

$$\hbar\omega = 0.021 \text{ eV}$$

$$A = 6.3 \times 10^{-5} \text{ cm}^3/\text{s}$$

High-T slope given by carrier capture activation energy

$$E_b = (E_t - E_r)^2 / (4E_r) = 0.27 \text{ eV}$$



# Henry and Lang MPE Theory

Assume a configuration coordinate  $\lambda$

Energies depend on charge  $q$  and  $\lambda$

Red Line:  $E_{q=0}(\lambda)$

Blue Line:  $E_{q=+1}(\lambda) + E_{CB}$

Black Line:  $E_{q=+1}(\lambda) + E_{VB}$

Physical Meaning of Parameters

Trap depth  $E_t$ : From DFT or independent measurements (DLTS)

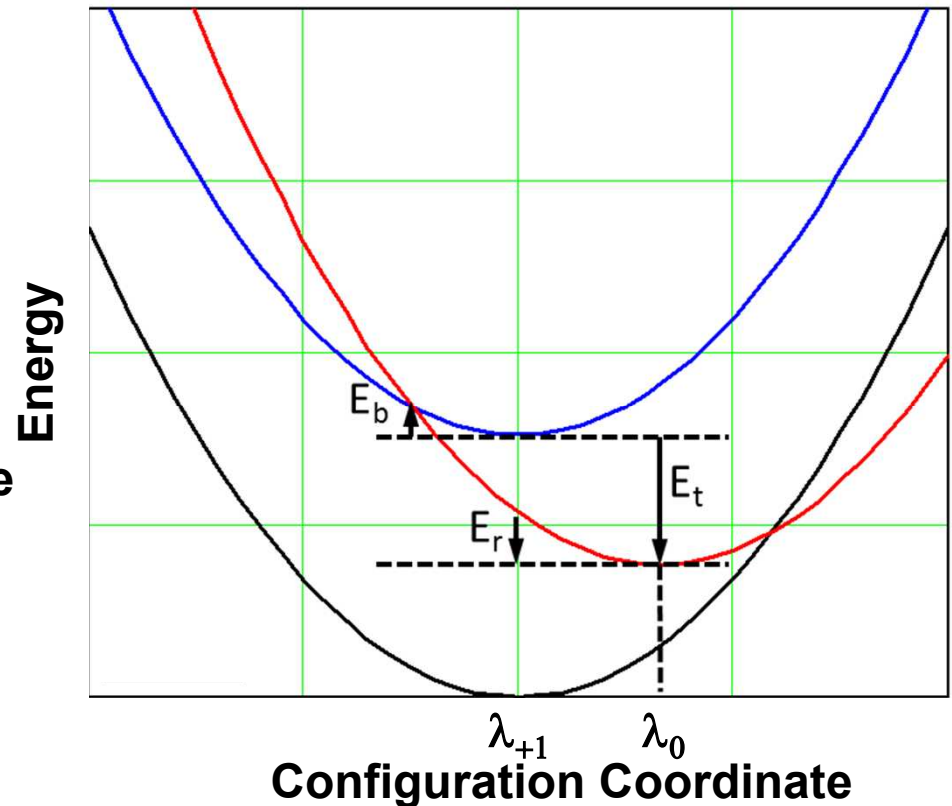
Relaxation energy  $E_r$ : From DFT

$$E_{q=0}(\lambda_{+1}) - E_{q=0}(\lambda_0)$$

$$E_{q=+1}(\lambda_0) - E_{q=+1}(\lambda_{+1})$$

Phonon frequency  $\hbar\omega$ : From DFT  
second derivative of  $E(\lambda)$

Prefactor  $A$ : Experiments suggest  $10^{-6}$  to  $10^{-4}$  cm<sup>3</sup>/s; Other groups (Alkauskas et al, L.-W. Wang et al) working on DFT-based calculations



Activation energy  $E_b$  is energy to reach crossing points where energy can be conserved during a non-radiative capture process

# DFT Calculations of the EL2 Relaxation Energy

## Computational Details:

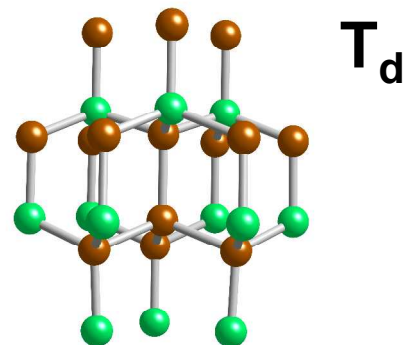
216 atom supercells

LDA and PBE functionals

Norm-conserving pseudopotentials

40 Rydberg plane-wave cutoff

Up to -4X-4X-4 k-point sampling



|     | $E_{q=0}(\lambda_{+1}) - E_{q=0}(\lambda_0)$ | $E_{q=+1}(\lambda_0) - E_{q=+1}(\lambda_{+1})$ | $E_b$        |
|-----|--|--|--------------|
| LDA | 0.076 eV                                     | 0.075 eV                                       | 1.49-1.52 eV |
| PBE | 0.077 eV                                     | 0.076 eV                                       | 1.47-1.49 eV |

Calculated EL2 relaxation energies differ by a factor of 3 from the value obtained by fitting to experiment:  $E_r = 0.24$  eV

As a result, the calculated activation energies differ by a factor of 6 from the experimental activation energy:  $E_b = 0.27$  eV



# Anharmonic, Multidimensional Theory for $E_b$

Can we generalize Henry and Lang's ideas to arbitrary energies  $E_{q=0}(\Lambda)$  and  $E_{q=+1}(\Lambda)$  in high dimensional configuration space  $\Lambda$ ?

The activation energy  $E_b$  controls high-T behavior, and thus getting an accurate  $E_b$  is essential to capturing experimental behavior

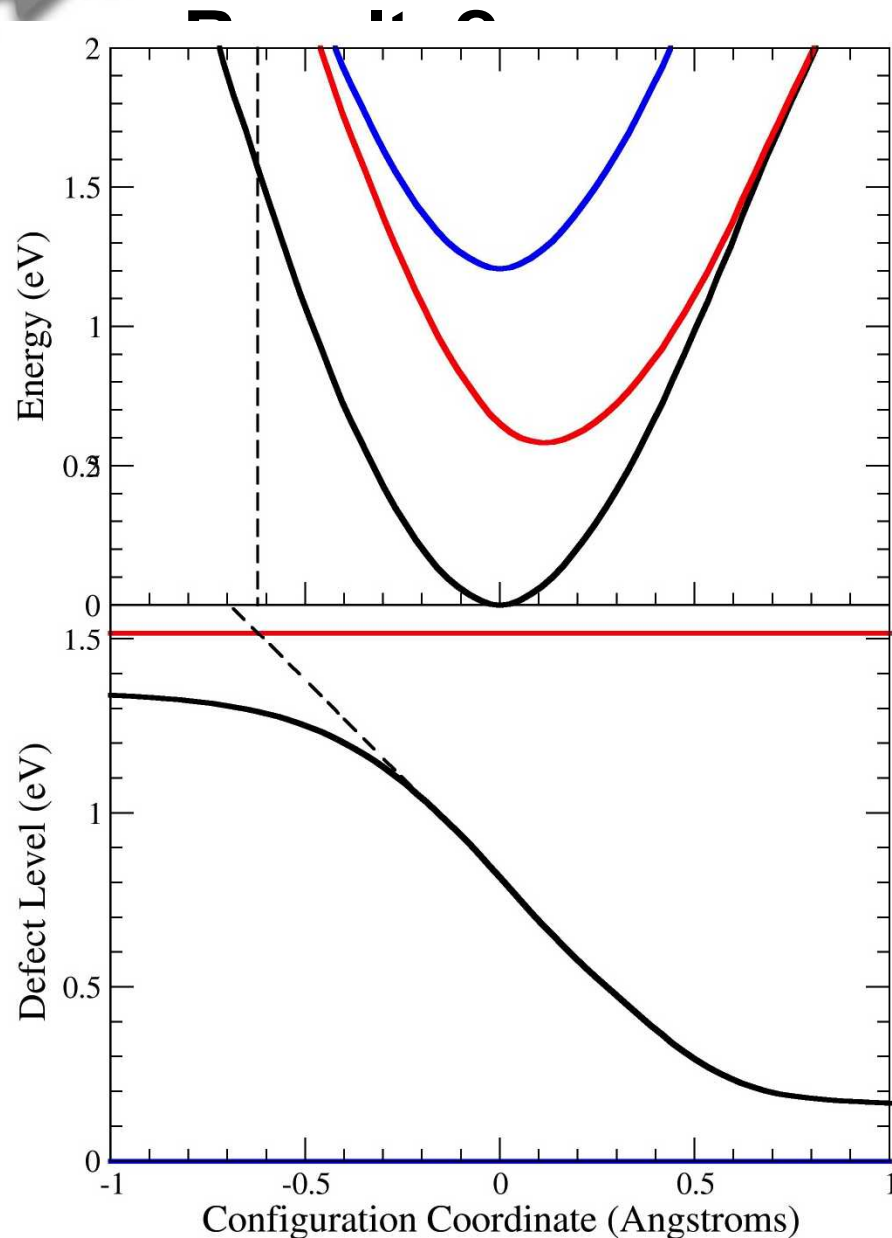
From the correspondence principle and transition state theory, non-radiative carrier capture at high-T should be dominated by a “carrier-capture transition state” defined as the lowest energy state where energy can be conserved during the capture process

Energy can be conserved during electron capture when the vertical defect level reaches the conduction band, i.e.,  $\Delta(\Lambda) = E_{q=0}(\Lambda) \square E_{q=+1}(\Lambda) = E_{CB}$

We have developed a new algorithm that does a constrained minimization of the energies  $E_{q=0}(\Lambda)$  and  $E_{q=+1}(\Lambda)$  over configurations  $\Lambda$  that are consistent with a given level  $\Delta(\Lambda)$

When we apply this algorithm to the level  $\Delta(\Lambda) = E_{CB}$ , we obtain the “carrier-capture transition state” and  $E_b$

# Does Anharmonicity Change Our EL2



**We have applied our anharmonic, multidimensional approach to E2 using the LDA functional**

**We calculated obtain an optimized configuration coordinate by varying the target  $\Delta(\Lambda)$**

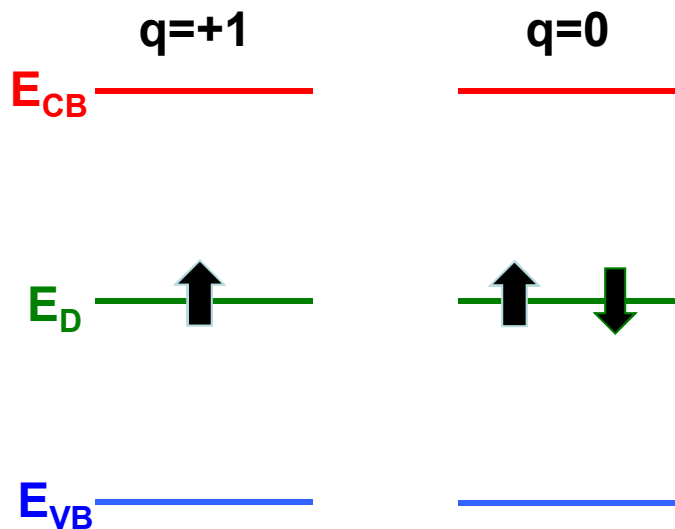
**The energies and defect levels are plotted along this coordinate**

**By processing the data to remove artifacts of the LDA band gap problem, we obtain  $E_b = 1.57$  eV**

**This is in good agreement with the harmonic theory (1.49 – 1.52 eV), but not with experiment**

# How About Hybrid Functionals?

## Hubbard U



## Computational Details:

216 atom supercells

-2X-2X-2 K-point sampling

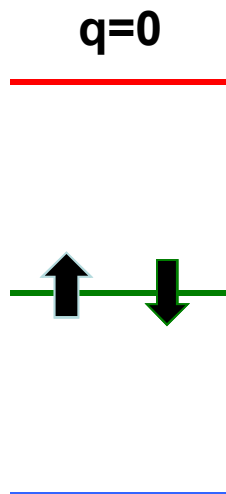
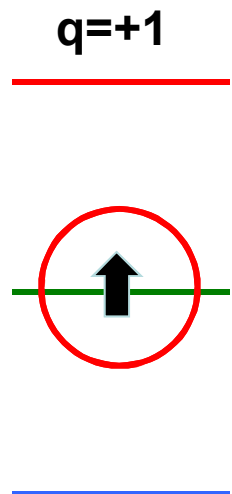
Atomic positions relaxed with the hybrid  
Sandia's Socorro software

Performed during Trinity Phase 1 Open  
Science Period at LANL

|        | $E_{q=0}(\lambda_{+1}) - E_{q=0}(\lambda_0)$ | $E_{q=+1}(\lambda_0) - E_{q=+1}(\lambda_{+1})$ | $E_b$        |
|--------|--|--|--------------|
| PBE23% | 0.122 eV                                     | 0.112 eV                                       | 0.81-0.91 eV |
| HSE06  | 0.114 eV                                     | 0.108 eV                                       | 0.89-0.95 eV |

The calculated  $E_r$  and  $E_b$  are closer to experiment ( $E_r = 0.24$  eV,  $E_b = 0.27$  eV), but still off by factors of about 2 and 4, respectively.

# How About Spin-Polarization?



|               | <b>E<sub>r</sub> Without Spin</b> | <b>E<sub>r</sub> With Spin</b> |
|---------------|-----------------------------------|--------------------------------|
| <b>LDA</b>    | <b>0.076 / 0.075 eV</b>           | <b>0.074 / 0.074 eV</b>        |
| <b>PBE</b>    | <b>0.077 / 0.076 eV</b>           | <b>0.074 / 0.073 eV</b>        |
| <b>PBE23%</b> | <b>0.122 / 0.112 eV</b>           | <b>0.109 / 0.099 eV</b>        |
| <b>HSE06</b>  | <b>0.114 / 0.108 eV</b>           | <b>0.102 / 0.094 eV</b>        |

**Including spin polarization for the q=+1 state actually lowers the calculated relaxation energy and increases the discrepancy with experiment**





# Conclusions and a Challenge

**These results leave us with a puzzle!**

**Possible resolutions:**

**Unexpected coupling between hybrids and anharmonicity?**

**An intermediate electronic state?**

**Something missing in our calculations?**

**Something missing in the Henry and Lang MPE theory?**

**EL2 is not what we think it is?**

**I challenge researchers working on DFT calculations for carrier capture to see whether their methods give better results for EL2!**

# DFT w/ Assumed Configuration Coordinate

$V_{\text{Ga}}$  -2/-3 level in GaN – Possibly associated with yellow luminescence

Extrapolated configuration coordinate from difference in relaxed structures

Upper Panel:

Blue Line:  $Q_D = -2$  with no carriers

Black Line:  $Q_D = -3$  with a hole

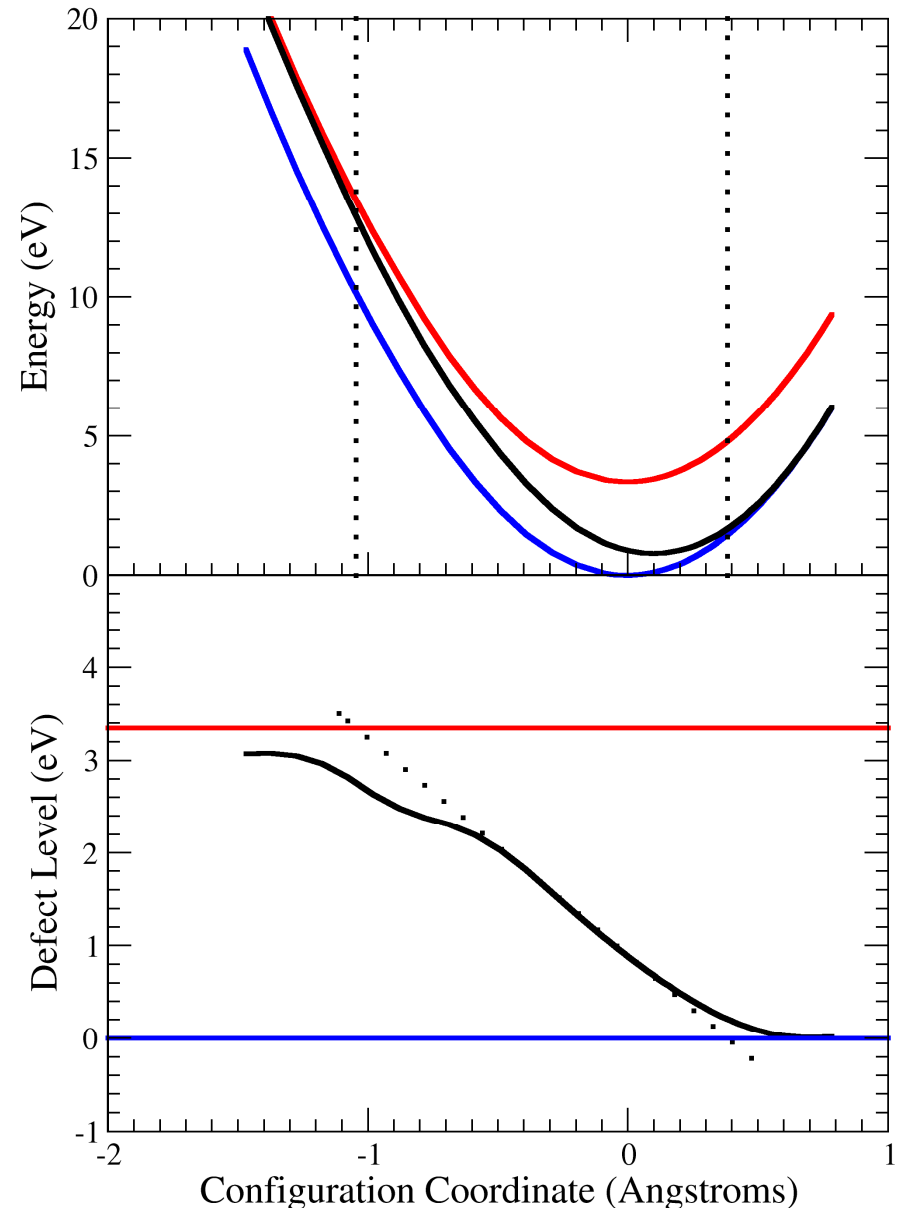
Red Line:  $Q_D = -2$  with hole and electron

Lower Panel:

Black Line: -3/-2 defect level

Blue Line: Lower Bound

Red line: Upper Bound



# DFT w/ Optimized Configuration Coordinate

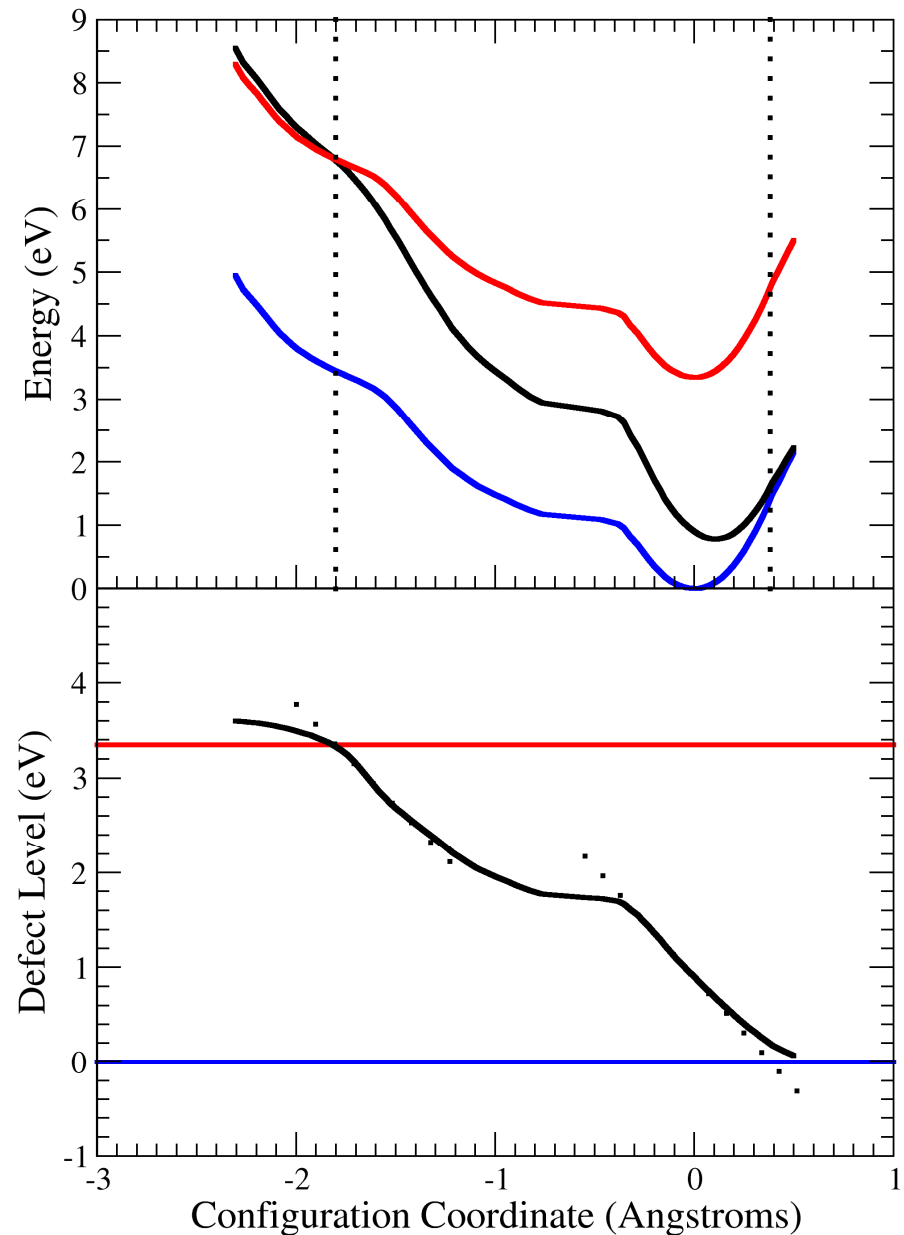
For hole capture, optimization doesn't change the results very much

For electron capture, a complex, symmetry breaking distortion is observed beyond about 0.4 Å

Extrapolation Gives:

3.45 eV activation energy for electron capture which is consistent with yellow luminescence – Good!

0.83 eV activation energy for hole capture – Maybe too large?





# Next Steps For This Work

**Study more defects looking for cases where direct comparison with experiment is possible**

**Extend our procedures to the full temperature range:  $S$  and  $\omega$  are easily calculated near the ground state configurations, but how do we smoothly patch together our anharmonic, multidimensional results for high temperature with the harmonic theory for low temperatures?**

**In general, we probably have to solve the anharmonic Schrodinger equation for vibration along the configuration coordinate.**



# **Do These Classical Activation Energies Generally Control the High Temperature Behavior?**

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**Henry and Lang obtained the same result from the high-T limit of the fully quantum expressions in the harmonic case**

**Quantum and classical results should agree at high-T based on the Correspondence Principle**

**We have checked two cases carefully:**

- **Does the classical barrier give the high-T slope for small relaxation energies? YES**
- **Does the classical barrier give the high-T slope when you consider thermal excitation of the band edge carriers? YES**

**Thanks for help from Audrius Alkauskas and Bill Wampler!**

# Model for Calculating Capture Cross-Sections

$Q$  is an ionic configuration coordinate

$U_v$  is the energy of neutral system with defect

$U_c$  is the energy with extra electron in CB

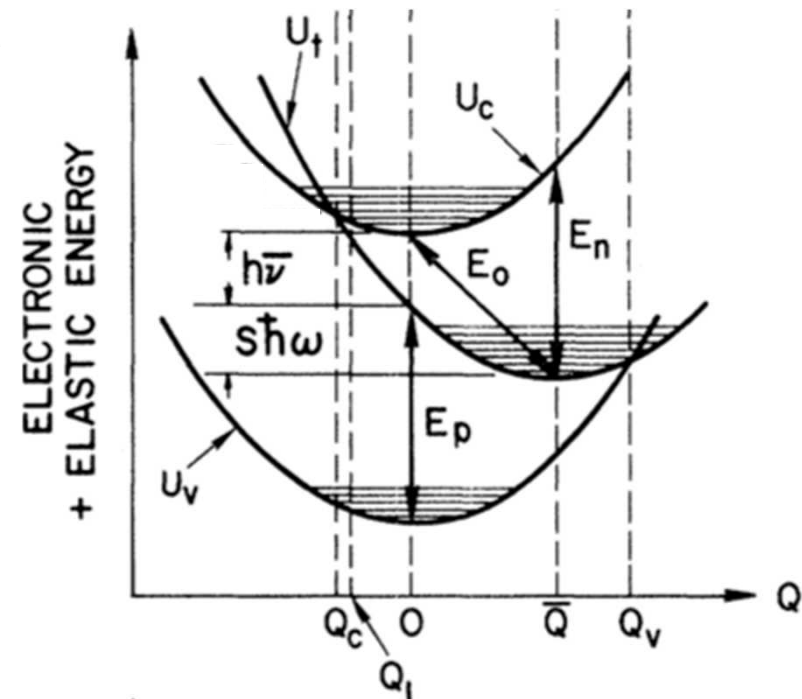
$U_t$  is the energy with extra electron on defect

$Q=0$  is the minimum for neutral defect

$Q=\bar{Q}$  is the minimum for charged defect

$U_c$  and  $U_t$  cross at  $Q=Q_c$

$U_v$  and  $U_t$  cross at  $Q=Q_v$



$E_p$  is the optical excitation threshold from VB

$E_n$  is the optical excitation threshold from defect

$h\nu$  is the zero-phonon emission line from CB

$s\hbar\omega$  is the relaxation energy for the charged defect

$E_0$  is the thermodynamic defect level (relative to the CB)

# Model for Calculating Capture Cross-Sections

Normally, the defect level is defined as the difference of relaxed charge state energies

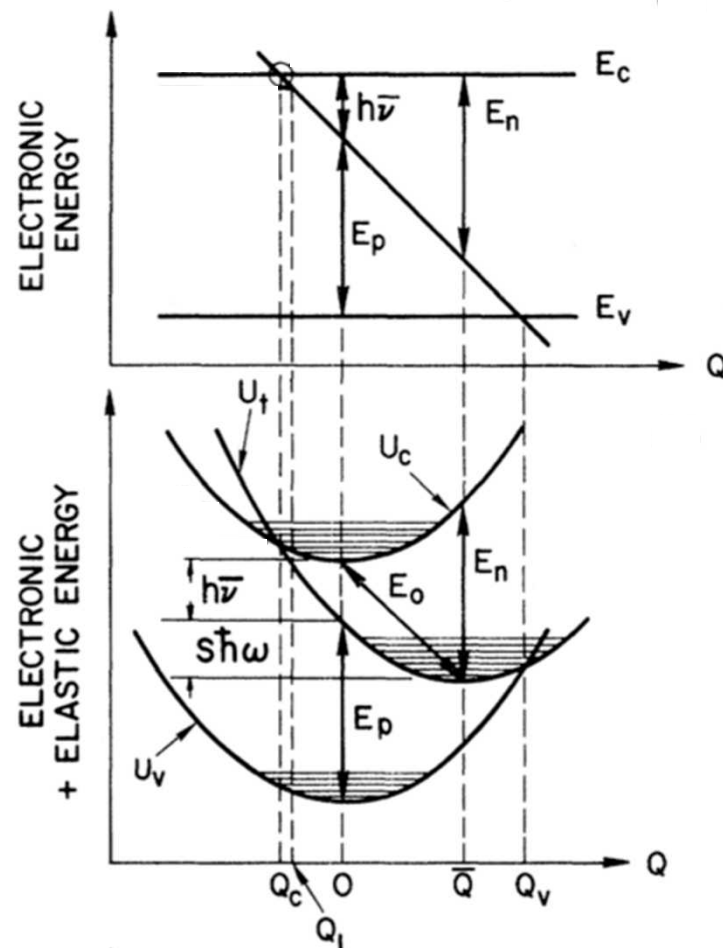
Alternatively, we can define a  $Q$  dependent defect level for charge states at the same  $Q$

This defect level crosses into the CB at  $Q=Q_c$  and the VB at  $Q=Q_v$

These level crossing regions dominate the thermal capture and emission of carriers

We can define  $Q_1$  as the point where the adiabatic approximation begins to break down

$Q_1$  is taken to be 0.06 eV from the crossing in Henry and Lang



# Calculating Nonradiative Capture Cross-sections

Perturbation theory for the transition rate gives

$$w = \frac{2\pi}{\hbar} \text{ave}_{n_c} \sum_{n_t} |\langle tn_t | \mathcal{H} | cn_c \rangle|^2 \delta(\mathcal{E}_c - \mathcal{E}_t)$$

Where  $|n_c\rangle$  and  $|n_t\rangle$  are vibrational states,

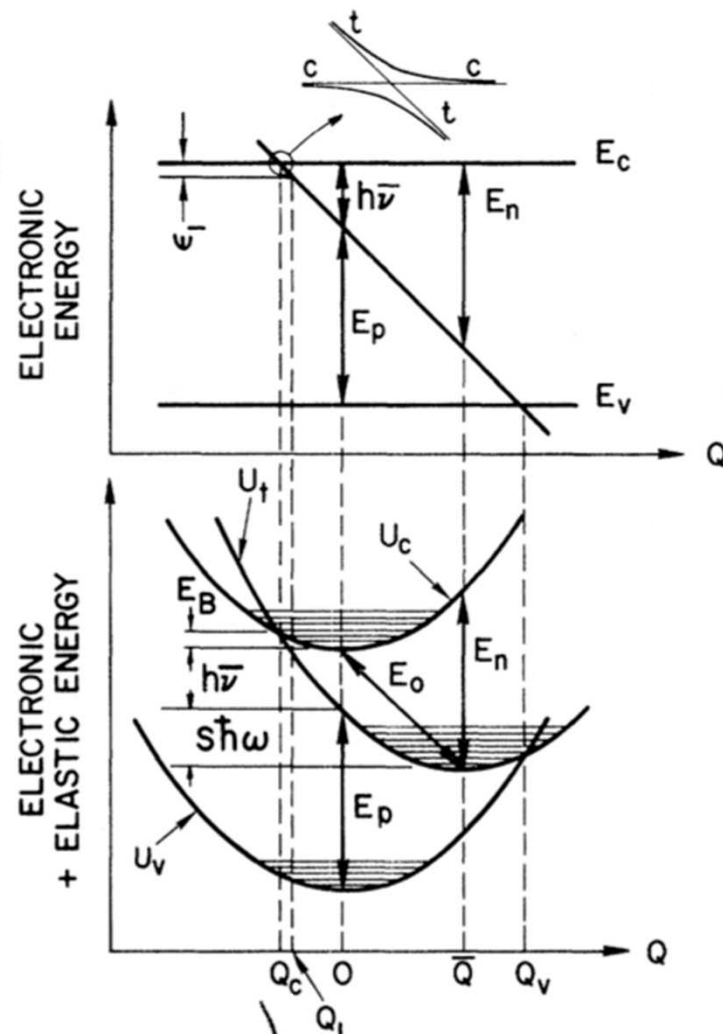
$$\mathcal{E}_c = n_c \hbar \omega, \text{ and } \mathcal{E}_t = n_t \hbar \omega - E_0$$

Result is dominated by level crossing region, so assume that  $|c\rangle$  and  $|t\rangle$  are constant

$$|c\rangle = u_c(r, Q_1), \quad |t\rangle = u_t(r, Q_1)$$

Then, defining  $\Delta V(r, Q) = V(r, Q) - V(r, Q_1)$

$$w = \frac{2\pi}{\hbar} |\langle t | \Delta V | c \rangle|^2 \left( \text{ave}_{n_c} \sum_{n_t} |\langle n_t | n_c \rangle|^2 \delta(\mathcal{E}_c - \mathcal{E}_t) \right)$$





# Calculating Nonradiative Capture Cross-sections

## Defining the “Normalized Line Shape”

$$f(h\nu) = \text{ave}_{n_c} \sum_{n_t} |\langle n_t | n_c \rangle|^2 \delta(\mathcal{E}_c - \mathcal{E}_t - h\nu)$$

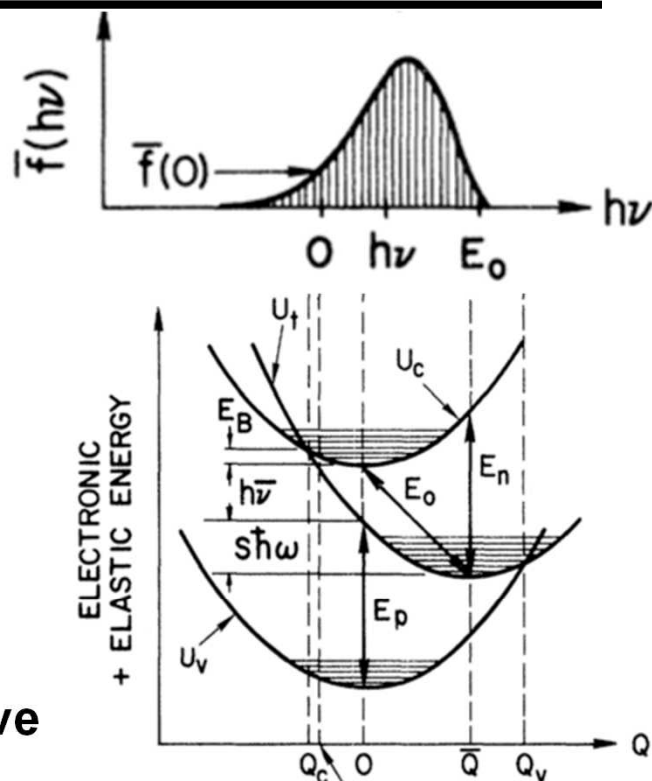
And using  $\sigma_c = (\Omega / \langle v \rangle) w$ , we get

$$\sigma_c = \underbrace{(2\pi\Omega / \hbar \langle v \rangle)}_{\text{Weakly Varying}} \underbrace{|\langle c | \Delta V | t \rangle|^2}_{\text{Strongly Varying}} f(0)$$

For Radiative Recombination, similar arguments give

$$\begin{aligned} \frac{dw^{\text{rad}}}{d(h\nu)} &= \frac{2\pi}{\hbar} \text{ave}_{n_c} \sum_{n_t} |\langle u_t n_t | H_{\text{rad}}(r) | u_c n_c \rangle|^2 \delta(\mathcal{E}_c - \mathcal{E}_t - h\nu) \rho_{\text{rad}} \\ &= \frac{2\pi}{\hbar} \rho_{\text{rad}} |\langle \varphi_t | H_{\text{rad}} | \varphi_c \rangle|^2 f(h\nu) \sim (h\nu)^3 f(h\nu) \end{aligned}$$

where  $\varphi_c$  and  $\varphi_t$  are evaluated at  $Q = 0$  (the Condon Approximation)



# Observations about the Normalized Line Shape

If we assume simple harmonic oscillators with the same frequency, we get the Huang and Rhys line shape, a function of  $E_0$ ,  $\hbar\omega$  and  $S$

$$f(h\nu) = \sum_{p=-\infty}^{\infty} \delta(E_0 - p\hbar\omega - h\nu) W_p$$

Where

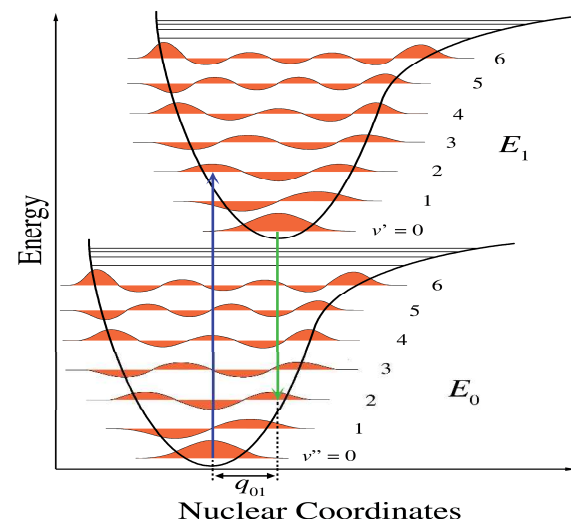
$$W_p = \exp[-(2n+1)S] \left[ \frac{\bar{n}+1}{\bar{n}} \right]^{p/2} \times I_p(2S[(\bar{n}+1)\bar{n}]^{1/2})$$

$I_p$  is a modified Bessel function, and

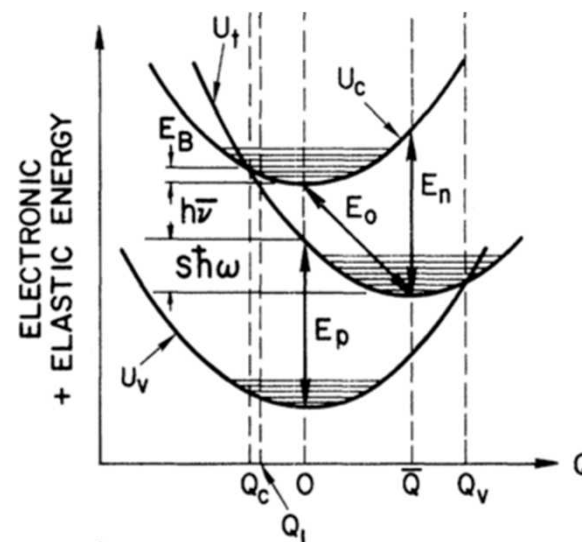
$$\bar{n} = [\exp(\hbar\omega/kT) - 1]^{-1}$$

At high temperature, we obtain

$$\sigma_c = \left( \frac{4\pi E_B}{kT} \right)^{1/2} \frac{\Omega |\langle t | \Delta V | c \rangle|^2}{\hbar \langle v \rangle \hbar \bar{\nu}} e^{-E_B/kT}$$



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# **Comparison of Our Calculated Barriers for $V_{Ga}$ to Experiments on Yellow Luminescence**

**Experiment suggests electron capture barrier  $> 0.6$  eV**

**We get 3.45 eV. Good!**

**However, the hole capture barrier 0.83 eV would suggest that non-radiative capture of holes would be slower than observed**

**Some possible explanations:**

**Convergence with respect to supercell size and/or BZ sampling**

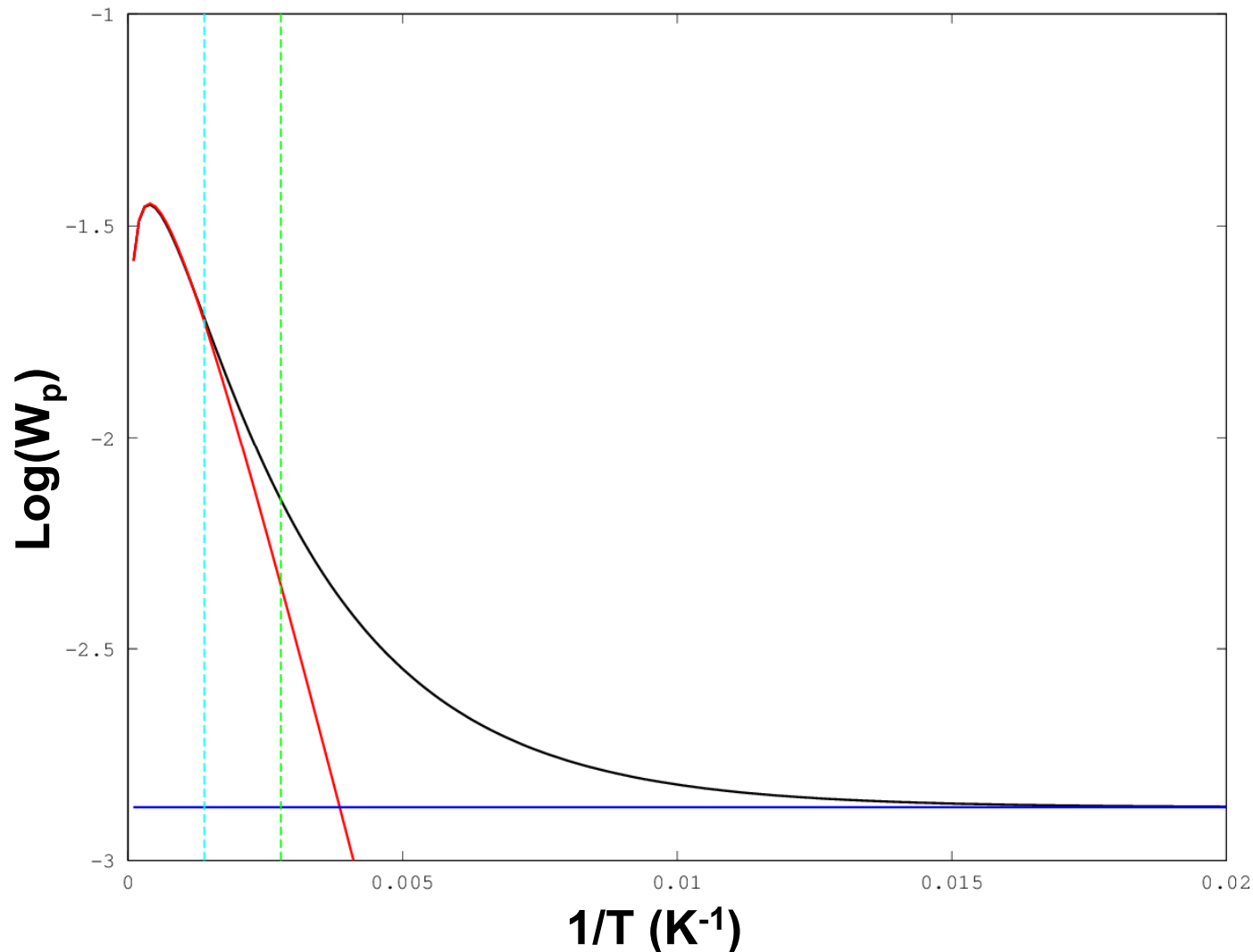
**Better functionals (e.g., hybrids) might be needed**

**$V_{Ga}$  must be in a defect complex to contribute to YL**

**Some other defect (e.g.,  $C_N$ ) is solely responsible for YL**

# Is This High Temperature Limit Generally Valid?

Evaluate exact  $W_p$  (Black) and Approximate  $W_p$  (Red) including the  $T^{-1/2}$  factor for  $E_0=0.31\text{eV}$ ,  $\hbar\omega=0.031\text{ eV}$ , and  $S=3.226$



# How Much Do Real Cross-Sections Vary?

Henry and Lang, PRB 1977 studied  
13 cross-sections in GaAs and GaP

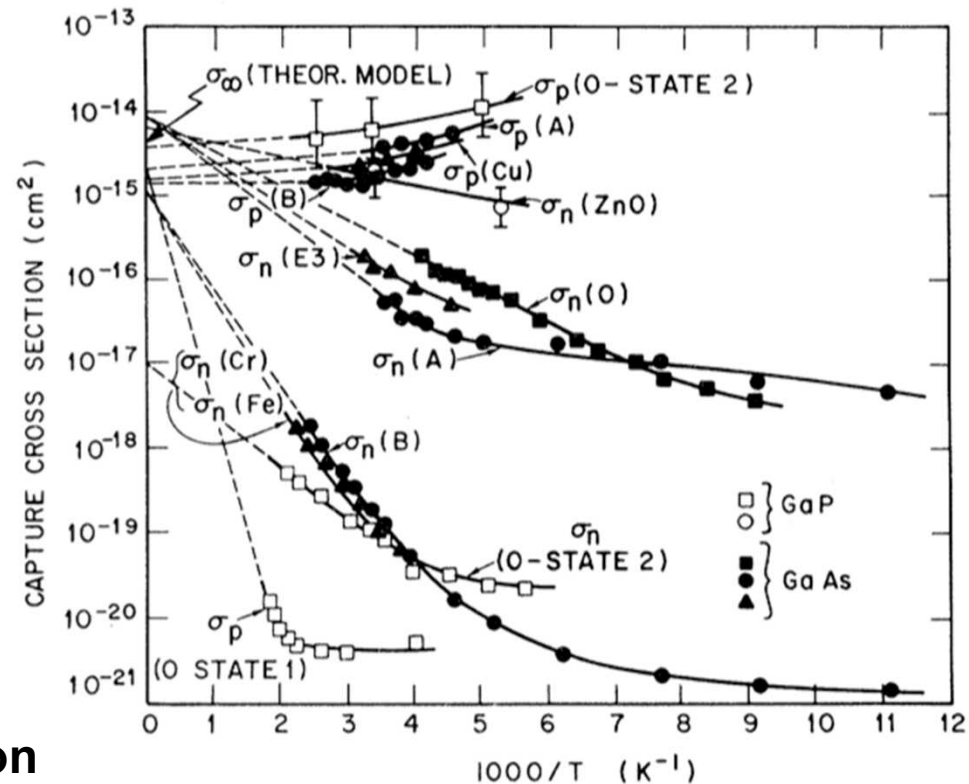
Wide variation at room-temperature

Extrapolate to similar values at high-T

Generally, have the high-T form

$$\sigma(T) = \sigma_{\infty} \exp(-E_B/kT)$$

Henry and Lang's theory of Multiphonon  
Emission explains this result



# Henry and Lang Theory of Carrier Capture

Assume a configuration coordinate  $\lambda$

Upper Panel:  $\lambda$ -Dependent Energies

Blue Line:  $Q_D=q$  with no carriers

Black Line:  $Q_D=q-1$  with a hole

Red Line:  $Q_D=q$  with hole and electron

Lower Panel:  $\lambda$ -Dependent Defect Level

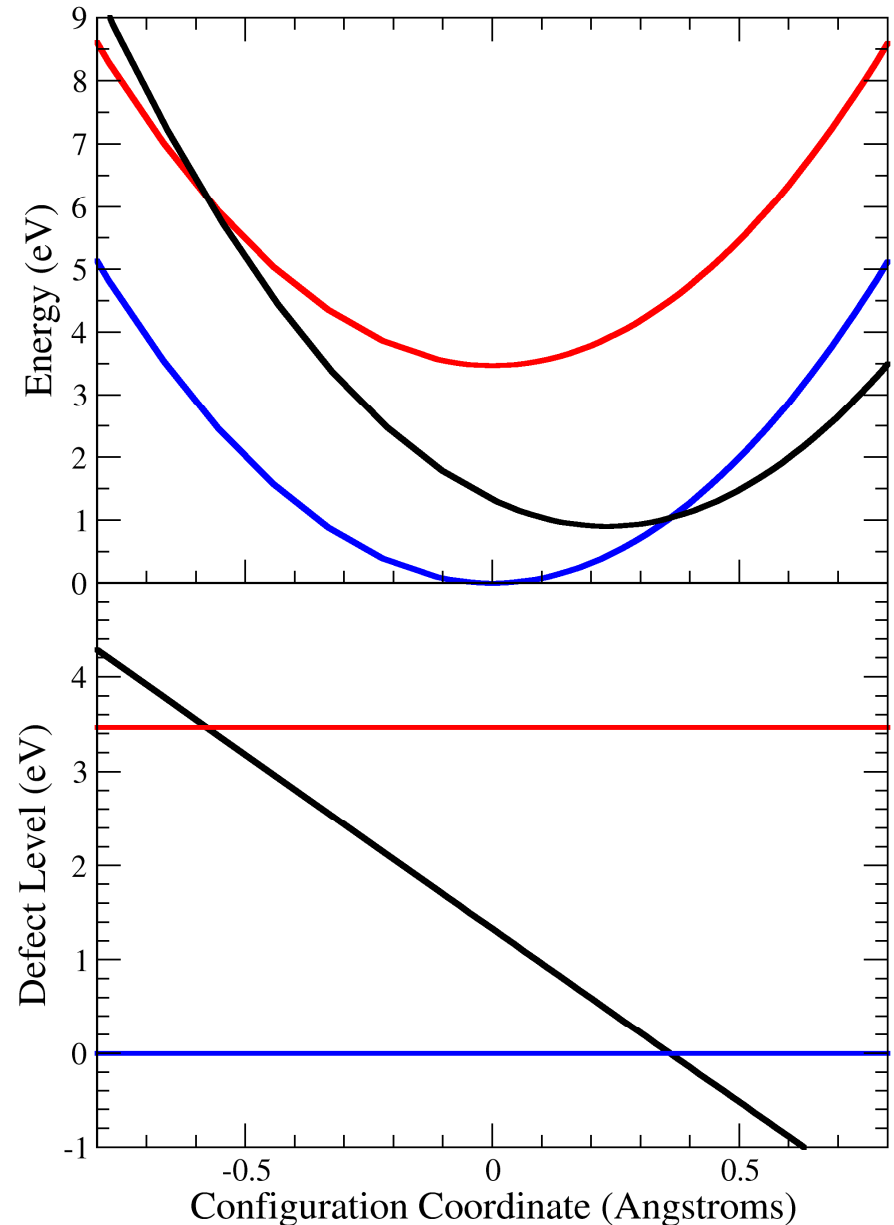
Black Line:  $q-1/q$  defect level

Blue Line: Valence band

Red line: Conduction band

Nonradiative capture/emission occurs  
near level crossings when energy  
can be conserved

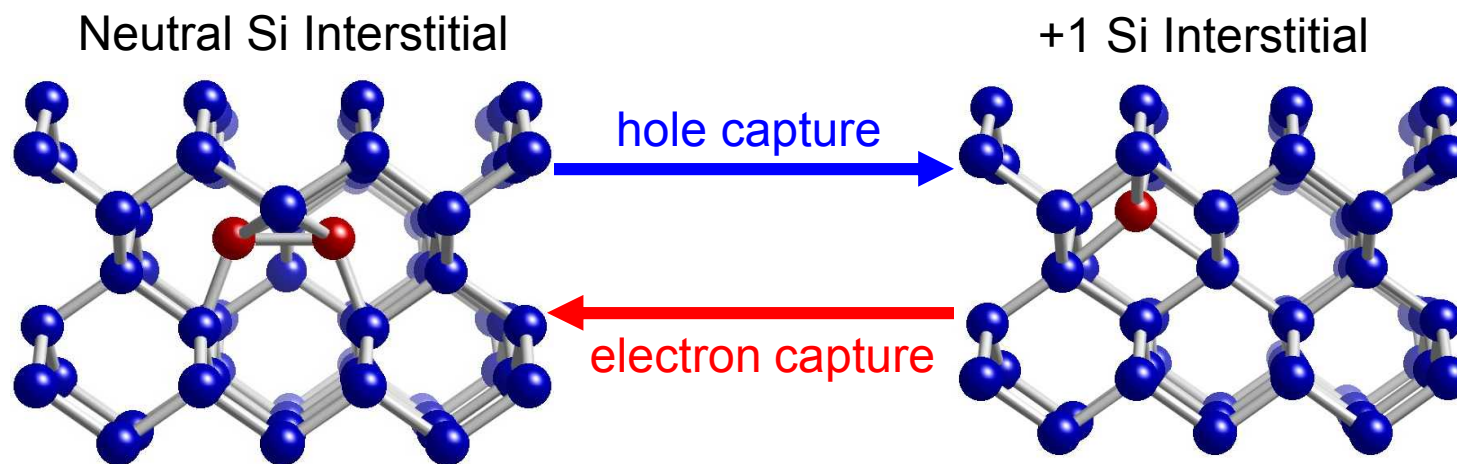
Activation energies given by differences  
between crossing-points and minima



# First-Principles Calculations of Carrier Capture

Detailed theories based on DFT perturbation theory (L. Shi and L.-W. Wang, PRL 2012; A. Alkauskas et al., PRB 2014) have been developed to calculate capture cross-sections when the harmonic approximation holds

Instead, we have considered the anharmonic case:



So far, we have focused on carrier capture activation energies and the high-T behavior